Swing up and Stabilization Control Experiments for a Rotary Inverted Pendulum– An Educational Comparison

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***Abstract*— This paper focuses on the swing up and stabilization control of a rotary inverted pendulum system with linear quadratic regulator (LQR), sliding mode control (SMC) and fuzzy logic control (FLC). The inverted pendulum, a popular control application exists in several forms and due to its widespread use for prototyping control schemes we present experimental results obtained on a rotary version of the pendulum. The paper develops the dynamical model and introduces the implementation of the considered schemes comparatively.**

Keywords—*Rotary inverted pendulum, LQR, fuzzy logic, sliding mode control*

I. Introduction

The inverted pendulum systems have two typical forms studies frequently in the literature. First is the classical one running on straight path to balance the pole yet the other is the rotary one which is capable of performing swing up. Both are underactuated mechanical systems and they have been studied numerously to demonstrate when a novel scheme is proposed or when a control teaching material is proposed. The nonlinear and coupled nature of the differential equations describing the system dynamics is found further appealing to those experimenting control laws in real time.

Underactuated systems have fewer actuators than the degrees of freedom. Because of this, the strategies developed for fully actuated systems may not be applied directly to underactuated systems During the last few decades, developing different control strategies for underactuated systems has drawn a great interest as most physical systems have an underactuated description, such as those in robotics, aerospace engineering, and marine engineering. Until recently, designing global controllers for underactuated systems was a challenge, yet, to overcome this problem, the control engineering framework now offers several tools based on quadratic optimization, hard boundary switching laws (sliding mode control) or soft boundary laws based on fuzzy set theory.

Aside from the underactuated nature of the dynamics, the difficulty of the control problem is the desire of balancing the pole around the unstable equilibrium point. Despite the dynamics is well known and it is possible to apply the feedback linearization scheme, the fact that the system is a nonminimum phase one makes the application of the scheme a tedious task [1]. This paper presents the results obtained with linear quadratic controller. Since the optimization problem can be expressed in terms of the tracking errors and the cost functional, it is possible to develop a linear quadratic controller based on the linearized model of the plant around the desired operating point. Despite the widespread applicability of the scheme, the fact that it necessitates the linearized plant dynamics is a drawback making the controller useless for he operating points that are dissimilar to the one used for linearization. Sliding mode control, a nonlinear and two sided witching type control scheme, is implemented to see the robustness and invariance properties. Lastly, the experiments are carried out to discover the performance of the fuzzy controllers based on sets labeled by linguistic qualifiers and with unsharp boundaries. Each scheme has pros and cons discussed in the paper.

So far, many approaches have been proposed to swing up and stabilization control for inverted pendulum systems. In [2-3], a swing up control algorithm is proposed and in [4], minimum time swing up by iterative impulsive control method is elaborated. Another technique, called energy based control, is studied in [5-6]. The LQR is presented to optimize the controller gains in [7], artificial neural networks and fuzzy logic systems are utilized for swing up process and stabilization in [8-10]. In [11], fuzzy swing up and fuzzy stabilization based on linear quadratic regulator approach for the rotary inverted pendulum is investigated. Conventional sliding mode control technique can not be applied for underactuated systems since they can not be decoupled and not every subsystem is equipped with an individual control channel. In [12], to overcome this problem, a modification to sliding surface is considered. A coupled sliding surface is highlighted for actuated and unactuated states and their time derivatives in [13]. Also an aggressive swing up by saturation functions and a state dependent coupling parameter in coupled sliding surfaces is introduced in [13-14]. Due to insufficiency of conventional methods a nonregular backstepping technique is proposed in [15]. Sliding mode control, designed by the use of fuzzy sliding surfaces, is introduced in [16-17].

This paper is organized as follows. The second section presents description of the rotary inverted pendulum system and derivation of the mathematical model. LQR and state feedback controller, SMC and FLC are described in the following sections, respectively, with real time experimental results. Finally, the concluding remarks are given at the end of the paper.

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II. Rotary Inverted Pendulum System

A. Description of the system

Rotary inverted pendulum system as known Furuta pendulum was first developed by Katsuhisa Furuta in Tokyo Institute of Technology. The testing apparatus showing the definitions of the angles are is depicted in Fig. 1.

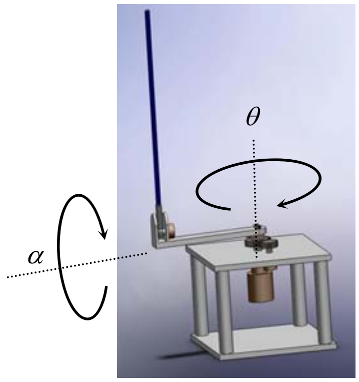

Figure1:Rotary inverted Pendulum System

The system consists of a servo system and a pendulum attached to the tip of a link appropriately as shown above. Angular displacement in the horizontal plane is denoted by θ and α denotes angular displacement of the pendulum in the vertical plane (See Fig. 1). The motivation of the problem is to design a controller that swings the pendulum up and balance it at the upright position which is the unstable equilibrium point of the system.

B. Mathematical Model of the Rotary Inverted Pendulum

Mathematical model of the system is obtained by using Euler Lagrange formulation. As a first step to obtain the dynamic equations of the system: the potential and kinetic energy is given in (1) and (2), respectively.

$$V = mgl\cos(\alpha) \tag{1}$$

$$\begin{aligned} T = {} & \frac{1}{2}J_{eq}.\dot{\theta}^2 + \frac{1}{2}m.(r\dot{\theta} - L\cos(\alpha).(\dot{\alpha}))^2 + \\ & \frac{1}{2}m(-L\sin\alpha.(\dot{\alpha}))^2 + \frac{1}{2}J_{cm}\dot{\alpha}^2 \end{aligned} \tag{2}$$

Lagrange equation is described by (3).

$$L = T - V \tag{3}$$

The output torque of the motor is given by (4)

$$T_{output} = \frac{\eta_m \eta_g K_t K_g (V_m - K_g K_m \theta)}{R_m} \tag{4}$$

where η_m is motor efficiency, η_g is gear efficiency of the pendulum and motor arm, K_t motor torque constant, K_m electromotor constant, R_m armature resistance and V_m is the input voltage. Euler equations to derive the dynamics are given in (5) and the dynamics of the system is described in (6).

$$\begin{aligned} & \frac{\delta}{\delta t}\left(\frac{\delta L}{\delta \dot{\theta}}\right) - \frac{\delta L}{\delta \theta} = T_{output} - B_{eq}\dot{\theta} \\ & \frac{\delta}{\delta t}\left(\frac{\delta L}{\delta \dot{\alpha}}\right) - \frac{\delta L}{\delta \alpha} = 0 \end{aligned} \tag{5}$$

$$\begin{aligned} & a\ddot{\theta} - b\cos(\alpha)\ddot{\alpha} + b\sin(\alpha)\dot{\alpha}^2 + G\dot{\theta} = \frac{\eta_m \eta_g K_t K_g}{R_m} V_m \\ & c\ddot{\alpha} - b\cos(\alpha)\ddot{\theta} - d\sin(\alpha) = 0 \end{aligned} \tag{6}$$

where $a = J_{eq} + mr^2$, $b = mLr$, $c = 4mL^2/3$, $d = mgL$, $E = ac - b^2$, $G = (\eta_m \eta_g K_t K_m K_g^2 + B_{eq} R_m)/R_m$, $k = (\eta_m \eta_g K_t K_g)/R_m$.

TABLE I. Parameters of The System

	Description	Value	Unit
L	Length to pendulum's center of mass	0.1675	m
r	Rotating arm length	0.215	m
J_{eq}	Equivalent moments of inertia	0.0036	kg.m^2
m	The mass of the pendulum	0.125	kg
R_m	Armature resistance	2.6	Ohms
B_{eq}	Equivalent viscous damping coef.	0.0040	kg.m^2

III. Linear Quadratic Regulator Design

Essentially, the LQR is a particular type of a state feedback method entailing a cost functional in a particular form (See (7)) with a constraint, the system dynamics, in the form of a linear ordinary differential equation as in (8).

$$J = \frac{1}{2}\int_0^{\infty}(x^{\mathrm{T}}Qx + V_m^2)\mathrm{d}t \tag{7}$$

$$\begin{bmatrix} \dot{\theta} \\ \dot{\alpha} \\ \ddot{\theta} \\ \ddot{\alpha} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 39.32 & -14.52 & 0 \\ 0 & 81.78 & -13.78 & 0 \end{bmatrix} \begin{bmatrix} \theta \\ \alpha \\ \dot{\theta} \\ \dot{\alpha} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 25.54 \\ 24.59 \end{bmatrix} V_m \tag{8}$$

where $x = (\theta \;\; \alpha \;\; \dot{\theta} \;\; \dot{\alpha})^{\mathrm{T}}$ and Q is a weighting matrix.

The states of the system to be controlled is assumed to be available and the design phase is involved with the design of a swing up controller and that responsible for maintaining the motion around the unstable equilibrium point, the upright position.

A proportional plus derivative controller with a positive feedback is implemented for the swing up phase. The used gains are given as K_p=100 and K_d=2.5, which are large enough to provoke instability and force the pendulum pass through the unstable equilibrium point. The results with such a swing up controller are shown in Fig. 2, where it is clearly seen that the positive feedback controller causes fast turns and the pendulum visits the desired position several times. The stabilizing controller will now be considered to stop when the pendulum approaches its desired point.

We choose $Q = diag(5 \;\; 20 \;\; 0.5 \;\; 0.5)$. This choice has been refined after a few iterations. The gain matrix K is accessed using the solution of Riccatti equation given by (9) and the control signal is constructed using (10).

$$KA - KBB^T K + Q + A^T K = 0 \tag{9}$$

$$V_m = -(B^T K)x \tag{10}$$

The gain K is obtained as given by K=(−3.32 26.77 −2.53 3.88). Using the control law above, it is observed that the unstable equilibrium point of rotary inverted pendulum remains stable and control performance was found adequate. The real time experimental results for stabilization of rotary inverted pendulum around unstable equilibrium point are given in Fig. 3 and the control signal is depicted in Fig. 4.

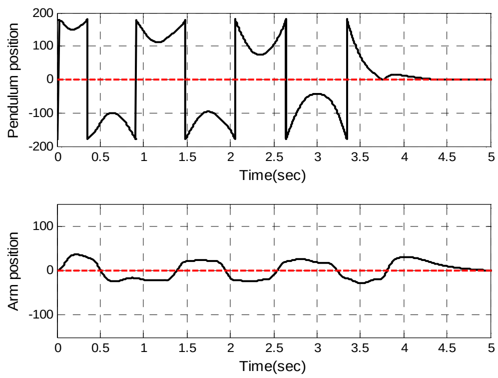

Figure 2:Swing up action with PD controller

IV. Sliding Mode Controller Design

Sliding mode control, also known as variable structure control, is a nonlinear control technique that is based on a two sided switching control law. The method is useful when there are uncertainties in the system dynamics and the parameter values. The uncertainties are assumed to be bounded and the closed loop system exhibits a certain degree of robustness against such uncertainties. The design starts with the definition of a particular subspace of the phase space called the sliding hypersurface (sliding line or switching subspace). The SMC scheme displays two fundamentally different responses called the reaching mode and the sliding mode. The error vector is forced towards the sliding subspace and when it hits the sliding surface, it remains on it thereafter. The period ending with the hitting the sliding subspace is called the reaching phase, and the motion afterwards is called the sliding mode. Since the error vector slides toward the origin along the sliding subspace, the control scheme is called the sliding mode control.

Consider the system in (6) in the following affine form.

$$x^{(n)} = F(x) + G(x)u \tag{11}$$

$$y = x \tag{12}$$

Defining e as the error in the output, choosing the switching variable as given in (13) and considering the Lyapunov function in (14) means that a control law ensuring $\dot{V} < 0$ guides all trajectories toward the subspace characterized by s = 0, the sliding subspace.

$$s = \left(\frac{de}{dt} + \lambda e\right)^{n-1} \tag{13}$$

$$V_S = \frac{1}{2}s^2 \tag{14}$$

The state variables can be treated as the states of two subsystems as in (15). Rotary inverted pendulum is a nonminimum phase underactuated system; therefore designing a variable structure controller is not straightforward for such a system.

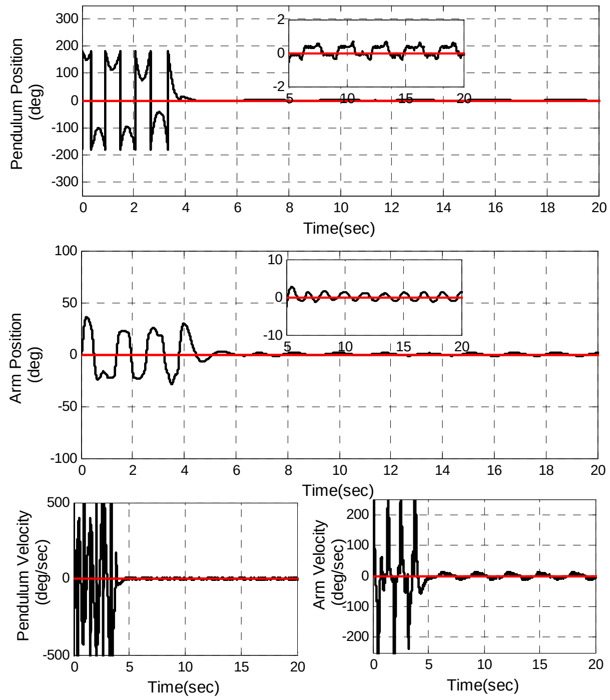

Figure 3: Stabilization with LQR in Real Time

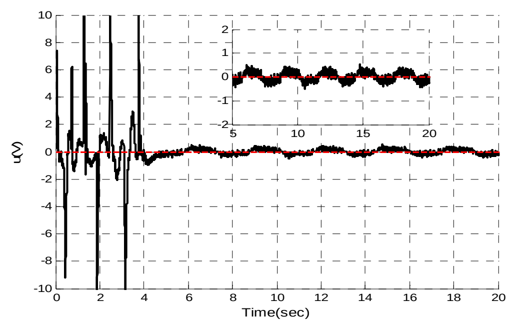

Figure 4: The Control Signal of LQR

$$\begin{aligned}\dot{x}_1 &= x_2\\ \dot{x}_2 &= f_1 + g_1 u\\ \dot{x}_3 &= x_4\\ \dot{x}_4 &= f_2 + g_2 u\end{aligned} \tag{15}$$

where

$$f_1 = \frac{ad\sin(\alpha) - b\cos(\alpha)\left(b\sin(\alpha)\dot{\alpha}^2 + G\dot{\theta}\right)}{ac - b^2\cos^2(\alpha)}$$

$$g_1 = \frac{ak}{ac - b^2\cos^2(\alpha)},\ f_2 = \frac{bd\cos(\alpha)\sin(\alpha) - bc\sin(\alpha)\dot{\alpha}^2 - Gc\dot{\theta}}{ac - b^2\cos^2(\alpha)}$$

$$g_2 = \frac{kb\cos(\alpha)}{ac - b^2\cos^2(\alpha)} \tag{16}$$

Zero dynamics of a system describe the internal behavior of the system when the output is kept at zero. As mentioned earlier designing a feedback linearization controller for a nonlinear system may not meet the performance goals because of the unstable zero dynamics. Thus the zero dynamics of the system should be analyzed also. In pendulum displacement as the output of the system, the relative degree of the system is obtained, r=2. We set the control law as described in (17)-(20)

$$\begin{bmatrix} \ddot{\theta} \\ \ddot{\alpha} \end{bmatrix} = \frac{1}{D}\begin{bmatrix} f_1 \\ f_2 \end{bmatrix} + \frac{1}{D}\begin{bmatrix} g_1 \\ g_2 \end{bmatrix} u \qquad (17)$$

$$u = \frac{v - f_2 / D}{g_2 / D} = \frac{Dv - f_2}{g_2} \qquad (18)$$

$$\begin{bmatrix} \ddot{\theta} \\ \ddot{\alpha} \end{bmatrix} = \frac{1}{D}\begin{bmatrix} f_1 + g_1 u \\ f_2 + g_2 u \end{bmatrix} = \frac{1}{D}\begin{bmatrix} f_1 + g_1 \dfrac{Dv - f_2}{g_2} \\ f_2 + g_2 \dfrac{Dv - f_2}{g_2} \end{bmatrix} = \begin{bmatrix} \dfrac{f_1}{D} + g_1 \dfrac{Dv - f_2}{Dg_2} \\ v \end{bmatrix} \qquad (19)$$

$$v = -k_1\alpha - k_2\dot{\alpha} - k_3 \operatorname{sgn}(\alpha) - k_4 \operatorname{sgn}(\dot{\alpha}) \qquad (20)$$

The control input given in (20) can not stabilize the pendulum since the zero dynamics given in (21) is unstable. Thus output feedback linearization type of a control law is not helpful for rotary inverted pendulum system.

$$\ddot{\theta} = -\frac{Gc}{ac - b^2}(\dot{\theta} - 1) \qquad (21)$$

Hence we have to design a controller taking the unstable zero dynamics into consideration. The sliding surfaces for the two subsystems are defined as in (22) and (23).

$$s_\alpha = \dot{\alpha} + c_1\alpha = 0 \qquad (22)$$

$$s_\theta = \dot{\theta} + c_2\theta = 0 \qquad (23)$$

The relation between two surfaces is linear as given in (24).

$$S_{pendulum} = s_\alpha + c_3 s_\theta \qquad (24)$$

Now consider a Lyapunov function as given in (25).

$$V_S = \frac{1}{2} S_{pendulum}^2 \qquad (25)$$

Thaing the time derivative of Spendulum and forcing $\dot{S}_{pendulum} := -k_1 \operatorname{sgn}\left(S_{pendulum}\right)$ yields the control law in (26).

$$u = -\left(\frac{f_1 + f_2}{g_1 + g_2}\right) - \frac{D}{g_1 + g_2}\left(k_1 \frac{S_{pendulum}}{\left|S_{pendulum}\right| + \varepsilon} + c_2\dot{\theta} + c_1\dot{\alpha}\right) \qquad (26)$$

The controller has been implemented with the parameters settings shown in Table II and the results obtained are depicted in Figs. 5-6, where the pendulum is stabilized at its unstable equilibrium and, as expected, the control signal contains some fast fluctuations due to the switching nature of the controller.

TABLE II. PARAMETER VALUES FOR SMC

Parameters used in Sliding Mode Control					
Parameter	***Value***	***Parameter***	***Value***	***Parameter***	***Value***
k_1	1.7	c_1	13	c_3	1.9
ε	0.05	c_2	4		

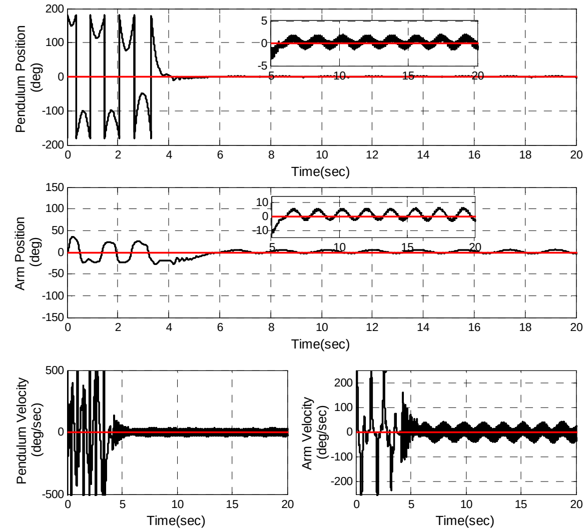

Figure 5. Stabilization Results with Sliding Mode Control

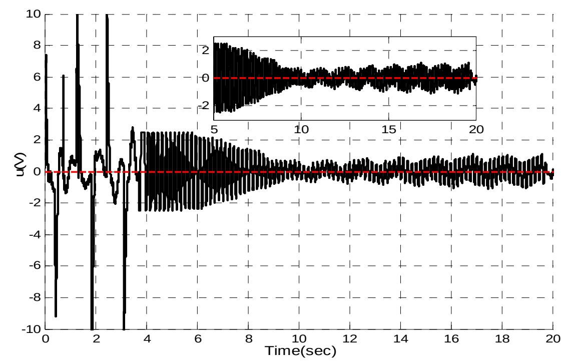

Figure 6. The Control Signal Produced by Sliding Mode Controller

V. FUZZY LOGIC CONTROL

Fuzzy logic control has been used in many control applications due to its suitability to the goals of a diverse set of feedback control problems. Fuzzy control is an approach to design controller based on expert knowledge and past experiences of a designer. The controllers based on fuzzy logic offers as a platform of decision making that does not rely on the mathematical models or hard computing. The control actions are defined by words and the fuzzy logic formalism provides the necessary means for expressing the verbally defined actions to a set of rules and inference engine based input-output relations.

The proposed fuzzy controller consists of two parts; first one is designed for the swing up action and second one is designed for stabilizing the pendulum around the unstable equilibrium point.

A. Swing up controller

The main objective of the controller is to take pendulum from pendent position to upward position.

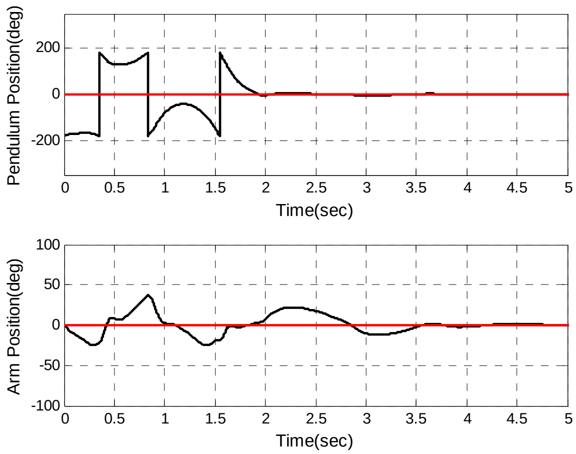

Figure 7. Swing up fuzzy controller

The proposed swing up action used the methodology of region of attraction and energy compensation methodology. Swing up consists of sequential motion of arm on positive and negative direction which means moving right and left consecutively. The fuzzy swing up controller is built via a Mamdani fuzzy inference system. The controller has four inputs and one output. Fuzzy logic based swing up rules are built on the two following basis:

- Control signal u is maximum, when all state variables are zero means pendulum and arm are stationary.
- When pendulum displacement is increasing one direction and pendulum velocity is zero, then control signal should be applied opposite direction.
- When the arm which is driven by servo system swung in either direction with a suitable frequency, the pendulum momentum increases with each swing.

The application results for a swing up fuzzy controller are shown in Fig. 7, whose details are skipped due to the space limit. The figure depicts a successful driving of the system toward the desired unstable equilibrium point.

B. Stabilization controller

The main objective of the stabilization controller is to keep the pendulum at the upright position and keep balancing continuously. We utilize a two part fuzzy controller, one dedicated to the arm and other dedicated to the pendulum. The input-output relation for such fuzzy models is given in (27) and the rule table is depicted in Figure 8, the implied control surface is also shown in Figure 10.

$$u = \frac{\sum_{i=1}^{R} y_i \prod_{j=1}^{m} \mu_{ij}(e_j)}{\sum_{i=1}^{R} \prod_{j=1}^{m} \mu_{ij}(e_j)} \qquad (27)$$

$\dot{e}$ \ e	NB	NS	Z	PS	PB
NB	NB	NB	NB	NS	Z
NS	NB	NB	NS	Z	PS
Z	NB	NS	Z	PS	PB
PS	NS	Z	PS	NB	PB
PB	Z	PS	PB	PB	PB

Figure 8. The Rule Based Fuzzy Stabilization Controller

- **If** the angular position error is negative big and the angular velocity error is negative big **then;** the control signal is negative big**.**
- **If** the angular position error is negative small and the angular velocity error is positive small **then;** the control signal is zero.

The triangular membership functions are shown in Fig. 9. The inputs of membership functions are in between (-3 3) for position error of pendulum and between (-15,15) for velocity error of pendulum as shown in Fig. 10. The defuzzifier parameters are chosen as in (28).

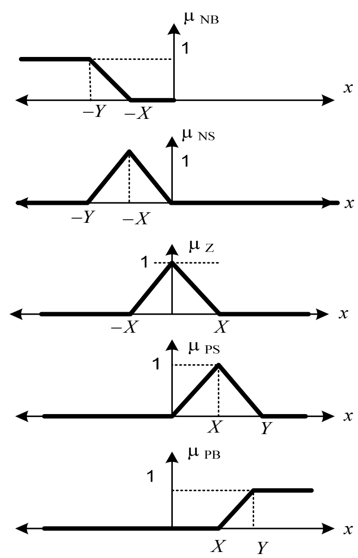

Figure 9. The Membership Functions of Fuzzy Stabilization Controller

$$y=[\ -1.32\ -1.21\ -1.08\ -0.91\ -0.83\ -0.72\ -0.61\ -0.562\ -0.462\ -0.33\ 0.15\ -0.05\ 0\ 0.08\ 0.165\ 0.314\ 0.45\ 0.57\ 0.62\ 0.715\ 0.80\ 0.94\ 1.07\ 1.21\ 1.32]^T \qquad (28)$$

The results of the implementation are illustrated in Figs. 11-12, where the pendulum is balanced at its desired position after a swing up phase lasting approximately 2 seconds.

VI. Concluding Remarks

This study analyzes the real time performance of different control techniques on a rotary inverted pendulum system. Laboratory system consists of three parts; rotary base with the DC motor, a pendulum, analog/digital interface and a personal computer. Control algorithms are prototyped in Matlab/Simulink® environment.

Both swing up and stabilization problem has been studied. Experimental tests have been done in real time. Three control schemes are elaborated, namely, linear quadratic controller, sliding mode control and fuzzy logic control. According to the results, shortest swing up phase is observed with the fuzzy controller. However, smoothest control signal is produced by the linear quadratic controller. In sliding mode control scheme, the control signal contains sharp fluctuations yet it provides robustness against parameter uncertainties.

Overall, the paper provides a comparative guide to those practicing the control laws on such a standard nonlinear and underactuated system.

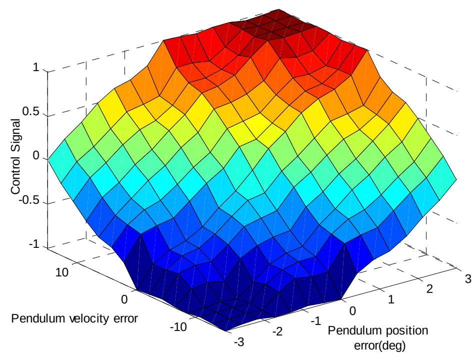

Figure 10. Rotary Inverted Pendulum Stabilization Control Surface for each Fuzzy Controller

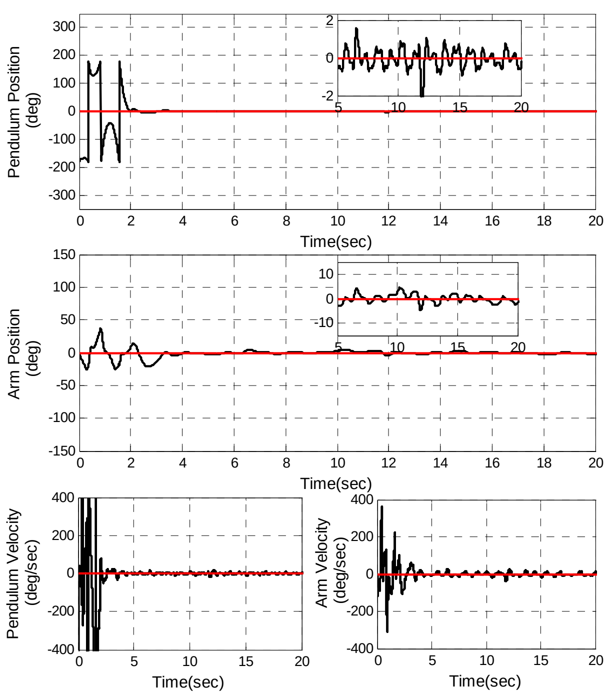

Figure 11. The Stabilization with Fuzzy Controller

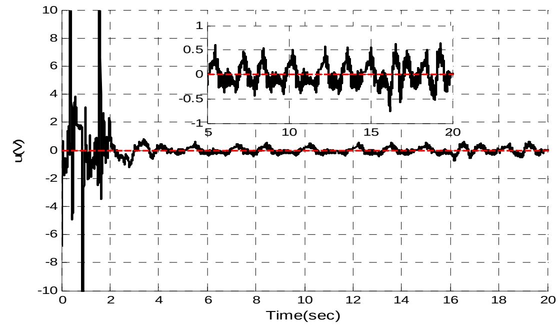

Figure 12. The Control Signal for Fuzzy Logic Controller

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