Symmetrical Components in the Time Domain and Their Application to Power Network Calculations

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***Abstract*—Although the Symmetrical Component Transformation has existed for 80 years, its application in the time-dependent form is practically restricted to the electric-machine theory. In the Power Systems field one uses the transformation applied to gteady-state ginusoidal phasors in a nonunitary form for fault calculations. For time-domain calculations the real equivalat, 0, α, β, is preferred, usually extended to 0, d, q-components. In network calculations, however, the application of time-dependent symmetrical components makes sense, since many net-component parameters are already available in this form. In this paper a short historical overview of the symmetrical-component transformation and the application of unitary and orthogonal transformations are presented. From these general transformations logic choices for base quantities necessary in per unit calculations will be derived. The relations between real and complex transformations, steady-state phasors and well-known sequence networks are given and illustrated through the use of some examples with asymmetrical faults.**

***Index Terms*—Power-invariant Complex and Real Transformations, Time Domain, Asymmetries, Complex Phasors, Instantaneous and Average Power, Per Unit calculation.**

I. Introduction

The application of Symmetrical Components dates from 1918 when Fortescue [1] introduced them as a decomposition of complex steady-state phasors. Although they were introduced for three-phase phasors of sinusoidal time functions, they are the basis for the transformation of arbitrary instantaneous variables. The first application of the Symmetrical Components to time-dependent variables was introduced by Lyon [2]. He no longer called it a decomposition but a transformation and used the transformation matrix that follows from the decomposition introduced by Fortescue:

$$\boldsymbol{S} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^{-1} & a^{-2} \\ 1 & a^{-2} & a^{-1} \end{bmatrix} \qquad \text{where } a = \exp\left[j\,\frac{2\pi}{3}\right]$$

j is the complex number [0, 1].

White and Woodson [3] extended this transformation to m-n winding machines and used the unitary form, which, from the point of view of electrical machines, offers the advantage that power and torque need no back transformation, since a unitary transformation is power invariant.

In [4], where transfomtations and per-unit (pu) systems are discussed in detail, it is concluded that the use of nonunitary transformations is preferable in electrical-machines theory. This thesis is still cited by many authors [5]–[7] today and its principles extended to the calculations of power systems. The main advantage is that, after transformation, the physical structure of machines, e.g., the winding turns and flux relationships, are unchanged. However, in network calculations the intemal relations of magnetic-field quantities and currents in the separate machines are of less importance. The proper relations between terminal voltages, currents and power may prevail. To maintain these proper relations between power, voltage and current, unitary and orthogonal transformations are used in this paper. Line, transformer, static load and asynchronous machine load models are implemented by their symmetrical components, while synchronous machines are represented in d, q-coordinates. To link rotating machines onto the grid, interfaces are used that bridge the difference in base quantities and usual pu parameters. The use of interfaces makes it possible to employ machine models with their usual (pu) data delivered by the manufacturer.

In the unitary fomn, the symmetrical-component transformation matrix is:

$$\boldsymbol{S} = \frac{1}{\sqrt{3}} \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^{-1} & a^{-2} \\ 1 & a^{-2} & a^{-1} \end{bmatrix} \qquad \text{with } \boldsymbol{S^{-1}} = \boldsymbol{S^{*T}} \tag{1}$$

The transformation is applied to both voltages and currents. The new variables $\mathbf{U}'$ and $\mathbf{I}'$ are connected to the old ones by:

$$\boldsymbol{U} = \boldsymbol{SU}' \quad \text{and} \quad \boldsymbol{I} = \boldsymbol{SI}' \tag{2}$$

In electrical machine theory, where the transformation was applied to both 3-phase and m-phase systems, many applications can be found [8]–[10]. [8], [9] show, for example, the remarkable property of this transformation in decomposing higher space harmonics into special groups.

II. Transformation of Time-Dependent Signals

Although the transformation can be applied to arbitrary time functions we start with the application of the symmetrical component transformation on a single-frequency sinusoidal function. Let $\mathbf{U}$ be a general asymmetrical three-phase voltage:

$$\boldsymbol{U} = \begin{bmatrix} u_a \\ u_b \\ u_c \end{bmatrix} = \begin{bmatrix} U_a\sqrt{2}\,\cos(\omega t + \alpha_a) \\ U_b\sqrt{2}\,\cos(\omega t + \alpha_b) \\ U_c\sqrt{2}\,\cos(\omega t + \alpha_c) \end{bmatrix} \tag{3}$$

where u denotes the instantaneous value and U the rms value of the phase voltage. Before it is transformed into symmetrical

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components (3) is written as the sum of 2 complex conjugated terms:

$$\boldsymbol{U}=\frac{\sqrt{2}}{2}\begin{bmatrix}\underline{U}_a e^{j\omega t}+\underline{U}_a^* e^{-j\omega t}\\ \underline{U}_b e^{j\omega t}+\underline{U}_b^* e^{-j\omega t}\\ \underline{U}_c e^{j\omega t}+\underline{U}_c^* e^{-j\omega t}\end{bmatrix}\quad \text{with}\quad \begin{aligned}\underline{U}_a&=U_a e^{j\alpha_a}\\ \underline{U}_b&=U_b e^{j\alpha_b}\\ \underline{U}_c&=U_c e^{j\alpha_c}\end{aligned} \tag{4}$$

$\underline{U}$ denotes the phasor of u. Transformation into symmetrical components by using $\mathbf{U}'=\mathbf{S}^{-1}\mathbf{U}$ yields:

$$\boldsymbol{U}'=\begin{bmatrix}u^0\\ u^+\\ u_-\end{bmatrix}=\frac{\sqrt{2}}{2}\begin{bmatrix}\underline{U}_0 e^{j\omega t}+\underline{U}_0^* e^{-j\omega t}\\ \underline{U}_1 e^{j\omega t}+\underline{U}_2^* e^{-j\omega t}\\ \underline{U}_2 e^{j\omega t}+\underline{U}_1^* e^{-j\omega t}\end{bmatrix} \tag{5}$$

where the phasors

$$\begin{aligned}\underline{U}_0&=\frac{1}{\sqrt{3}}\left(\underline{U}_a+\underline{U}_b+\underline{U}_c\right)\\ \underline{U}_1&=\frac{1}{\sqrt{3}}\left(\underline{U}_a+a\underline{U}_b+a^2\underline{U}_c\right)\\ \underline{U}_2&=\frac{1}{\sqrt{3}}\left(\underline{U}_a+a^2\underline{U}_b+a\underline{U}_c\right)\end{aligned} \tag{6}$$

In literature the time-dependent components are usually expressed as u_0, u^+, u^-, [3], [8], [9], while the steady-state phasors are written as $\underline{U}_0$, $\underline{U}_1$, $\underline{U}_2$, [5]–[7]. From the result in (5) and (6) the following conclusions can be drawn:

- The zero-sequence component u^0 is always real.
- The negative-sequence component u^- is the complex conjugate of the positive-sequence component u^+ and is therefore superfluous. However, the negative component phasor $\underline{U}_2$ is completely independent of $\underline{U}_1$.
- Starting with three real variables u_a, u_b, u_c, we obtain after transformation three new real variables: u_0, $\Re[u^+]$ and $\Im[u^+]$.
- The steady-state sequence-component phasors are already incorporated in the time function u^+, which contains all the information of the phasors $\underline{U}_1$ and $\underline{U}_2$.
- The same transformation also holds for phasors.

For a symmetrical three-phase voltage, where

$$\begin{aligned}\underline{U}_a&=Ue^{j\alpha}=\underline{U}\\ \underline{U}_b&=Ue^{j(\alpha-\frac{2\pi}{3})}=a^{-1}\underline{U}\quad \text{with}\quad \underline{U}=Ue^{j\alpha}\\ \underline{U}_c&=Ue^{j(\alpha-\frac{4\pi}{3})}=a^{-2}\underline{U}\end{aligned} \tag{7}$$

we obtain

$$\boldsymbol{U}'=\begin{bmatrix}u^0\\ u^+\\ u^-\end{bmatrix}=\frac{\sqrt{2}}{2}\begin{bmatrix}0\\ \underline{U}_1 e^{j\omega t}\\ \underline{U}_1^* e^{-j\omega t}\end{bmatrix};\quad \begin{aligned}\underline{U}_0&=0\\ \underline{U}_1&=\sqrt{3}\underline{U}\\ \underline{U}_2&=0\end{aligned} \tag{8}$$

Eq. (8) clearly shows that only positive phasors exist for symmetrical three-phase signals with phase sequence a-b-c. When the phase sequence is a-c-b only negative sequence phasors exist. Of course, the expressions (3)–(8) also hold for currents, where U is exchanged by I and u by i.

A widely used transformation for time-dependent signals is the Park transformation $\mathbf{P}$ [3], [7], [11], being the product of a phase transformation $\mathbf{A}$, the orthogonal Clarke transformation [12], and an angle transformation $\mathbf{B}$ (see Fig. 1).

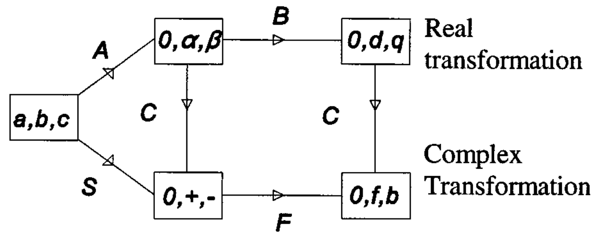

Fig. 1. Overview of transformations.

The transformation matrices $\mathbf{A}$ and $\mathbf{B}$ are:

$$\boldsymbol{A}=\sqrt{\frac{2}{3}}\begin{bmatrix}\frac{1}{2}\sqrt{2} & 1 & 0\\ \frac{1}{2}\sqrt{2} & -\frac{1}{2} & \frac{1}{2}\sqrt{3}\\ \frac{1}{2}\sqrt{2} & -\frac{1}{2} & -\frac{1}{2}\sqrt{3}\end{bmatrix};$$

$$\boldsymbol{B}=\begin{bmatrix}1 & 0 & 0\\ 0 & \cos\theta & \sin\theta\\ 0 & -\sin\theta & \cos\theta\end{bmatrix}$$

$$\text{with } \boldsymbol{A^{-1}}=\boldsymbol{A^T} \text{ and } \boldsymbol{B^{-1}}=\boldsymbol{B^T}$$

The orthogonal Park transformation $\mathbf{P}=\mathbf{AB}$ is:

$$\boldsymbol{P}=\frac{2}{\sqrt{3}}\begin{bmatrix}\frac{1}{2}\sqrt{2} & \cos(\theta) & -\sin(\theta)\\ \frac{1}{2}\sqrt{2} & \cos\left(\theta-\frac{2\pi}{3}\right) & -\sin\left(\theta-\frac{2\pi}{3}\right)\\ \frac{1}{2}\sqrt{2} & \cos\left(\theta-\frac{4\pi}{3}\right) & -\sin\left(\theta-\frac{4\pi}{3}\right)\end{bmatrix}$$

$$\text{with } \boldsymbol{P^{-1}}=\boldsymbol{P^T} \tag{9}$$

θ is an arbitrary time-dependent angle. In electric-machine equations θ is usually chosen as the electric rotor-position angle. The relation between a, b, c and 0, d, q-variables is given by:

$$\boldsymbol{U_{a,b,c}}=\boldsymbol{PU}_{0,d,q}\quad \text{and}\quad \boldsymbol{I_{a,b,c}}=\boldsymbol{PI}_{0,d,q} \tag{10}$$

This transformation is the real equivalent of a complex transformation, which is the product of the symmetrical component transformation $\mathbf{S}$ and the forward–backward transformation $\mathbf{F}$:

$$\boldsymbol{F}=\begin{bmatrix}1 & 0 & 0\\ 0 & e^{j\theta} & 0\\ 0 & 0 & e^{-j\theta}\end{bmatrix}\qquad \text{with } \boldsymbol{F^{-1}}=\boldsymbol{F^{*T}} \tag{11}$$

and related to the Park transformation, see Fig. 1, through:

$$\boldsymbol{SF}=\boldsymbol{ABC}=\boldsymbol{PC} \tag{12}$$

where the complex transformation $\mathbf{C}$ is:

$$\boldsymbol{C}=\frac{1}{\sqrt{2}}\begin{bmatrix}\sqrt{2} & 0 & 0\\ 0 & 1 & 1\\ 0 & -j & j\end{bmatrix}\qquad \text{with } \boldsymbol{C^{-1}}=\boldsymbol{C^{*T}} \tag{13}$$

In Fig. 1 the relations between the a, b, c and the other transformed variables are depicted. These relations enable the construction of an interface between the network in $0, +, -$ variables and synchronous generators in $0, d, q$ components.

Applying the Park transformation to a symmetric voltage (3), where $U_a = U_b = U_c = U$ and $\alpha_a = \alpha_b - \alpha_c = 0$, yields:

$$u_0 = 0, \quad u_d = U\sqrt{3}\cos(\omega t - \theta), \quad u_q = U\sqrt{3}\sin(\omega t - \theta)$$
$$\text{with } \theta = \theta_0 + \omega t \text{ and } \delta = \frac{\pi}{2} + \theta_0$$
$$\text{we obtain } u_d = U\sqrt{3}\sin\delta; \quad u_q = U\sqrt{3}\cos\delta \tag{14}$$

where δ is the rotor angle of the concerned synchronous generator. After the Clarke transformation the new variables become:

$$u_0 = 0, \qquad u_\alpha = U\sqrt{3}\cos\omega t, \qquad u_\beta = U\sqrt{3}\sin\omega t$$

Since the angle transformation $\mathbf{B}$ has no influence on the amplitudes, the α, β and d-q-variables have the same amplitudes.

III. Instantaneous and Average Power

The general expression for instantaneous power related to an arbitrary voltage and current is:

$$p(t) = \boldsymbol{U}^{*T}\boldsymbol{I} = u_a i_a + u_b i_b + u_c i_c \tag{15}$$

and after transformation into symmetrical components:

$$\begin{aligned} p(t) &= \boldsymbol{U}'^{*T}\boldsymbol{I}' \\ &= u^{0^*} i^0 + u^{+*} i^+ + u^{-*} i^- \\ &= u^{0*} i^0 + 2\,\Re\left[u^{+*} i^+\right] \end{aligned} \tag{16}$$

or with (5) expressed in phasors:

$$\begin{aligned} p(t) = \Re\big[&\underline{U}_0^* \underline{I}_0 + \underline{U}_1^* \underline{I}_1 + \underline{U}_2^* \underline{I}_2 \\ &+ (\underline{U}_0 \underline{I}_0 + \underline{U}_2 \underline{I}_1 + \underline{U}_1 \underline{I}_2)\, e^{2j\omega t}\big] \end{aligned} \tag{17}$$

The average power, P, is the time-independent part in (17) and can directly be calculated from the phasors.

$$P = \Re[\underline{U}_0^* \underline{I}_0 + \underline{U}_1^* \underline{I}_1 + \underline{U}_2^* \underline{I}_2] \tag{18}$$

For a symmetrical three-phase voltage and current, see (8), where $\underline{U} = U$ and $\underline{I} = Ie^{j\phi}$, the instantaneous power becomes:

$$p(t) = \Re[\underline{U}_1^* \underline{I}_1] = 3UI\cos\phi \tag{19}$$

The instantaneous power is the same as the one that would be obtained from the a, b, c-quantities since the transformation is power invariant. For a steady-state symmetrical case the instantaneous power $p(t)$ is identical to the average power P. The expressions for the instantaneous power in 0, α, β and 0, d, q components are:

$$\begin{aligned} p(t) &= \boldsymbol{U}^T\boldsymbol{I} = u_0 i_0 + u_\alpha i_\alpha + u_\beta i_\beta \\ p(t) &= \boldsymbol{U}^T\boldsymbol{I} = u_0 i_0 + u_d i_d + u_q i_q \end{aligned} \tag{20}$$

which yields, for a symmetrical three-phase voltage and current, the same result as calculated in (19).

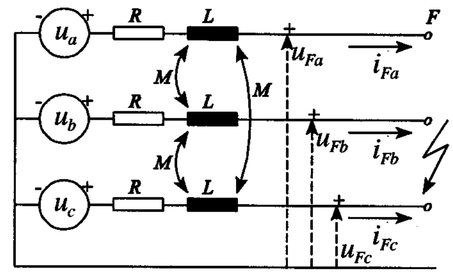

Fig. 2. Simple faulted network.

Example 1: Fig. 2 depicts an arbitrary three-phase voltage source connected to a symmetric three-phase line, which is represented by its series resistance and inductance only. The resistance and inductance matrices are given by:

$$\boldsymbol{R} = \begin{bmatrix} R & 0 & 0 \\ 0 & R & 0 \\ 0 & 0 & R \end{bmatrix}; \qquad \boldsymbol{L} = \begin{bmatrix} L & M & M \\ M & L & M \\ M & M & L \end{bmatrix} \tag{21}$$

At the terminals, F, of the line in Fig. 2, a disturbance in the form of a short circuit between the phases b and c is applied.

The voltage equations of the undisturbed network are:

$$\boldsymbol{U} = \boldsymbol{RI} + \boldsymbol{L}\frac{d}{dt}(\boldsymbol{I}) + \boldsymbol{U_F} \tag{22}$$

Applying the symmetrical-component transformation (1) yields respectively:

$$\begin{aligned} \boldsymbol{SU}' &= \boldsymbol{RS}\,\boldsymbol{I}' + \boldsymbol{L}\frac{d}{dt}(\boldsymbol{SI}') + \boldsymbol{SU}'_{\boldsymbol{F}} \\ \boldsymbol{U}' &= \boldsymbol{S}^{-1}\boldsymbol{RS}\,\boldsymbol{I}' + \boldsymbol{S}^{-1}\boldsymbol{LS}\frac{d}{dt}(\boldsymbol{I}') + \boldsymbol{U}'_{\boldsymbol{F}} \\ \boldsymbol{U}' &= \boldsymbol{R}'\boldsymbol{I}' + \boldsymbol{L}'\frac{d}{dt}(\boldsymbol{I}') + \boldsymbol{U}'_{\boldsymbol{F}} \end{aligned} \tag{23}$$

where

$$\boldsymbol{R}' = \boldsymbol{R}; \qquad \boldsymbol{L}' = \boldsymbol{S}^{-1}\boldsymbol{LS} = \mathrm{diag}[L_0 \;\; L_1 \;\; L_2]$$

with

$$L_0 = L + 2M; \qquad L_1 = L_2 = L - M \tag{24}$$

Because the transformation $\mathbf{S}$ is time independent it is allowed that $\mathbf{S}$ is placed in front of the d/dt operator in the term $d/dt(\mathbf{SI})$. The final equation has, in this case, the same form as the original one (22). As the line is symmetrical, the transformed voltage equations are disconnected, which means that the 0 and $+$ components can be solved separately.

Note that the transformed parameters are the same as those obtained from the nonunitary transformation.

In components (23) can be written as:

$$\begin{aligned} u_0 &= Ri^0 + L_0\frac{di^0}{dt} + u_F^0 \\ u^+ &= Ri^+ + L_1\frac{di^+}{dt} + u_F^+ \\ \Big(u^- &= Ri^- + L_1\frac{di^-}{dt} + u_F^-\Big) \end{aligned} \tag{25}$$

Note that the third equation is superfluous!

The two-phase short circuit can be introduced by the following constraints at place F:

$$i_{Fa} = 0; \qquad i_{Fb} + i_{Fc} = 0; \qquad u_{Fb} = u_{Fc}$$

After transformation we obtain for the instantaneous variables:

$$\begin{aligned} i_F^0 = 0; \qquad i_F^+ + i_F^- = 0 \quad \text{or} \quad \Re[i_F^+] = 0 \\ u_F^+ = u_F^- \text{ or } \Im[u_F^+] = 0 \end{aligned} \tag{26}$$

As no other load is connected to the network it holds that

$$\boldsymbol{I}'_F = \boldsymbol{I}'$$

Combining (25) and (26) yields the voltage equations for the faulted case:

$$\begin{aligned} u_F^0 = u_0; \qquad u_F^+ &= \Re[u^+] \\ Ri_F^+ + L_1 \frac{di_F^+}{dt} &= \Im[u^+] \end{aligned} \tag{27}$$

For a short circuit between phases b and c, the equation for the positive-sequence component splits up into a real part which is not disturbed, and an imaginary part that faces the short circuit. These separation into a disturbed and not disturbed part appears for all kinds of asymmetric faults.

The steady-state phasor solution can be obtained by substitution of the general expressions (5) in (27), but whereas (27) is very suitable for numerical calculations, the determination of the steady-state solution by hand is easier when using the equations in (25) together with the constraints in (26). We obtain:

$$\begin{aligned} u_F^0 &= u^0 \\ u^+ - u^- &= 2Ri_F^+ + 2L_1 \frac{di_F^+}{dt} \end{aligned} \tag{28}$$

When the voltages and the current as depicted in their general form in (5) are substituted into (28) it yields, after splitting up into frequencies:

$$\begin{aligned} \underline{U}_{F0} &= \underline{U}_0 \\ \underline{U}_1 - \underline{U}_2 &= 2(R + j\omega L_1)\underline{I}_{F1} \\ \underline{U}_1 - \underline{U}_2 &= -2(R + j\omega L_1)\underline{I}_{F2} \Rightarrow \underline{I}_{F2} = -\underline{U}_{F1} \end{aligned} \tag{29}$$

which is the same result as would be obtained with a steady-state phasor approach. The sequence networks for the transformed time and phasor solutions are depicted in Fig. 3.

For a simple calculation of the instantaneous and average power, the inductance L in the circuit is ignored while the source voltage is supposed to be symmetrical as in (7)–(8) with $\alpha = 0$. From (28) and (29) the time-domain current and current phasor are calculated respectively as:

$$i_F^+ = \frac{u^+ - u^-}{2R}; \qquad \underline{I}_{F1} = \frac{\underline{U}_1}{2R}$$

The time-domain current is imaginary as also followed from (26). With the relations for a symmetric source voltage:

$$u^+ = \sqrt{\frac{3}{2}}\, U e^{j\omega t}; \qquad \underline{U}_1 = \sqrt{3}U$$

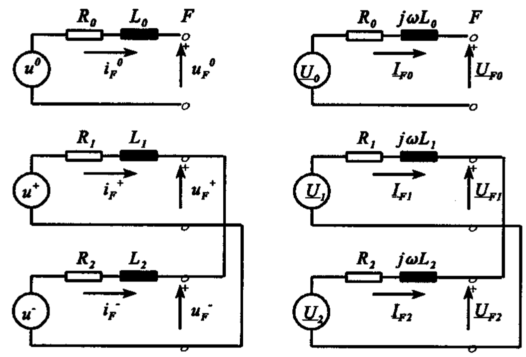

Fig. 3. Time-domain and phasor-sequence networks.

the instantaneous power is:

$$\begin{aligned} p(t) &= 2\,\Re\left[u^{+^*} i_F^+\right] \\ &= \frac{3}{2} U^2\, \Re\left[e^{-j\omega t}\, \frac{e^{j\omega t} - e^{-j\omega t}}{R}\right] \\ &= \frac{3U^2}{2R}\,[1 - \cos(2\omega t)] \end{aligned}$$

while the average power calculated from the phasors yields:

$$P = \Re[\underline{U}_1^* \underline{I}_1] = \frac{\underline{U}_1^* \underline{U}_1}{2R} = \frac{3U^2}{2R}$$

which is the constant part of the instantaneous power.

IV. Per-Unit Calculations

The use of per-unit quantities offers several well-know advantages which are described in [4]. The choice of base units is arbitrary. However, in calculations, it is preferable to choose the base units in such a way that simple relations exist between pu and physical units; for example, 1 pu corresponds to the rated value. This requirement can be satisfied for single components. For networks containing lots of components with various power rates it is impossible to meet this constraint. One base power is chosen for the whole network, while rated base voltages are taken for circuits with different voltage levels.

In steady-state phasor calculations the rated rms values are a logical choice, while in the time domain peak values could make sense. However, in network calculations where the a, b, c-variables are transformed into positive-sequence components ($+$ or 1) and synchronous machines into d-q components, also logical choices for base quantities can be made in such a way that the pu values of the transformed variables have a simple relation with the physical values in the a, b, c-domain.

0, 1, 2 Components: After transformation the steady-state voltage and current phasors become $\underline{U}_1 = \sqrt{3}\underline{U}$ and $\underline{I}_1 = \sqrt{3}\underline{I}$, where U is the rms-phase voltage and I the rms-line current. The apparent power $S = |\underline{U}_1|\,|\underline{I}_1| = 3UI$, which is the three-phase apparent power.

In this domain it makes sense to take the voltage between lines $U\sqrt{3}$ as base voltage. With the rated three-phase base power S the base current will be $I\sqrt{3}$, where I is the rated-line current.

In this case the apparent power is 1 pu when U_1 and I_1 are 1 pu. See (18).

$$P = \Re[\underline{U}_1^* \underline{I}_1]$$

$0, \alpha, \beta$ and $0, d, q$ Components: In $0, \alpha, \beta$ and $0, d, q$ components it makes sense to choose also $U\sqrt{3}$ and $I\sqrt{3}$ as base units. In that case the power relation is fulfilled. The power is 1 pu when the voltage and current are 1 pu. See (20).

$0, +, -$ Components: In complex time-domain calculations we are not able to meet the former power relation when maintaining the proper power-voltage-current relation.

When we take the rated apparent power and

$$u^+ = \sqrt{\frac{3}{2}}\, U_{rated}$$

as base units, we obtain $S = 1$ pu, $u^+ = 1$ pu, but the rated-circuit current is $i^+ = 0.5$ pu because we need to fulfill (16):

$$p(t) = 2\,\Re[u^{+^*} i^+] \tag{30}$$

In a network simulation program, synchronous generators can be modeled in their own (factory) base units. Through an interface the generator models are connected to the network described in time-dependent symmetrical components. In the transformed network one should note, because of (30), the fact that:

A current of 0.5 pu corresponds to the ***rated-circuit current*** *belonging to the* ***system base MVA*** *and to* ***rated-terminal voltage*** *of the circuit concerned.*

The pu values for power and voltage correspond to the base unit values.

In time-domain calculations it is fruitful to introduce a time base $\tau = \omega t$ to obtain system equations which are dimensionless. This option will not be clarified further in this context.

V. Synchronous Machine Network Interface

To maintain the generator base units with the associated pu parameters provided by the factory data, it is necessary to introduce an interface between the generator and the network equations. The relation between $0, d, q$ and $o, +, -$ variables is given, see Fig. 1, by $\boldsymbol{U_{0,d,q}} = \boldsymbol{CF^{-1}U_{0,+,-}}$ which yields:

$$u_d = \frac{1}{\sqrt{2}}\left[u^+ e^{-j\theta} + u^- e^{j\theta}\right] = \sqrt{2}\,\Re\left[u^+ e^{-j\theta}\right]$$
$$u_q = \frac{1}{\sqrt{2}}\left[-ju^+ e^{-j\theta} + ju^- e^{j\theta}\right] = \sqrt{2}\,\Re\left[-ju^+ e^{-j\theta}\right]$$

and the back transformation:

$$u^+ = \frac{1}{\sqrt{2}}\,[u_d + ju_q]e^{j\theta}$$

Upon introducing the rotor angle δ, which appears in the equations of the synchronous machine, the relations become:

$$u_d = \sqrt{2}\,\Re\left[ju^+ e^{-j(\delta+\omega t)}\right]$$
$$u_q = \sqrt{2}\,\Re\left[u^+ e^{-j(\delta+\omega t)}\right]$$
$$u^+ = -\frac{j}{\sqrt{2}}\,[u_d + ju_q]e^{(\delta+\omega t)}$$

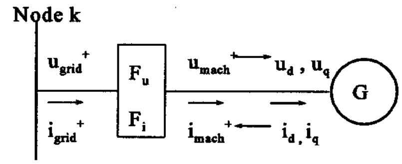

Fig. 4. Generator interface.

For currents the same relations hold. Besides the relations between the different transformed variables, the difference in base units has to be taken into account. Defining the machine and network base units with subscript "mach" and "grid" respectively, we can write:

$$u_{grid} = F_u u_{mach}: \qquad i_{grid} = F_i i_{mach}$$

where

$$F_u = \frac{U_{b,\,mach}}{U_{b,\,grid}} \quad \text{and} \quad F_i = \frac{I_{b,\,mach}}{I_{b,\,grid}} \,.\, N_{gen}$$

where N_{gen} is the number of coherent generators.

With these relations the interface can be depicted in Fig. 4.

VI. Application to a Simple Network

In the next example the use of the earlier introduced pu values and their simple relation with physical units is demonstrated.

In Fig. 5 a simple network is depicted consisting of a slack node feeding a 10MW, 10kV load and connected to five 1.125 MW, 0.4 kV synchronous generators through a transformer and a cable. At node 2 an additional load "SCLOAD" is placed to simulate an asymmetric or symmetric short circuit, but it is switched off in the load-flow calculation. Below Fig. 5 the input data and output results of the simulation program SIMNET are given.

The input data are depicted in physical values and in pu. The network components such as lines and transformers, but also static and motor loads, are modeled in symmetrical components, synchronous machines in d, q variables. For transformers the variables at the primary side are depicted. The second part gives the result of the load-flow calculation in transformed variables. The generator produces about rated power at rated MVA. $V \approx 1.06$, $I = 0.95$ pu.

$P = -0.79$ and $Q = -0.62$ pu (Base = 1,125MVA) The negative values appear since synchronous machines are modeled as loads. As voltage and power are simply related to their physical values, 1 pu voltage is rated voltage and 1 pu power is rated power, we will focus on the currents:

The generator rated current is 1.624 kA, which yields the actual current $I(\mathrm{rms}) = 0.95 * 1.624 = 1.54$ kA. As five coherent generators are connected to the bus, the total current is 7.7 kA.

The current in line 3 is 0.267 pu. As for all components except for synchronous machines it holds that 0.5 pu corresponds with the rated-circuit current, we find: $I(\mathrm{rms}) = 0.267/0.5 * 14.434 = 7.7$ kA. This is equal to the current provided by the generators.

The load current is about 0.5 pu, so it is equal to the rated-circuit current (0.577 kA), and it is also equal to the rated load current in this case.

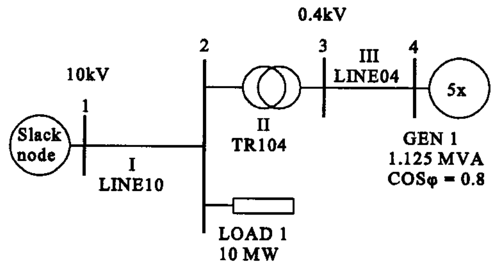

Fig. 5. One-line diagram.

```
***** PROGRAM SIMNET VERSION (98.04.09) ******* 199809241001
****** Network *****case1b.TXT*******
BASE POWER=      10.00 (MVA) NOM.FREQU.=      50.00 Hz

                    LINE DATA
LIN NOD NOD  NAME   U1(kV) U2(kV)  RK(ohm)   XK(ohm)   Icircuit (kA)
 1   1   2   LINE10 10.000 10.000  0.400E+00 0.400E+00  0.577
 2   2   3   TR104  10.000  0.400  0.200E+00 0.114E+01  0.577
 3   3   4   LINE04  0.400  0.400  0.100E-03 0.100E-03 14.434

                    LOAD DATA
LOAD NODE NAME    UL(kV)  S(MVA)   COSphi  I(kA)   R(ohm)  X(ohm)
 1     2  SCLOAD  10.000  1000.000  0.999  57.735  0.100   0.004
 2     2  LOAD1   10.000  10.000    0.999   0.577  9.990   0.447

              SYNCHRONOUS MACHINES DATA
  Gen Name  Nod  UL(kV)  Tnom  S(MVA)  Cos(phi)  I(rated) (kA)
   1 HC634K  4   0.400   5730. 1.125     0.8      1.624
 Xdh   Xqh   Xss    Xfs    XDs    XQs    rs     rf     rD     rQ     H
2.472 1.432 0.0680 0.1507 0.1941 0.1576 0.0122 0.0025 0.0617 0.1687 2.0000

                    P.U. DATA
 Ubase net components and ASYN.Motors = U(phase)*SQRT(3/2)
    Ubase Synchr. Machines = U(phase)*SQRT(3)

LINE NAME   UB(kV) IB(kA)   ZB(ohm)   RK        XK
 1   LINE10 7.071  1.41421  0.500E+01 0.800E-01 0.800E-00
 2   TR104  7.071  1.41421  0.500E+01 0.400E-01 0.228E+00
 3   LINE04 0.283 35.35534  0.800E-02 0.125E-01 0.125E-01

                  P.U. LOAD DATA
LOAD NODE NAME    UBL(kV) IBL(kA) ZBL(ohm) RL     XL     FLI   ON
 1     2  SCLOAD  7.0711  1.4142  5.0000   0.0200 0.0009 1.000 1
 2     2  LOAD1   7.0711  1.4142  5.0000   1.9980 0.0894 1.000 1

G NODE  NAME   UBL(kV) IBL(kA) ZBL(ohm) FLI    FLU    Tnom   num ON
1  4    HC634K 0.4000  2.8125  0.1422   0.3977 1.4142 0.8000  5   1


  result case1b.001    is used as input
  Netvoltage is 102.000 % , phase angle is  0.000   * PI
  LOAD  1 IS OFF   0
  STATIC LOAD  1 is:  10000.00000 %  with power factor    0.99900
  STATIC LOAD  2 is:    100.00000 %  with power factor    0.99900
SYNCHR. GEN.  1: MECH. POWER  -100.00 %,  EXITATION 315.00 %

        RESULTS LOADFLOW CALCULATION (PU)
                      REAL     IMAG.    MAGNITUDE
NODE 1  VOLTAGE=  1.02000  0.00000   1.02000
NODE 2  VOLTAGE=  1.00661 -0.03311   1.00715
NODE 3  VOLTAGE=  1.05206  0.00860   1.05209
NODE 4  VOLTAGE=  1.05673  0.00922   1.05677
                      REAL     IMAG   .MAGNITUDE
LINE 1  CURRENT=  0.29065  0.12321  0.31568  P=  0.59292  Q= -0.25135
LINE 2  CURRENT= -0.21141  0.16225  0.26650  P= -0.43636  Q= -0.31265
LINE 3  CURRENT= -0.21141  0.16225  0.26650  P= -0.44205  Q= -0.34503

  LOAD 1  V,I,P,Q:      1.00715  0.00000  0.00000  0.00000
  LOAD 2  V,I,P,Q:      1.00715  0.50358  1.01334  0.04535
  GEN 1  Vref,EFD,OMref,Tu: 0.00000 3.15000 0.00000 -0.80000
         V,I,P,Q,Delta,Deltag: 1.05677 0.94755 -0.78902 -0.61655 0.52929 0.52057

****************DYNAMIC RESPONSE**************************
 Start calculation t1=    0.00000
  End calculation t1=     0.04010
   SCLOAD     IS SWITCHED ASYM
 Start calculation t1=    0.04010
  End calculation t1=     0.10000
   SCLOAD     IS SWITCHED SYM
 Start calculation t1=    0.10000
  End calculation t1=     0.15000
   SCLOAD     IS SWITCHED OFF
 Start calculation t1=    0.15000
  End calculation t1=     0.25008
******************************************************************************
```

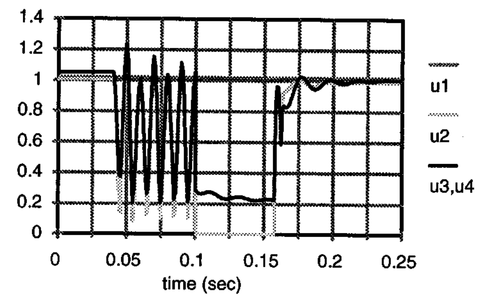

Fig. 6. Node voltages $|u^+|$ in pu.

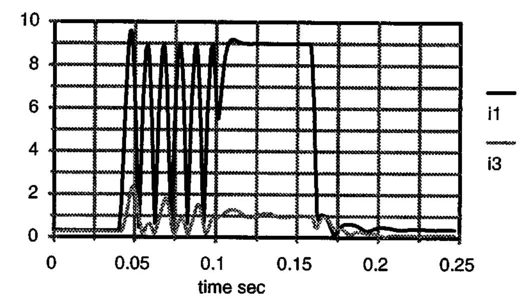

Fig. 7. Line currents $|i^+|$ in pu.

Note that we refer to the *rated-circuit current* and not to the *base current* in these relations between pu and kA.

Of course the network and load currents can also be obtained through back transformation and multiplication with their base currents.

The use of time-domain symmetrical components suggests transient calculations. The exernple in Fig. 5 is therefore used to calculate the application of a two-phase short circuit initiated at $t = 0.04$ sec at node 2 between the phases b and c, leading to an ungrounded three-phase short circuit at $t = 0.1$ sec.

At $t = 0.16$ sec the short circuit is isolated. The steady-state load-flow calculation, which already includes the 7^{th}-order generator model, is used to provide the initial values.

The node voltages and line currents are depicted in Figs. 6 and 7. In these graphs the absolute value of the u^+ and i^+ components are depicted. They represent the magnitude of the rotating phasor. See Fig. 8 where the in time rotating phasor i^+ is depicted in the complex plane. If the three-phase voltage or current are symmetrical, the curves are smooth, while oscillations appear if they are asymmetrical. The latter appears during the two-phase short circuit, $0.04 < t < 1$, where a big inverse component is present.

In Fig. 8 the real part of the phasor is about zero in this time interval and the phasor only moves along the imaginary axis, which is in accordance with example 1. The initial magnitudes of the voltage and the current curve correspond to the values obtained from the load flow. The advantage of working with complex variables is that the magnitude of the phasors can easily be depicted. They are representative for the instantaneous maximal value of the three-phase currents or voltages and in steady-state and symmetric conditions they are constant, which yields a perfect circle in the complex plane.

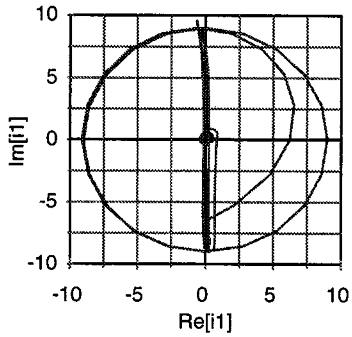

Fig. 8. Time phasor i_1^+ in pu.

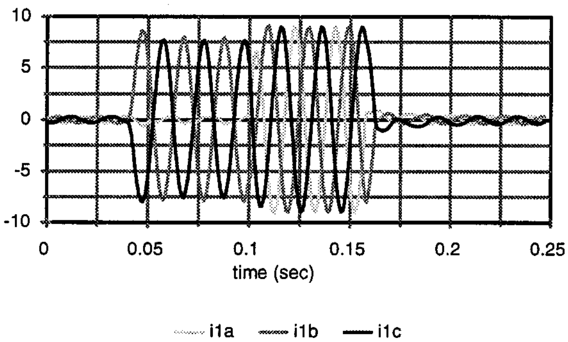

Fig. 9. Line current 1 in pu.

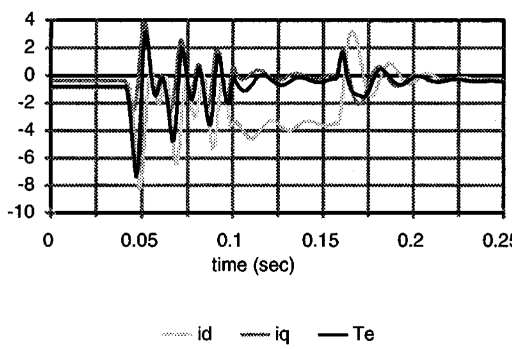

Fig. 10. Stator currents and torque of GEN 1 in pu.

In Fig. 9 the a, b, c currents in line 1 are depicted. The back transformation is performed via $i_a = \Re[i_1^+]$, etc., without multiplication with the usual numerical factor, to maintain the simple relation between pu and physical values in the a, b, c domain as well.

In Fig. 10 the stator currents i_d and i_q and the electromagnetic torque of GEN 1 are depicted to illustrate the response of the generator to the two-phase and the subsequent three-phase short circuit. The difference between a symmetric and asymmetric short circuit is clearly observable.

VII. Conclusions

Power-independent transformations are useful tools for network calculations, where many components with different power ratings are involved. Their advantage is that in all transformed stages the power is the same. Application of proper base values facilitate the introduction of transformed variables in pu which are simply related to physical values in the a, b, c domain.

The use of time-dependent symmetrical components in network calculations has several advantages:

- Network-component data are usually available in these coordinates.
- The simple relation with their steady-state phasors facilitates the interpretation of calculation results by the well-known steady-state phasor theory, for example, in case of asymmetric faults.
- The use of time-dependent complex phasors in equations and results provides a simple relation to their rms values in the a, b, c domain, which, for example, can be used for perusal and for graphical output.

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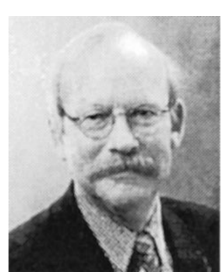

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