Symmetries in observer design: review of some recent results and applications to EKF-based SLAM

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- Symmetries have been much used in control theory for feedback design
- It has been applied to mobile robot control, passive walking ... but suprisingly much less for observer design
- They generally allow to reduce the complexity of the control problem. As a matter fact it was realized recently it also reduces the complexity of observer design for non-linear systems possessing symmetries.

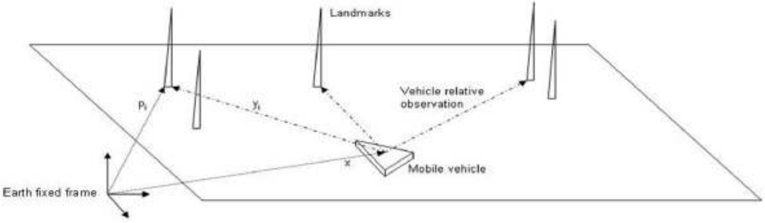
Outline

Introductive example: SLAM

According to the tutorial article of Hugh Durrant-Whyte and Tim Bailey (IEEE 2006):

- The simultaneous localization and mapping (SLAM) problem asks if it is possible for a mobile robot to be placed at an unknown location in an unknown environment and for the robot to incrementally build a consistent map of this environment while simultaneously determining its location within this map.
- A solution to the SLAM problem has been seen as a holy grail for the mobile robotics community as it would provide the means to make a robot truly autonomous.

Introductive example: SLAM - usual model



- The vehicle state is defined by the position $x \in \mathbb{R}^2$ and the orientation of the vehicle axis θ in the reference frame.
- The landmarks are modeled as points and represented by their position $p_i \in \mathbb{R}^2$ where $1 \leq i \leq N$.
- measurements = relative position to environmental landmarks.

$$z_i = R_{-\theta}(p_i - x) \quad 1 \leq i \leq N$$

Introductive example: SLAM

The trusted equations of motion are based on non-holonomic constraints

$$\frac{d}{dt}x = u\, R_\theta e_1, \quad \frac{d}{dt}\theta = uv, \quad \frac{d}{dt}p_i = 0 \quad 1 \leq i \leq N$$

where $e_1 = (1,0)^T$ and R_θ is the rotation matrix of angle θ. The measurements are

$$z_i = R_{-\theta}(p_i - x) \quad 1 \leq i \leq N$$

In a stochastic independent white Gaussian state measurements noises are added.

Introductive example: SLAM

- Simultaneous localization and mapping principle : Estimate x, θ, p_i, $1 \leq i \leq N$ at the same time.
- Extended Kalman Filter SLAM: Consider the estimation problem above as an observer design problem and build an EKF, i.e.

$$\frac{d}{dt}\hat{x} = uR_{\hat{\theta}}e_1 + \sum_1^N L_x^k(\hat{z}_k - z_k),$$

$$\frac{d}{dt}\hat{\theta} = uv + \sum_1^N L_\theta^k(\hat{z}_k - z_k),$$

$$\frac{d}{dt}\hat{p}_i = \sum_1^N L_i^k(\hat{z}_k - z_k), \quad 1 \leq i \leq N$$

where $\hat{z}_k = R_{-\hat{\theta}}(\hat{p}_k - \hat{x})$ and where the L_i's are tuned via the usual EKF equations.

Introductive example: SLAM

What is wrong with the update ?

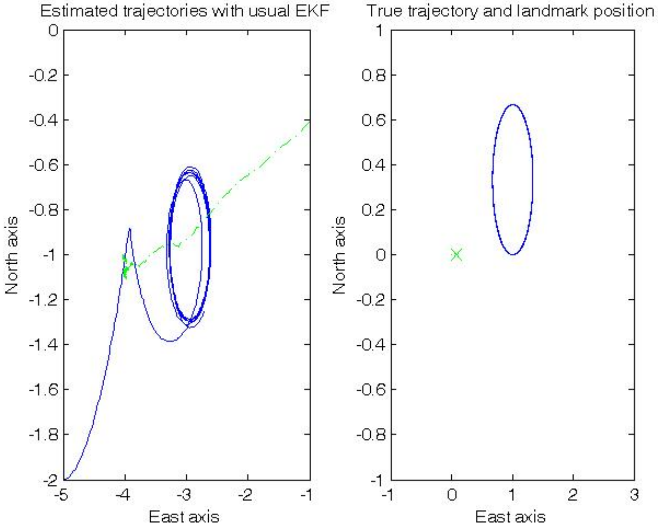
$$\frac{d}{dt}\hat{x} = uR_{\hat{\theta}}e_1 + \sum_1^N L_x^k \boxed{(\hat{z}_k - z_k)}$$

where $z_k = R_{-\theta}(p_k - x)$.

- It is not natural to “correct" variables expressed in the reference frame with measurements in the vehicle frame
- Do the $L_i's$ adapt automatically (one way or another the $L_i's$ must depend on θ) ?
  - Yes but the gain matrix (hence the covariance matrix) can not converge ...

Introductive example: SLAM

Simulation[1] results illustrate this shortcoming of EKF SLAM:



[1] Estimated and true trajectory and landmark for one landmark and a car moving over a circular path with a 20% measurment noise.

Introductive example: SLAM

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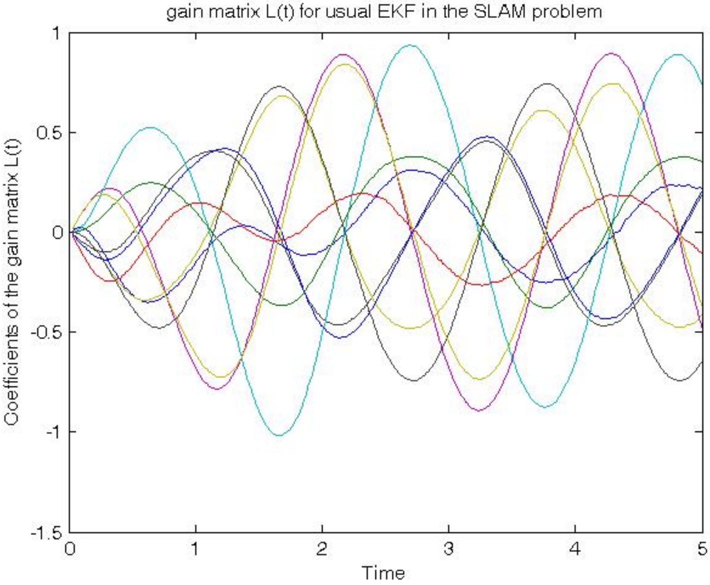


Introductive example: SLAM

- A natural way to circumvent those drawbacks would be to modify the updates

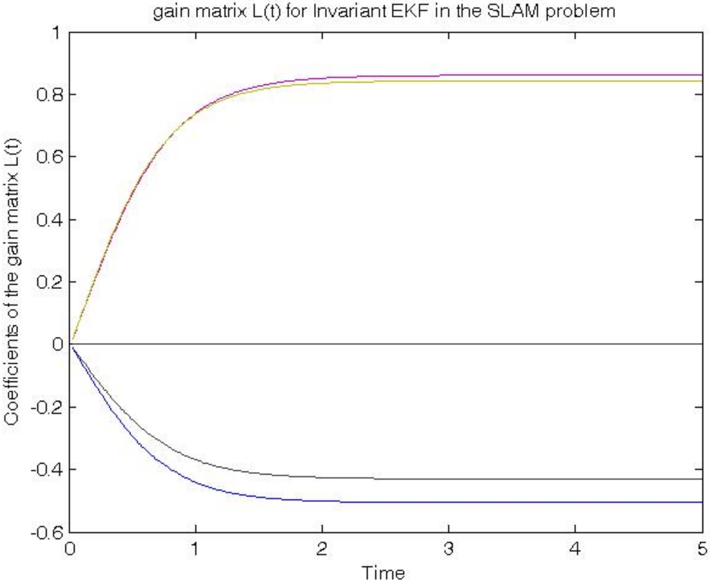
$$\frac{d}{dt}\hat{x} = uR_{\hat{\theta}}e_1 + \sum_1^N L_x^k R_{\hat{\theta}} \boxed{(\hat{z}_k - z_k)}$$

where $z_k = R_{-\theta}(p_k - x)$.

- Those kinds of heuristic modifications are encompassed in the general framework of symmetry-preserving observers presented in this talk.
- The theory allows to find judicious modifications leading to the following results (see next slide).

Introductive example: SLAM

A symmetry-based modification of the EKF algorithm leads to stability



Linear and non-linear observer design: Luenberger observer, Extended Kalman Filter.

The linear case:

$$\frac{d}{dt}x = Ax + Bu \qquad y, u \text{ known signals}$$
$$y = Cx + Du$$

Linear observer

A stable filter mixing the input and output signals $u(t)$ and $y(t)$:

$$\frac{d}{dt}\hat{x} = A\hat{x} + Bu(t) - L(C\hat{x} + Du(t) - y(t))$$

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Is $\hat{x}$ a good estimate of x?

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Is $\hat{x}$ a good estimate of x?

Error system for $e := \hat{x} - x$

$$\frac{d}{dt}e = (A - LC)e$$

The error system is autonomous (separation principle...)

The linear case:

$$\frac{d}{dt}x = Ax + Bu \qquad y, u \text{ known signals}$$
$$y = Cx + Du$$

Linear observer

A stable filter mixing the input and output signals $u(t)$ and $y(t)$:

$$\frac{d}{dt}\hat{x} = A\hat{x} + Bu(t) - L(C\hat{x} + Du(t) - y(t))$$

The celebrated Kalman Filter admits M, N as “tuning" parameters (Gaussian noise covariances chosen by the user) and we get

$$L = -PC^T N$$
$$\frac{d}{dt}P = AP + PA^T + M^{-1} - PC^T NCP$$

The nonlinear observation problem:

For a system with dynamics described by

$$\frac{d}{dt}x = f(x, u)$$

equipped with sensors yielding measurements

$$y = h(x, u)$$

We focus on observers such that the evolution of $\hat{x}(t)$ is given by :

$$\frac{d}{dt}\hat{x} = F(\hat{x}, y, u)$$

Examples[2]

Observers of the form ?

$$\frac{d}{dt}\hat{x} = F(\hat{x}, y, u)$$

Estimator, observer, filter, etc:

$$\frac{d}{dt}\hat{x} = f(\hat{x}, u) - L(\hat{x}, y) \cdot \left(h(\hat{x}, u) - y\right)$$

- Luenberger observer, gain scheduling, high gains, ...
- Extended Kalman Filter

[2] See, e.g., G. Besançon (Ed.): Nonlinear Observers and Applications; Springer(2007). J.P. Gauthier, I. Kupka: Deterministic Observation Theory and Applications (2001).

Extended Kalman Filter

$$\frac{d}{dt}\hat{x} = f(\hat{x}, u) - L(\hat{x}, y) \cdot \left(h(\hat{x}, u) - y\right)$$

Let $e = \hat{x} - x$. We have up to second order terms in e

$$\frac{d}{dt}e = (A(t) - L(t)C(t))e$$

M, N are tuning parameters and the EKF is based on

$$A(t) = \frac{\partial f}{\partial x}(\hat{x}, u) \qquad L = -PC^T N$$

$$C(t) = \frac{\partial h}{\partial x}(\hat{x}, u) \qquad \frac{d}{dt}P = AP + PA^T + M^{-1} - PC^T NCP$$

- Tuning? Domain of convergence? Computational cost?

Symmetry-preserving observers: theory and recent results

Symmetry group of a system of differential equations

- A thing is symmetrical if one can subject it to a certain operation and it appears exactly the same after the operation
- How can a dynamical model be "symmetrical" ?
- What operation can we do to a an experiment, and leave the result the same ?

Symmetry group of a system of differential equations

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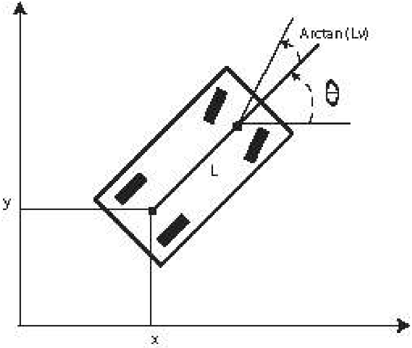
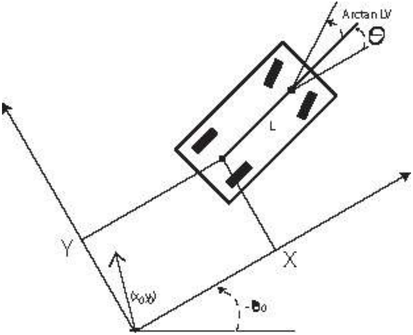




Symmetry group of a system of differential equations

Let G be a group, and M be a set.

Definition

A group action is defined on M if to any $g \in G$ on can associate a diffeomorphic transformation $\phi_g : M \to M$ such that

$$\phi_{gh} = \phi_g \circ \phi_h, \quad (\phi_g)^{-1} = \phi_{g^{-1}}$$

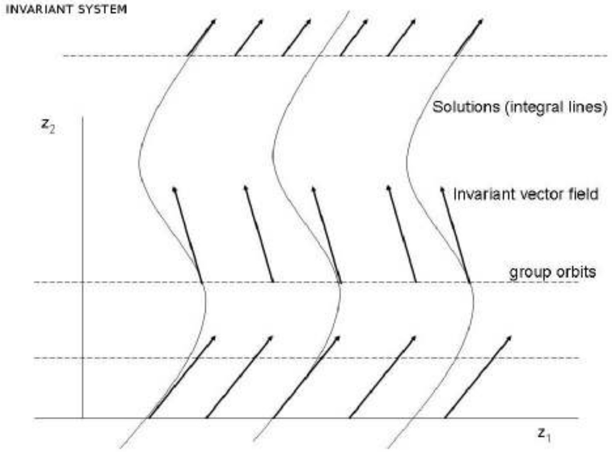
Definition

G is a symmetry group of a system of differential equations defined on M if it maps solutions to solutions. In this case we say the system is invariant.

Definition

A vector field w on M is said invariant if the system $\frac{d}{dt}z = w(z)$ is invariant.

Symmetry group of a system of differential equations



Illustrative example. $M = \mathbb{R}^2$, and the symmetry group is made of horizontal translations. We have $\phi_g(z_1, z_2) = (z_1 + g, z_2)^T$ where $g \in G = \mathbb{R}$.

Symmetry group of a system of differential equations

Another way to see invariance is the following :

Proposition

If the system

$$\frac{d}{dt}z = w(z)$$

is invariant, then for any $g \in G$ if we let

$$\phi_g(z) = Z$$

we have

$$\frac{d}{dt}Z = w(Z)$$

Definition

A ***scalar invariant*** *is a function $I : M \rightarrow \mathbb{R}$ such that $I(\phi_g(z)) = I(z)$ for all $g \in G$. In other words*

$$I(Z) = I(z)$$

Symmetry group of a system of differential equations

Consider now the general non-linear system

$$\frac{d}{dt}x = f(x, u)$$

Consider also the local group of transformations on $\mathcal{X} \times \mathcal{U}$

$$\phi_g(x, u) = \big(\varphi_g(x), \psi_g(u)\big), \qquad (1)$$

Definition

The system $\frac{d}{dt}x = f(x, u)$ is said invariant if it is invariant to the group action (1)*.*

It means

$$\frac{d}{dt}X = f(X, U)$$

where $(X, U) = (\varphi_g(x), \psi_g(u))$ for all $g \in G$.

Symmetry-preserving observers

We would like the observer to be an invariant system for the *same* symmetry group.

Definition

The observer

$$\frac{d}{dt}\hat{x} = F(\hat{x}, u, y)$$

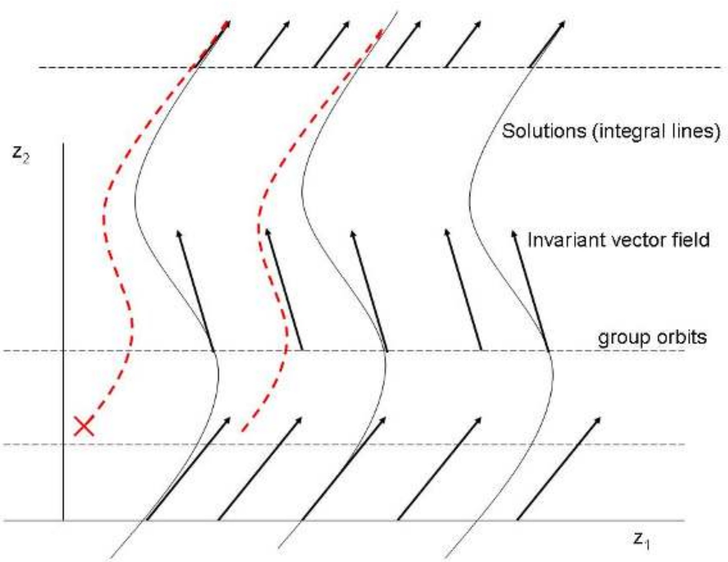
is invariant *or "symmetry-preserving" if it is an invariant system for the group action*
$(\hat{x}, x, u, y) \mapsto \big(\varphi_g(x), \varphi_g(\hat{x}), \psi_g(u), h(\varphi_g(x), \psi_g(u))\big)$.
The observer is unchanged by the transformation, i.e.

$$\frac{d}{dt}\hat{X} = F(\hat{X}, U, Y)$$

where $(\hat{X}, U, Y) = \big(\varphi_g(x), \varphi_g(\hat{x}), \psi_g(u), h(\varphi_g(x), \psi_g(u))\big)$.

Symmetry-preserving observers

Invariant system and invariant observer in the phase space.



Symmetry-preserving observers

Theorem: Every invariant candidate observer reads[3]

$$\frac{d}{dt}\hat{x} = f(\hat{x}, u) + W(\hat{x})L\Big(I(\hat{x}, u), E(\hat{x}, u, y)\Big)E(\hat{x}, u, y)$$

- $E(\hat{x}, u, y)$ invariant output error
- $W(\hat{x}) = \big(w_1(\hat{x}), .., w_n(\hat{x})\big)$ invariant frame
- $I(\hat{x}, u)$ invariant
- $L(I, E)$ freely chosen $n \times p$ gain matrix

Definition

The smooth map $(\hat{x}, u, y) \mapsto E(\hat{x}, u, y) \in \mathbb{R}^p$ is an invariant output error (invariant counterpart of $\hat{y} - y$) if

- $E\big(\hat{X}, U, Y\big) = E(\hat{x}, u, y)$ *for all* $\hat{x}, u, y$ *(invariant)*
- *the map* $y \mapsto E(\hat{x}, u, y)$ *is invertible for all* $\hat{x}, u$ *(output)*
- $E\big(\hat{x}, u, h(\hat{x}, u)\big) = 0$ *for all* $\hat{x}, u$ *(error)*

[3] Bonnabel, Martin, Rouchon: Symmetry-preserving observers (IEEE-TAC, 2008).

Symmetry-preserving observers: Illustration with the SLAM example

Non-linear invariant system (no noise):

$$\frac{d}{dt}x = u\,R_\theta e_1, \quad \frac{d}{dt}\theta = uv, \quad \frac{d}{dt}p_i = 0 \quad 1 \leq i \leq N$$

Symmetry group SE(2): rotations and translations in the reference frame.

$$\varphi_g(x, p_1, \cdots, p_N) = (R_{\theta_0}x + x_0, R_{\theta_0}p_1 + x_0, \cdots, R_{\theta_0}p_N + x_0)$$

- Invariant output error : $\hat{z}_i - z_i$
- Every invariant observer reads

$$\frac{d}{dt}\hat{x} = uR_{\hat{\theta}}e_1 + R_{\hat{\theta}}(\sum_1^N L_x^k(\hat{z}_k - z_k)), \; \frac{d}{dt}\hat{\theta} = uv + \sum_1^N L_\theta^k(\hat{z}_k - z_k),$$

$$\frac{d}{dt}\hat{p}_i = R_{\hat{\theta}}(\sum_1^N L_i^k(\hat{z}_k - z_k))$$

So what?

- So far, building invariant observers essentially amounts to writing the correction terms in the same frame as the estimated vectors.
- Does the theory goes beyond?
- The answer is yes. The error equation has very interesting properties[4].
- The most striking results are obtained when there are as many symmetries as the dimension of the state space; i.e. the state space coincides with its symmetry group.
- This should not apply to EKF SLAM as the state space is $2N + 3$ dimensional whereas the symmetry group is of dimension 3.

[4] see Bonnabel, Martin, Rouchon (IEEE TAC 2008).

Some applications

Before going into more theoretical properties, let us present some applications.

The method was successfully used for data fusion of GPS and IMU as an alternative to EKF. Three main groups worked on the use of those observers for data fusion applications :

- in Australia: see e.g. Mahony, Hamel, Pflimlin (CDC 2005, IEEE-TAC 2008)
- in Portugal: e.g. Vasconcelos, Silvestre and Oliveira (CDC 2008)
- in France: e.g. Martin, Salaun (CDC 2008), Bonnabel, Rouchon (Springer 2005)

Some applications at Ecole des Mines

- An invariant observer with constant gains was designed for quadrotor applications. The low computational cost allowed an implementation on a cheap (5$) 8-bit micro-controller. [5]
- Fusion of odometry/GPS/IMU was experimented on a car[6].

[5] See PhD Thesis of Erwan Salaün.

[6] Bonnabel, Salaun, Control Engineering Practice, 2011.

Particular case where the state space is a Lie group

Let $\mathcal{X} = G$. The system is assumed to be invariant to left multiplications i.e.

$$\frac{d}{dt}X = X\Omega(t).$$

The considered group action is

$$\varphi_g(x) = gX$$

where $g, X \in G$. The system is invariant to the transformation as

$$\frac{d}{dt}(gX) = (gX)\Omega(t)$$

In the SLAM problem the motion of the car $\dot{x} = uR_\theta e_1, \dot{\theta} = uv$ can be viewed as a left-invariant system on the Lie group SE(2):

$$X = \begin{pmatrix} R_\theta & x \\ 0_{1\times 2} & 1 \end{pmatrix}, \quad \Omega = \begin{pmatrix} \omega_x & ue_1 \\ 0_{1\times 2} & 0 \end{pmatrix}, \text{ with } \omega_x = \begin{pmatrix} 0 & -uv \\ uv & 0 \end{pmatrix}$$

Particular case where the state space is a Lie group

Consider the natural (right) invariant state error $\eta = \hat{X}X^{-1}$. Suppose there is an invariant output error $E(\eta)$. Consider the invariant observer:

$$\frac{d}{dt}\hat{X} = \hat{X}\Omega + L(E(\eta))\hat{X}$$

Particular case where the state space is a Lie group

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$$\frac{d}{dt}\hat{X} = \hat{X}\Omega + L(E(\eta))\hat{X}$$

We have

$$\frac{d}{dt}X^{-1} = -X^{-1}\dot{X}X^{-1} = -X^{-1}X\Omega X^{-1} = -\Omega X^{-1}$$

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Using this property, the error equation is

$$\frac{d}{dt}\eta = \hat{X}\Omega X^{-1} + L(E(\eta))\eta - \hat{X}\Omega X^{-1} = L(E(\eta))\eta$$

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$$\frac{d}{dt}\eta = \hat{X}\Omega X^{-1} + L(E(\eta))\eta - \hat{X}\Omega X^{-1} = L(E(\eta))\eta$$

The error equation is completely autonomous !

Particular case where the state space is a Lie group

The linearized system around any trajectory reads

$$\frac{d}{dt}\delta\eta = LC\delta\eta$$

with fixed C. Several benefits:

- A constant gain observer is easily tuned for (at least) local convergence around every trajectory with a good local behavior (interesting practical property)[7].
- For several ground and aerial vehicles, a (local) separation principle on Lie groups holds around the trajectories around which the linearized system is time-invariant (connection to the work of Bullo and Murray, Samson and Morin, etc.) [8].

[7] Bonnabel, Martin, Rouchon (IEEE TAC 2009)

[8] Bonnabel, Martin, Rouchon, Salaun (IFAC, 2011)

Particular case where the state space is a Lie group

The covariance update of the Extended Kalman Filter behaves as if the system were linear and time-invariant[9]:
$\dot{x} = 0, \; y = CX$.

$$L = -PC^T N$$
$$\frac{d}{dt}P = M^{-1} - PC^T NCP$$

- Autonomy is the key for numerous powerful convergence results for observers on Lie groups[10].
- Even when the state space is not a Lie group, there is a large set of trajectories around which the error equation is autonomous.

[9] Bonnabel (CDC 2007); Bonnabel, Martin, Salaun (CDC 2009)

[10] Bonnabel, Martin, Rouchon (CIFA 2006). Lageman, Trumpf, Mahony (MTNS 2008), Vasconcelos, Silvestre and Oliveira (CDC 2008)

Application to EKF SLAM

Application to EKF SLAM

Consider the following matrix representation:

$$X = \begin{pmatrix} R_\theta & x \\ 0_{1\times 2} & 1 \end{pmatrix}, \quad P_i = \begin{pmatrix} R_\theta & p_i \\ 0_{1\times 2} & 1 \end{pmatrix},$$
$$\Omega = \begin{pmatrix} \omega_x & ue_1 \\ 0_{1\times 2} & 0 \end{pmatrix}, \quad \Omega_i = \begin{pmatrix} \omega_x & 0 \\ 0_{1\times 2} & 0 \end{pmatrix}$$

The equations of the system

$$\frac{d}{dt}x = u\, R_\theta e_1, \quad \frac{d}{dt}\theta = uv, \quad \frac{d}{dt}p_i = 0 \quad 1 \leq i \leq N$$

can be written

$$\frac{d}{dt}X = X\Omega, \; \frac{d}{dt}P_i = P_i\Omega_i, \; 1 \leq i \leq N$$

and the system can be viewed as a left-invariant dynamics system on the (huge) Lie group $G \times \cdots \times G$.

Application to EKF SLAM

Let $\eta_X = \hat{X}X^{-1}, \eta_i = \hat{P}_i P_i^{-1}$ be the invariant state error.

Relative position to landmarks (measurements)

$$E_i = R_{\hat{\theta}}(\hat{z}_i - z_i)$$

is a function of $(\eta_i - \eta_X)$.

Consider
the following invariant observer (invariant to right multiplications)
$\frac{d}{dt}\hat{X} = \hat{X}\Omega + L_X(E_1, \cdots, E_N)\hat{X}, \quad \frac{d}{dt}\hat{P}_i = \hat{P}_i\Omega_i + L_i(E_1, \cdots, E_N)\hat{P}_i$

The (non-linear) error equation is completely autonomous !

Application to EKF SLAM

With the initial notation, the observer is

$$\frac{d}{dt}\hat{\theta} = uv + \mathcal{L}_\theta(E), \quad \frac{d}{dt}\hat{x} = uR_{\hat{\theta}}e_1 + \mathcal{L}_\theta(E)e_3 \wedge \hat{x} + \mathcal{L}_x(E),$$
$$\frac{d}{dt}\hat{p}_i = \mathcal{L}_\theta(E)e_3 \wedge \hat{p} + \mathcal{L}_i(E)$$

where $E = \{R_{\hat{\theta}}(\hat{z}_i - z_i), 1 \leq i \leq N\}$. The invariant state error is $\eta = (\tilde{\theta}, \tilde{x}, \tilde{p}_1, \cdots, \tilde{p}_n)$ where

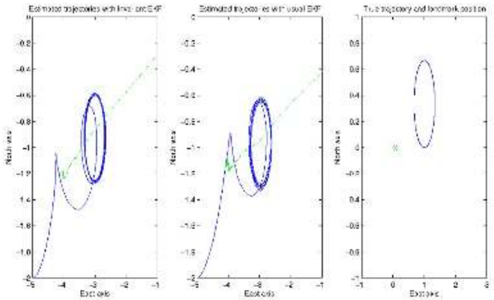
$$\tilde{\theta} = \hat{\theta} - \theta, \; \tilde{x} = \hat{x} - R_{\tilde{\theta}}x, \; \tilde{p}_i = \hat{p}_i - R_{\tilde{\theta}}p_i$$

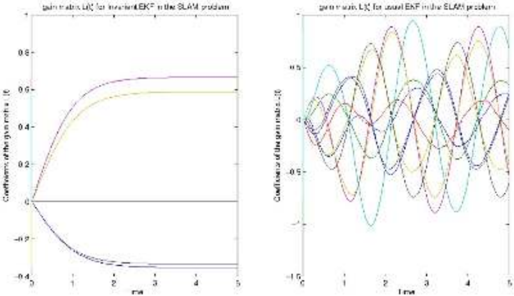
The error equation is autonomous. The linearized error equation writes

$$\frac{d}{dt}\delta\eta = LC\delta\eta$$

where L can be freely chosen and C is a fixed matrix.

Application to EKF SLAM: simulations with a 20% noise





Linearized system

$$\frac{d}{dt}\delta\eta = (LC)\delta\eta$$

Kalman filtering

$$L = -PC^TN$$

$$\frac{d}{dt}P = M^{-1} - PC^TNC$$

with M, N tuning matrices.

Application to EKF SLAM

- The covariance matrix converges as if the system was linear, time-invariant and stationnary.
- Compute the gain matrix offline ! The computational burden is lowered and a lot more landmarks can be included in the map.

Now let us propose a special tuning of the gains having very interesting properties.

Application to EKF SLAM

The following constant gain observer is globally convergent

Proposition

Consider the SLAM problem without noise. The following observer

$$\frac{d}{dt}\hat{\theta} = uv, \quad \frac{d}{dt}\hat{x} = uR_{\hat{\theta}}e_1, \quad \frac{d}{dt}\hat{p}_i = k_i\ R_{\hat{\theta}}(\hat{z}_i - z_i)$$

with $k_i > 0$ *is such that* $\boxed{\frac{d}{dt}(R_{\hat{\theta}}(\hat{z}_i - z_i)) = -k_i\ R_{\hat{\theta}}(\hat{z}_i - z_i)}$

Proof: we have $\frac{d}{dt}(\hat{\theta} - \theta) = uv - uv = 0$. And $R_{\hat{\theta}}(\hat{z}_i - z_i) = (\hat{p}_i - \hat{x}_i) - R_{\hat{\theta}-\theta}(p_i - x_i)$. Thus

$$\frac{d}{dt}(R_{\hat{\theta}}(\hat{z}_i - z_i)) = k_i\ R_{\hat{\theta}}(\hat{z}_i - z_i) + -uR_{\hat{\theta}}e_1 + R_{\hat{\theta}-\theta}uR_{\theta}e_1.$$

The blue term = 0.

Application to EKF SLAM

This new algorithm as several advantages:

- The algorithm possesses convergence properties as the landmarks estimation errors $\hat{z}_i - z_i$ go to zero exponentially
- Such global properties generally allow a great robustness to noise.
- The estimation of the landmarks are decoupled, and the k_i's must be chosen accordingly to the level of noise associated with the i-th observation.
- The algorithm complexity is linear in N whereas in Kalman filtering scales in N^2. The number of landmarks, typically $N = 1000$, can be much increased !

Conclusion

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- The theory of symmetry-preserving observers offers a versatile geometric framework to “put" some physics in the design of non-linear observers in robotics problems
- The simplicity of those observers as well as the hope for convergence properties has made them a challenger to usual EKF for UAV state estimation
- The application to SLAM looks promising but still needs to be checked againts experiments (implementation, robustness to uncorrect data associations etc.)
- It works in 3D.
- Future work includes application to visual SLAM (bearing only measurements).
- A proper theory of invariant noises must be elaborated

Thank you, any questions?