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Symmetry Breaking in the $SU(3)$ Chiral Dynamics

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A generalization of the $SU(2)$ nonlinear chiral dynamics[1] to the $SU(3)$ has been achieved on the basis of the quark model.[2] In this letter we take the $SU(3)$ symmetry breaking into account.

In the $SU(3)$ linear chiral dynamics based on the quark model, the Lagrangian is given by

$$L=-\bar{q}\gamma_\mu\partial_\mu q+g\bar{q}(\sigma+i\gamma_5\pi)q$$
$$-\tfrac{1}{2}(\partial_\mu\pi\partial_\mu\pi+\partial_\mu\sigma\partial_\mu\sigma) \qquad (1)$$

for the quark q, the scalar meson σ and the pseudoscalar meson π, which is invariant under linear chiral transformation. We proceed to nonlinear chiral dynamics by the Weinberg transformation,

$$q=\frac{1+i\gamma_5\phi}{\sqrt{1+\phi^2}}\psi\,. \qquad (2)$$

Here we identify the χ and $\varphi=\phi/\alpha$ as actual fields for the $SU(3)$ nonlinear chiral dynamics, (see (7)). We require that the second term of (1) becomes the mass term of ψ with mass difference, that is,

$$\bar{q}(\sigma+i\gamma_5\pi)q=\bar{\psi}\Sigma\psi\,, \qquad (3)$$

where

$$\Sigma=-\frac{m}{g}\begin{pmatrix}1 & & 0\\ & 1 & \\ 0 & & 1+\lambda\end{pmatrix}=-\frac{m}{g}(1+\lambda\Lambda_{33}), \qquad (4)$$
$$\lambda=\delta m/m\,.$$

From (2) and (3) we can define σ and π as follows:

$$\sigma+i\pi=U^*\Sigma U^*, \qquad (5)$$

where

$$U=\frac{1+i\phi}{\sqrt{1+\phi^2}}\,. \qquad (6)$$

The proportionality coefficients α for ϕ are determined under the normalization condition that the free Lagrangian for mesons becomes $-\frac{1}{2}\partial_\mu\varphi\partial_\mu\varphi$. The α's of (7) are different for each field due to symmetry breaking. The η-η' mixing effect can also be taken into account by diagonalization of the free Lagrangian for mesons. Thus, when we define the coefficients α by

$$\phi_\pi=\alpha_\pi\varphi_\pi\,,\ \phi_K=\alpha_K\varphi_K\,,\ \phi_\eta=\alpha_\eta\varphi_\eta\,,\ \phi_{\eta'}=\alpha_{\eta'}\varphi_{\eta'}\,, \qquad (7)$$

we find that

$$\alpha_\pi=\frac{g}{2m}\,,\ \frac{\alpha_\pi}{\alpha_K}=1+\frac{\lambda}{2}\,,\ \frac{\alpha_\pi}{\alpha_\eta}=1\,,\ \frac{\alpha_\pi}{\alpha_{\eta'}}=1+\lambda. \qquad (8)$$

(i) *baryon-meson interactions*

We obtain an information on baryon-meson interactions, by substituting (2) into the first term of (1). By sandwiching the quark-meson interaction between the 56-dimensional wave functions of the $SU(6)$, the P-wave baryon-meson interactions are given by

$$g_A\,\mathrm{Tr}\,(i\bar{b}\gamma_5\gamma_\mu\partial_\mu\phi b+\frac{1}{5}i\bar{b}\gamma_5\gamma_\mu b\partial_\mu\phi), \qquad (9)$$

where b is the baryon octet, $g_A=(5/3)\gamma$, and $\gamma\approx1/\sqrt{2}$ is the renormalization of π-quark vertex. In order for (9) to explain the π-N vertex, we find that

$$\alpha_\pi=\sqrt{2}/F_\pi\,,\quad F_\pi=2m_Ng_A/G_{\pi N}\,. \qquad (10)$$

From (7), (8), (9) and (10) we obtain

$$\frac{G^2_{KN\Lambda}}{4\pi}=\frac{G^2_{\pi N}}{4\pi}\left(\frac{m_N+m_\Lambda}{2m_N}\right)^2\left(\frac{3\sqrt{3}}{5}\right)^2\frac{1}{(1+(\lambda/2))^2}\,,$$
$$\frac{G^2_{KN\Sigma}}{4\pi}=\frac{G^2_{\pi N}}{4\pi}\left(\frac{m_N+m_\Sigma}{2m_N}\right)^2\left(\frac{1}{5}\right)^2\frac{1}{(1+(\lambda/2))^2}\,. \qquad (11)$$

By (8) the coupling constants for vertices relating to K and η' become smaller than the symmetric case by a factor $1/(1+(\lambda/2))^2$ and $1/(1+\lambda)^2$, respectively. However, the coupling constants relating to π and η are the same as for the symmetric case. In Table I the values of $G^2_{KN\Lambda}/4\pi$ and $G^2_{KN\Sigma}/4\pi$ are shown for $\lambda=0$, 1/3 and 2/3. The case $\lambda=1/3$ is consistent with the values of Kim.[3)]

Table I. Coupling constants.

λ	0	1/3	2/3
$G^2_{KN\Lambda}/4\pi$	16	14	11
$G^2_{KN\Sigma}/4\pi$	0.60	0.56	0.43

(ii) *mass difference of spin-1 meson*[4)]

The chiral invariant Lagrangian has previously been obtained,[2)] and it contains terms such as

$$-\tfrac{1}{2}m_\rho^2\,\mathrm{Tr}(V_\mu^2+P_\mu^2), \tag{13}$$

$$-\frac{2m_\rho^2}{F_\pi^2}\,\mathrm{Tr}[(V_\mu^2+P_\mu^2)(\sigma^2+\pi^2) - V_\mu\sigma V_\mu\sigma+P_\mu\sigma P_\mu\sigma], \tag{14}$$

where V_μ is the vector meson and P_μ is the axial-vector meson. (13) is determined so as to fix the mass term of ρ. Now, the term (13) may well be modified by introducing an adjustable parameter n

$$-\frac{1}{2}m_\rho^2\left(\frac{8}{F_\pi^2}\right)^n\mathrm{Tr}(V_\mu^2+P_\mu^2)(\sigma^2+\pi^2)^n \tag{15}$$

since $\sigma^2+\pi^2=F_\pi^2/8$. First, we consider the case $n=1$. Substituting (5) into (14) and (15), we obtain the mass terms with the symmetry breaking:

$$-\tfrac{1}{2}m_\rho^2\,\mathrm{Tr}(V_\mu^2+2P_\mu^2) -\tfrac{1}{2}m_\rho^2(2\lambda+\lambda^2)\mathrm{Tr}(V_\mu^2+P_\mu^2)\Lambda_{33} -\tfrac{1}{4}m_\rho^2\,\mathrm{Tr}[\lambda^2V_\mu^2\Lambda_{33}-\lambda^2V_\mu\Lambda_{33}V_\mu\Lambda_{33} +(4\lambda+\lambda^2)P_\mu^2\Lambda_{33}+\lambda^2P_\mu\Lambda_{33}P_\mu\Lambda_{33}]. \tag{16}$$

We take into account ω-ϕ mixing and D-E mixing by diagonalization of (16). Finally we obtain

$$\begin{aligned} m^2_{A1}&=2m_\rho^2\\ m^2_{K^*}&=\left(1+\lambda+\frac{3}{4}\lambda^2\right)m_\rho^2,\\ m^2_{K_A}&=\left(2+2\lambda+\frac{3}{4}\lambda^2\right)m_\rho^2,\\ m_\omega^2&=m_\rho^2,\quad m_D^2=2m_\rho^2,\\ m_\phi^2&=(1+\lambda)^2m_\rho^2,\\ m_E^2&=2(1+\lambda)^2m_\rho^2. \end{aligned} \tag{17}$$

Predicted mass levels for $\lambda=1/3$ are shown in Table II, and are consistent with experiment. The case $n=0$ is inconsistent with experiment; for example, $m_\omega=m_\phi=m_\rho$.

Table II. Mass levels.

	$\lambda=1/3$ (MeV)	exp. (MeV)
m_{K^*}	905	890
m_ω	760	780
m_ϕ	1013	1019
m_{K_A}	1260	1320
m_D	1074	1285
m_E	1430	1420

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