Sustainable maintenance strategy under uncertainty in the lifetime distribution of deteriorating assets

Alireza Daneshkhah, Amin Hosseinian-Far and Omid Chatrabgoun

Abstract In the life-cycle management of systems under continuous deterioration, studying the sensitivity analysis of the optimised preventive maintenance decisions with respect to the changes in the model parameters is of a great importance. Since the calculations of the mean cost rates considered in the preventive maintenance policies are not sufficiently robust, the corresponding maintenance model can generate outcomes that are not robust and this would subsequently require interventions that are costly. This chapter presents a computationally efficient decision-theoretic sensitivity analysis for a maintenance optimisation problem for systems/structures/assets subject to measurable deterioration using the Partial Expected Value of Perfect Information (PEVPI) concept. Furthermore, this sensitivity analysis approach provides a framework to quantify the benefits of the proposed maintenance/replacement strategies or inspection schedules in terms of their expected costs and in light of accumulated information about the model parameters and aspects of the system, such as the ageing process. In this paper, we consider random variable model and stochastic Gamma process model as two well-known probabilistic models to present the uncertainty associated with the asset deterioration. We illustrate the use of PEVPI to perform sensitivity analysis on a maintenance optimisation problem by using two standard preventive maintenance policies, namely age-based and condition-based maintenance policies. The optimal strategy of the former policy is the time of replacement or repair and the optimal strategies of the later policy are the inspection time and the preventive maintenance ratio.

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These optimal strategies are determined by minimising the corresponding expected cost rates for the given deterioration models' parameters, total cost and replacement or repair cost. The robust optimised strategies to the changes of the models' parameters can be determined by evaluating PEVPI's which involves the computation of multi-dimensional integrals and is often computationally demanding, and conventional numerical integration or Monte Carlo simulation techniques would not be helpful. To overcome this computational difficulty, we approximate the PEVPI using Gaussian process emulators.

Keywords: Deterioration models, Partial Expected Value of Perfect Information, Gaussian process, optimised maintenance, cost-benefit

1 Introduction

The resilience of an asset/machinery component or networked infrastructure system is greatly dependent on an efficient cost effective life-cycle management. This can be achieved by determining optimal maintenance and rehabilitation scheduling schemes. Maintenance costs for an asset or a networked infrastructure systems including rail, water, energy, bridge etc. are swiftly rising, but current estimates suggest that ontime optimized maintenance schedules could save one trillion dollars per year on infrastructure costs [15].

The maintenance strategies have generally been divided into two categories: Corrective Maintenance (CM); and Preventative Maintenance (PM). The former includes repairing failed components and systems, while the latter involves systematic inspection and correction of initiative failures, before they progress into major faults or defects. In the recent years, an increasing dominance of PM has been clearly observed with overall costs illustrated to be lower than CM strategy. The preventive maintenance is extensively applied to lessen asset deterioration and mitigate the risk of unforeseen failure. This maintenance strategy can be further classified into two methods: Time-Based Maintenance (TBM), and Condition-Based Maintenance (CBM). In the TBM, maintenance activities take place at predetermined time intervals, but in the CBM, interventions are immediately carried out based on the information collected through condition sensing and monitoring processes (either manual or automated). Both TBM and CBM are widely used for asset/infrastructure life-cycle management decision making, and extensively studied in [1, 14, 23, 34].

The main difficulty to make informed PM decisions is that predicting the time to first inspection, maintenance intervention, or replacement is confounded by model parameters' uncertainties associated with the failure, deterioration, repair, or maintenance distributions. As a result, studying sensitivity of the model output with respect to the changes in the model parameters/inputs is very essential to determine an optimal maintenance strategy under these uncertainties. One of the main aims of this paper is to investigate sensitivity analysis of the optimised maintenance with respect to the changes in the model's inputs when the aforementioned preventive maintenance strategies are considered for the asset/infrastructure which is under continuous deterioration. The optimal maintenance decision under this maintenance policies is normally considered as the inspection interval and the preventive maintenance ratio that would minimize the expected total cost of the maintenance strategy. This type of maintenance strategy is known to practically be more useful, particularly for larger and more complex systems [1, 35]. This is mainly because it eliminates the need to record component ages. It should be noted that finding the optimal decision under a CBM policy for a deteriorating component involves solving a two-dimensional optimisation problem, while for the TBM case the aim is to determine the critical age as a single strategy variable.

As mentioned above, the PM policy cost function is dominated by the deterioration and repair distribution's parameters. Therefore, the computation of a mean cost rates for a specific PM policy is not sufficiently robust, and the corresponding maintenance model can generate results that are not robust. In other words, the determination of an optimal maintenance intervention will be sensitive to the parameters creating uncertainty as to the optimal strategy. The uncertainty around the optimal PM maintenance can be mitigated by gathering further information on some (or all) of the model parameters/inputs. In particular, Partial Expected Value of Perfect Information (PEVPI) computations provide an upper bound for the value (in terms of cost-benefit) that can be expected to be yielded from removing uncertainty in a subset of the parameters to the cost computation. The PEVPI provides a decision-informed sensitivity analysis framework which enables researchers to determine the key parameters of the problem and quantify the value of learning about certain aspects of the system [27, 35]. In maintenance studies ([16, 12]), this information can play a crucial role, where we are interested in not only determining an optimal PM strategy, but also in collecting further information about the system features, including the deterioration process to make more robust decisions.

The identification of PEVPI requires the computation of high-dimensional integrals that are regularly expensive to evaluate, and typical numerical integration or Monte Carlo simulation techniques are not practical. This computational burden can be overcome, by applying the sensitivity analysis through the use of Gaussian process (GP) emulators [27, 35]. In this paper, we adopt and execute this sensitivity analysis approach for determining robust optimised PM strategies for a component/system which is under continuous deterioration.

We use the random variable model (RV) to probabilistically model the deterioration of a component/system of interest. Under the RV model [28], the probability density and distribution functions of the lifetime are respectively given by

$$f_T^R(t) = \frac{(\delta/\rho)}{\Gamma(\eta)}(\frac{\rho}{\delta t})^{\eta+1} e^{-\rho/(\delta t)}, \tag{1}$$

and

$$F_T^R(t) = 1 - \mathscr{G}(\rho/t;\eta,\delta) = 1 - \mathscr{G}(\rho;\eta,\delta t), \tag{2}$$

where η and δ are respectively the shape and scale parameters, $\mathscr{G}(\rho/t;\eta,\delta)$ denote the gamma cumulative distribution function with the same shape and scale parameters for $X = \rho/T$, and $\rho = (r_0 - s) > 0$ is called the available design margin or a *failure threshold*. In the last expression, r_0 is the initial resistance of the component against the load effect, s. Thus, a failure is defined as the event at which the cumulative amount of deterioration exceeds the deterioration threshold (ρ). The threshold ρ, s and r_0 are assumed to be deterministic constants for simplicity of discussion (see [28] for further details).

The rest of the chapter is organised as follows. In Section 2, we discuss how this probabilistic deterioration model links to TBM and CBM maintenance optimisation problems. We formulate uncertainty quantification of the optimised PM policies using the decision-informed sensitivity analysis in Section 3. The GP emulator required to compute PEVPI's as within the context of decision-theoretic sensitivity analysis is also briefly discussed in Section 3. Section 4 is dedicated to derive the robust optimised maintenance decisions for TBM and CBM policies using several illustrative settings of different complexity. We conclude by discussing the implications of our approach and identify opportunities for future work.

2 Optimal Preventive Maintenance Policy

The central objective of a preventive maintenance (TBM or CBM) optimisation model is to determine the value of the decision variable T (replacement time or inspection time) that optimizes a given objective function amongst the available alternative maintenance decisions. For instance in a TBM policy, the optimisation problem is usually defined over a finite time horizon $[0,t]$, and the objective function, denoted by $C(t)$, is the long-term average cost. There are various ways to define these costs [3, 13]. The long-term mean cost per unit of time is normally defined in terms of the length of two consecutive replacements (or life cycle) as follows:

$$\mathscr{C}(T) = \frac{C(T)}{L(T)}. \tag{3}$$

The following formula is an example of the expected cost per unit of a component under a general TBM policy

$$\mathscr{C}(T) = \frac{c_1 F(T) + c_2 R(T)}{T \cdot R(T) + \int_0^T t f(t) \mathrm{d}t + \tau}, \tag{4}$$

where $F(T)$ is the failure distribution function of a system at time T (or probability of unplanned replacement due to an unexpected failure), $R(T) = 1 - F(T)$ is the probability of planned replacement at time T, c_1 is the cost of a corrective maintenance, c_2 is the cost of planned replacement and τ is the expected duration of replacement.

The objective is then to identify the optimal strategy T^* that corresponds to the minimum cost rate (cost per unit of time), that is;

$$T^* = \arg\min_{T>0} \{\mathscr{C}(T)\}. \tag{5}$$

A similar methods is used to determine the optimised CBM strategy. The cost function in this policy is the mean cost rate which is defined as:

$$\mathscr{K}(t_I, \upsilon) = \frac{E[C(t_I, \upsilon)]}{E[L(t_I, \upsilon)]}, \quad (6)$$

where $E[C(t_I, \upsilon)]$ is the renewal cycle cost, $E[L(t_I, \upsilon)]$ is the renewal cycle length, t_I is the inspection time interval and υ is the PM ratio. The details of numerator and denominator of the mean cost rate will be given in Section 4.

The objective is then to find t_I^* and υ^* so that $\mathscr{K}(t_I^*, \upsilon^*)$ becomes the minimal cost solution.

2.1 Uncertainty quantification using via decision-theoretic sensitivity analysis

The optimal maintenance strategies derived by minimizing the expected cost rate is influenced by characteristics such as the deterioration process or failure behaviour of the system and the characteristics of maintenance tasks (including repair/replacement policy, maintenance crew and spare part availability etc.). These characteristics are subject to uncertainty, prompting study of the sensitivity of an optimal maintenance strategy with respect to changes in the model parameters and other inflecting uncertain inputs. Such an analysis improves understanding of the 'robustness' of the derived inferences or predictions of the model, and, offers a tool for determining the critical influences on model predictions [33, 11]. Zitrou et al. [35] summarise the main sensitivity measures and discuss their values and applications in an extensive sensitivity analysis. They conclude that a simple yet effective method of implementing sensitivity analysis is to vary one or more parameter inputs over some plausible range, whilst keeping the other parameters fixed, and then examine the effects of these changes on the model output. Although this method is straightforward to implement and interpret, it becomes inconvenient where there are large numbers of model parameters or when the model is computationally intensive.

In order to resolve this difficulty, we use a variance-based method for sensitivity analysis [33]. This approach can capture the fractions of the model output variance which are explained by the model inputs. In addition, it can also provide the total contribution to the output variance of a given input - i.e. its marginal contribution and its cooperative contribution. The contribution of each model's input to the model output variance serves as an indicator of how strong an influence a certain input or parameter has on model output variability. However, within a decision-making context like the maintenance optimisation problem, we are primarily interested in the effect of parameter uncertainty on corresponding utility or loss. To achieve this objective, we use the concept of the Expected Value of Perfect Information (EVPI) as a measure of parameter importance [27, 35]. The EVPI approach allows the application of sensitivity analysis to the maintenance optimisation model and identifies the model parameters for which collecting additional information (learning) prior to the maintenance decision would have a significant impact on total cost.

3 Decision-theoretic sensitivity analysis

3.1 Expected Value of Perfect Information

As discussed in the previous sections, the objective function of interest to us is the expected cost function (e.g., the cost rate function given in Equation (4) for TBM or the mean cost rate given in (6) for CBM). These cost functions take reliability and maintenance parameters as uncertain inputs (denoted by θ) and a decision parameter, T (which could be critical age or periodic inspection interval). A strategy parameter (which is fixed) needs to be selected in the presence of unknown reliability and maintenance parameters. These unknown parameters can be modelled by a joint density function, $\pi(\theta)$. In the maintenance optimisation setting, the decision maker can choose the strategy parameter T (from a range or set of positive numbers) where each value of T corresponds to a maintenance decision. The decision T is selected so that the following utility function is maximised

$$U(T,\theta) = -\mathscr{C}(T;\theta), \tag{7}$$

where $\mathscr{C}(T;\theta)$ is a generic cost function per unit of time given the unknown parameters θ.

Suppose that we need to make a decision now, on the basis of the information in $\pi(\theta)$ only. The optimal maintenance decision (known as *baseline* decision), given no additional information, has expected utility

$$U_0 = \arg\max_{T>0} E_\theta\left[U(T,\theta)\right], \tag{8}$$

where

$$E_\theta\left[U(T,\theta)\right] = -\int_\theta \mathscr{C}(T;\theta)\pi(\theta)\mathrm{d}\theta. \tag{9}$$

Now suppose that we wish to learn the precise value of a parameter θ_i in θ before making a decision (e.g., through exhaustive testing; new evidence elicited from the domain expert). Given θ_i, we are still uncertain about the remaining input parameters, $\theta_{\underline{i}} = (\theta_1,\ldots,\theta_{i-1},\theta_{i+1},\ldots,\theta_n)$, and so we would choose the maintenance strategy to maximise

$$E_{\theta|\theta_i}\left[U(T,\theta)\right] = -\int_{\theta_{\underline{i}}} \mathscr{C}(T;\theta)\pi(\theta \mid \theta_i)\mathrm{d}\theta_{\underline{i}}. \tag{10}$$

The expected utility of learning θ_i is then given by:

$$U_{\theta_i} = E_{\theta_i}\left[\arg\max_{T>0} E_{\theta|\theta_i}\left\{U(T,\theta)\right\}\right]. \tag{11}$$

Therefore, learning about parameter θ_i before any maintenance decision being taken will benefit the decision-maker by:

$$\text{EVPI}_{\theta_i} = E_{\theta_i}[U_{\theta_i}] - U_0. \tag{12}$$

Therefore, the quantity EVPI_{θ_i}, known as the partial Expected Value of Perfect Information (partial EVPI or PEVPI), is a measure of the importance of parameter θ_i in terms of the cost savings that further learning (data collection) will achieve.

EVPI's allow for sensitivity analysis to be performed in a decision-theoretic context. However, the computation of partial EVPIs as in (12) requires the evaluation of expectations of utilities over many dimensions. Whereas the one-dimensional integral $E_{\theta_i}[U_{\theta_i}]$ can be evaluated efficiently using Simpson's rule, averaging over the values of multiple parameters is computationally intensive. One way to approximate these expectations is to use a Monte Carlo numerical method. However, the Monte Carlo based integration methods require a large number of simulations which make the computation of the PEVPI's impractical. Zitrou et al. [35] propose an alternative method for resolving this problem by utilizing a GP emulator based sensitivity analysis to the objective function of interest. This method enables computation of the multi-dimensional expectations at a limited number of model evaluations with relative computational ease. We develop this method further for the purposes mentioned above.

3.2 Gaussian Process Emulators

An emulator is an approximation of a computationally demanding model, referred to as the *code*. An emulator is typically used in place of the code, to speed up calculations. Let $\mathscr{C}(\cdot)$ be a code that takes as input a vector of parameters $\theta \in \mathscr{Q} \subset \mathbb{R}^q$, for some $q \in \mathbb{Z}_+$, and has output $y = \mathscr{C}(\theta)$, where $y \in \mathbb{R}$. As we will see later on, this is not a restrictive assumption, and we will let $y \in \mathbb{R}^s$, for some $s \in \mathbb{Z}_+$. For the time being, let $\mathscr{C}(\cdot)$ be a deterministic code, that is for fixed inputs, the code produces the same output each time it 'runs'.

An emulator is constructed on the basis of a sample of code runs, called the *training set*. In a GP emulation context, we regard $\mathscr{C}(\cdot)$ as an unknown function, and use a $q-$ dimensional GP to represent prior knowledge on $\mathscr{C}(\cdot)$, i.e.

$$\mathscr{C}(\cdot) \sim N_q(m(\cdot), v(\cdot,\cdot)). \tag{13}$$

We subsequently update our knowledge about $\mathscr{C}(\cdot)$ in the light of the training set, to arrive at a posterior distribution of the same form.

Expression (13) implies that for every $\{\theta_1,\ldots,\theta_n\}$ output $\{\mathscr{C}(\theta_1),\ldots\mathscr{C}(\theta_n)\}$ has a prior multivariate normal distribution with mean function $m(\cdot)$ and covariance function $v(\cdot,\cdot)$. There are many alternative models for the mean and covariate functions $m(\cdot)$. Here, we use the formulation in line with [25], and assume

$$m(\theta) = h(\theta)^{\mathsf{T}}\beta \tag{14}$$

for the mean function, and

$$v(\theta,\theta') = \sigma^2 c(\theta,\theta') \tag{15}$$

for the covariance function. In 14, $h(\cdot)$ is a vector of q known regression functions of θ and β is a vector of coefficients. In (15), $c(\cdot,\cdot)$ is a monotone correlation function on $\mathbb{R}^+$ with $c(\theta,\theta) = 1$ that decreases as $|\theta-\theta'|$ increases. Furthermore, the function $c(\cdot,\cdot)$ must ensure that the covariance matrix of any set of outputs is positive semi-definite. Throughout this paper, we use the following correlation function which satisfies the aforementioned conditions and is widely used in the Bayesian Analysis of Computer Code Outputs (BACCO) emulator ([24, 27]) for its computational convenience:

$$c(\theta,\theta') = \exp\{-(\theta-\theta')^{\mathsf{T}}B(\theta-\theta')\}, \tag{16}$$

where B is a diagonal matrix of positive smoothness parameters. B determines how close two inputs θ and θ' need to be such that the correlation between $k(\theta)$ and $\mathscr{C}(\theta')$ takes a particular value. For further discussion on such modelling issues [19]. To estimate parameters β and σ, we use a Bayesian approach as in [26]: a normal inverse gamma prior for (β,σ^2) is updated in the light of an observation vector $y = (\mathscr{C}(\theta_1),\ldots,\mathscr{C}(\theta_n))^{\mathsf{T}}$, to give a GP posterior (see [35] for the details of how the posterior distribution of the function of interest can be derived). Note that y is obtained by running the initial code $\mathscr{C}(\cdot)$ n times on a set of design points $(\theta_1,\theta_2,\ldots,\theta_n)^{\mathsf{T}}$.

In many cases, input parameters are subject to uncertainty and are modelled as random variables. The input of the code is now a random vector θ with probability distribution F, implying that the code output $Y = \mathscr{C}(\theta)$ is also a random variable. The uncertainty in the output is epistemic, arising from the uncertainty of the input parameters. But there is also uncertainty due to the incomplete knowledge of the model output, called *code uncertainty* (we are not running the actual code, but just an approximation). We can quantify code uncertainty on the basis of the covariance function (15) and control its effect by modifying the number of design points.

Emulators are useful tools for uncertainty and sensitivity analysis [24, 21]. For GP emulators in particular, this is due to the fact that Bayesian quadrature as described in [25] allows one to take advantage of the emulator's analytical properties to evaluate the expected value $E[Y]$ and the variance $Var[Y]$ relatively quickly. In particular, since Y is a GP, the integral

$$E[Y] = \int_{\theta} \mathscr{C}(\theta)\pi(\theta)\mathrm{d}\theta \tag{17}$$

has a normal distribution. In this particular context, we are interested in computing partial EVPIs as in (12). By using an emulator to approximate utility $U(T,\theta)$, expectations of the utility can be computed rapidly, considerably reducing the computational burden of sensitivity analysis. Emulators perform better than standard Monte-Carlo methods in terms of both accuracy of model output and computational effort [19, 24].

The objective of the maintenance optimisation problem described here is to identify decision T (time where PM is performed) that maximises the Utility given in (7). Essentially, there are two approaches to the problem:

A assume that T belongs to a finite set $\mathscr{S}$ comprising s options $T_1, T_2, \ldots, T_s$, or
B assume that T can take any value in $(T_{\min}, T_{\max})$.

Under Approach A, to identify the optimal decision we need to determine the vector of outputs $Y = (Y_1, \ldots, Y_s)$, where $Y_j = U(T_j, \theta)$ $(j = 1, \ldots s)$ is described in (7). As discussed in [9], there are essentially two procedures for emulating a code with a multi-dimensional output like this one: the *Multi-Output emulator (MO)* and the *Many Single Outputs (MS)* emulator. Alternatively, under Approach B, we need to determine output $Y(T) = U(T,\theta)$ as a function of decision variable T. To do so, one can use a *Time Input (TI)* emulator [9].

The MO emulator is a multivariate version of the single output emulator, where the dimension of the output space is s. This process allows for the representation of any correlations existing among the multiple outputs. The MS emulator procedure treats $\{Y_1, \ldots, Y_s\}$ as independent random variables, and emulates each output Y_j separately. This means that s separate GP emulators are built, each describing the utility for each decision $T \in \mathscr{S}$. Finally, the TI emulator is a single-output emulator that considers decision variable T as an additional input parameter. The advantage of this approach is that S does not have to be a finite space, and utility $U(T,\theta)$ can be determined for any value of T over some interval $(T_{\min}, T_{\max})$. For the maintenance optimisation problem examined here, the TI emulator has a very important advantage over the other two: it allows for decision T to be a continuous variable. Expectations $E[Y]$ are continuous functions of T, and the utilities of the optimal strategies are calculated without restricting the decision-maker to choose amongst a pre-determined, finite number of options. We believe that this feature outweighs the more general correlation structure provided by the MO emulator. We note, however, that this may not be the case in dynamic codes, where the representation of the temporal structure of some physical process is key.

3.3 The TI Emulator

Suppose that the optimal decision T in a maintenance optimisation problem (critical age or periodic interval) belongs to an infinite set $S = (T_{\min}, T_{\max})$. We consider T as a code input and we are interested in building a single-output emulator to approximate code

$$k(T,\theta) = U(T,\theta) = -\mathscr{C}(T;\theta), \tag{18}$$

where $\mathscr{C}(T;\theta)$ is the cost rate function. This will allow us to calculate expected utilities $E_\theta[U(T,\theta)]$ and $E_{\theta|\theta_i}[U(t,\theta)]$ for $T \in \mathscr{S}$ - see Relationships (8) and (10) - fast and efficiently. We note that this setting is an extension of the decision problem considered in [27], where the decision-maker must choose among a small set of options i.e. where the optimal decision T belongs to a finite set S.

To estimate the hyper-parameters of the TI emulator, we generate training set $\mathscr{T}$ consisting of code outputs

$$y_1 = k(x_1), \ldots, y_N = k(x_N),$$

where $(x_1, x_2, \ldots, x_N)^\intercal$ are design points. We have

$$x_l = (T_i, \theta_j), \quad l = 1, 2, \ldots, N = s \times n,$$

where T_i is a maintenance decision ($i = 1, \ldots, s$) and θ_j are (reliability, maintainability) parameter values ($j = 1, \ldots, n$).

The choice of design points affects how well the emulator is estimated. Here, we choose equally spaced points $\{T_1, \ldots, T_s\}$ so that interval S is properly covered. Points $(\theta_1, \theta_2, \ldots, \theta_n)^\intercal$ are generated using Latin hypercube sampling (see [22]), which ensures that the multidimensional parameter space is sufficiently covered.

As mentioned earlier, building a TI emulator requires the inversion of an $N \times N$ matrix. Given the size of the training set, this can be computationally challenging. Essentially, there are two ways to build the TI emulator: (1) fit a GP directly to the whole training set $\mathscr{T}$ obtained as described above; (2) separate $\mathscr{T}$ and fit two GPs: one on the set of design points $(\theta_1, \theta_2, \ldots, \theta_n)$ and one on the time input data points $\{T_1, \ldots, T_s\}$ [31, 9].

Zitrou et al. [35] present the methodology to fit these two GPs to a similar decision-informed optimidation problem, including estimating the roughness (or smoothness) parameters and other hyper-parameters. They showed that however the firs approach based on fitting a single GP to to the whole training set $\mathscr{T}$ takes longer, but it would produce more accurate results. They showed that the relative mean squared error of the posterior predictive mean of the first model (based on fitting a single GP) is much smaller than when fitting two GP. We therefore follow their suggestion and fit a single GP to the full training set.

The baseline maintenance strategy is then chosen as a value of T that maximises expected utility as

$$U_0 = \max_{T \in \mathscr{S}} E_k\{E_\theta\left[k(T,\theta)\right]\} \tag{19}$$

and the utility of the optimal strategy in (10), after learning the value of θ_i, becomes

$$U_{\Theta_i} = \max_{T \in \mathscr{S}} E_k\{E_{T,\theta|\Theta_i}\left[k(T,\theta)\right]\}. \tag{20}$$

Bayesian quadrature [25] allows us to compute these expectations relatively quickly based on the fitted GP and calculate PEVPIs for the different parameters as in (12).

The details of the approximation of this type of integral (expectation) in terms of the fitted GP can be found in [11].

We use R and GEM-SA packages to fit the GP to the training points and then approximate the expected utilities and their corresponding uncertainty bounds. To calculate the aforementioned expected utilities, the calculations are carried out based on the discretisation of the interval $\mathscr{S}$ (maintenance decision) and the support of the joint prior distribution of the parameters $\pi(\theta)$. It is apparent that the computation of these expectation can become quite expensive by choosing a finer discretisation. The following section presents two illustrative examples. The focus here is on the way emulators can be used to perform sensitivity analysis based on EVPI, providing a resource efficient method for maintenance strategy identification and identifying targets for institutional learning (uncertainty reduction). In the first example we build an emulator for a TBM optimisation problem and in the second example finding a robust CBM of a civil structure using emulator-based sensitivity analysis is presented.

4 Numerical examples

4.1 Sensitivity analysis of Time-based maintenance decisions

Under the TBM policy (also known as age-based replacement), the system or component under study is in one out of two operating conditions; working or failed. System failure is identified immediately and corrective maintenance (CM) actions are undertaken to restore the system to its original condition. Regardless of the system condition, the system is renewed when it reaches a predetermined time (or age) T^*. In the TBM optimisation problem, the main challenge is to identify the optimal time to maintenance to minimise overall maintenance costs. This optimisation problem is usually defined over a finite horizon $[0,t]$, and we seek to minimise the objective cost function $\mathscr{C}(t)$ over this time interval.

Using the renewal theory ([3, 13]), the expected cost rate per unit of time can be mathematically expressed as:

$$\mathscr{C}_R(t;\theta_1) = \frac{C_F F_T(t;\theta_1) + C_P[1 - F_T(t;\theta_1)]}{\int_0^t [1 - F_T(t;\theta_1)]dt}, \tag{21}$$

where C_F is the total cost associated with all the consequences of system failure (particularly, structural failure due to deterioration), C_P is the cost associated with a preventive maintenance action, $F_T(t;\theta_1)$ is the system lifetime distribution function given in (2), and θ_1 is the set of the lifetime distribution parameters.

The cost rate function given in (21) can be developed further (see [3, 2]) to

$$\mathscr{C}_R(t;\theta) = \frac{C_1 F_T(t;\theta_1) + C_2[1 - F_T(t;\theta_1)]}{\int_0^t [1 - F_T(t)]dt + \tau_r(\theta_2)}, \tag{22}$$

where $\tau_r(\theta_2)$ is the expected duration of the maintenance action, and is defined by

$$\tau_r(\theta_2) = \int_0^\infty t g_T(t;\theta_2)dt, \tag{23}$$

where $g_T(t;\theta_2)$ is the time to repair (or replacement) distribution, θ_2 is the set of repair distribution parameters, and $\theta = (\theta_1, \theta_2)$. Without loss of generality, $g_T(t;\theta_2)$ is assumed to follow a Gamma distribution with α and β as shape and scale parameters respectively.

For numerical illustration, we set $C_F = 50$ and $C_P = 10$. As discussed in Section 1, the lifetime distribution of a deteriorating component using the RV model follows an inverted Gamma distribution with the density function given in (1) In this distribution, η and δ are respectively the shape and scale parameters of the lifetime distribution. Figure 1 illustrates how the expected cost rates change over the decision variable T for specific values of parameters, $\theta = (\eta, \delta, \alpha, \beta)$.

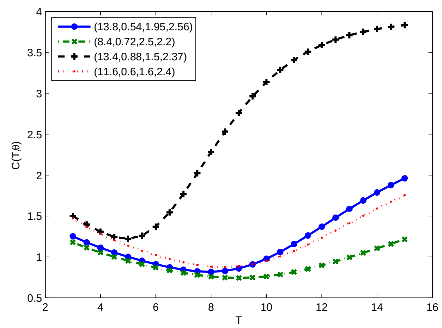

Fig. 1: Total long-run average costs per unit time function for different values of $\theta = (\eta, \delta, \alpha, \beta)$

The decision-maker proposes the following prior distribution on θ

$$\pi(\theta) = \pi_1(\eta)\pi_2(\delta)\pi_3(\alpha)\pi_4(\beta), \tag{24}$$

where each of these parameters individually is uniformly distributed as follows

$$\eta \sim U(3,14),\ \delta \sim U(0.15,0.95),\ \alpha \sim U(1,3),\ \beta \sim U(2,3),$$

where $U(a_1,b_1)$ denote a uniform density function defined over (a_1,b_1).

It can be shown that the cost function in (22) has a unique optimal solution (according to Theorem 1 given in [2]). When the uncertainty in input parameters θ are included, the optimal maintenance decision will lie in the interval, $I = [3,15]$ [35].

In order to lower the computational load of computing the value of information measures (PEVPIs) as the sensitivity analysis index, a TI emulator is fitted to the cost rate function $\mathscr{C}_R(t;\theta)$, given in (22). The total training data-points to build this emulator is 1500 selected as follows. We first generate 60 design points from the joint prior distribution, θ, using the Latin hypercube design [32]. We then calculate the cost rate function (as a computer code) at each design point for 25 values of T (i.e., $T = 3, 3.5, 4, \ldots, 15$).

Using the fitted GP, the baseline optimal decision is derived at $T = 8.5$ where the corresponding maximum utility is $U_0 = 0.896$. So, if there is no additional information available on individual input parameters, apart from the prior information, the optimal time to maintenance is at 8.5 time units resulting in an almost 90% cost saving compare the corrective maintenance. Further saving can be achieved if additional information about the values of the parameters can be provided before making any decision. For example, suppose that α is known before making a decision. Table 1 provides the detailed information about the optimal decisions for the different values of η. For instance, when the shape parameter of the lifetime distribution of a component under study takes values in $(3, 3.825)$, then the cost rate is minimum for $T = 10.2$, but if $\eta \in (8.255, 8.775)$, then the optimal maintenance decision is $T = 7.80$. Tables 2 to 4 shows the optimal maintenance decisions for the different values of δ, α and β, respectively.

The values of the PEVPI's along with the uncertainty intervals are given in Table 5. From these results, it can be concluded that η and δ (the shape and scale parameters of the lifetime distribution) are the most important factors. Note that knowning η prior to the decision shows the most substantial differentiation between optimal strategies. Thus, this parameter is ‘important’ in the sense that reducing uncertainty about its value is likely to result in a different optimal strategy. This conclusion is further supported in Figure 3 which summaries the sensitivity analysis of the cost rate function with respect to the changes of the model input parameters. In this figure, the variance contribution of each parameter to the total variance of the cost rate at $T = 8.5$ is shown. The variance contribution of η and δ are almost 30% and 26% based on only 60 data-points at T=8.5, respectively, while α and β covers only 6% and 1% of total variances, respectively. In addition to the individual impacts of η and δ, their interaction which covers 37% of total variance is considered as an important factor influencing the maintenance cost at the chosen optimal replacement time. The analysis also exposes the behaviour of the expected cost at a specific time for different values of the parameters. Figure 3 illustrates how expected cost $E_{\theta|\theta_i}[-\mathscr{C}_R(t;\theta)]$ when T=8.5 changes with different values of the parameters (i.e., $(\eta, \delta, \alpha, \beta)$)), along a 95% uncertainty bound (the thickness of the band).

Range	T	Range	T
(3,3.825)	10.2	(8.225,8.775)	7.80
(3.825,4.375)	11.4	(8.775,9.875)	7.2
(4.375,4.925)	13.8	(9.875,10.975)	7.80
(4.925, 5.475)	10.80	(10.975,11.525)	6.6
(5.475,6.025)	7.20	(11.525, 12.075)	7.2
(6.025, 7.125)	6.6	(12.075,13.175)	13.2
(7.125, 7.675)	7.80	(13.175, 13.725)	10.8
(7.675, 8.225)	8.40	(13.725, 14)	9

Table 1: Optimal decisions when parameter η is known prior the maintenance decision.

Range	T	Range	T
(0.15,0.29)	7.2	(0.61,0.73)	9
(0.29,0.43)	7.8	(0.73,0.93)	9.6
(0.43,0.61)	8.4	(0.93,0.95)	10.2

Table 2: Optimal decisions when parameter δ is known prior the maintenance decision.

Range	T	Range	T
(1,1.10)	8.52	(2.30,2.34)	8.28
(1.10,1.34)	8.28	(2.34,2.38)	8.52
(1.34,1.50)	8.04	(2.38,2.42)	9
(1.50,1.66)	7.8	(2.42,2.46)	9.48
(1.66,1.82)	7.56	(2.46,2.50)	9.96
(1.82,2.14)	7.32	(2.50,2.54)	10.20
(2.14,2.22)	7.56	(2.54,2.62)	10.44
(2.22,2.26)	7.80	(2.62,2.78)	10.68
(2.26,2.30)	8.04	(2.78,3)	10.92

Table 3: Optimal decisions when parameter α is known prior the maintenance decision.

Range	T	Range	T
(2,2.03)	14.76	(2.19,2.21)	9.48
(2.03,2.05)	13.80	(2.21,2.23)	9.24
(2.05,2.07)	13.32	(2.23,2.27)	9
(2.07,2.09)	12.60	(2.27,2.33)	8.76
(2.09,2.11)	11.64	(2.33,2.41)	8.52
(2.11,2.13)	10.68	(2.41,2.55)	8.28
(2.13,2.15)	10.20	(2.55,2.77)	8.04
(2.15,2.17)	9.96	(2.77,2.91)	8.28
(2.17,2.19)	9.72	(2.91,3)	8.52

Table 4: Optimal decisions when parameter β is known prior the maintenance decision.

θ_i	$PEVPI_i$	$C.I$
η	0.109	(0.102, 0.116)
δ	0.0268	(0.0199, 0.0338)
α	0.0075	(0.0235, 0.0213)
β	0.009	(0.00216, 0.0159)

Table 5: The PEVPI's estimated based on the fitted GP process for the parameters of the RV model for the TBM policy.

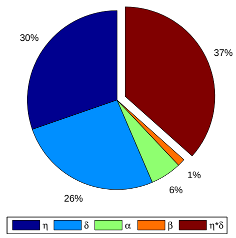

Fig. 2: The variance contribution of each input parameter to the cost rate function of the time-based maintenance policy at $T = 8.5$ for the RV deterioration model.

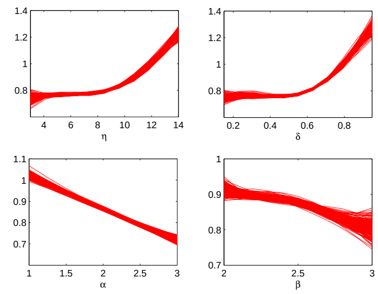

Fig. 3: Expected utilities and 95% uncertainty bounds for $T = 8.5$ when the parameters are completely known before the maintenance decision.

4.2 *The condition-based maintenance policy*

Under the RV deterioration model, the following CBM policy is considered, based on the periodic inspection of a component at a fixed time interval t_I with cost, C_I (see also [28])

1. If $X(t_I) \leq \upsilon\rho$, no maintenance action should be taken until the next inspection with the cost, C_I.
2. If $\upsilon\rho < X(t_I) \leq \rho$, the maintenance action will be taken at the cost C_P to renew the system.
3. If the system fails between inspections ($X(t_I) > \rho$), a corrective maintenance will restore the system at the cost of C_F

where $0 < \upsilon < 1$ is called PM ratio, and $\upsilon\rho$ is the threshold for the PM which is a fraction of the failure threshold.

According to renewal theory ([2, 29]), the mean cost rate for the CBM policy under the RV deterioration model is given by

$$\mathcal{K}_R(t_I, \upsilon; \theta) = \frac{E[\mathcal{C}_U(t_I, \upsilon; \theta)]}{E[\mathcal{L}_D(t_I, \upsilon; \theta)] + \tau_r}, \tag{25}$$

where

$$\begin{aligned} E[\mathcal{C}_U(t_I, \upsilon; \theta)] &= (C_I + C_P)Pr(X(t_I) \leq \rho) + C_F Pr(X(t_I) > \rho) \\ &= (C_I + C_P - C_F)\mathcal{G}(\rho/t_I; \eta, \delta) + C_F, \end{aligned}$$

$$
\begin{aligned}
E[\mathscr{L}_D(t_I,\upsilon;\theta)] &= \int_0^{t_I} Pr(X(t)<\rho)dt + \int_{t_I}^{\infty} Pr(X(t)<\upsilon\rho)dt \\
&= \int_0^{t_I} \mathscr{G}(\rho/t;\eta,\delta)dt + \int_{t_I}^{\infty} \mathscr{G}(\upsilon\rho/t;\eta,\delta)dt,
\end{aligned}
$$

τ_r is the expected duration of the maintenance action as given in (23), and $\theta = (\eta,\delta,\alpha,\beta)$.

In [29, 28], it was discussed that the optimal inspection time (t_I) is unique and will lie in an interval with the details given in these works (e.g., $t_I \in [3,16]$ in the following numerical example). The optimal inspection interval and PM ratio, (t_I,υ) apparently depends on the parameter values, θ. Thus, the robust optimal inspection interval can be derived through performing a proper decision-based sensitivity analysis described above.

The PM ratio, υ is considered as an extra parameter and included into the uncertain parameters, that is, $\phi = (\eta,\delta,\alpha,\beta,\upsilon)$. The joint prior distribution given in (24) will be revised as follows:

$$
\pi(\phi) = \pi_1(\eta)\pi_2(\delta)\pi_3(\alpha)\pi_4(\beta)\pi_5(\upsilon), \tag{26}
$$

where

$$
\eta \sim U(3,14),\ \delta \sim U(0.15,0.95),\ \alpha \sim U(1,3),\ \beta \sim U(2,3),\ \upsilon \sim U(0,1)
$$

To compute the PEVPI's associated with these input parameters, the TI emulator is then fitted to the mean cost rate, $\mathscr{K}_R(t_I,\phi)$ based on 2160 training data points. These data points consists of 80 design points generated from the joint prior distribution of ϕ (using the Latin hypercube design) and then evaluating the mean cost rate at each of these design points for 27 selected values for t_I, that is, $t_I = 3, 3.5, \ldots, 16$.

The maximum benefit of using the information available in $\pi(\phi)$ is $U_0 = 0.642$ which attains at the optimal inspection time interval $T = 6.62$. Tables 8 to 11 in Appendix A show the optimal inspection interval decisions when the values of γ,ξ,α and β are learned prior to making any decision about the inspection time. Table 6 illustrates the optimal t_I when the values of PM ration υ is learned prior to the decision making. For instance, when the PM ratio lies in (0.625, 0.925), then the mean cost rate is minimum at $t_I = 5.1$, but when υ takes values closer to its median, that is, $\upsilon \in (0.475, 0.525)$, then $t_I = 5.65$.

The values of the PEVPI's along with their 95% confidence bounds over the considered interval for optimal inspection time (i.e., $I_2 = [3,16]$) are presented in Table 7. The parameters with the highest PEVPI's are δ and η (the scale and shape parameters of the RV deterioration model) and the PM ratio parameter, υ. Learning about these parameters prior to making when to inspect the component of interest, will allow the decision-maker to implement a strategy that will result in the higher cost saving. That also means these parameters are essential in the sense that, reducing uncertainty about their values, is likely to change the optimal inspection interval.

Range	T	Range	T
(0,0.025)	8.95	(0.375, 0.475)	6.20
(0.025,0.075)	8.40	(0.475, 0.525)	5.65
(0.075,0.175)	7.85	(0.525, 0.625)	4
(0.175,0.275)	7.30	(0.625, 0.925)	5.10
(0.275,0.375)	6.75	(0.925, 1)	4

Table 6: Optimal decisions when PM ratio parameter, υ is known prior the maintenance decision.

θ_i	$PEVPI_i$	$C.I$
η	0.0117	(0.005, 0.018)
δ	0.0174	(0.01, 0.0243)
α	0.008255	(0.0014, 0.015)
β	0.0085	(0.0017, 0.015)
υ	0.01184	(0.004, 0.02)

Table 7: The estimated PEVPI's based on the fitted GP emulator for the parameters of the RV model for the CBM policy.

Figure 4 also illustrates the sensitivity analysis of $\mathscr{K}_R(t_I, \phi)$ with respect to the changes of ϕ at $t_I = 6.6$. Based on the variance contribution fractions, it can be similarly concluded that υ, δ and η (23%) are the most influencing factors affecting the mean cost rate at $t_I = 6.6$.

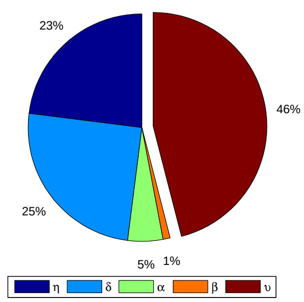

Fig. 4: The variance contribution of each input parameters to the cost rate function of the CBM policy at $t_I = 6.6$ for the RV deterioration model.

5 Conclusions and Discussion

Strategic planning for information systems implementation requires a thorough assessment of the to-be system's lifecycle, resilience and overall sustainability. This is of no exception for cloud services and big data analytic systems. The overall aim is to reduce the energy concumption of the system, enhance system's environmental friendliness, align the system with social aspects of sustainability, reduce the cost, prolong system's endurance or a combination of all of the above. There are numerous facets to information systems strategic engineering. Hosseinian-Far & Chang [18] assessed the information system's sustainability e.g. Strategic Information Systems (SIS)'s endurance by quantifying seven factors, two of which were contextualised for the specific SIS's under study. However, the research presented in this paper, surpasses that by focusing on the preventive maintenance strategies by which the system's lifecycle can be prolonged. This is in line with the economy domain of the three pillar generic sustainability model(i.e. People, Planet and Profit [17], as it entails less costs as opposed to post issue maintenance strategies i.e. Corrective Maintenance (CM) strategies.

In this chapter, we have demonstrated how the life-cycle management of an asset (system) under continuous deterioration can be efficiently and effectively improved by studying the sensitivity of optimised PM decisions with respect to changes in the model parameters. The novelty of this research is the development of a computationally efficient decision-theoretic sensitivity analysis for a PM optimisation problem for infrastructure assets subject to measurable deterioration using the PEVPI concept. This approach would enable the decision-maker to select a PM strategy amongst an infinite set of decisions in terms of expected costs and in terms of accumulated information of the model parameters and aspects of the system, including deterioration process and maintenance models. We use an RV deterioration model to present the uncertainty associated with asset deterioration across both age-based and condition-based PM policies. The computation of PEVPI's is very demanding, and conventional numerical integration or Monte Carlo simulation techniques would not be as helpful. To overcome this computational challenge, we introduced a new approach which approximates the PEVPI using GP emulators; a computationally efficient method for highly complex models which require fewer model runs than other approaches (including MCMC based methods). The method is illustrated on worked numerical examples and discussed in the context of analytical efficiency and, importantly, organisational learning.

The EVPI-based sensitivity analysis presented here can be used for other maintenance optimisation problems including problems with imperfect maintenance [30], or delay-time maintenance [10]. In this case, it is considered as one of the more effective preventive maintenance policies for optimising inspection planning. An efficient condition-based maintenance strategy which allows us to prevent system/component failure by detecting the defects via an optimised inspection might be identified using the sensitivity analysis proposed in this paper to determine a robust optimal solution for delay-time maintenance problems and the expected related costs, when the cost function parameters are either unknown or partially known.

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Appendix A: Optimal CBM maintenance decisions' Tables

Range	t_I	Range	t_I
(3,3.335)	10.97	(7.585,7.95)	9.13
(3.335,3.7)	12.07	(7.95, 8.315)	8.77
(3.70,4.385)	12.80	(8.315, 8.585)	8.4
(4.385,5.015)	12.43	(8.5855, 8.95)	8.03
(5.015,5.385)	11.70	(8.95, 9.415)	7.67
(5.385,5.75)	11.3	(9.415, 9.785)	7.3
(5.75,6.115)	10.97	(9.785, 10.15)	6.93
(6.115,6.485)	10.6	(10.15,10.885)	6.57
(6.485,6.85)	12.23	(10.885, 11.615)	6.20
(6.485,7.215)	9.9	(11.615, 14)	5.83
(7.215,7.585)	9.5		

Table 8: Optimal decisions when parameter η is known prior the maintenance decision.

Range	t_I	Range	t_I
(0.15,0.1758)	11.33	(0.457,0.483)	9.87
(0.1758,0.19)	15	(0.483, 0.51)	9.5
(0.19,0.217)	14.63	(0.51, 0.537)	8.77
(0.217,0.243)	13.17	(0.537, 0.563)	8.40
(0.243,0.297)	12.80	(0.563, 0.59)	8.03
(0.297,0.323)	12.43	(0.59, 0.617)	7.77
(0.323,0.35)	12.07	(0.617, 0.643)	7.30
(0.35,0.377)	11.70	(0.643,0.67)	6.93
(0.377,0.403)	11.33	(0.67, 0.723)	6.57
(0.403,0.43)	10.97	(0.723, 0.803)	6.20
(0.43,0.457)	10.23	(0.803,0.937)	5.83
		(0.937, 0.95)	5.47

Table 9: Optimal decisions when parameter δ is known prior the maintenance decision.

Range	t_I
(1,3)	6.64

Table 10: Optimal decisions when parameter α is known prior the maintenance decision.

Range	t_I
$(2,2.15)\cup(2.83,2.95)$	6.42
$(2.15,2.41)\cup(2.59,2.83)$	6.64
(2.41,2.59)	6.86
(2.95, 3)	6.20

Table 11: Optimal decisions when parameter β is known prior the maintenance decision.