Sustainable Passenger Transportation:
Dynamic Ride-Sharing

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ABSTRACT AND KEYWORDS

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Abstract

Ride-share systems, which aim to bring together travelers with similar itineraries and time schedules, may provide significant societal and environmental benefits by reducing the number of cars used for personal travel and improving the utilization of available seat capacity. Effective and efficient optimization technology that matches drivers and riders in real-time is one of the necessary components for a successful ride-share system. We formally define dynamic ride-sharing and outline the optimization challenges that arise when developing technology to support ride-sharing. We hope that this paper will encourage more research by the transportation science and logistics community in this exciting, emerging area of public transportation.

1 Introduction

Finite oil supplies, rising gas prices, traffic congestion, and environmental concerns have recently increased the interest in services that allow people to use personal automobiles more wisely. The demand for ride-sharing services, which aim to bring together travelers with similar itineraries and time schedules, has increased sharply in recent years [Saranow, 2006]. Ride-share providers across the globe are offering online notice boards for potential carpoolers, whether for daily commutes or for one-time trips to festivals, concerts, or sports events. Some online services, such as Nuride, provide incentives like restaurant coupons, gift certificates, or retail sales discounts to participants. Ride-sharing has generated much interest, and recent media coverage can be found in the Wall Street Journal [Saranow, 2006], Newsweek [Levy, 2007], Business Week [Walters, 2007], ABC News [Bell, 2007], The NY Times [Wiedenkeller, 2008], among many others.

Private car occupancy rates (the number of travelers per vehicle trip) are relatively low; average car occupancies in Europe range from 1.8 for leisure trips to 1.1 for commuters [EEA, 2005]. Similar occupancy rates are also found in the US [BTS, 2001]. The large demand for automobile transportation at peak-hours together with low occupancies leads to traffic congestion in many urban areas. The annual cost of congestion in the US in terms of lost hours and wasted fuel was estimated to be $78 billion in 2007 [Schrank and Lomax, 2007]. Private automobile usage is also the dominant transportation mode producing carbon dioxide emissions [Hensher, 2008]. Vehicle emissions give rise to problems both on a local and global scale. Locally, the health effects of air pollution represent a serious problem in many of the most densely populated regions worldwide [Brunekreef and Holgate, 2002, Kunzli et al., 2000]. Globally, carbon dioxide emissions are associated with climate change and global warming.

Effective usage of empty car seats by ride-sharing may represent an important opportunity to increase occupancy rates, and could substantially increase the efficiency of urban transportation systems, potentially reducing traffic congestion, fuel consumption, and pollution. Moreover, ride-sharing allows users to share car-related expenses, which can be substantial, especially since the price of oil has doubled over the past five years [IEO, 2009]. While ride-sharing is not a new idea, recent technological advances may increase its popularity, as we will explain. Certainly, ride-sharing must be easy, safe, flexible, efficient and economical before it will be widely adopted.

To broaden its appeal, ride-sharing must be able to compete with one of the greatest advantages of private car usage: immediate access to door-to-door transportation. Technological advances, both hardware and software, may be key enablers. In the US, 63 million people already use mobile Internet, a third on a daily basis [Lardinois, 2009]. The growing ubiquity of Internet-enabled mobile devices partially enables practical *dynamic* ride-sharing [Hartwig and Buchmann, 2007, Hartmann, 2008].

By dynamic ride-sharing, we refer to a system where an automated process provided by a ride-share provider matches up drivers and riders on very short notice or even en-route. Recent startups like Carticipate, EnergeticX, Avego, and Piggyback offer dynamic ride-sharing applications that allow drivers with spare seats to connect to people wanting to share a ride. They provide applications that run on (location-aware) Internet-enabled mobile phones. To ease the fear of sharing a ride with a potential stranger, these services use reputation systems (see *e.g.*, PickupPal) or can be linked with social network tools like Facebook (see *e.g.*, GoLoco and Zimride).

The ability of a dynamic ride-share provider to successfully establish ride-shares on short notice depends on the characteristics of the environment in terms of participant geographic density, traffic patterns, and the available roadway and transit infrastructure. Hall and Qureshi [1997] analyze the likelihood that a person will be successful in finding a ride-match, given a pool size of potential ride matches. Using a probabilistic analysis, they conclude that in theory ride-sharing is viable since a congested freeway corridor should offer sufficient potential ride-matches. The authors also observe that there are many obstacles, primarily in terms of communication, so that the chance of finding a ride match in practice may in fact be small. Fortunately, technological advances have greatly reduced this communication obstacle.

Although the enabling technology is available, ride-sharing success stories are still in short supply. The development of algorithms for optimally matching drivers and riders in real-time may only play a small role in the ultimate success of ride-sharing, but it is central to the concept, and the transportation research community has largely ignored this research area. We believe that this is unfortunate given the potential benefits of ride-sharing, and the interesting optimization challenges created when trying to match riders and drivers in real-time. However, it is not too late. Ride-sharing is still in its infancy and an opportunity exists to influence its success. By introducing and formally defining dynamic ride-sharing problems, and by illustrating and outlining the optimization challenges that arise when developing technology to support ride-sharing, we hope to encourage more research by the transportation science and logistics community in this exciting, emerging area of public transportation.

The remainder of the paper is structured as follows. In Section 2, we explain and characterize the dynamic ride-sharing concept and introduce several relevant planning issues that arise in this context. In Section 3, we compare the ride-share concept with other modes of passenger transportation such as on-demand transportation and scheduled transit. In Section 4, we present a more formal definition of the basic ride-share problem and its various variants. In Section 5, we present the multi-modal version of the ride-sharing problem. In Section 6, we discuss a number of important challenges that must be addressed in order to make dynamic ride-sharing systems successful in practice. To illustrate the optimization problems that arise in dynamic ride-share, we provide various illustrative examples throughout the paper. Finally, in Section 7, we summarize our main insights and discuss directions for future research.

2 Problem Characteristics

2.1 Features of Dynamic Ride-Sharing

In this paper, we use the term dynamic ride-sharing to describe an automated system that facilitates drivers and riders to share one-time trips close to their desired departure times. The concept is also known as real-time ride-sharing, ad-hoc ride-sharing, and instant ride-sharing. We characterize this concept by the following features:

Dynamic The ride-share can be established on short-notice, which can range from a few minutes to a few hours before departure time. The growing use of Internet-enabled mobile phones allows people to offer and request trips whenever they want, wherever they are. Thus, communication technology is a key enabler to dynamic, on-demand ride-sharing.

Independent The drivers which provide the rides are independent private entities. This is different from most traditional forms of passenger transportation where a central organization owns vehicles and/or employs drivers.

Cost-sharing The variable trip-related costs are reallocated among the ride-share participants in a way that makes it beneficial for them to participate from the perspective of cost reduction. The variable trip cost minimally includes fuel expense, but may also take into account wear and tear on vehicles, parking costs or road fees such as tolls.

Non-recurring trips Dynamic ride-sharing focuses on single, non-recurring trips. This distinguishes it from traditional carpooling or vanpooling, both of which require a long-term commitment among two or more people to travel together on recurring trips for a particular purpose, often for traveling to work. Single-trip ride-sharing is more flexible because it does not require rigid time schedules or itineraries over time.

Prearranged The trips are prearranged which means that the participants agree to share a ride in advance, typically while they are not yet at the same location. This is different from the spontaneous, so-called casual ride-sharing (see *e.g.*, Kelley [2007]) in which riders and drivers establish a ride-share on the spot, similar to hitch-hiking or hailing a taxi on the side of the street. In casual ride-sharing, drivers and riders line up at established locations to share rides to other established locations to take advantage of high occupancy vehicle lane time-savings or toll savings. The main limitation of casual ride-sharing is the inflexibility of its routes, which does not allow door-to-door transportation.

Automated matching To establish ride-shares in a way that requires minimal effort from the participants, ride matching should be automated in a dynamic setting. This means that a system matches up riders and drivers and communicates the matches to the participants. We do not include in our definition simple (online) notice boards where riders and drivers can post desired or planned trips and choose to contact potential ride-share partners themselves.

2.2 The Ride-Share Process

To facilitate a discussion on the planning issues in dynamic ride-sharing, we briefly explain the process of Avego, an Ireland-based software company that currently offers a dynamic ride-share application for Internet-enabled mobile phones. The service that they offer is quite generic and similar to that of other existing ride-share providers such as Carticipate, Piggyback, and EnergeticX.

With the Avego Shared Transport software application, users can offer a ride as a *driver* or a request for transportation as a *rider*. To facilitate easy trip specification, the application lets users store and select pre-defined locations such as home, work, and the grocery store. With a GPS-enabled phone, users can set their current location as the origin of their trip, even en-route. If a ride-share match is established, Avego proposes the arrangement to the participants. If the driver and the rider agree on the proposed arrangement, the driver picks up the rider at the agreed time and location. Avego sends the driver the rider's photo and personal identification number, which allows him to verify the rider's identity.

Avego will guide the driver to an appropriate pickup location and from thereon to the rider's destination via the incorporated navigation system. When the driver is in range for the pickup, the application will notify the rider in real-time. Avego automatically assesses a trip fee to the rider, of which the company receives a fixed percentage.

2.3 Ride-sharing System Objectives

Ride-sharing allows people to save on travel-related expenses by sharing trip costs. A ride-share provider, either private or public, helps people to establish ride-shares on short-notice by automatically matching up drivers and riders. If the system is private and operated for profit, the value provided by the ride-share provider is to reduce the total costs of all participants by the largest amount possible; by enabling this economy, the provider receives as payment a percentage of the savings generated. Private ride-share providers typically charge a commission per successful ride-share, either a fixed fee or proportional to the trip cost. As a result, the objective of the provider is mostly in line with the goals of the participants.

This is also true for a public system with a societal objective, such as the reduction of pollution and congestion. The objectives of the ride-share provider and ride-share users are aligned because both the total travel costs of the users and the external costs to society relate to the total system-wide vehicle-miles. Note that we implicitly assume here that a rider has a car at his disposal and will use that car to reach his destination if not matched up for a ride. This seems a reasonable assumption, especially in many US settings where 9 out of 10 people own a car [Nielsen Company, 2007]. Enhancing the mobility of system users without cars can be thought of as an important additional societal benefit.

Given these system objectives, it seems likely that a ride-share provider should consider one of the following objectives when determining ride-share matches:

- *Minimize system-wide vehicle-miles* The system-wide vehicle-miles represent the total vehicle-miles driven by all participants traveling to their destinations, either in a ride-share or driving alone. This objective is important from a societal point of view since it helps to reduce pollution (emissions) and congestion. This objective is also compatible with *minimizing total travel costs*, which is an important consideration for the participating drivers and riders and directly related to the revenues of the ride-share provider.

- *Minimize the system-wide travel time* The travel time is the time spent in the vehicle while actually traveling between origin and destination. From a societal perspective, this is an important measure since vehicle emissions not only relate to vehicle-miles but also to vehicle speeds. Obviously, time is also an important convenience consideration for the participants.

- *Maximize the number of participants* This objective maximizes the number of satisfied drivers and riders in the system. This objective may be beneficial for a private ride-share provider whose revenues are linked to the number of successful ride-share arrangements. Moreover, the matching success-rate may also be an important performance indicator for users of a particular ride-share service, and a high success rate may spur larger participant pools in the future.

2.4 Constraints on Matches

When determining matches between drivers and riders in a ride-share system, a number of constraints on the feasibility of matches must be observed. The timing of rides is probably the most important consideration since time tends to be a more constraining factor than the availability of spare seats.

Both riders and drivers must provide information on their time schedule preferences. Many of the currently available and proposed dynamic ride-share applications simply let each potential participant specify a desired departure time. The provider then attempts to find an assignment with a departure time that is as close as possible to this desired departure time. This approach minimizes the information that participants must supply, but, at the same time, provides only limited information regarding a participant's time preferences and flexibility. A time window representation may capture a participant's time preferences more accurately. One could, for example, let a participant specify an earliest possible departure time and latest possible arrival time (see Figure 1). Furthermore, it may be beneficial to allow limits on the actual time that users may spend traveling on a given trip, for example by allowing each participant to specify the maximum excess travel time (over the direct travel time for his origin to destination) he is willing to accept.

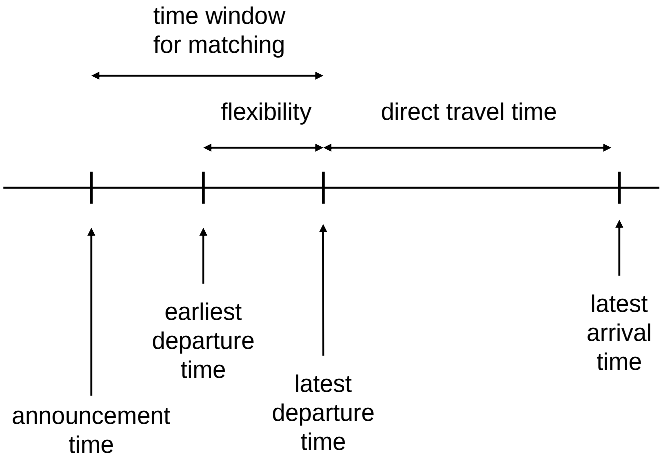

Figure 1: Time schedule information in ride-sharing

Although closely related, such a time constraint is not the same as an out-of-distance constraint as vehicle speed varies by road type and by time of the day. It may, for example, not be time feasible to perform a pickup at a certain location during rush-hour and still reach destinations on time.

In addition to time, there are other important feasibility considerations that determine whether a particular ride-share match is one that the participants would accept. For example, female participants may not feel safe sharing a ride alone with a male stranger [Levin et al., 1977], while smoking may be another critical issue. The user may only feel comfortable sharing a ride with certain groups of people, where the group preferences may be motivated by personal safety or social considerations. For example, one may not be willing to share a ride with a complete stranger and may only want to share rides with friends and colleagues. Of course, the more restrictions a potential user places on his pool of potential ride-share partners, the more difficult it will be to find successful matches for that user [Dailey et al., 1999].

Lastly, ride-share users may choose to participate primarily to reduce their travel costs. Therefore, it is probably also necessary to include constraints that restrict feasible matches to those that reduce the travel costs of each ride-share participant. Of course, determining whether or not a user reduces his costs via a ride-share depends on how costs are shared in such systems, which is the subject of the next section.

2.5 Cost Considerations

People may choose to participate in ride-sharing primarily for potential cost-savings; trip-related expenses, such as fuel and tolls, are shared. Thus, ride-shares should only be established if they reduce the cost of each individual participant. Although large cost savings may eventually come from riders giving up automobiles, freeing themselves from the capital and insurance costs associated with owning or leasing a car, ride-sharing is unlikely to reduce private car ownership in the near future. In the short term, individuals will not give up their cars, making the costs of ownership essentially fixed and thus not pertinent to the travel decisions. Therefore, we choose to focus in this paper on variable travel costs that are proportional to vehicle-miles. Most of the ideas that we discuss here can be generalized fairly readily.

When travel costs are roughly proportional to distance traveled, cost reduction is only possible when the length of a ride-share trip is shorter than the sum of the lengths of the separate trips. Note that a complete ride-share trip should be defined as all travel required to move each participant from his origin to his destination. For example, in the case where a driver shares a ride with a single rider this would include travel from the driver's origin to the rider's origin, then onto to the rider's destination, and finally onto the driver's destination.

If the cost of ride-share trip is less than the sum of the costs of individual trips of its participants, it is always feasible to allocate the cost savings among the participants such that each individual receives cost savings. Each driver can reduce his trip cost by receiving compensation that is greater than the marginal cost required to accommodate the rider(s), *i.e.*, the marginal travel cost required by detours necessary to serve the riders. For the ride-share to be beneficial for a rider, the compensation he pays to the driver(s) should be lower than the cost of driving themselves with their own car. The cost-savings threshold for participation may differ from person to person. Some people may only participate if their trip cost is reduced by at least x percent, whereas for others the social and environmental benefits of ride-sharing may be reason enough to participate (they may even be willing to accept a small increase in their trip costs). As an additional note, we should point out that regulations may need to be updated in some regions to allow cost-sharing on a per trip basis (see *e.g.*, http://save.pickuppal.com/).

There are various ways to divide the trip costs between the ride-share partners. A natural way to allocate the costs of the joint trip is proportional to the distances of the separate trips. We illustrate this proportional scheme by considering one driver sharing a ride with one rider. In this case, user i pays $\frac{c_i}{c_i+c_j}c_{ij}$, where c_i is the cost for user i to travel from his origin to his destination and c_{ij} is the cost of the joint trip of user i and j together. In all subsequent examples, we will use this simple cost-sharing mechanism as an illustration.

2.6 System Versus User Benefits

It is important to recognize that a system-wide optimal solution aimed at minimizing the external societal costs may not necessarily optimize the cost-savings of all individual ride-share participants. Consider a system with two drivers d_1 and d_2, two riders r_1 and r_2, and locations and distances given in Figure 2. Each driver can accommodate a single rider. When minimizing system-wide vehicle-miles, the optimal solution has value 20 and matches d_1 with r_1 and d_2 with r_2; represented by bold paths in Figure 2. When costs are allocated proportionally, each driver and rider pays the costs of 5 miles, *i.e.*, $\frac{6}{12}$ of the joint trip length of 10 miles. However, driver d_1 and rider r_2 could reduce their trip costs even more by establishing a ride-share on their own, since the joint trip length would be 9 miles of which each would pay for 4.5 miles. In this case, driver d_2 and rider r_1 would be without a ride-share since their joint trip length would be 17 miles. In the terminology of cooperative game theory, the system-wide solution is not stable.

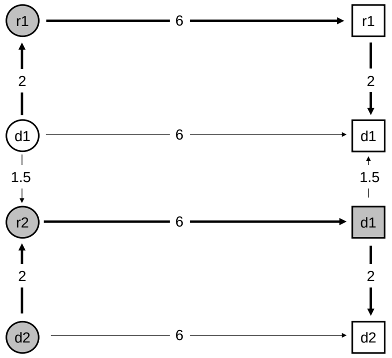

Figure 2: Riders (grey) and Drivers (white) traveling from Origin (circle) to Destination (square)

3 Comparison with Other Modes of Passenger Transportation

This section both serves an overview of the related passenger transportation literature as well as a systematic comparison of ride-sharing with other modes of passenger transportation. The main dimensions of the different types of passenger transportation systems are summarized in Table 1.

Table 1: Comparison of dynamic ride-share with other forms of public transport

	Scheduled Transit	**On-Demand Transit**	**Dynamic Ride-share**
Supply			
Product	A-to-B transportation	Door-to-door transportation	Door-to-door transportation
Capacity	Fleet of company vehicles and drivers	Fleet of company vehicles and drivers	Individual private vehicles and drivers
Route	Scheduled	On-demand	On-Demand/ Scheduled
Service Quality	Legal working and driving time restrictions	Legal working and driving time restrictions	Individual driver's preferences
Dynamics	Scheduled fleet	Scheduled fleet	Exogenous arrival of new drivers
Demand			
Revenues	Ticket fee	Taxi fare	Cost-sharing
Booking	No booking required	Few weeks up to a few minutes in advance	Few hours up to a few minutes in advance
Service Quality	Excess travel time	Excess travel time	Excess travel time, success rate, cost-savings
Dynamics	Exogenous arrival of new passengers	Exogenous arrival of new requests	Exogenous arrival of new requests

Dynamic ride-sharing is established on-demand, similar to other on-demand types of passenger transit such as taxis and dial-a-ride services like airport shuttles. The key planning tasks in on-demand transportation are the assignment of passengers (riders) to vehicles (drivers) and the design of the associated routes for pickup and delivery. Such planning problems are typically special cases of the so-called pickup and delivery problem, which has been studied extensively in the operations research literature; see *e.g.*, Savelsbergh and Sol [1995]. These problems involve the construction of vehicle routes and schedules to satisfy transportation requests between origins and destinations. A fleet of vehicles with a given capacity is available to operate the routes, typically based at one or more depot locations. It is also usually assumed that the pickup and delivery of each individual request is made by one vehicle. Recently, Cortes et al. [2009] showed that allowing transfers from one vehicle to another can be beneficial in some settings.

The dial-a-ride problem is a special case of the pickup and delivery problem that focuses on the transportation of passengers [Cordeau and Laporte, 2007, Berbeglia et al., 2007]. Consequently, passenger convenience considerations become important. Passenger service quality may be measured, for example, in terms of the ratio of actual drive time and direct drive time, the waiting time, the number of stops while on board, and the difference between actual and desired delivery times [Paquette et al., 2009]. These criteria may be treated as constraints or may be incorporated into the objective function. Note that it is often passenger convenience rather than physical capacity that keeps taxis from serving multiple passengers simultaneously.

Dynamic ride-sharing differs from conventional on-demand transportation primarily with regards to the supply of drivers and vehicles. Instead of being employed by a company, drivers in a ride-sharing system are private independent entities. Like riders, they arise dynamically over time at various locations in a process that may be difficult to predict with certainty. Since they are independent, they are not obligated to accept ride-share arrangements that they do not like. Therefore, in addition to rider preferences, driver preferences need to be accounted for when matching drivers and riders in a ride-sharing system. Driver preferences may include a maximum deviation from the direct trip duration, a maximum number of simultaneous riders, and a maximum number of stops.

How far in advance transportation requests are known and how transportation requests are handled is a crucial aspect of on-demand transit. Dial-a-ride services for elderly and disabled people generally employ booking deadlines, so that all transportation requests that have to be served are known at the time vehicle routes and schedules are constructed. On the other hand, a transportation request for an urban taxi typically arrives only a short time before the desired departure [Lee et al., 2004] and vehicle routes and schedules are updated each time a new transportation request arrives. The dynamic ride-sharing environment is more likely to resemble an urban taxi environment in terms of the arrival process of transportation requests, *i.e.*, rides, but also has the added complexity of an arrival process of transportation resources, *i.e.*, drivers.

Another difference between a dial-a-ride system and a ride-share system is that in a dial-a-ride system all vehicles typically operate out of one or more depot locations, whereas in a ride-share system each driver may have a unique origin and destination. This implies that in a ride-share system, routing decisions are represented and evaluated as deviations from a driver's direct path from origin to destination. Deviations from a given path are also at the heart of a Mobility Allowance Shuttle Transport (MAST) service, in which a vehicle has a predefined route but is allowed to deviate from this route to pick-up and drop-off passengers at preferred locations within a certain service area [Quadrifoglio et al., 2008, Zhao and Dessouky, 2008]. In addition, customers who board the vehicle at a scheduled stop can request a drop off location that is within half a mile from the predefined route [Zhao and Dessouky, 2008]. The MAST concept aims at combining the flexibility and convenience of on-demand transportation with the cost-efficiency of fixed route transit. Los Angeles County operates a MAST during the night hours. Passengers located within half a mile off the route may call-in for pick-ups at off-route locations.

In scheduled passenger transportation, the other end of the spectrum, the sequence of stops and the time-table of stops is fixed over a period of time. Scheduled passenger transportation does not typically provide door-to-door transportation. Scheduled public transit optimization problems include line-planning [Borndorfer et al., 2008, Murray, 2003, Wu and Murray, 2005], time-tabling [Kroon et al., 2009], fare-planning and duty scheduling [Grotschel et al., 2003].

To increase public transit usage, Liaw et al. [1996] and Lee et al. [2005] have proposed on-demand taxis to serve as a feeder for scheduled transit. Liaw et al. [1996] consider the integration of paratransit dial-a-ride vehicles with fixed-route buses, in a system where transportation bookings are made in advance. They show that the combination of on-demand vehicles and scheduled transit allows for an increase of the number of accommodated requests while at the same time decreasing the number of required taxis. Lee et al. [2005] consider the integration of dial-a-ride taxis with a metropolitan rapid transit line. They study a highly dynamic environment where new transportation requests continuously arrive and are to be assigned to taxis en-route. They propose a dispatch strategy and determine an optimal required fleet-size taking into account passenger waiting and travel time, number of satisfied requests, and system costs.

4 Basic Ride-Sharing Problems

Matching drivers offering a ride and riders requesting a ride is at the heart of ride-sharing. In this section, we describe optimization models that can be used to address these matching problems. We limit our attention here to what we will denote as *static* ride-sharing variants, where it is assumed that all driver and rider requests are known in advance prior to the execution of a matching process. In subsequent sections, this restriction will be relaxed as we examine the more relevant problems of dynamic matching.

Drivers offering a ride may want to take a *single* rider or may be willing to take *multiple* riders. Similarly, riders requesting a ride may want a ride with a *single* driver or may be willing ride with *multiple* drivers and transfer from one to another en route to their destinations. Thus, we can distinguish four basic ride-sharing system variants as shown in Table 2. As will be shown shortly, optimally matching drivers offering a ride and riders requesting a ride is easy for the static variant in which a single driver takes along a single rider. This variant is easy because there are a polynomial number of potential matches, and determining the optimal route sequence for a given potential match is simple. In all other variants, determining the best route sequence for a given match, which may involve multiple drivers and riders, can be more complicated.

Table 2: Ride-Share Variants

	Single Rider	**Multiple Riders**
Single Driver	Matching of pairs of drivers and riders: *Easy*	Routing of drivers to pickup and deliver riders: *Difficult*
Multiple Drivers	Routing of riders to transfer between drivers: *Difficult*	Routing of riders and drivers: *Difficult*

4.1 Modeling Framework and Notation

To be able to properly discuss research opportunities for the transportation science and logistics community, we start by formally defining the basic ride-sharing problem variants using a time-space network modeling framework for illustration.

We are given a set of locations P and travel time t_{ij} and travel distance d_{ij} between each pair of locations $i, j \in P$. Furthermore, we are given a set of drivers D and a set of riders R. Each driver $d \in D$ (rider $r \in R$) wants to travel from his origin $v(d) \in P$ ($v(r) \in P$) to his destination $w(d) \in P$ ($w(r) \in P$). Each driver $d \in D$ (rider $r \in R$) has an earliest time $e(d)$ ($e(r)$) at which he can depart from his origin $v(d)$ ($v(r)$) and a latest time $l(d)$ ($l(r)$) at which he can arrive at his destination $w(d)$ ($w(r)$). Each driver d has $q(d)$ spare seats available.

To accommodate time, we model the ride-sharing problem using a time-space network as opposed to a flat network. The set of nodes N consists of pairs (p, t) where $p \in P$ is a location and $t \in \{0, 1, ..., T\}$ is a time epoch for some appropriate discretization of the planning horizon T, where it is assumed that time 0 corresponds to the minimum earliest time e over all participants and time T corresponds to the maximum latest time l. The set of arcs A consists of arcs $((p, t), (p, t+1))$ representing the possibility to wait at location p from time t to $t+1$, and arcs $((p_1, t_1), (p_2, t_1 + t_{p_1,p_2}))$ representing the possibility to travel from location p_1 to location p_2 at time t_1, which implies an arrival time at p_2 of $t_1 + t_{p_1,p_2}$. A solution to the ride-sharing problem partitions the set of riders into two groups: a group of riders that are not matched with drivers (and therefore that drive themselves) and a group of riders that shares a ride with a driver or drivers. Let y^r be a binary variable that indicates whether rider r drives himself ($y^r = 1$) or not ($y^r = 0$). When we consider the drivers together with the riders that are picked up and dropped of by a driver, we can think of a solution as consisting of paths for the drivers and paths for these riders. Therefore, we introduce binary variables x^d_{ij} indicating whether driver d uses arc (i, j) ($x^d_{ij} = 1$) or not ($x^d_{ij} = 0$), and x^r_{ij} indicating whether rider r uses arc (i, j) ($x^r_{ij} = 1$) or not ($x^r_{ij} = 0$).

To ensure that each rider gets from his origin to his destination, we define constraints

$$\sum_j x^r_{(v(r),e(r)),j} + y^r = 1 \quad \forall r \in R$$

and

$$\sum_i x^r_{i,(w(r),l(r))} + y^r = 1 \quad \forall r \in R$$

and

$$\sum_j x^r_{ji} = \sum_j x^r_{ij} \quad \forall r \in R, i \in N \setminus \{(v(r), e(r)), (w(r), l(r))\}.$$

To ensure that each driver gets from his origin to his destination, we define constraints

$$\sum_j x^d_{(v(d),e(d)),j} = 1 \quad \forall d \in D$$

$$\sum_i x^d_{i,(w(d),l(d))} = 1 \quad \forall d \in D$$

$$\sum_j x_{ji}^d = \sum_j x_{ij}^d \quad \forall d \in D, i \in N \setminus \{(v(d), e(d)), (w(d), l(d))\}.$$

To ensure that when a rider does not drive himself he is always with a driver, we define constraints

$$\sum_r x_{ij}^r \leq \sum_d q(d) x_{ij}^d \quad \forall (i,j) \in A.$$

Under the assumption that the objective is to minimize total system-wide travel time incurred by all users, the objective can be written as

$$\min \sum_r t_{v(r),w(r)} y^r + \sum_d \sum_{ij} t_{ij} x_{ij}^d,$$

where we abuse notation slightly by using t_{ij} to mean the travel time between the location associated with node i and the location associated with node j.

Note that for each driver and rider, only arc-flow variables corresponding to feasible arcs need to be introduced, where an arc is feasible if there is at least one time-feasible path that departs the origin as early as possible and arrives at the destination as late as possible in which the arc appears. In other words, it should be time-feasible to reach the tail of the arc when departing the origin as early as possible and it should be time-feasible to reach the destination as late as possible when departing the head of the arc.

Many practical considerations can easily be accommodated in this type of formulation. For example, if a driver $d \in D$ has only a certain willingness to deviate from the direct route from his origin to his destination to pick up and drop-off riders along the way, *i.e.*, the driver wants his travel time to be at most $\rho t_{v(d),w(d)}$ for some $\rho > 1$, then we can simply add constraint

$$\sum_{ij} t_{ij} x_{ij}^d \leq \rho t_{v(d),w(d)}.$$

Or, if a rider does not want to be picked up or dropped off at his home for security and/or privacy reasons, the rider's home location can be represented by a set of possible pick-up or drop-off locations within walking distance of his home, say $H(r) \subseteq P$, and we can introduce pseudo-node $\bar{h}(r)$ and arcs $(\bar{h}(r), h)$ for all $h \in H(r)$ and replace the constraints that ensure that the rider gets from his origin or to his destination by

$$\sum_j x_{(\bar{h}(r),e(r)),j}^r + y^r = 1 \quad \forall r \in R,$$

in case his origin is his home, and

$$\sum_i x_{i,(\bar{h}(r),l(r))}^r + y^r = 1 \quad \forall r \in R,$$

in case his destination is his home.

4.2 Single Rider, Single Driver Arrangements

If a driver would like to share a ride with at most a single rider, then at most one pickup and delivery take place during his trip. Thus, if driver d and rider r are matched, then their joint trip length is $d_{v(d),v(r)} + d_{v(r),w(r)} + d_{w(r),w(d)}$. By comparing the vehicle-miles of the joint trip with the two separate trips, we can easily calculate the potential savings for each driver-rider match. If we want to match drivers and riders in the system in a way that minimizes the total system-wide vehicle-miles, the driver-rider match optimization problem can be represented as a maximum-weight bipartite matching problem.

The bipartite graph consists of two disjoint sets of vertices, a set representing drivers D and a set representing riders R. An edge between a driver and a rider exists if the match is feasible, with a weight that represents the positive savings in distance when traveling together compared to when each of them drives separately. More formally, a constraint on positive cost savings implies a necessary condition for the feasibility of a match between driver d and rider r: only if $d_{v(d),w(d)} + d_{v(r),w(r)} - (d_{v(d),v(r)} + d_{v(r),w(r)} + d_{w(r),w(d)}) > 0$. Moreover, matches must also be time feasible, where both the rider's and the driver's travel windows are respected. In the case where participants are willing to adopt either the role of driver or rider, we can also determine the optimal role assignment for each feasible potential ride-share pair upfront.

Note that when a single driver travels with at most a single rider and riders do not transfer, the total system-wide vehicle-miles traveled by participants can be reduced by no more than 50%. The reason for this is that the length of the joint trip can not be smaller than the larger of the individual trips of the ride-share partners. Again when transfers are not allowed and a driver can ride with at most $q(d)$ passengers, we can save at most $1/q(d)$ of the system-wide vehicle-miles by ride-sharing.

4.2.1 Illustrative Example 1

Consider three drivers d_1, d_2 and d_3 and two riders r_1 and r_2 and the distance network in Figure 3. Each rider and driver has an earliest departure and latest arrival time as given in the figure at the origin and destination nodes. The travel time between locations is proportional to the distance and for convenience we consider a constant vehicle speed of 60 mph. If all potential participants drove themselves, the system-wide vehicle-miles is 31.

Figure 4 depicts the associated bipartite graph for matching with the savings for each feasible ride-share match. The graph indicates that four of the six possible matches are feasible. In Figure 3, we observe that if rider r_2 shares a ride with driver d_1, the system-wide vehicle-miles increases. The reason for this is that their joint trip of 18 miles is longer than the sum of their individual trips of 16 miles. As a result, this match does not provide any savings to the system. We also see that it is not time feasible for driver d_3 to accommodate rider r_1 as the driver is unable to reach the rider's destination in time. That is, $e_{d_3} + t_{v(d_3),v(r_1)} + t_{v(r_1),w(r_1)} > l_{r_1}$.

Given the feasible matches, we observe that an optimal solution will match rider r_1 with driver d_1 or d_2, and rider r_2 with driver d_2 or d_3. The greatest individual savings for a pairing is associated with the match between d_2 and rider r_1. However, the maximum system-wide savings is achieved by matching driver d_1 with rider r_1 and driver d_2 with rider r_2. This results in total savings of 5.2 vehicle-miles, which represents a 17% reduction in travel distance compared to the situation without ride-sharing.

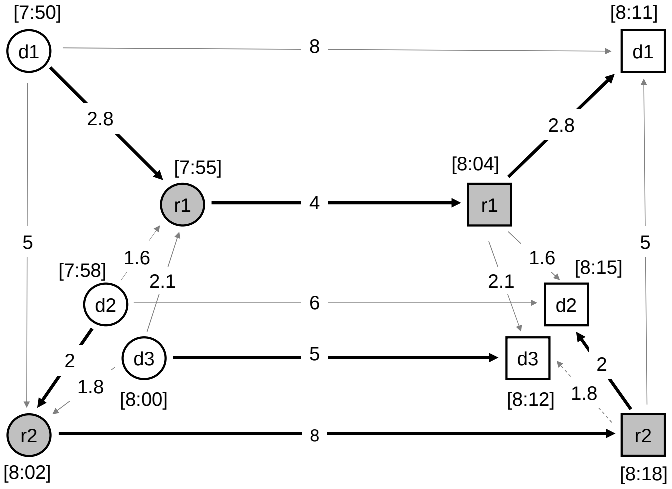

Figure 3: Riders (grey) and Drivers (white) traveling from Origin (circle) to Destination (square)

4.3 Single Driver, Multiple Rider Arrangements

If drivers have sufficient time flexibility, they may be willing to provide rides to several riders on a trip, either one after the other or simultaneously for portions of the time. The pickup and drop-off of multiple riders in a single trip gives rise to more complex routing decisions.

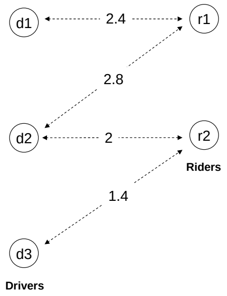

Figure 4: Bipartite graph associated with the problem

The carpool problem is a special case of this ride-sharing variant. In the carpool problem workers, partitioned into riders and drivers, want to go to their common work location from their homes. The objective is to assign riders to drivers and construct feasible routes for drivers to minimize the travel costs plus a penalty associated with unserved riders. Each worker has an earliest time he can leave home and a latest time he can arrive at work. Furthermore, each driver has a maximum time he is willing to spend driving from home to work.

Baldacci et al. [2004] address the *to-work* variant of the carpool problem separately from the *return-from-work* variant. They propose both an exact and heuristic method to solve the problem based on two integer programming formulations. They solve several instances, some based on real-world data, with the number of workers ranging from 50 to 250. Calvo et al. [2004] study the problem using a model that allows different network travel times at different times of the day. They develop a heuristic approach to solve the problem based on construction and local search. In a computational study using real-life carpool data, the authors investigate the impact of varying the ratio of drivers to riders and show that the total system-wide travel time increases with the ratio between driver and riders.

4.4 Single Rider, Multiple Driver Arrangements

If we allow riders to transfer between drivers, a rider may travel with more than one driver to reach his final destination. Gruebele [2008] describes such a multi-hop ride-share system in detail. Potential transfer points could include public transport stops, shopping malls, or park-and-ride lots.

Note that in the model presented in Section 4.1, a rider could be picked up by one driver and dropped off by another driver, *i.e.*, transfers are allowed. By adding additional constraints, the number of transfers can be limited. For example, the constraints

$$\sum_{t \in \{e(r),\ldots,l(r)-t_{v(r),w(r)}\}} x^r_{(v(r),t)(w(r),t+t_{v(r),w(r)})} + y^r = 1 \quad \forall r \in R$$

eliminate the use of transfer points.

Alternatively, the constraints

$$\sum_t \sum_{i \neq v(r) \wedge i \neq w(r)} \sum_{j \neq v(r) \wedge j \neq w(r)} x^r_{(i,t)(j,t+t_{i,j})} \leq F - 1 \quad \forall r \in R$$

allow up to F transfers per rider.

5 The Multi-Modal Ride-Sharing Problem

Instead of providing door-to-door transportation, the ride-share concept could be integrated with other modes of transportation, such as public transit. Ride-sharing may provide a very effective means to increase the use of a scheduled public transportation system if it can be used as a feeder service. In such a setting, a driver would first take a rider from the rider's origin to a public transport stop, then he would use public transit to get close to his destination, and finally he would walk or use another ride-share driver to travel from the transit stop to his destination. Aktalita, a project currently in development in Guadalajara, Mexico (www.aktalita.com) aims at developing such an integrated ride-share system.

A scheduled public transit system can be represented fairly readily within a time-space network. Let $\bar{P}$ be a set of locations where riders can board and alight transit, referred to as the stops. Each stop $p \in \bar{P}$ has a set of next stops $N(p) \subseteq \bar{P}$ that can be reached directly. Because we are considering a scheduled system, there is a known set of departure times and arrival times at each stop for services between i and $j \in N(i)$. The nodes (p,t) of the time-space network correspond to a stop and an arrival and/or departure time at the stop. The arcs $((p,t),(p',t+t_{p,p'})$ correspond to scheduled transportation between stops, where (p,p') with $p' \in N(p)$ is a physical link in the transit service network.

By integrating such a representation of a public transportation system and the time-space network representing a ride-sharing system, we can easily develop a combined multi-modal ride-sharing network representation. The only changes that must be made to the constraints of the model are that when a rider uses a public transport arc, there is no requirement that he needs to be with a driver and that drivers cannot use a public transport arc.

5.1 Illustrative Example 2

Consider the transportation network in Figure 5, where we have two drivers d_1 and d_2 and one rider r_1 and a transit line with three successive stops. A commuter train departs every 10 minutes from the first stop and the travel time between successive stops is 10 minutes. Again, the earliest departure and latest arrival times of the driver and riders are depicted at the corresponding origins and destination. The drivers and the public transit line both travel at speed 60 mph.

The system optimal solution is one where driver d_1 picks up the rider at his origin and brings him to the first stop of the transit line. The rider arrives at this stop at 8:15 and waits 5 minutes to board the 8:20 train. The rider then travels to the third stop, where he is picked up by driver d_2 at 8:40 to travel together to their final destination.

It is also possible for driver d_1 to drop off rider r_1 at the second transit stop, which would allow the rider to catch an earlier train. However, this would increase the travel distance for driver d_1 and thus the system as a whole. Additionally, the rider would not reduce his travel time since he would need to wait at the third stop for driver d_3 to arrive.

It is interesting to note that by integrating the public transit option, the possibility to establish feasible ride-shares increases. Without public transport, a ride-share between rider r_1 and either one of the drivers are not time-feasible in this specific example.

6 Challenges

In this section, we discuss a number of important challenges that must be addressed in order to make dynamic ride-sharing systems successful in practice. We believe that each challenge below is important to the development of automated ride-share matching methods, and in some cases these challenges may require our community to significantly extend our capabilities for providing real-time decision support in general.

6.1 Challenge 1: Dynamics

In any practical dynamic ride-share implementation, new riders and drivers continuously enter and leave the system. A driver enters the system by announcing a planned trip and offering a ride,

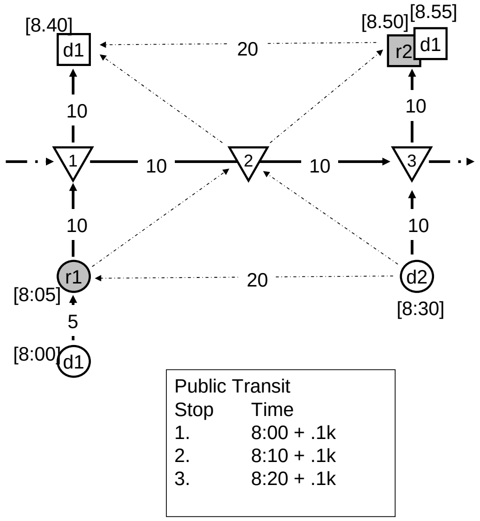

Figure 5: Riders (grey) and Drivers (white) traveling from Origin (circle) to Destination (square) with Triangles to represent Transit Stops

while a rider enters the system by announcing a planned trip and requesting a ride. Drivers and riders leave the system when a ride-share arrangement has been planned and accepted, or when their planned trips "expire," *i.e.*, when the latest possible departure time of a planned trip occurs before a successful arrangement can be found.

To avoid one potential worry for potential participants, it may be better to have each participant specify a trip expiration time in addition to (and which may be earlier than) his latest possible departure time. If the announcement time of a trip for driver d (rider r) is denoted $a(d)$ $(a(r))$ and the expiration time of the trip by $b(d)$ $(b(r))$, then the window for matching driver d (rider r) is $[a(d), b(d)]$ $([a(r), b(r)])$. Due to these matching windows, a match between driver d and rider r (assuming they have otherwise compatible trips) can be established only in the time interval $[\max\{a(d), a(r), \min\{b(d), b(r)]$.

Since new drivers and riders continuously arrive, not all relevant offers and requests may be known at the time the ride-share provider executes an algorithm for planning ride-sharing arrangements. A common approach for dealing with these types of planning uncertainties is to use a rolling horizon solution approach. A critical decision then is the re-optimization frequency, *i.e.*, how frequently the algorithm for finding arrangements is to be executed in practice. The ride-share provider may choose to re-optimize each time a new trip is announced (or after some batch of announcements), or may choose to re-optimize at fixed time intervals. The best choice is not obvious. Re-optimizing at fixed time intervals allows for some accumulation of trips, which may lead to larger cost savings, but on the other hand it may also lead to missed matches due to expiration of requests. Of course, it is also important to consider how much computation time will be available to determine ride-sharing arrangements, since this may also affect the solution approach that can be used for a given re-optimization frequency choice.

In a rolling horizon approach, the optimization problem to be solved includes all of the offered rides (drivers) and requested rides (riders) that are known at the time of execution and that have not yet been matched. The ride-share provider may decide to immediately notify drivers and riders of the matches identified by the optimization, or may decide to wait before notifying so as to find improved matches at the next execution time. For example, if the next optimization is scheduled at time t, then a match between driver d and rider r with $\min\{b(d), b(r)\} \geq t$ can be postponed without negatively impacting cost savings.

Although the issues of re-optimization frequency and solution commitment are not unique to ride-sharing, ride-share systems offer a highly dynamic environment in which to study them. The setting discussed so far in this paper implicitly assumes that matches must be identified before a driver departs from his origin. However, it is not hard to envision systems where matches are found after a driver has begun his planned trip. Mobile technology clearly exists to monitor the location of drivers in the system. Practical issues, of course, arise in this extension. Performing an optimization and communicating any identified matches to drivers (and riders) requires time. During that time, the locations of drivers in-transit change. Such changes must be anticipated and included in the optimization.

After ride-share systems have been in operation for a while, it is likely that some information about future unknown ride offers and ride requests may become available. Instead of myopically optimizing for the offered trips and requested trips that are known, it may of course be possible to incorporate information that partially describes the stochastic future into a modeling and solution approach in order to improve system-wide cost savings. Again, although these issues are not unique, ride-share systems are an interesting application area within which to study them.

6.2 Challenge 2: Safety and Reliability

A ride-share system must be safe and reliable for long-term viability. Safe implies that people will be willing to participate and will not believe that ride-sharing is significantly less safe than driving alone, while reliable means that potential riders will be confident that they will be matched and that identified arrangements will be executed as planned, without delays or last-minute cancelations.

Reputation systems, which are commonly used to support online sales transactions (the online marketplace eBay uses perhaps one of the most widely known reputation systems), could help to establish trust among participants and encourage reliable system behavior. A ride-share reputation system could provide drivers and riders the opportunity to rate each other. Such ratings could be converted into a feedback score. In addition, the ride-share provider could rate participants by monitoring cancelations, no-shows, and late arrivals. Such ratings could be converted into a reliability score. The reliability and feedback scores could then be used in the matching optimization to favor matches involving participants with high scores. Favoring in the matching process would mean that these participants would be more likely to be matched, and that if matched they would be more likely to receive a larger cost savings.

System reliability is also essential for participants to be confident in their ability to complete round trips via ride-sharing. It is likely that the very large majority of ride-share participants will need to plan round trips. In a dynamic ride-share system with a sufficient amount of capacity, the rider should be able to arrange the trips separately shortly before departure. However, some riders may not feel comfortable going to certain destinations without a guarantee that they will be able to find a ride back (for example, because the alternatives may be very costly). In fact, this concept is key to typical casual car-pooling arrangements; often, users are picked up at transit stops for drives into downtown areas, and the return trip is made by transit if no car-pooling in the reverse direction is available. The need for round trip planning may necessitate that systems allow riders to place two transportation requests at the same time, and only accept a ride-share if both transportation requests are matched. Of course, the return trip does not have to be with the same driver that provides the first trip. To reduce the risks for riders, the ride-share provider could also consider offering various backup services for riders, for example by using taxis.

Some people may only feel safe sharing rides with friends and acquaintances, and possibly also with friends of friends. The specification of acceptable subsets of ride-share partners can be supported by social or professional network tools, such as Facebook or Linkedin. It is unclear what kind of social network characteristics are conducive to successful ride-sharing systems, *e.g.*, geographic densities of the network, the average number of connections of network participants, etc. Modeling and understanding any relation between ride-share social network characteristics and ride-share system success seems to be a fertile and important area for research, especially given the popularity and growth of social networks.

6.3 Challenge 3: Incentives

The financial benefits of sharing trip-related expenses may motivate people to participate in ride-sharing. Rising fuel costs, pay-per-mile auto insurance and congestion pricing may further increase the cost of private car use in the future, and thus strengthen the advantages of ride-sharing. However, without a sufficient number of drivers and riders, the chance of finding a ride, especially one close to the desired departure time, may be very small, and thus the inconvenience may outweigh the financial benefits. To achieve the required density, *i.e.*, the number of necessary drivers and riders for a sustainable ride-share system, local governments and businesses may need to subsidize ride-share initiatives. These subsidies can be used to reward ride-share participants, either on a per-trip basis or a per-offer basis. Subsidizing commercial urban taxis to act as drivers may also be advantageous especially in a start-up phase. Subsidized ride-share systems may provide a relatively inexpensive way to increase the capacity and efficiency of the transportation system, a potentially interesting alternative to the capital investments required to build or expand the road network or expand public transportation.

6.4 Challenge 4: Choices and Behavior

A good understanding of participant behavior and participant preferences will be essential when designing of a dynamic ride-sharing system. If ride-share matches do not satisfy participant preferences, the match may not be accepted, or the participant may not make use of the ride-share system in the future. Unfortunately, providing comprehensive preferences may be difficult and time-consuming for participants, partly because preferences may be interdependent and may change from one day to the next. For example, a driver's time flexibility may depend on the day of the week, the financial benefits, and the specific rider. Moreover, some participants may be hesitant or unwilling to disclose certain preferences for privacy reasons.

Rather than being notified of a specific single ride-share match, participants may prefer to choose from a menu of available ride-share options. However, the selection process must not take too much time. Minimally, then, the ride-share provider should present only the best options and only the most relevant information regarding these options, which may include the pickup and drop-off times, the travel time, the financial benefits, but also person specific information, such as gender, age, professional profile, and feedback and reliability scores. Providing a menu of ride-share options introduces various system synchronization issues. If driver trips (rider trips) may appear as an option for several riders (drivers) simultaneously, there is a chance that preferred options clash, *i.e.*, the same driver trip or the same rider trip is chosen multiple times. Designing a selection-based matching process is non-trivial and would likely pose interesting new additional challenges for the underlying matching optimization engine.

6.5 Challenge 5: Multi-Modal Design

The effective integration of different modes of transportation, operated by different service providers, is often a huge challenge. This is true not only in passenger transportation, but also in freight transportation. Different modes of transportation are typically owned and managed by different entities and thus collaborative arrangements must be in place before any coordination of transportation activities can occur.

An effective integration of a ride-share system and a scheduled public transit system will potentially increase, in a sense, the coverage area of the public transit system, which has many societal and environmental benefits. However, the transfer from one mode of transportation to another must be seamless and efficient, and without long waiting times, before large numbers of people will make use of the integrated system. Consistent seamless and efficient mode transfers will only be possible with effective optimization technology.

An interesting area of research related to multi-modal passenger transportation systems also concerns system design. Rather than focusing only on effectively coordinating the operations of the different transportation systems, another important focus could be on whether different public transit system designs would be more effective in cases where ride-sharing systems are serving as feeders than in cases where they are not.

6.6 Challenge 6: Optimization

The integer programming model given in Section 4 is included in this paper primarily to provide a formal statement of a basic ride-share planning optimization problem. It may be unlikely that realistic-size instances of the model can be solved fast enough to be of use in a matching engine of an actual, sustainable ride-share system. In a major metropolitan area, thousands of riders and drivers travel between thousands of origins and destinations during the same time periods, which leads to very large optimization problems that may have to be solved very quickly as often as once every few minutes. Thus there is a clear need for fast solution approaches producing high-quality results.

Another area of research that should be of interest to the transportation science and logistics community is the design of de-centralized ride-share matching techniques. Centralized ride-share matching may not be practical for many larger metropolitan areas, or may not be computationally feasible. In such cases, effective decomposition approaches will be necessary. A simple decomposition based on a geographic partition is likely to be challenging since driver and rider trips involve both an origin and a destination location, and these locations may often be separated by significant distances. The existence of trip requests for which the origin location is in one subregion and the destination location is in another subregion is therefore quite likely. It is also not clear whether a static partition of the region suffices or whether the partition of the region should be adjusted dynamically based on the set of driver and rider trip requests that need to be matched.

7 Concluding Remarks

New dynamic ride-sharing systems have the potential to provide huge societal and environmental benefits. The development of algorithms for optimally matching drivers and riders in real-time is at the heart of the ride-sharing concept. We have formally defined dynamic ride-sharing, and have highlighted many of the interesting optimization challenges that arise when developing technology to support dynamic ride-sharing. We believe these challenges provide great research opportunities for the transportation science and logistics community and we hope that the introduction that we provide in this paper in part leads to many future contributions in this exciting, emerging area of public transportation.

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