

Symplectic embeddings and continued fractions: a survey^{*}

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Abstract. As has been known since the time of Gromov's Non-squeezing Theorem, symplectic embedding questions lie at the heart of symplectic geometry. After surveying some of the most important ways of measuring the size of a symplectic set, these notes discuss some recent developments concerning the question of when a 4-dimensional ellipsoid can be symplectically embedded in a ball. This problem turns out to have unexpected relations to the properties of continued fractions and of exceptional curves in blow ups of the complex projective plane. It is also related to questions of lattice packing of planar triangles.

Keywords and phrases: symplectic embedding, symplectic capacity, continued fractions, symplectic ellipsoid, symplectic packing, lattice points in triangles

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