Symbol time offset estimation in coherent OFDM systems

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Symbol Time Offset Estimation in Coherent OFDM Systems

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***Abstract*—This paper presents a symbol time offset estimator for coherent orthogonal frequency division multiplexing (OFDM) systems. The estimator exploits both the redundancy in the cyclic prefix and available pilot symbols used for channel estimation. The estimator is robust against frequency offsets and is suitable for use in dispersive channels. We base the estimator on the maximum-likelihood estimator for the additive white Gaussian noise channel. Simulations for an example system indicate a system performance as close as 0.6 dB to a perfectly synchronized system.**

***Index Terms*—Communication system, delay estimation, multicarrier, orthogonal frequency division multiplexing (OFDM), synchronization, time estimation.**

I. INTRODUCTION

MOST coherent orthogonal frequency-division multiplexing (OFDM) systems, such as the Digital Video Broadcast (DVB) system [1] and the Broadband Radio Access Network (BRAN) [2], use pilot symbols to estimate the channel [3]. In this paper, we present a method to use these pilot symbols for symbol time synchronization in conjunction with the redundancy present in the cyclic prefix [4]. Though the synchronization algorithm in [4] performs well without pilots, performance can be improved by using channel estimation pilots also for synchronization, yielding a more accurate estimate of the time offset. Synchronization is a critical problem in OFDM systems, and the effects of synchronization errors are documented in, e.g., [5]–[7].

First, we derive the maximum-likelihood (ML) estimator for a symbol time offset in coherent OFDM systems. It is based on a suitably chosen model of the OFDM symbol, emphasizing the cyclic prefix redundancy and the presence of pilots, but disregarding channel dispersion, frequency offset, and signal correlation. This estimator's performance is, however, very sensitive to variations in the carrier frequency. Based on knowledge about how to jointly estimate time and frequency offsets when not using pilots [4] we make an *ad hoc* extention of the ML estimator that is robust against frequency offsets and suitable for practical systems.

II. SIGNAL MODEL

In OFDM systems, the data are modulated in blocks by means of a discrete Fourier transform (DFT). By inserting a cyclic prefix in the OFDM symbol, intersymbol interference (ISI) and intercarrier interference (ICI) can be avoided [7]. Most coherent OFDM systems transmit pilot symbols on some of the subcarriers to measure the channel attenuations. Both the cyclic prefix and the channel estimation pilots contain information that can be used to determine the symbol start.

Assume that one transmitted OFDM symbol consists of N subcarriers of which N_p are modulated by pilot symbols. Let Υ denote the set of indexes of the N_p pilot carriers. We separate the transmitted signal in two parts. The first part contains the $N - N_p$ data subcarriers and is modeled by

$$s(k) = \frac{1}{\sqrt{N}} \sum_{n \in \{0,\ldots,N-1\} \setminus \Upsilon} x_n e^{j2\pi kn/N} \tag{1}$$

where x_n is the data symbol transmitted on the nth subcarrier, using some constellation with average energy $\sigma_x^2 = E\left\{|x_n|^2\right\}$. The second part contains the N_p pilot subcarriers, modeled by

$$m(k) = \frac{1}{\sqrt{N}} \sum_{n \in \Upsilon} p_n e^{j2\pi kn/N} \tag{2}$$

where p_n is the pilot symbol transmitted on the nth subcarrier. We assume $E\left\{|p_n|^2\right\} = \sigma_x^2$, although some systems use boosted pilots [1]. (It is straightforward to extend our method to accommodate boosted pilots.)

In the following,we assume an additive white Gaussian noise (AWGN) channel, not introducing any time dispersion, and we model the received signal $r(k)$ as

$$r(k) = s(k-\theta) + m(k-\theta) + w(k) \tag{3}$$

where θ represents the unknown integer-valued time offset and $w(k)$ is additive complex white zero-mean Gaussian receiver noise with variance σ_w^2. Two properties of the received signal allow for the estimation of θ: the statistical properties of $s(k)$ and the knowledge of $m(k)$. We simplify the statistical properties of $s(k)$ so that we can derive a tractable estimator. First we assume that the time-domain signal $s(k)$ is a Gaussian process with variance $\alpha\sigma_x^2$, where $\alpha = (N - N_p/N)$. In an OFDM system with a reasonably large number of data-carrying subcarriers $(N_p \ll N)$, $s(k)$ has statistical properties similar to a discrete-time Gaussian process (by the Lindeberg theorem [8, pp. 368–369]). Secondly, as in [4], we make simplifying assumptions about the statistical properties of the correlation of $s(k)$. In systems employing a cyclic prefix, the tail L samples of the N-sample $(L < N)$ transmitted signal $s(k)+m(k)$ are copied, i.e., $s(k) = s(k+N)$, and $m(k) = m(k+N)$, for $k \in [0, L-1]$. The length of one OFDM symbol is thus $N + L$ samples of which L samples constitute the cyclic prefix. Therefore, $s(k)$ is not white but contains pairwise correlations between samples spaced N samples apart within two sets. Furthermore, we ignore the correlation between successive time symbols in $s(k)$. For most practical systems this correlation will be small if the

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number of pilots is small. So, while we do model the correlation due to the cyclic prefix, we disregard any other correlation between time-symbols.

Since the noise is zero-mean Gaussian and the pilot signal $m(k)$ is a deterministic signal which is known at the receiver, the modeled received signal $r(k)$ is also Gaussian with time-varying mean $m(k)$ and variance $\alpha\sigma_x^2$. Based on the simplifying assumptions above, the autocorrelation becomes

$$c_r(k,l) = \begin{cases} 1, & k = l \\ \rho, & k-l=-N, k \in [\theta, \theta+L-1] \\ \rho, & k-l=N, l \in [\theta, \theta+L-1] \\ 0, & \text{otherwise} \end{cases} \tag{4}$$

where

$$\rho = \frac{\alpha\sigma_x^2}{\alpha\sigma_x^2+\sigma_w^2} = \frac{\alpha \mathrm{SNR}}{\alpha \mathrm{SNR}+1} \tag{5}$$

and $\mathrm{SNR} = \sigma_x^2/\sigma_w^2$ is the signal-to-noise ratio.

Based on this correlation structure and on the knowledge of the time-varying mean $m(k)$, we now derive an estimator of the time offset θ, using data from one received OFDM symbol.

III. Time Offset Estimation

We derive the ML estimator of the time offset θ by investigating the log-likelihood function of θ, i.e., the joint probability of the received samples $r(\cdot)$ given θ,

$$\Lambda(\theta) = \Pr\{r(\cdot)|\theta\}. \tag{6}$$

The ML time offset estimate θ is obtained by maximizing the log-likelihood function over all possible values of θ

$$\widehat{\theta}_{\mathrm{ML}} = \arg\max_{\theta}\{\Lambda(\theta)\}. \tag{7}$$

In Appendix A, $\Lambda(\theta)$ is shown to be

$$\Lambda(\theta) = \rho\Lambda_{cp}(\theta) + (1-\rho)\Lambda_p(\theta) \tag{8}$$

where

$$\Lambda_{cp}(\theta) = \mathrm{Re}\left\{\sum_{k=\theta}^{\theta+L-1} r^*(k)r(k+N)\right\} - \frac{\rho}{2}\sum_{k=\theta}^{\theta+L-1} |r(k)|^2 + |r(k+N)|^2$$

reflects the redundancy in the received signal due to the cyclic prefix and

$$\Lambda_p(\theta) = (1+\rho)\mathrm{Re}\left\{\sum_k r^*(k)m(k-\theta)\right\} - \rho\mathrm{Re}\left\{\sum_{k=\theta}^{\theta+L-1}(r(k)+r(k+N))^* m(k-\theta)\right\}$$

reflects the information carried by the pilot symbols, with ρ according to (5).

The estimator (7) weights the information carried by the signal's redundancy and the pilot information depending on the value of ρ, which is based on the SNR and the number of pilots. Fig. 1 illustrates this for an example OFDM system with 128 subcarriers and a cyclic prefix of 16 samples. Every fifth subcarrier contains a pilot symbol. For an SNR of 8 dB, Fig. 1 shows the contributions $\Lambda_{cp}(\theta)$, $\Lambda_p(\theta)$ and the log-likelihood function $\Lambda(\theta)$. The function $\Lambda_{cp}(\theta)$ essentially correlates samples spaced N samples apart, thus identifying the position of the cyclic prefix, and its contribution gives an unambiguous but coarse estimate. While the function $\Lambda_p(\theta)$ contains a filter matched to the pilots and its contribution has very distinct peaks, by itself it would yield an ambiguous estimate because the evenly spaced pilots result in many correlation peaks. Together, however, the properly weighted contributions yield an unambiguous and distinct peak in the log-likelihood function. The peaks of $\Lambda_p(\theta)$ fine-tune the coarse estimate based on $\Lambda_{cp}(\theta)$.

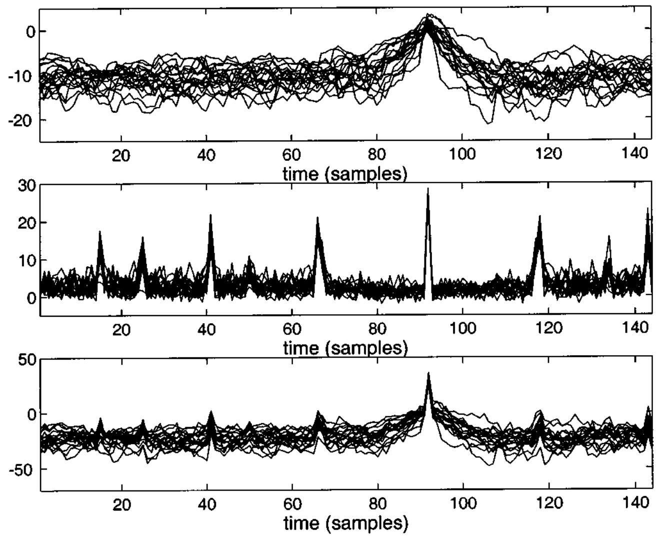

Fig. 1. The ML estimator statistics in an AWGN channel. Contribution from the cyclic prefix $\Lambda_{cp}(\theta)$ (top), contribution from the pilots $\Lambda_p(\theta)$ (middle), and the resulting log-likelihood function $\Lambda(\theta)$ (bottom). One OFDM symbol $(N+L)$ is 144 samples and the SNR is 8 dB.

For a large SNR ($\rho \approx 1$), the estimate is mainly based on the cyclic prefix redundancy, whereas for a low SNR ($\rho \approx 0$) the estimate relies more on the pilot symbols. If the transmitted signal does not contain any pilot symbols, then $N_p = 0, m(\cdot) = 0$, and $\rho = \mathrm{SNR}/\mathrm{SNR}+1$. In this case, the ML estimator (8) reduces to the estimator in [4], which only exploits the cyclic prefix redundancy.

A. A Robust Estimator

Most communication systems experience some fine error in the estimate of the carrier frequency [7]. That is, the received signal $r(k)$ will have the form

$$r(k) = (s(k-\theta)+m(k-\theta))\, e^{j2\pi\epsilon k/N} + n(k) \tag{9}$$

where $|\epsilon| < 0.5$. Frequency offsets or channel phase variations will cause the phases of the complex-valued moving sums in (8) to appear in a random manner and result in an increased variance in the estimator. Therefore, we propose a robust estimator based on the ML estimator (7) which we modify in two ways. First, as in [4], we take the absolute value of the terms in the log-likelihood function instead of the real part, thus preserving the constructive contribution of the peaks in $\Lambda(\theta)$. Secondly, since

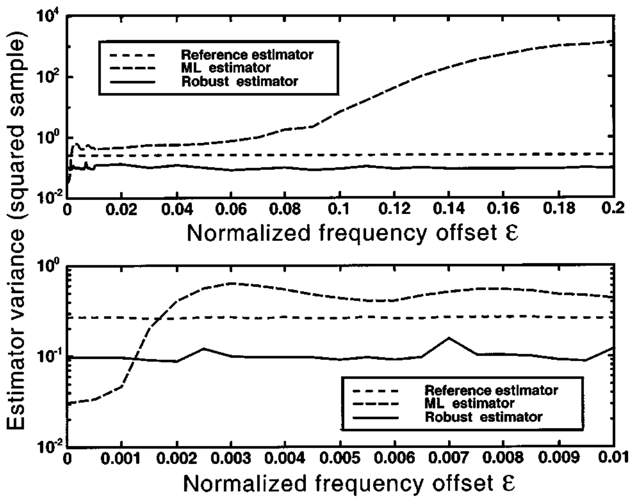

Fig. 2. Sensitivity to frequency offsets in an AWGN channel. $\mathrm{SNR} = 10$ dB, $N = 128, L = 16$, 1 pilot every 32nd subcarrier (total: 4 pilot subcarriers), and $\widetilde{\mathrm{SNR}} = 5$ dB. The reference estimator [4] (coarsely dashed), the ML estimator (7) (fine dashed line), and the robust estimator (10) (solid line). The bottom figure is a magnification of a part of the top figure.

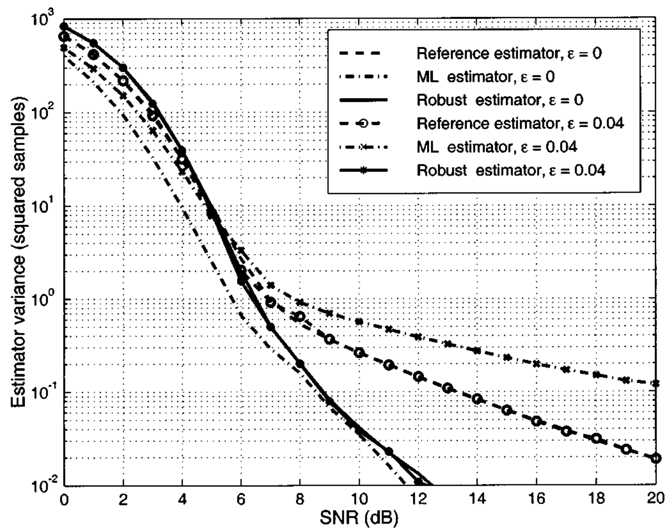

Fig. 3. The performance in an AWGN channel, with and without a 4% frequency offset, $N = 128, L = 16$, 1 pilot every 32nd subcarrier (total: 4 pilot subcarriers), and $\widetilde{\mathrm{SNR}} = 9$ dB. Note that the curves for the robust and reference estimator are almost on top of each other.

the SNR may not be known at the receiver, we design a generic estimator assuming a fixed SNR, which we denote with $\widetilde{\mathrm{SNR}}$. Our robust estimator then becomes

$$\widehat{\theta}_{\mathrm{ML}} = \arg\max_{\theta} \left\{ \tilde{\rho}\tilde{\Lambda}_{cp}(\theta) + (1-\tilde{\rho})\tilde{\Lambda}_{p}(\theta) \right\} \tag{10}$$

where $\tilde{\rho}$ is a fixed design parameter $\tilde{\rho} = \alpha\widetilde{\mathrm{SNR}}/\alpha\widetilde{\mathrm{SNR}}+1$, and

$$\tilde{\Lambda}_{cp}(\theta) = \left| \sum_{k=\theta}^{\theta+L-1} r^*(k)r(k+N) \right| - \frac{\tilde{\rho}}{2} \sum_{k=\theta}^{\theta+L-1} |r(k)|^2 + |r(k+N)|^2$$

$$\begin{aligned}\tilde{\Lambda}_{p}(\theta) =& (1+\tilde{\rho}) \left| \sum_{k} r^*(k)m(k-\theta) \right| \\ & - \tilde{\rho} \left| \sum_{k=\theta}^{\theta+L-1} (r(k)+r(k+N))^* \, m(k-\theta) \right|.\end{aligned}$$

The estimator differs from the estimator in (7) only in the absolute values and the generic choice of the weighting factor, $\tilde{\rho}$. Note that the robust estimator is not ML, although it has borrowed ideas from ML estimators (see (7) and [4]).

IV. Simulations

We use simulations to evaluate the estimators' performance, both in the AWGN channel (one-tap channel with Gaussian noise) and in a dispersive channel, showing the variance of the estimates and uncoded symbol error rate. In all simulations, we use the estimator from [4], which is based only on the cyclic prefix part Λ_{cp} of (8), as our reference estimator. The simulations are based on $N = 128$ subcarriers, the design SNR, $\widetilde{\mathrm{SNR}} = 9$ dB, and a simulation length of 432 000 OFDM symbols. For the AWGN channel, we have used a cyclic prefix of $L = 16$ samples, 1 pilot every 32nd subcarrier (4 pilot subcarriers in total), and for the dispersive channel we have 1 pilot every fifth subcarrier (25 pilot subcarriers in total).

Fig. 2 shows how a carrier frequency offset (normalized to the subcarrier spacing) ϵ affects the performance of the estimators, in an AWGN channel at $\mathrm{SNR} = 10$ dB. Even for small carrier frequency offsets, the performance of the ML estimator (dashed line in Fig. 2) decreases significantly. The ML estimator is so sensitive to this distortion that it is of little value in many practical systems. For example, the estimator from [4] (reference estimator, dash-dotted line) performs better under frequency offsets larger than 0.2%. The estimator (10) is robust against frequency offsets. In Fig. 2 (solid line), we see that its variance is almost constant, and lower compared to the other estimators, for a large range of frequency offsets.

Fig. 3 shows the variance of the estimators in an AWGN channel with two different fractional carrier frequency offsets. With no frequency offset ($\epsilon = 0$), the ML estimator and the *ad hoc* estimator, using pilots, have superior performance compared to the reference estimator. As expected, in this environment, the ML estimator performs best, but the robust estimator has only a small performance loss. For SNR values larger than 5 dB, the robust estimator is within 1 dB of the ML estimator with $\epsilon = 0$.

When applying the estimators in an environment with a frequency offset, the performance of the ML estimator decreases significantly, as also previously seen in Fig. 2, while the proposed robust estimator does not. For SNR values above 7 dB, the proposed estimator is substantially better then the other estimators for a frequency offsets of $\epsilon = 4\%$.

Though the robust estimator was derived assuming an AWGN channel, it will be used in practice in a dispersive channel. The symbol error rate of a system employing the estimators in a dispersive channel is shown in Fig. 4. The system uses a 4-PSK signal constellation and has the same parameters as in the AWGN simulations but with a cyclic prefix of $L = 8$ samples and 1 pilot every 5th subcarrier. The channel is exponentially decaying with an rms value of 5 samples and a length of 8 samples and is fading according to Jakes' model [9], it is quasi-static so that it is constant over each symbol. We assume perfect channel knowledge and perfect compensation

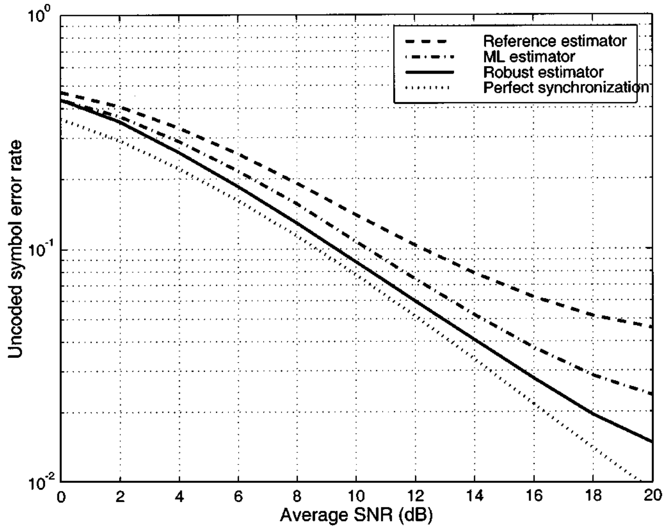

Fig. 4. Performance of the 4-PSK system in a dispersive channel, $N = 128$, $L = 8$, 1 pilot every 5th subcarrier, and $\widetilde{\mathrm{SNR}} = 9$ dB. The channel consists of 8 taps (independently fading according to Jakes' model) with an exponentially decaying power-delay profile and rms value of 5 taps.

for the phase rotations of the signal constellation due to time offsets. Thus, we isolate the effect of synchronization errors from possible performance loss due to nonideal channel estimation. To see the effect of the dispersive channel, we choose $\epsilon = 0$ (no frequency offset). The performance loss with respect to perfect synchronization shown in Fig. 4 is due to ISI and ICI caused by synchronization errors.

We see that the robust estimator now is superior to the others. In this simulation, the cyclic prefix and the channel impulse response have the same length. Under these tight synchronization requirements, the robust estimator has a 0.6-dB loss compared with a perfectly synchronized system at a 10-dB working SNR. For the ML estimator and the reference estimator, this loss is 1.7 dB and 3.5 dB, respectively.

V. Discussion and Conclusion

As seen in Fig. 1, the $\Lambda_p(\theta)$ is an ambiguous function with periodic peaks when the pilots are evenly spaced. We have observed (in simulations not shown in this paper) that the peaks surrounding the symbol start can be lowered by not having the pilots evenly spaced. Therefore, the pilot pattern is an interesting design parameter and system design could benefit from taking synchronization aspects into account when designing the channel estimation pilot pattern.

We draw two conclusions from our investigation. First, it is possible to extend the analytic techniques earlier employed in [4] to derive an ML time offset estimator for coherent OFDM systems. Secondly, it is possible to improve the synchronization performance when also taking the channel estimation pilots into account.

Appendix
Log-Likelihood Function (6)

The log-likelihood function can be written as [4]

$$\Lambda(\theta) = \sum_{k=\theta}^{\theta+L-1} \log\left(\frac{f(r(k), r(k+N))}{f(r(k))\, f(r(k+N))}\right) + \sum_{k} \log f(r(k)) \tag{11}$$

where $f(\cdot)$ denotes the probability density function of the variables in its argument. The two-dimensional density $f(r(k), r(k+N))$ is given in (12), shown at the bottom of the page, where the constant ρ is as defined in (5). The one-dimensional density $f(r(k))$ in (11) is given by

$$f(r(k)) = \frac{1}{\pi(\sigma_s^2 + \sigma_w^2)} \exp\left(-\frac{|r(k) - m(k-\theta)|^2}{(\sigma_s^2 + \sigma_w^2)}\right). \tag{13}$$

In three steps, the first term in (11) is now calculated. First, substitution of (12) and (13) yields a sum of a squared form. In the second step, we expand and simplify this form by noting that

$$m(k-\theta) = m(k+N-\theta), \quad k \in [\theta, \theta+L-1] \tag{14}$$

due to the cyclic prefix. In the third step, we ignore the terms $\sum_{k=\theta}^{\theta+L-1} |m(k-\theta)|^2$ and $\sum_{k=\theta}^{\theta+L-1} \log(1-\rho^2)$ because they are constants and are not relevant to the maximizing argument of the log-likelihood function. The first term is now proportional to

$$\begin{aligned} &\sum_{k=\theta}^{\theta+L-1} \log\left(\frac{f(r(k), r(k+N))}{f(r(k))\, f(r(k+N))}\right) \\ &\quad \propto \mathrm{Re}\left\{\sum_{k=\theta}^{\theta+L-1} r(k) r^*(k+N)\right\} \\ &\quad\quad - \frac{\rho}{2} \sum_{k=\theta}^{\theta+L-1} \left(|r(k)|^2 + |r(k+N)|^2\right) \\ &\quad\quad - (1-\rho)\,\mathrm{Re}\left\{\sum_{k=\theta}^{\theta+L-1} [r^*(k) + r^*(k+N)]\, m(k-\theta)\right\}. \end{aligned} \tag{15}$$

Similarly, the second term in (11) can be calculated, again noting that some terms in the expansion are independent of θ and do not affect the maximizing argument of the log-likelihood

$$\begin{aligned} &f(r(k), r(k+N)) \\ &= \frac{\exp\left(-\frac{|r(k)-m(k-\theta)|^2 - 2\rho \cdot \mathrm{Re}\{(r(k)-m(k-\theta))(r(k+N)-m(k+N-\theta))^*\} + |r(k+N)-m(k+N-\theta)|^2}{(\sigma_s^2+\sigma_w^2)(1-\rho^2)}\right)}{\pi^2(\sigma_s^2+\sigma_w^2)^2(1-\rho^2)} \end{aligned} \tag{12}$$

function. From these calculations, the log-likelihood function consists of the three terms in (15) and the additional term

$$\frac{1-\rho^2}{\rho}\mathrm{Re}\left\{\sum_k r(k)m^*(k-\theta)\right\}. \tag{16}$$

Expression (8) now follows readily.

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