SURFACE WAVE CHARACTERISTICS OF CIRCULAR CYLINDRICAL CORRUGATED AND UNIFORM DIELECTRIC ROD EXCITED IN E_0-MODE

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ABSTRACT

The paper presents a report of theoretical study of the surface wave characteristics, viz., decay coefficient, guide wave-length, attenuation constant, constant percentage power contour as functions of physical parameters of corrugated and uniform dielectric rods excited in E_0 – mode. The corrugated rod shows better surface wave characteristics than uniform rod.

1. INTRODUCTION

The present work is a part of the investigations [1-10] on surface wave characteristics of electromagnetic structures that are being conducted in the laboratory. An extensive bibliography on surface wave study have been given by Barlow and Brown [11] in their book on "Radio surface waves" which deals with the fundamental properties of surface wave and its associated problems in an illuminating manner. Wait[12-14] has done considerable significant theoretical work on the propagation of surface waves and has written an excellent comprehensive paper on electromagnetic surface waves.

It is the object of the paper to present the results of theoretical study of the field distribution on the uniform and corrugated dielectric rods excited in E_0 – mode. The variation of propagation constant, guide wavelength and radial field decay, attenuation constant as functions of structure parameters are reported. It has been shown that the corrugated dielectric rod can support a surface wave only for a certain definite combinations of the structure parameters. It is also shown that the corrugated rod shows better surface wave characteristics than an uniform dielectric rod both of them being excited in E_0 – mode.

2. Uniform Dielectric Rod

2.1 Field Components

The field components in the two media (See Fig. 1) are

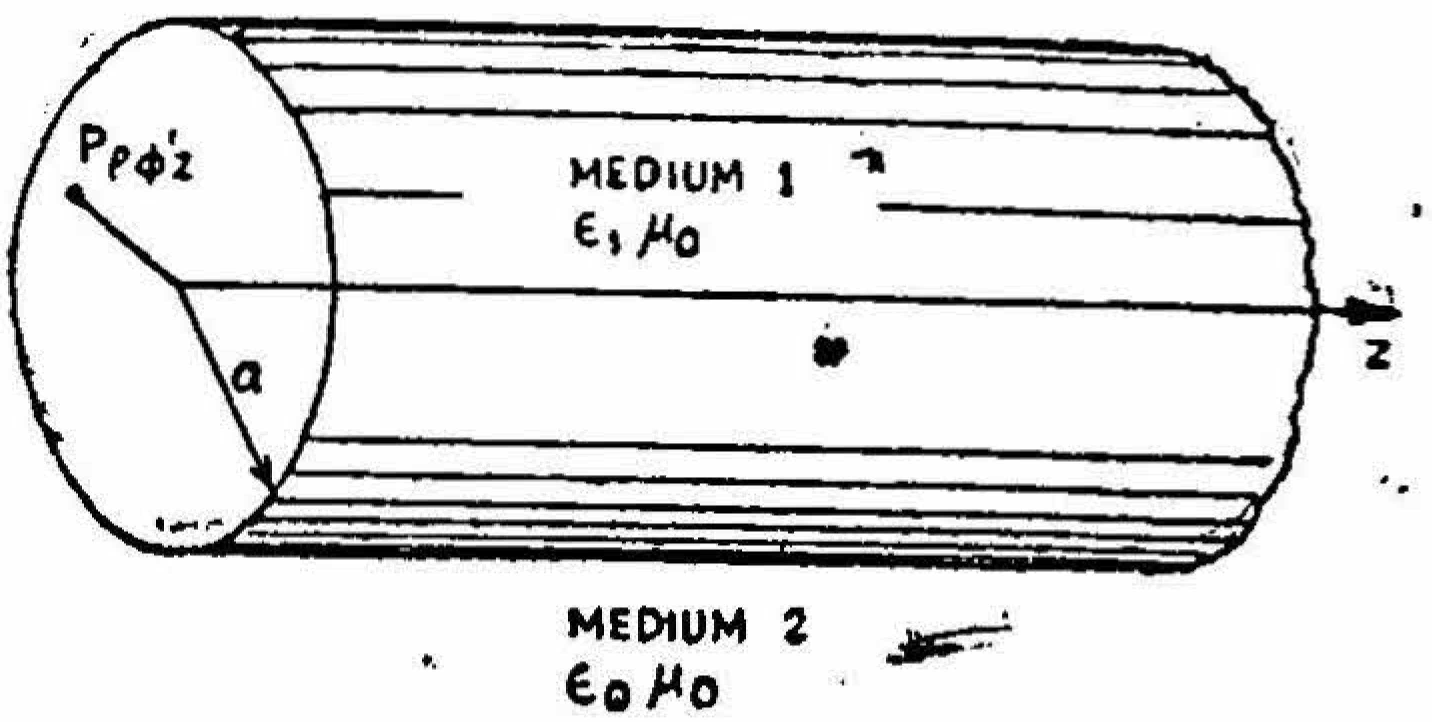


FIG. 1

Circular cylindrical co-ordinate system for uniform dielectric rod

ϵ_1 = Permittivity of the dielectric rod material ϵ_0 = Permittivity of the free space

μ_0 = Permeability of the free space $\rho\,\phi'\,z$ = Cylindrical co-ordinates

a = Radius of the rod

Medium 1 = Dielectric rod, $\rho \leqslant a$

Medium 2 = Free space, $\rho \geqslant a$

Medium 1 : $\rho \leqslant a$

$$E_{z1} = B\ [(k'_1)^2/j\omega\epsilon_1]\ J_0(k'_1\rho)\ \exp(-j\beta z)$$
$$E_{\rho 1} = B\ [k'_1\beta/\omega\epsilon_1]\ J_1(k'_1\rho)\ \exp(-j\beta z)$$
$$H_{\phi' 1} = B\ k'_1\ J_1(k'_1\rho)\ \exp(-j\beta z) \qquad [1]$$

Medium 2 : $\rho \geqslant a$

$$E_{z2} = D\ [(k'_2)^2/j\omega\epsilon_0\ H_0^{(1)}(k'_2\rho)\ \exp(-j\beta z)$$
$$E_{\rho 2} = D\ [k'_2\beta/\omega\epsilon_0]\ H_1^{(1)}(k'_2\rho)\ \exp(j\beta z)$$
$$H_{\phi' 2} = D\ k'_2\ H_1^{(1)}(k'_2\rho)\ \exp(-j\beta z) \qquad [2]$$

The time dependence is of the form $\exp(j\omega t)$. The radial propagation constants k'_1 and k'_2 are related to the axial phase constant β as follows

$$k'^2_1 = -\beta^2 + \omega^2\mu_0\epsilon_1 \qquad [3]$$
$$k'^2_2 = -\beta^2 + \omega^2\mu_0\epsilon_0 \qquad [4]$$

which yield

$$k'^2_1 - k'^2_2 = k^2(\epsilon_{r1} - 1) \qquad [5]$$

where, $\epsilon_{r1} = \epsilon_1/\epsilon_0$ dielectric constant of the material of the rod

$k^2 = \omega^2 \mu_0 \epsilon_0$

For E_0-surface wave field, k_2' must be positive imaginary. Therefore, $\beta^2 > \omega^2 \mu_0 \epsilon_0$, where β is real. At X-band, $\omega^2 \mu_0 \epsilon_1 >> \omega^2 \mu_0 \epsilon_0$ as $\epsilon_1 > \epsilon_0$, so only positive real values of k_1' are considered.

2.2 *Characteristic Equation*

The characteristic equation is formulated by using the appropriate field components and the following boundary conditions which have to be satisfied at the interface $\rho = a$ between the regions 1 and 2.

$$\begin{aligned} E_{z1} &= E_{z2} \\ \epsilon_1 E_{\rho 1} &= \epsilon_0 E_{\rho 2} \\ H_{\phi r_1} &= H_{\phi 2} \end{aligned} \qquad [6]$$

at $\rho = a$. By using the above boundary conditions and appropriate field components, the following characteristic equation is obtained

$$x_1 [J_0(x_1)/J_1(x_1)] = x_2 \epsilon_{r1} [H_0^{(1)}(x_2)/H_1^{(1)}(x_2)] \qquad [7]$$

where, $x_1 = k_1' a$, $x_2 = k_2' a$

and $$x_1^2 + \left(\frac{x_2}{j}\right)^2 = \left(\frac{2\pi a}{\lambda_0}\right)^2 (\epsilon_{r1} - 1) \qquad [8]$$

2.3 *Numerical Computation*

The roots of the characteristic equation (equation 7), x_1 and x_2 are obtained, as a fuuction of the rod radius 'a' with the help of a digital computer (Type Elliot 803). The numerical values of the different constants used for computation are $\epsilon_{r1} = 2.56$ and $\lambda_0 = 3.14$ cm.

The variation of k_1', k_2', β and λg as functions of 'a' is shown in Figures 2, 3 and 4 respectively. β and λg are determined from the following relations

$$\beta = [\omega^2 \mu_0 \epsilon_1 - (x_1/a)^2]^{1/2} \qquad [9]$$

$$\lambda g = \lambda_0 / [\epsilon_{r1} - x_1^2 (\lambda_0 / 2\pi a)^2]^{1/2} \qquad [10]$$

The theoretical radial field decay curves are shown in Fig. 5.

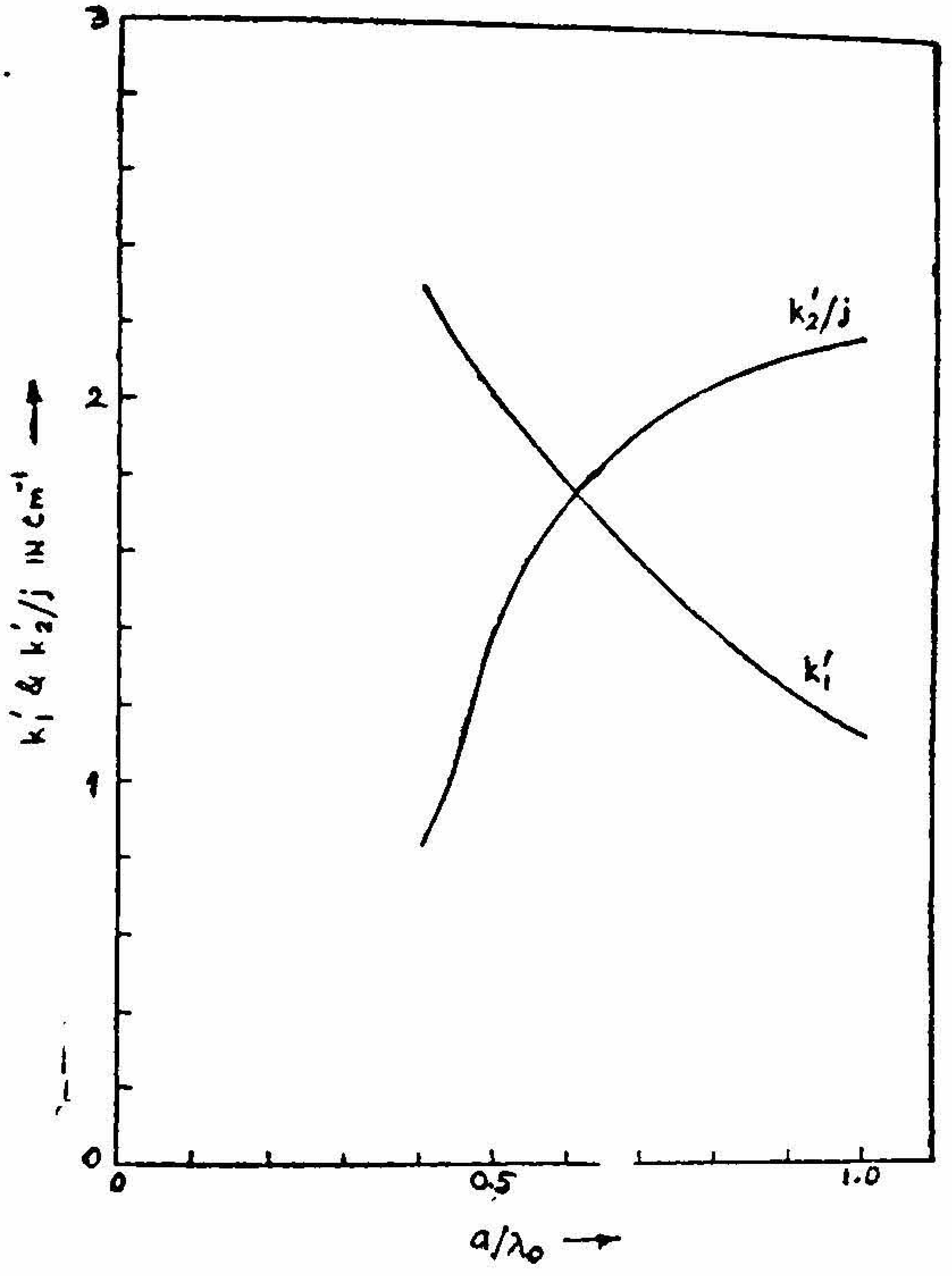


FIG. 2

Variation of radial propagation constants with a/λ_0 for uniform dielectric rod

k'_1 = Radial propagation constant inside the rod

k'_2 = Radial propagation constant outside the rod

a = Radius of the rod

$\lambda_0 = 3.14$ cm.

3. CORRUGATED DIELECTRIC ROD

The corrugated rod (Fig. 6) consists of two media. The first medium is ($\rho \leq a$) is characterised by ϵ_1, μ_0 and the second medium ($a \leq \rho \leq h$) by ϵ_2, μ_0 respectively.

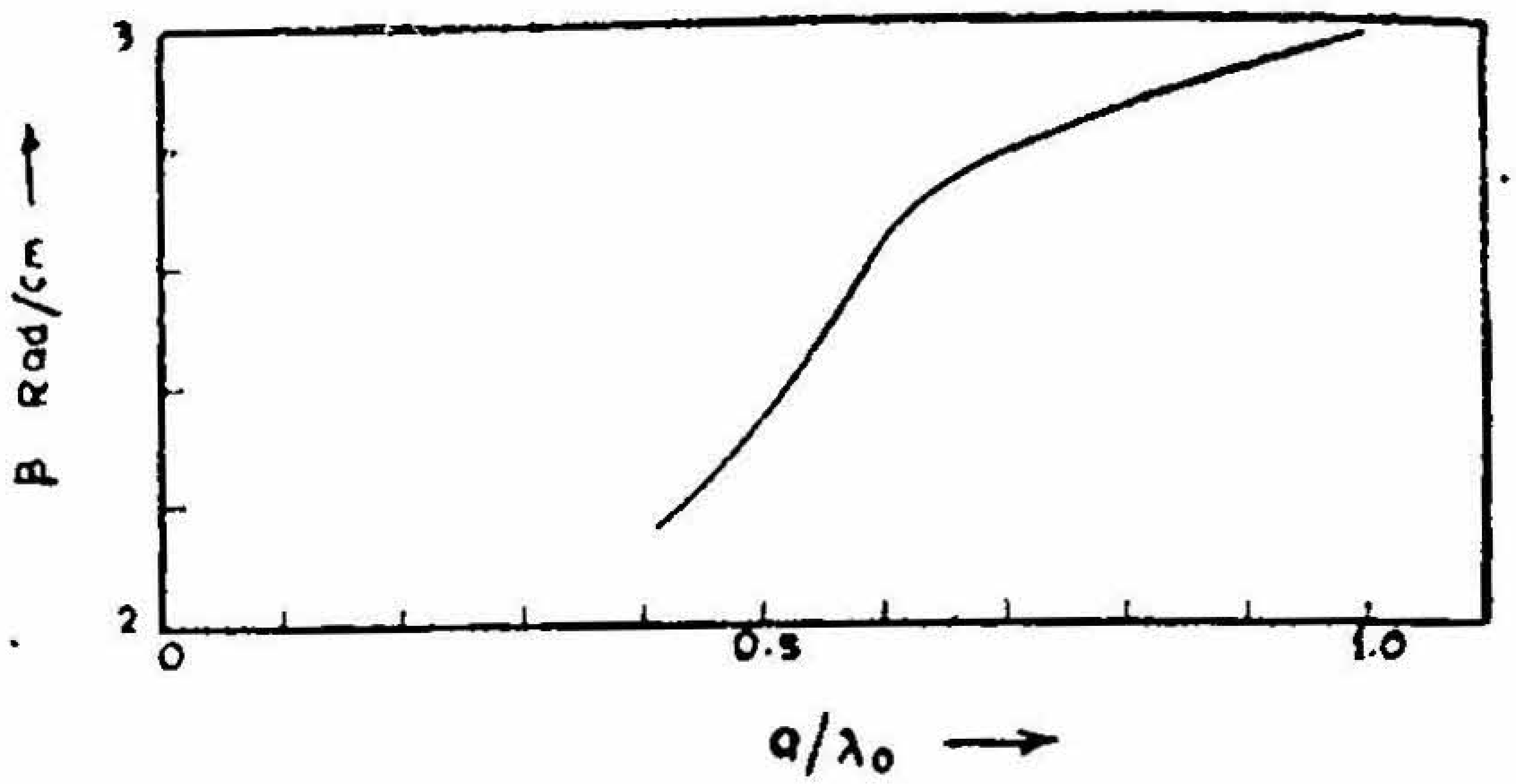


FIG. 3

Variation of axial propagation constant, β with a/λ_0 for uniform dielectric rod

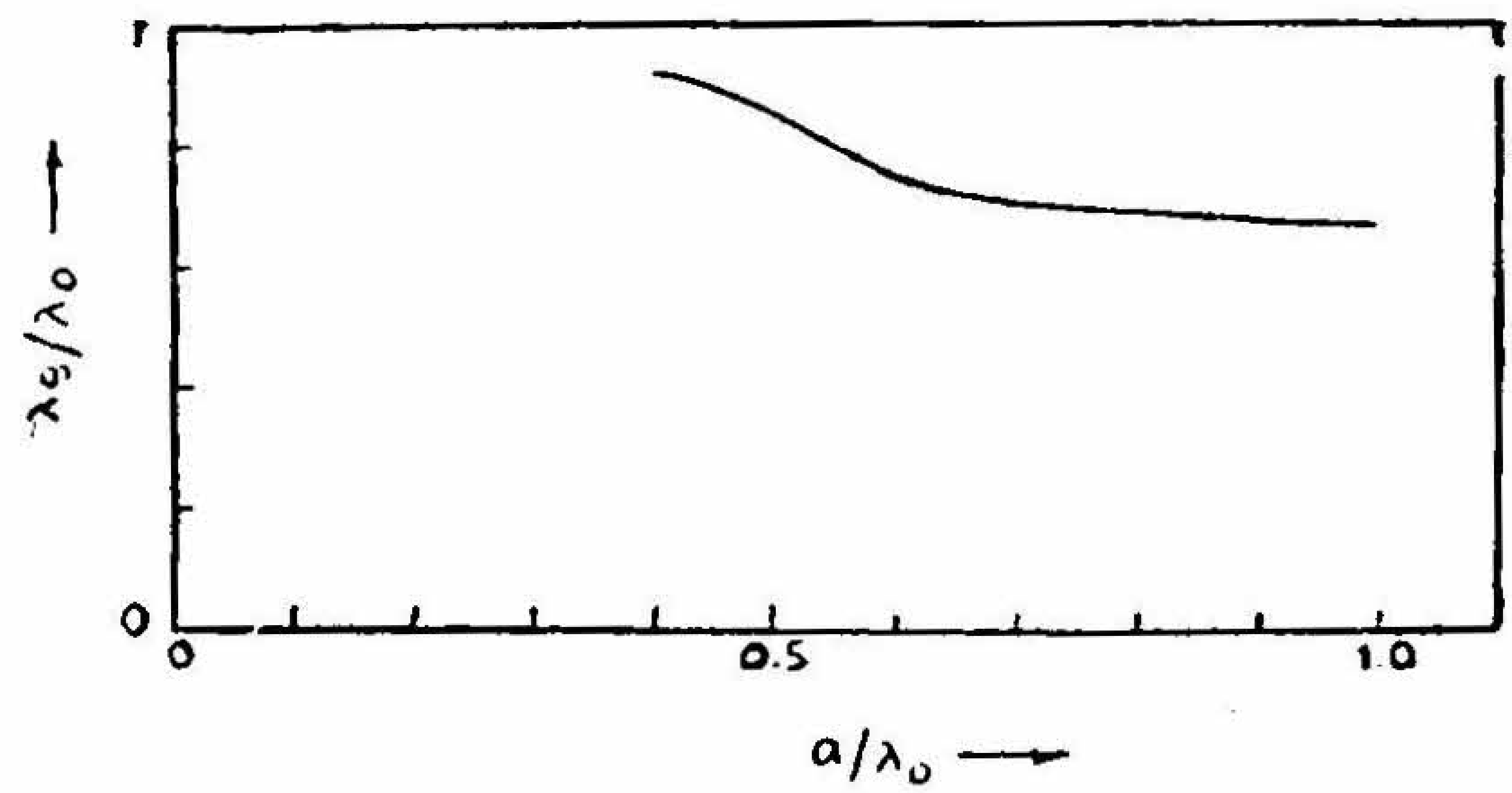


FIG. 4

Variation of λ_g/λ_0 with a/λ_0 for uniform dielectric rod

λ_g=Guid wavelength, a=Radius of the rod, λ_0=3.14 cm

3.1 *Effective Dielectric constant*

The second medium of the corrugated rod consisting of the grooved region with uniformly spaced disc is assumed to be a homogeneous medium having effective dielectric constant

$$\epsilon_{r2} = [(\sqrt{\epsilon_{r1}} \cdot t+s)/(t+s)]^2 \qquad [11]$$

which is obtained by equating the optical path lengths and assuming plane wave propagation in the medium.

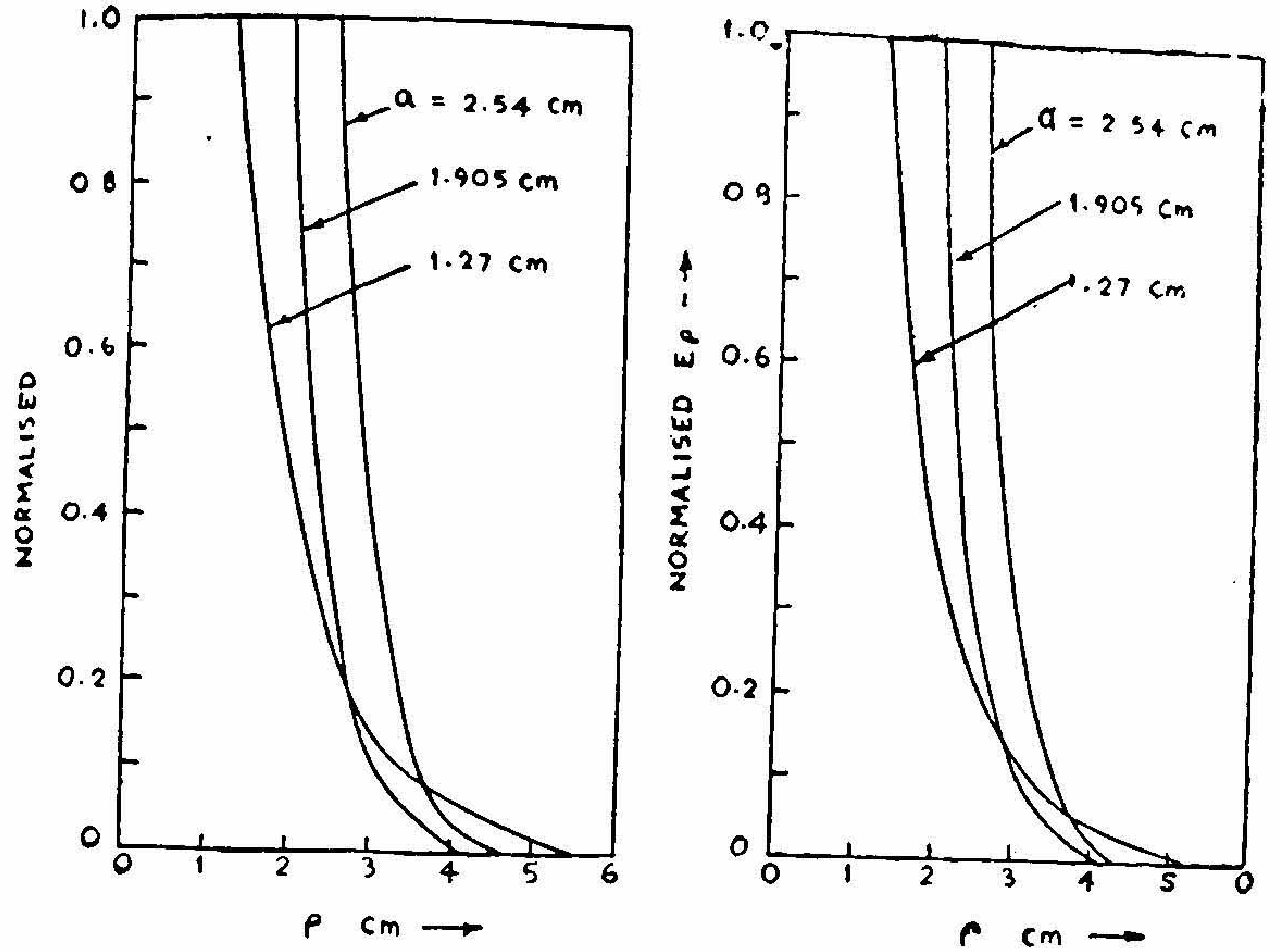


FIG. 5

Radial field decay curves for uniform dielectric rod. a = Radius of the rod

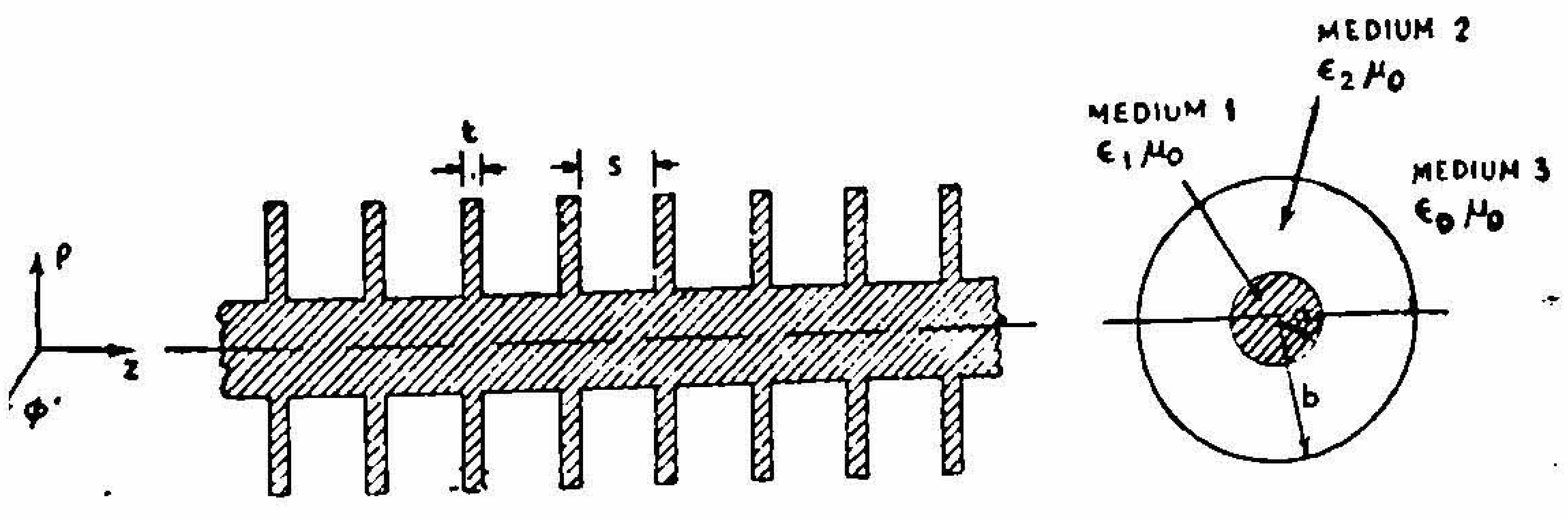


FIG. 6

Corrugated dielectric rod

a = Radius of the inner rod
b = Radius of disc
t Thickness of disc
s = Spacing between discs

ϵ_1 = Permittivity of medium 1
ϵ_2 = Permittivity of medium 2
ϵ_0 = Permittivity of free space (mediun 3)
μ_0 = Permeability of free space

Medium 1 : $\rho \leq a$
Medium 2 : $a \leq \rho \leq b$
Medium 3 : $\rho \geq b$

3.2 *Field Components*

The field components in the three media are

Medium 1 : $\rho \leq a$

$$E_{z1} = A_1 J_0(k_1 \rho) \exp(-j\beta z)$$

$$H_{\phi'1} = A_1 \frac{j\omega\epsilon_1}{k_1} J_1(k_1 \rho) \exp(-j\beta z)$$

$$E_{\rho 1} = A_1 \frac{j\beta}{k_1} J_1(k_1 \rho) \exp(-j\beta z) \quad [12]$$

Medium 2 : $a \leq \rho \leq b$

$$E_{z2} = [A_2 J_0(k_2 \rho) + A_3 Y_0(k_2 \rho)] \exp(-j\beta z)$$

$$H_{\phi'2} = \frac{j\omega\epsilon_2}{k_2} [A_2 J_1(k_2 \rho) + A_3 Y_1(k_2 \rho)] \exp(-j\beta z)$$

$$E_{\rho 2} = \frac{j\beta}{k_2} [A_2 J_1(k_2 \rho) + A_3 Y_1(k_2 \rho)] \exp(-j\beta z) \quad [13]$$

Medium 3 : $\rho \geq b$

$$E_{z3} = A_4 H_0^{(1)}(k_3 \rho) \exp(-j\beta z)$$

$$H_{\phi'3} = A_4 \frac{j\omega\epsilon_0}{k_3} H_1^{(1)}(k_3 \rho) \exp(-j\beta z)$$

$$E_{\rho 3} = A_4 \frac{j\beta}{k_3} H_1^{(1)}(k_3 \rho) \exp(-j\beta z) \quad [14]$$

The excitation constants A's are evaluated by using boundary conditions.

3.3 *Boundary Conditions*

The boundary conditions that have to be satisfied $\rho = a$ and $\rho = b$ are

At $\rho = a$,

$$E_{z1} = E_{z2} \qquad H_{\phi'1} = H_{\phi'2} \qquad \epsilon_1 E_{\rho 1} = \epsilon_2 E_{\rho 2} \quad [15]$$

and $\rho = b$,

$$E_{z2} = E_{z3} \qquad H_{\phi'2} = H_{\phi'3} \qquad \epsilon_2 E_{\rho 2} = \epsilon_0 E_{\rho 3} \quad [16]$$

3.4 *Characteristic Equation*

The following four equations are obtained by using appropriate field components and proper boundary conditions.

$$A_1 J_0(k_1 a) - A_2 J_0(k_2 a) - A_3 Y_0(k_2 a) = 0$$

$$A_1 \frac{J_1(k_1 a)}{k_1} - A_2 \frac{\epsilon_2}{\epsilon_1} \frac{J_1(k_2 a)}{k_2} - A_3 \frac{\epsilon_2}{\epsilon_1} \frac{Y_1(k_2 a)}{k_2} = 0$$

$$A_2 J_0(k_2 b) + A_3 Y_0(k_2 b) - A_4 H_0^{(1)}(k_3 b) = 0$$

$$A_2 \frac{\epsilon_2}{\epsilon_0} \frac{J_1(k_2 b)}{k_2} + A_3 \frac{\epsilon_2}{\epsilon_0} \frac{Y_0(k_2 b)}{k_2} - A_4 \frac{H_1^{(1)}(k_3 b)}{k_3} \qquad [17]$$

The following determinantal equation is obtained from equation [17]

$$x_1 \frac{J_0(x_1)}{J_1(x_1)} = \frac{\begin{vmatrix} -J_0(Dx_3) & -Y_0(Dx_3) & 0 \\ J_0(x_3) & Y_0(x_3) & -H_0^{(1)}(x_4) \\ \dfrac{-B_1 J_1(x_3)}{x_3} & -B_1 Y_1(x_3) & \dfrac{H_1^{(1)}(x_4)}{x_4} \end{vmatrix}}{\begin{vmatrix} -\dfrac{A}{D} \dfrac{J_1(Dx_3)}{x_3} & -\dfrac{A}{D} \dfrac{Y_1(Dx_3)}{x_3} & 0 \\ J_0(x_3) & Y_0(x_3) & -H_0^{(1)}(x_4) \\ -B_1 \dfrac{J_1(x_3)}{x_3} & \dfrac{-B_1 Y_1(x_3)}{x_3} & \dfrac{H_1^{(1)}(x_4)}{x_4} \end{vmatrix}} \qquad [18]$$

where, $x_1 = k_1 a \qquad A = \epsilon_{r2}/\epsilon_{r1}$

$x_3 = k_2 b \qquad B_1 = \epsilon_{r2}$

$x_4 = k_3 b \qquad D = a/b$

The axial propagation Constant β is related to the radial propagation constants k_1, k_2, k_3 as follows

$$k_1^2 = \left(\frac{x_1}{a}\right)^2 = -\beta^2 + \omega^2 \mu_0 \epsilon_1$$

$$k_2^2 = \left(\frac{x_3}{b}\right)^2 = -\beta^2 + \omega^2 \mu_0 \epsilon_2$$

$$k_3^2 = \left(\frac{x_4}{b}\right)^2 = -\beta^2 + \omega^2 \mu_0 \epsilon_0 \qquad [19]$$

In order that the structure may support an unattenuated surface wave, the axial phase constant β must be real which requires that the following inequalities must be satisfied

$$\omega^2 \mu_0 \epsilon_1 > k_1^2$$

$$\omega^2 \mu_0 \epsilon_2 > k_2^2$$

and k_3 must be positive imaginary. k_1 and k_2 are considered positive and real.

3.5 *Nnmerical Computation*

The characteristic equation [18] has been solved with the aid of a digital computer (Type 803, Elliot) to determine propogation constant as function of 't', 's', 'a' and 'b'.

3.5 (1) *Radial Propagation Constants*

The radial propagation constants k_1, k_2 and k_3 obtained from the solution of the characteristic equation are plotted as functions of s/λ_0, a/λ_0 and b/λ_0 in Figures 7, 8 and 9.

3.5 (2) *Guide Wavelength*

The variation of guide wavelength λg as a function of s/λ_0, a/λ_0 and b/λ_0 is shown in Figures (10—12).

3.5 (3) *Surface Reactance*

The surface reactance of the corrugated structure is given by

$$z = j X_S = E_z'/H_\phi'|_{\rho=b} = [k_3 / j\omega\epsilon_0] \; [H_0^{(1)}(k_3 b)/H_1^{(1)}(k_3 b)] \qquad [20]$$

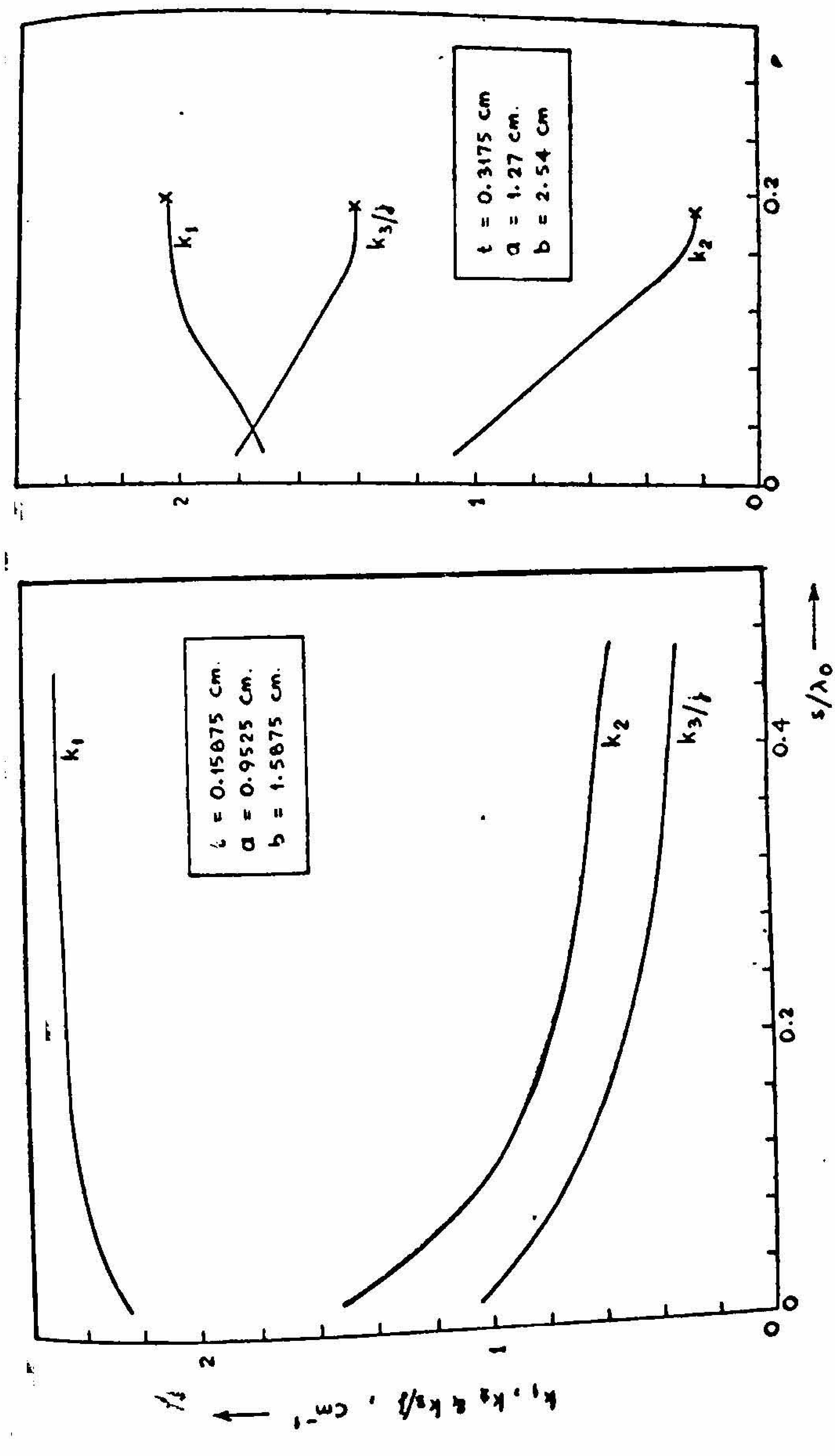
which is plotted as a function of s/λ_0, a/λ_0, b/λ_0 and $\lambda g/\lambda_0$ in Figs. (13—16).

3.5 (4) *Radial Field Decay*

The radial field decay curves for various combinations of the structure parameters are shown in Figures (17) and (18), The values of E_ρ is normalised with respect to the field at the surface $\rho = b$.

4.1 *Uniform Dielectric Rod*

(i) k_1' decreases with increasing a/λ_0 and k_2' increases with increasing a/λ_0 and tends to become constant for large values of a/λ_0 (See Fig. 2). This indicates a decreasing influence of large a/λ_0 on the radial field spread.



(*a*) (*b*)

FIG. 7

Variation of radial propagation constants with s/λ_0 for corrugated dielectric rod

t = Disc thickness a = Inner rod radius λ_0 = 3.14 cm.

s = Disc spacing b = Disc radius x denotes cut off value

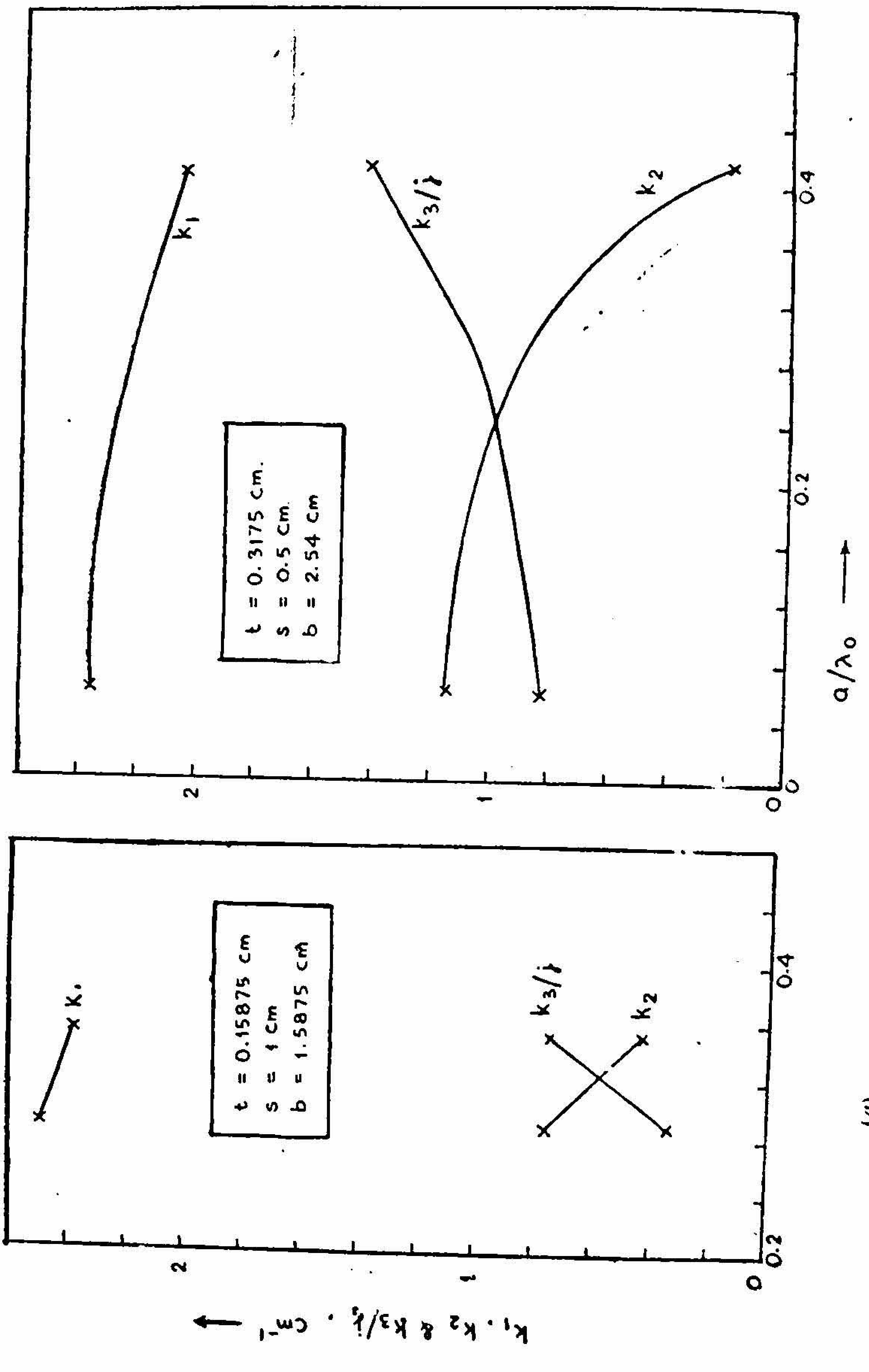


FIG. 8

Variation of radial propagation constants with a/λ_0 for corrugated dielectric rod

t – Disc thickness
s – Disc spacing
a – Inner rod radius
b – Disc radius
λ_0 – 3.14 cm.
x denotes cut off value

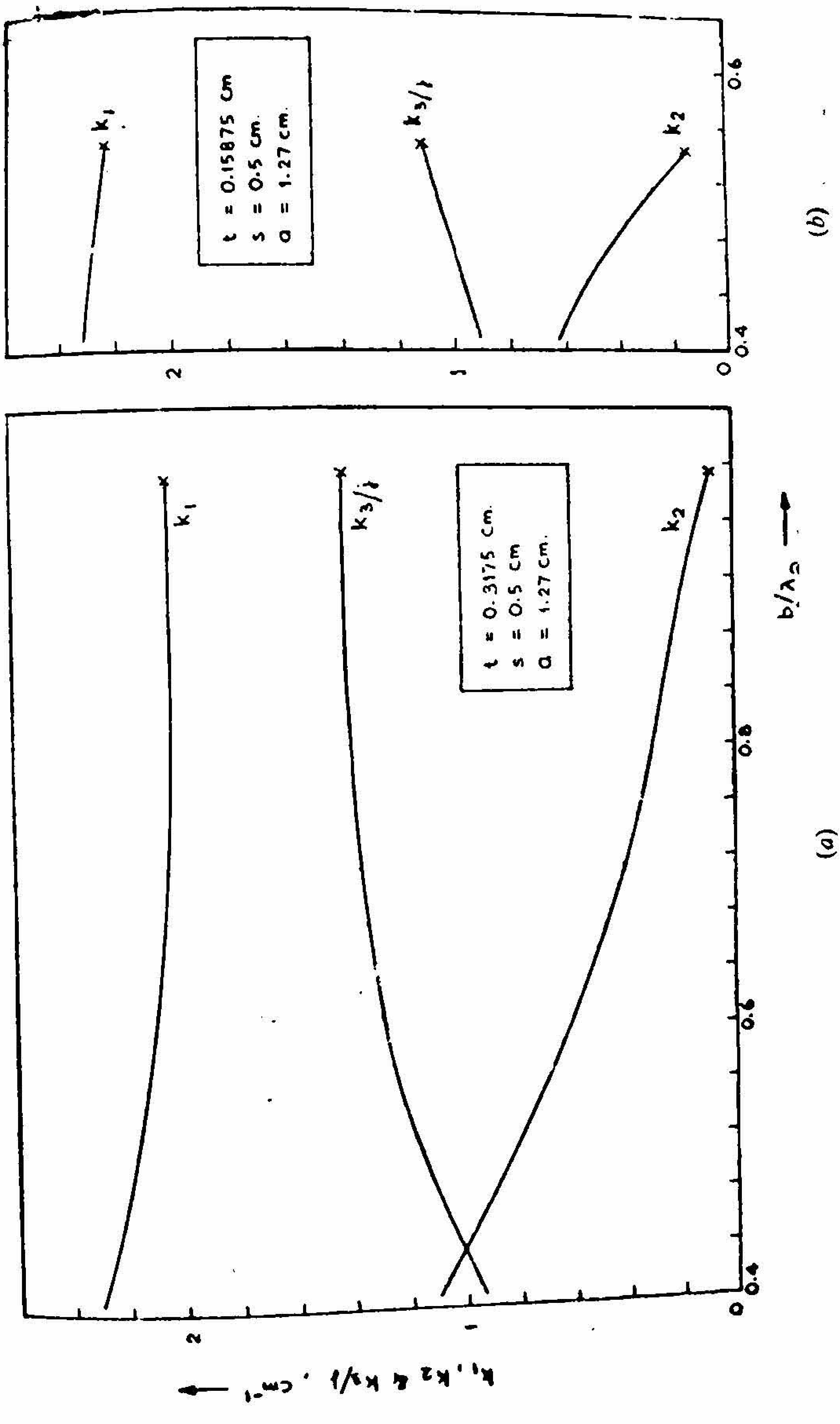


FIG. 9

Variation of radial propagation constants with b/λ_0 for corrugated dieletric rod

t – Disc thickness; s – Disc spacing; a – Inner rod radius; b – Disc radius; λ_0 = 3.14 cm. x den otes cut off value

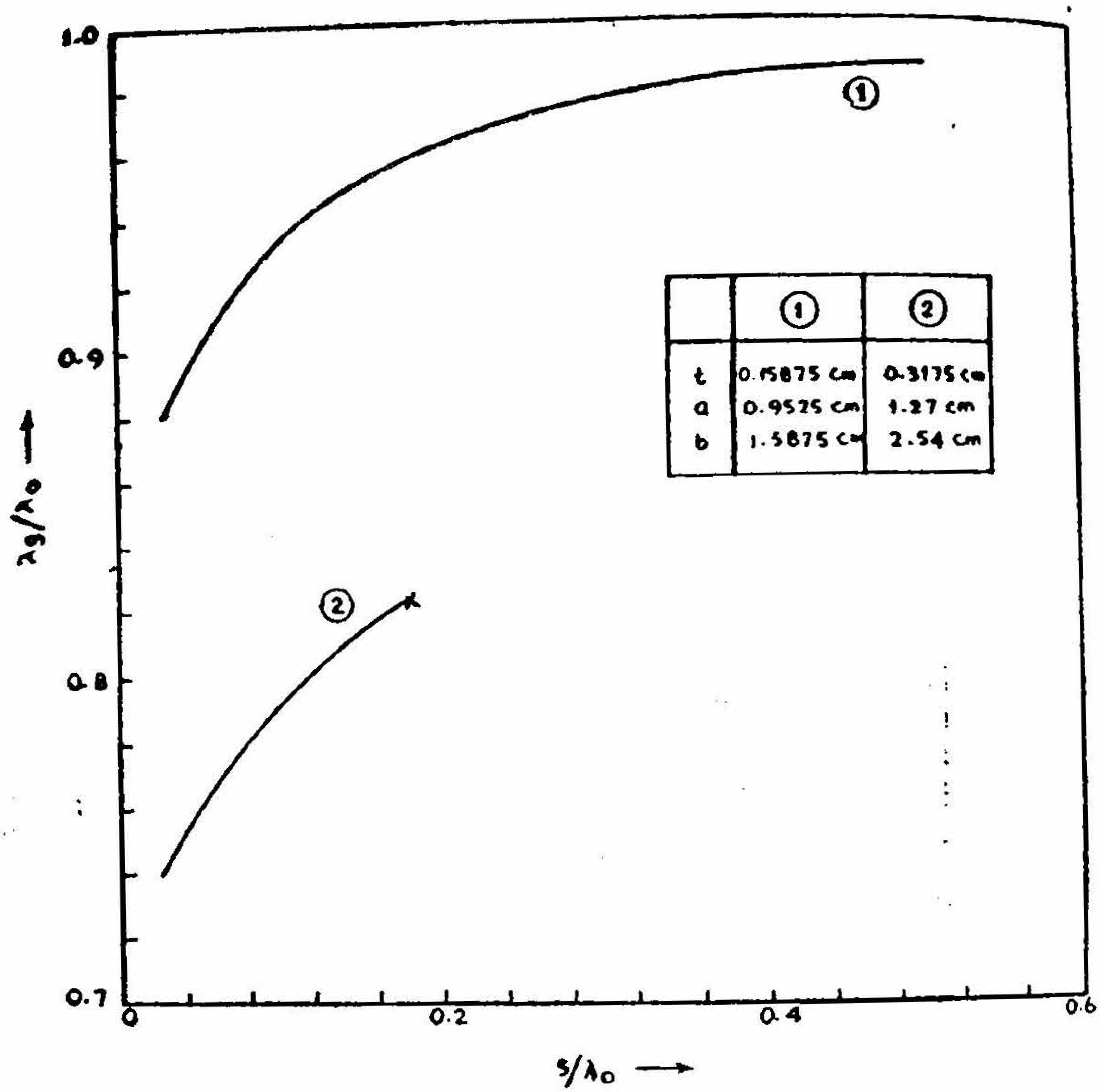


FIG. 10

Variation of λ_g/λ_0 with s/λ_0 for corrugated dielectric rod

t = Disc thicknees — λ_g = Guide wavelength
s = Disc spacing — $\lambda_0 = 3.14$ cm.
a = Inner rod radius — x denotes cut off value
b = Disc radius

(ii) The axial phase constant increases at first with increasing a/λ_0 and tends in the limit to the value corresponding to a plane wave propagating in an infinite medium having μ and ϵ same as that of the dielectric rod.

(iii) The value of $\lambda g/\lambda_0$ decreases wilh a/λ_0 and finally approaches asymptotically the value $1/\sqrt{\epsilon_{r_1}}$. This corresponds to the propagation of the wave through an infinite medium having a dielectric contant ϵ_{r_1}.

(iv) The uniform dielectric rod excited in E_0−mode has a cut-off radius below which it cannot support a surface wave. For $\lambda_0 = 3,14$ cm and $\epsilon_{r1} = 2.56$, the cut-off radius is about 0.9624 cms.

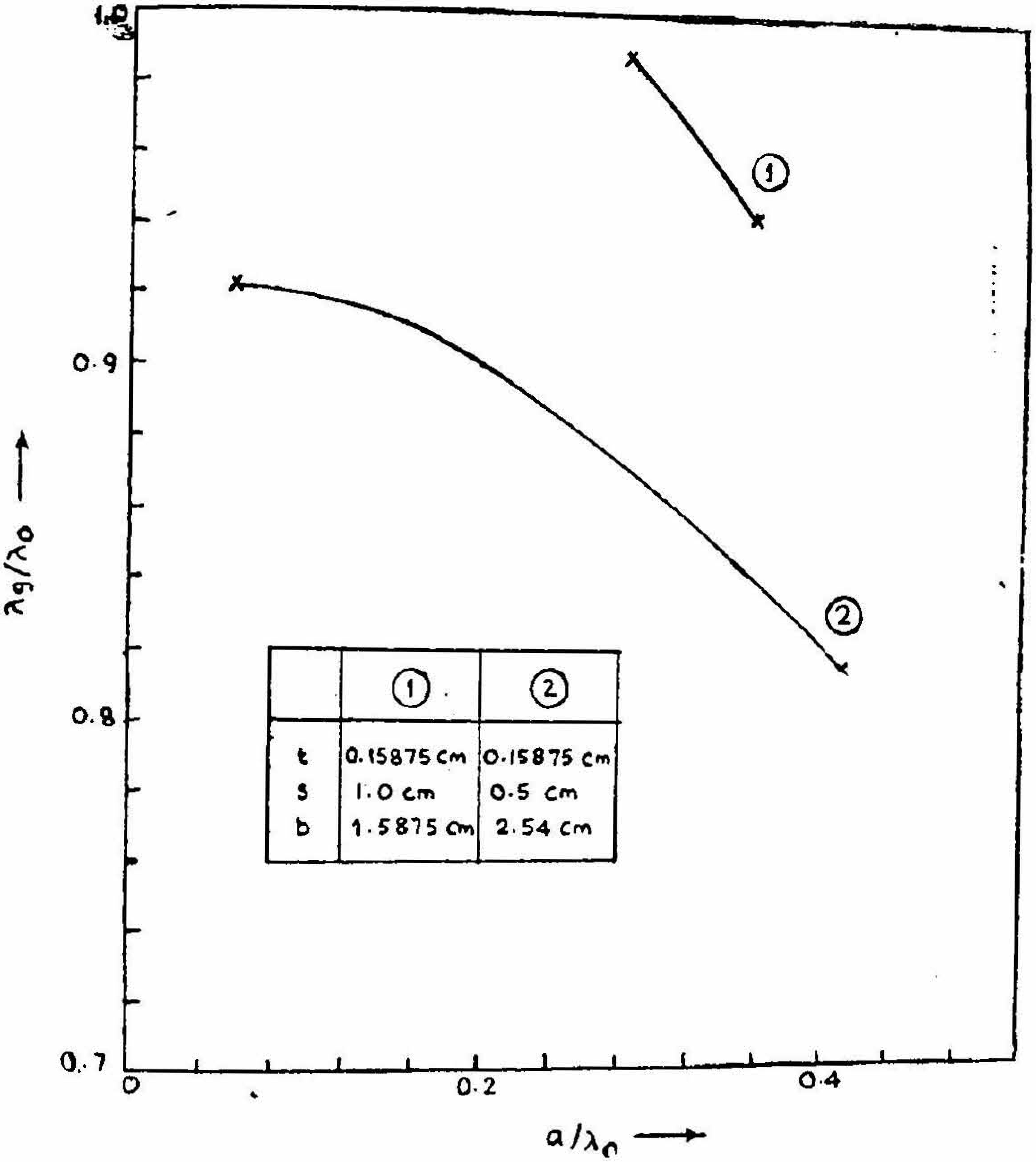


FIG. 11

Variation of λ_g/λ_0 with a/λ_0 for corrugated dielectric rod

t – Disc thickness	λ_g – Guide wavelength
s – Disc spacing	λ_0 – 3.14 cm.
a – Inner rod radius	x denotes cut off value
b – Disc radius	

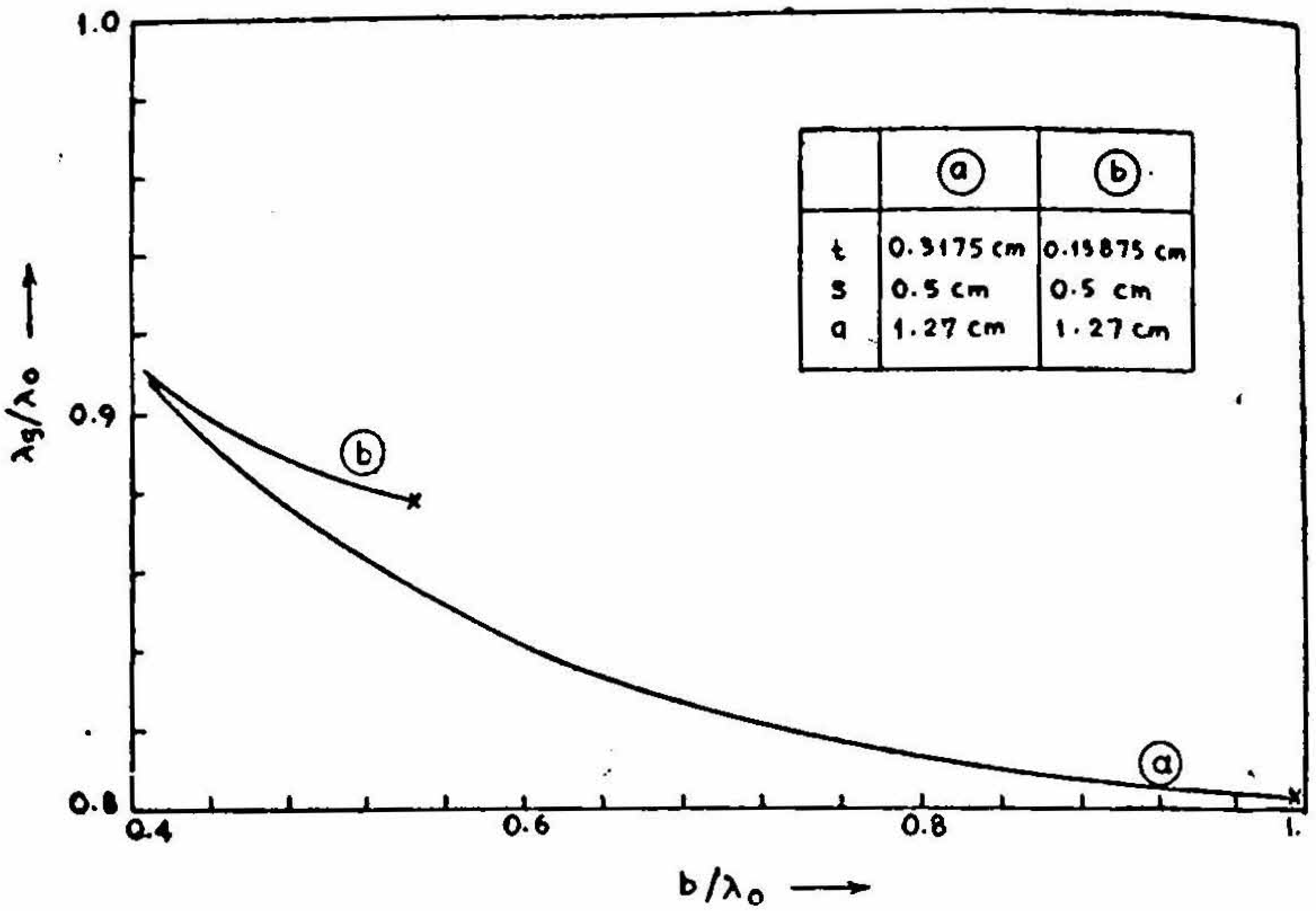


FIG. 12

Variation of λ_g/λ_0 with b/λ_0 for corrugated dielectric rod

t = Disc thickness
s = Disc spacing
a = Inner rod radius
b = Disc radius
λ_g = Guide wavelength
λ_0 = 3.14 cm.
x denotes cut off value

4.2 *Corrugated Dielectric Rod*

(i) Table 1. illustrates some combinations of the structure parameters for which the surface wave root does or does not exist. The combinations are not unique.

(ii) The corrugation on an uniform dielectric rod influence the cut-off condition. The surface wave root exists for the value of 'a' as low as 0.2 cm, whereas, the cut-off value of 'a' in case of uniform dielectric rod is about 0.9624 cm.

(iii) The effect of structure parameters on the field decay coefficient can be studied from the radial field decay curves. It is found that the corrugated rod possesses better surface wave characteristics than a uniform rod.

(iv) The variation of surface reactance with guide wavelength and structure parameters is shown in Figures 13—16. For the combinations of the structure parameters used, the surface impedance is positive imaginary *i.e.*, the surface behaves as an inductive surface. As the surface reactance of the structure increases, the guide wavelength decreases, or in other words, the structure supports a tightly bound surface wave.

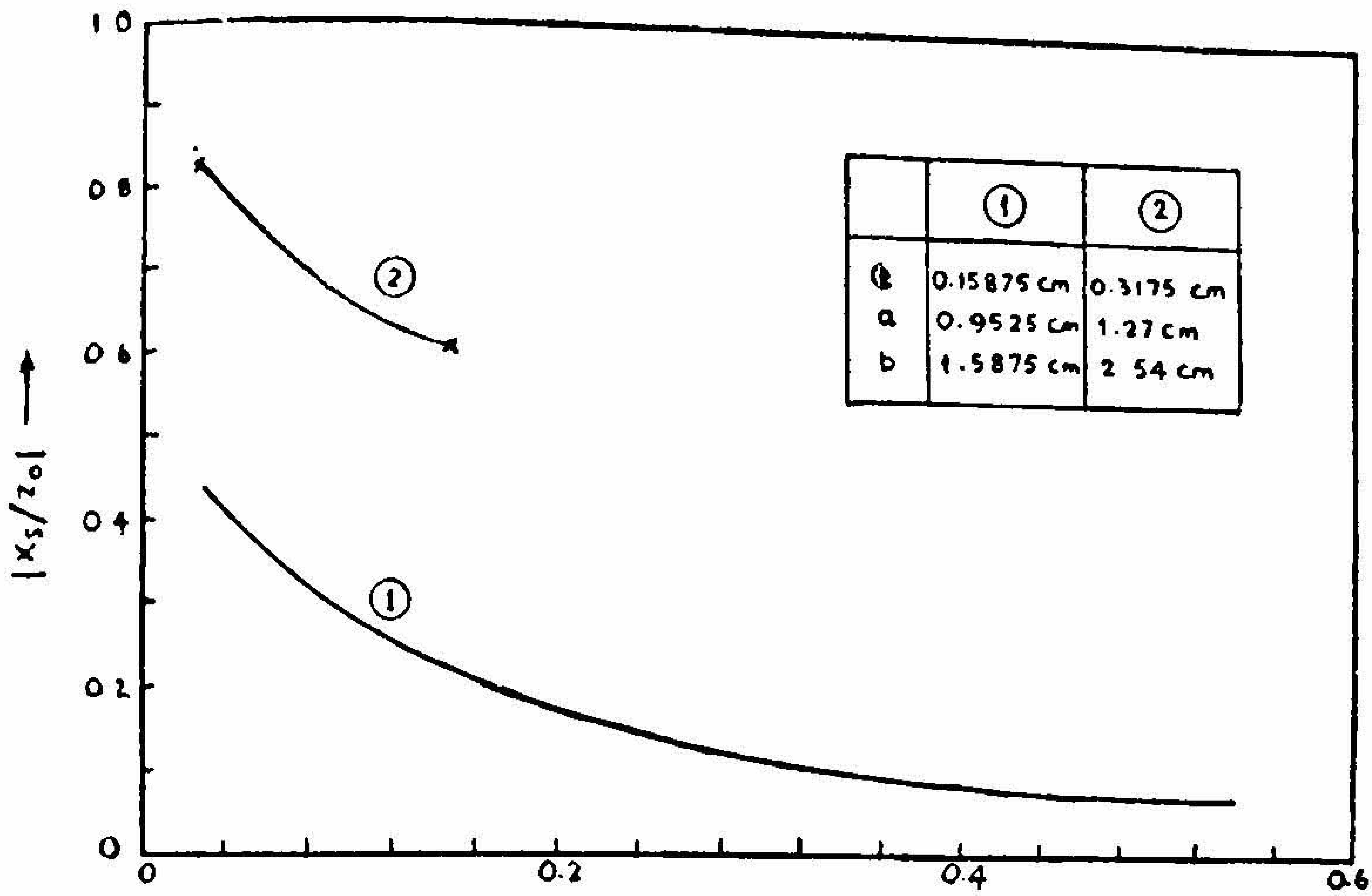


FIG. 13

Variation of surface reactance with s/λ_0 for corrugated dielectric rod

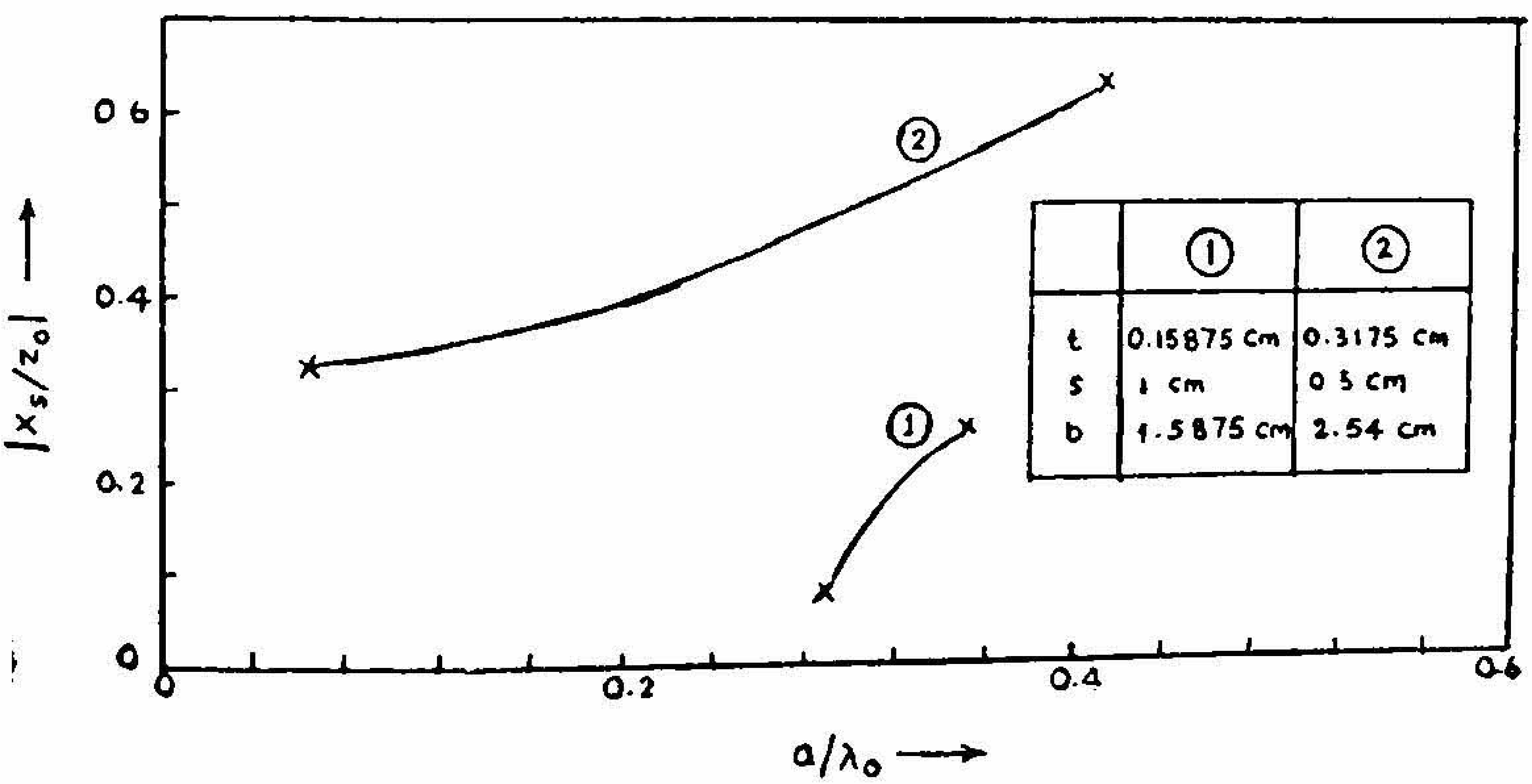


FIG. 14

Variation of surface reactance with a/λ_0 for corrugated dielectric rod

t – Disc thickness
s – Disc spacing
a – Inner rod radius
b – Disc radius

Z_0 = Free space impedence = 377Ω
λ_0 = 3.14 cm.
x denotes cut off value

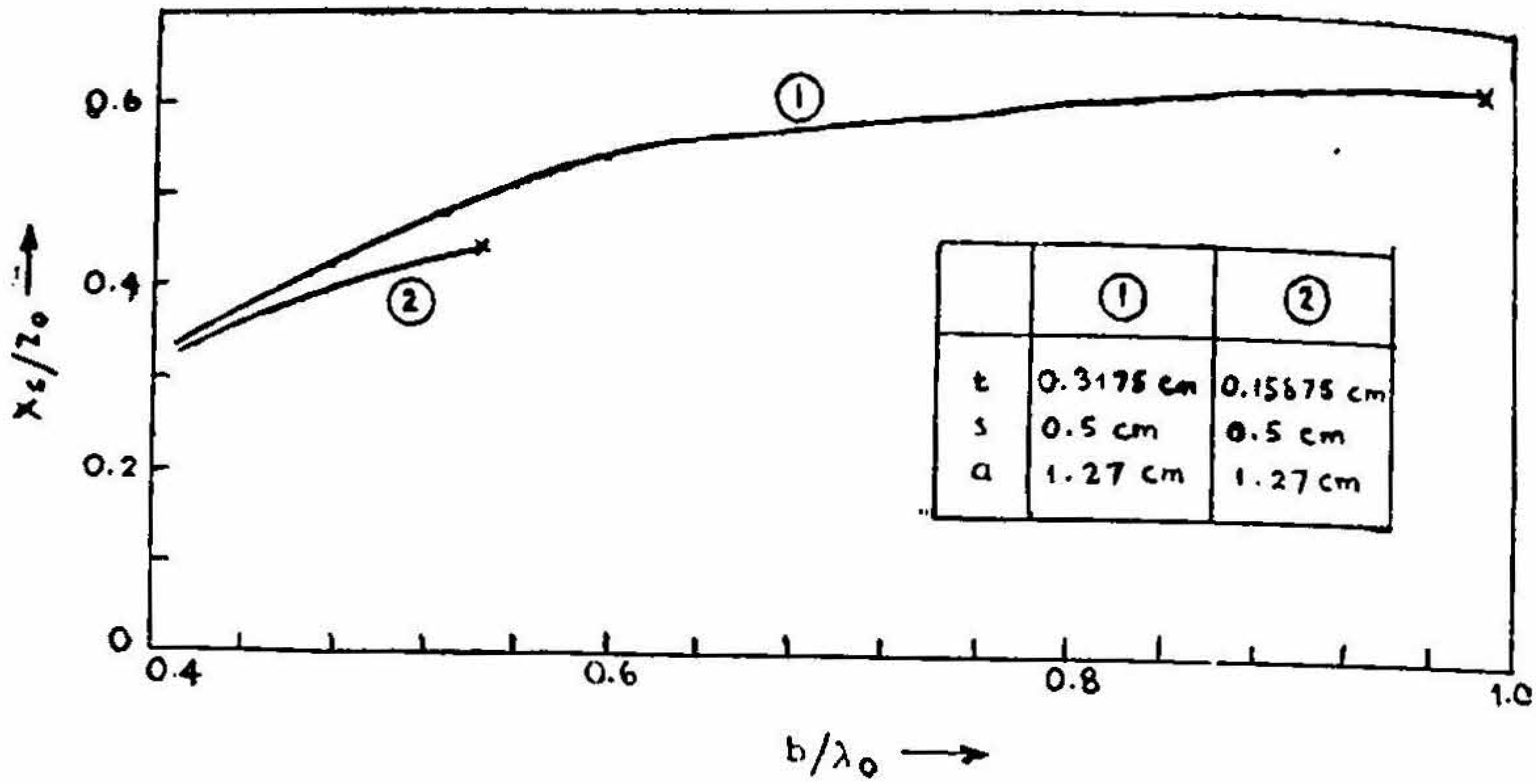


FIG. 15

Variation of surface reactance with b/λ_0 for corrugated dielectric rod

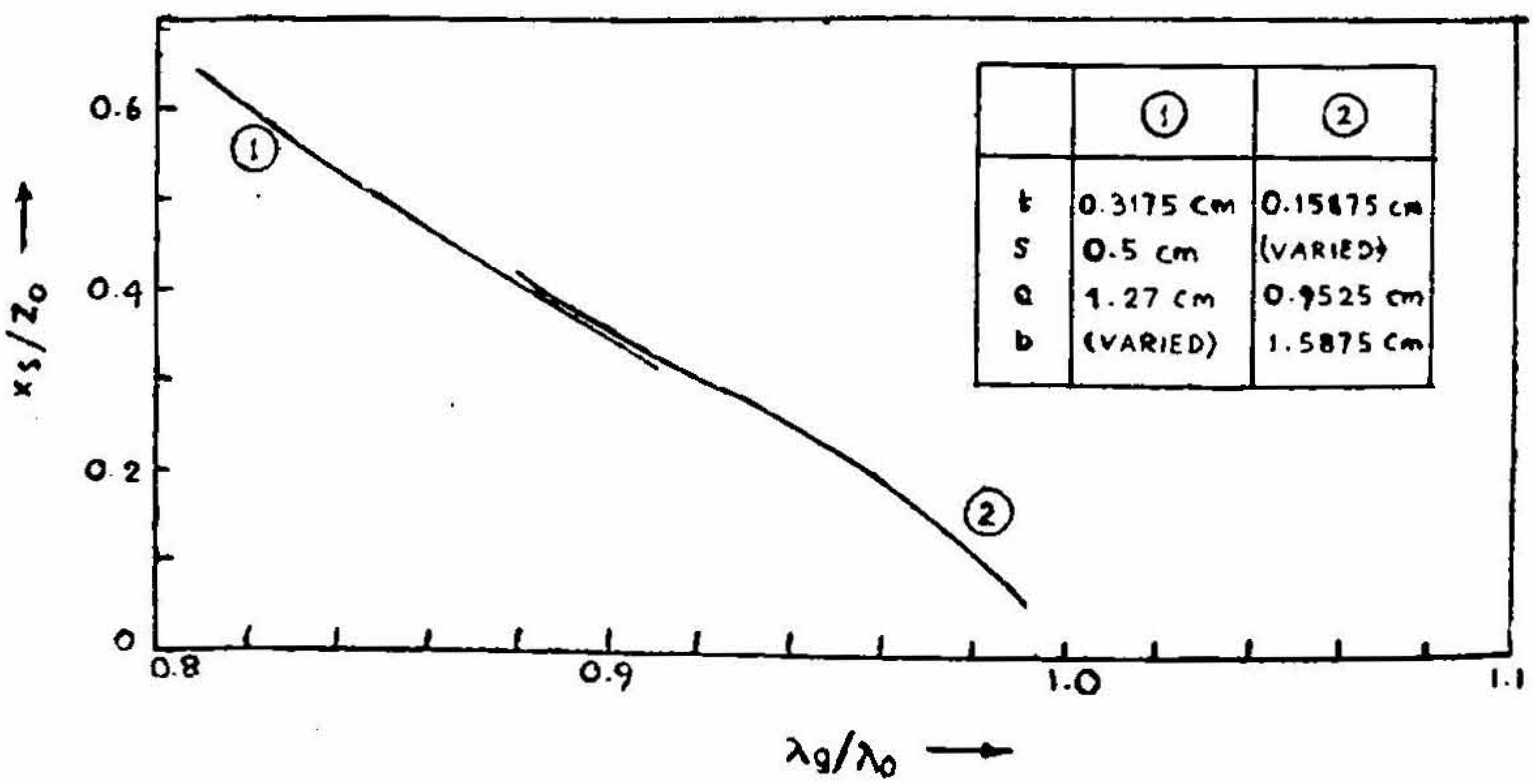


FIG. 16

Variation of surface reactance with λ_g/λ_0 for corrugatted dielectric rod

t = Disc thickness
s = Disc spacing
a = Inner rod radius
b = Disc radius

Z_0 = Free space impedance = 377 Ω
λ_g = Guide wavelength
λ_0 = 3.14 cm.

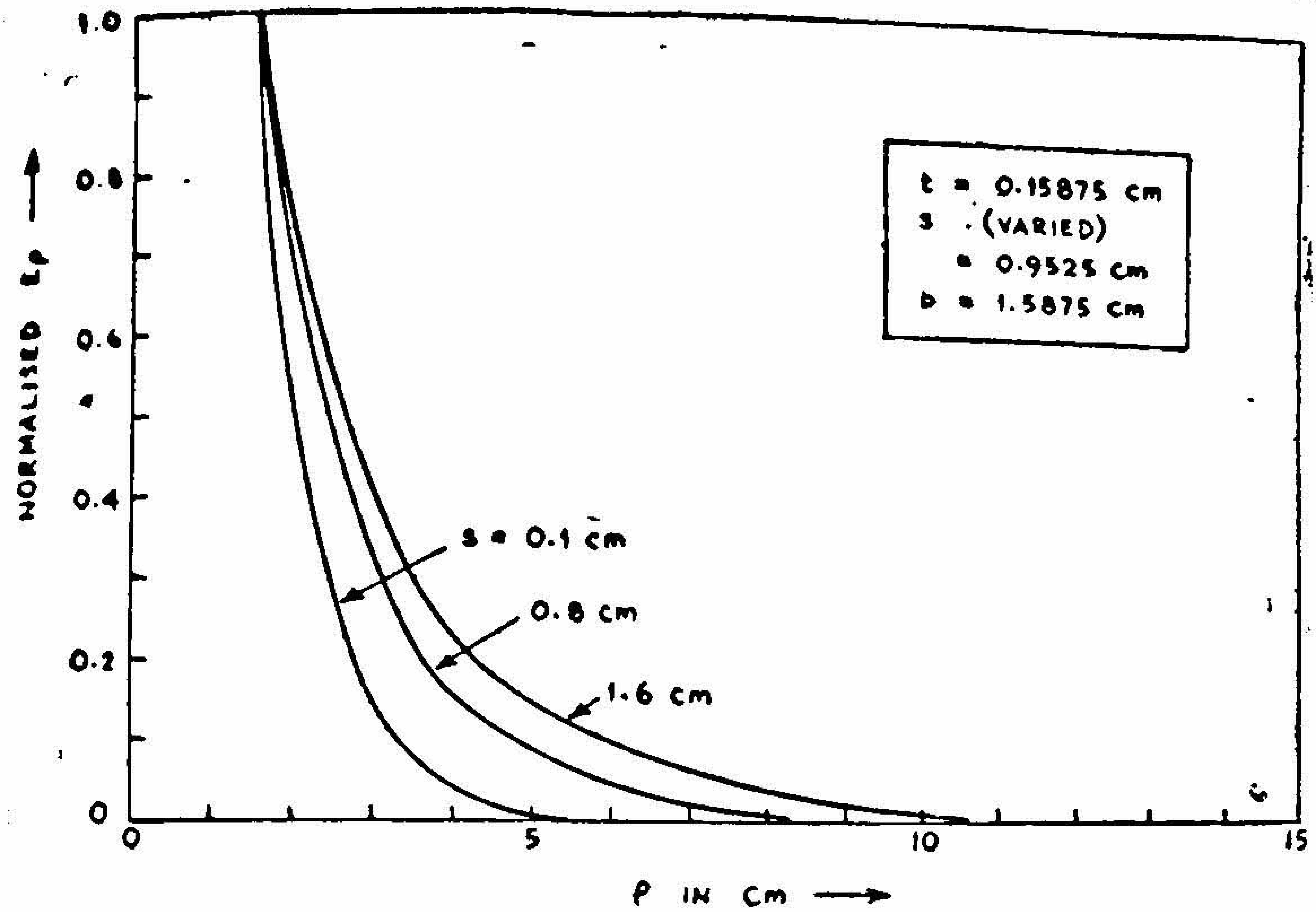


FIG. 17
Radial field decay curves for corrugated dielectric rod

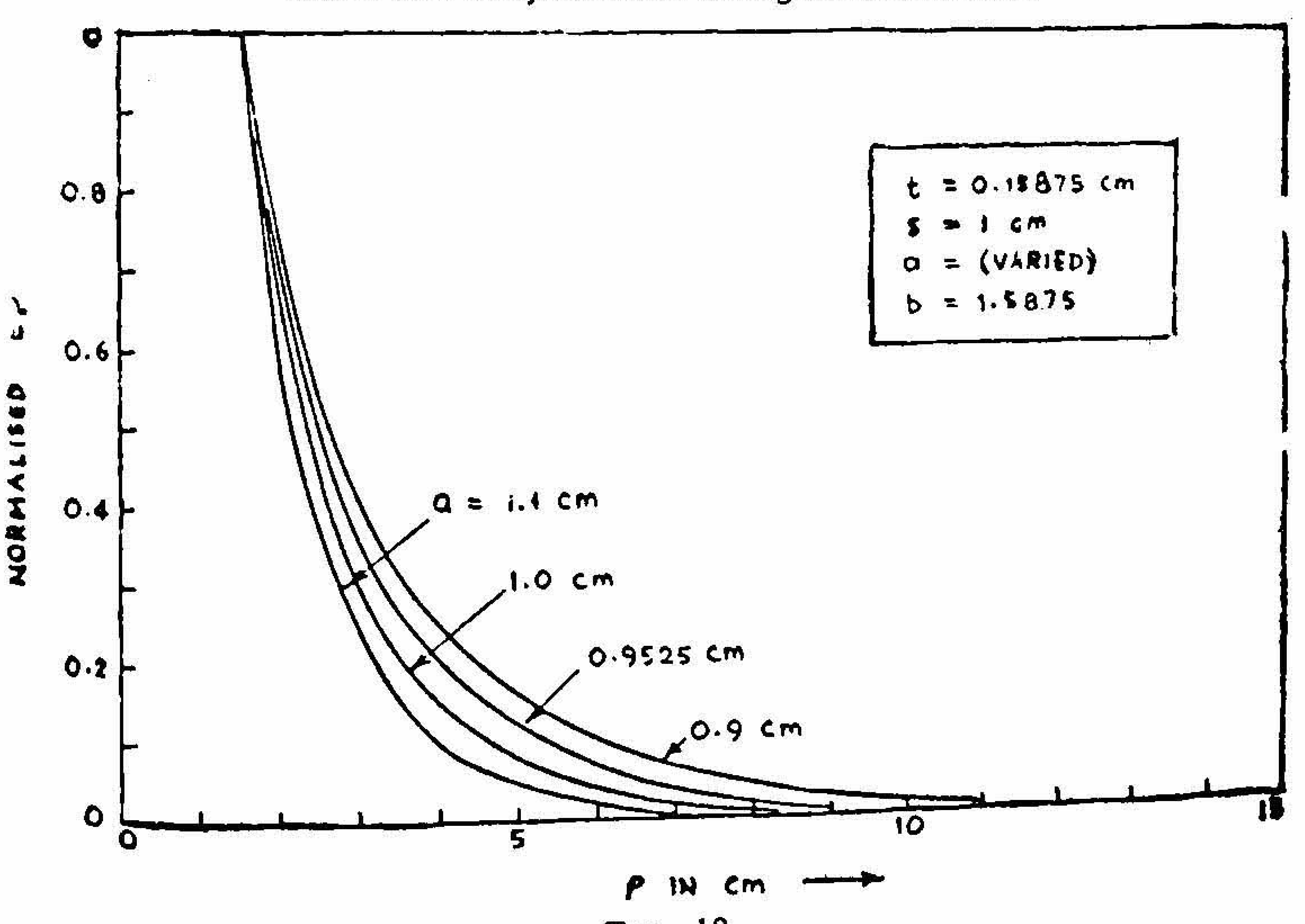


FIG. 18
Radial field decay curves for corrugated dielectric rod
t–Disc thickness
s–Disc spacing
a–Inner rod radius
b–Disc radius

TABLE 1

Effect of structure Parameters on the Surface Wave Root

	Structure Parameters (cm.)				Parameters Varied	Existence of Surface Wave Roots
	t	s	a	b		
1.	.15875	–	.9525	1.5875	s is varied from 0.1 cm to 2 cm.	Surface wave root exists throughout the range of 's'.
2.	0.15875	–	.9525	1.905	s is varied from 0.1 cm to 2 cm.	Surface wave root exists througout the range of 's'.
3.	0.3175	–	1.27	2.54	s is varied from 0.1 cm to 1.6 cm	(i) Surface wave root exists for all values of s from $s=0.1$ cm. to $s=0.5$ cm. (ii) for $s > 0.5$ cm surface wave root does not exist.
4.	.15875	1	–	1.5875	a is varied from 0.1 cm to 1.5 cm.	(i) from $1=0.1$ cm to 0.8 cm surface wave root does not exist (ii) from $a=0.9$ to 1.1 cm. the surface wave root exists. (iii) For $a > 1.2$ cm surface wave root does not exist.
5.	.15875	1	–	1.27	a is varied from 0.1 cm to 1.2 cm	Surface wave root does not exist for $a=0.1$ cm to 0.8 cm.
6.	0.3175	0.5	–	2.54	a is varied from 0.1 cm to 2.5 cm	(i) Surface wave root does not exist for $a=0.1$ cm to 0.18 cm. (ii) from $a=0.2$ cm 1.3 cm. surface wave root exists. (iii) for a 1.3 cm surface wave root does not exist.

TABLE 1 – (*contd.*)

Effect of structure Parameters on the Surface Wave Root

	Structure Parameters (cm.)				Parameters Varied	Existence of Surface wave Roots
	t	s	a	b		
7.	.3175	1.6	–	3.175	a is varied from 0.1 cm to 3 cm.	(i) from $a=0.1$ cm to 0.7 cm surface wave root does not exist. (ii) from $a=0.8$ cm to 1 cm surface wave root exists. (iii) from $a=1.1$ cm. to 2 cm. surface wave root does not exist. (iv) from $a=2.1$ cm. to 2.4 cm. surface wave root exists. (v) For $a>2.4$ cm. surface wave root does not exist.
8.	0.3175	0.5			b is varied from 1.3 cm to 6 cm	(i) surface wave root exists from $b=1.3$ cm to 3 cm. (ii) For $b>3$ cm surface wave root does not exist.

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