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ORIGINAL PAPER

Synchronization behavior of coupled neuron circuits composed of memristors

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Abstract The fluctuation of intracellular and extracellular ion concentration induces the variation of membrane potential, and also complex distribution of electromagnetic field is generated. Furthermore, the membrane potential can be modulated by time-varying electromagnetic field. Therefore, magnetic flux is proposed to model the effect of electromagnetic induction in case of complex electrical activities of cell, and memristor is used to connect the coupling between membrane potential and magnetic flux. Based on the improved neuron model with electromagnetic induction being considered, the bidirectional coupling-induced synchronization behaviors between two coupled neurons are investigated on Spice tool and also printed circuit board. It is found that electromagnetic induction is helpful for discharge of neurons under positive feedback coupling, while electromagnetic induction is necessary to enhance synchronization behaviors of coupled neurons under negative feedback coupling. The frequency analysis on isolate neuron confirms that the frequency spectrum is enlarged under electromagnetic induction, and self-induction effect is detected. These experimental results can be helpful for further dynamical analysis on synchronization of neuronal network subjected to electromagnetic radiation.

Keywords Memristor · Neuron · Synchronization · Electromagnetic induction

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1 Introduction

Since the pioneering work carried out by Hodgkin–Huxley, which nonlinear analysis was processed on the sampled time series for membrane potential of squid large axon neurons by using patch clamp technique, it is believed that the Hodgkin–Huxley neuron model [1] can reproduce the main properties of electrical activities with the effect of ion channels being considered. It is confirmed that the fluctuation of membrane potential depends on the changes of intracellular and extracellular ion concentration, and external forcing currents. This ion hypothesis has been verified its reliability within biological experiments and can give important guidance for computational neuroscience that blocking and activation agent of ion channels can be used to change the modes of electrical activities in neurons [2,3]. Furthermore, neuronal networks designed with different topological connections, for example regular network with nearest-neighbor connection, small-world network composed of local regular connection and long-range connection with different probability, have been proposed to the dynamical response, phase transition of collective behaviors of neuronal activities, and pattern formation and selection in brain. For example, Gong et al. [4] investigated the effect of ion channel blocking on firing pattern in stochastic Hodgkin–Huxley neuronal network. Liu et al. [5] confirmed that the stability of spiral wave in the neuronal network depends on the ion channel and blocking in channels can break the regularity of spirals in the media. Reference [6] suggested that ion channel can be used to adjust and regulate the collective behaviors of neuronal network with small-world connection. Furthermore, Ref. [7] synchronization transition was discussed in the time-delayed stochastic neuronal network. Sun et al. [8] reported that optimal blocking in channels can be helpful for regularity in electrical activities of neuronal network. Shuai et al. [9] suggested that the improved Langevin approximation method can be effective to approach statistical properties in electrical activities of stochastic Hodgkin–Huxley neuron model. In fact, researchers prefer to detect the transition in electrical activities and many simplified neuron models have been presented for dynamical analysis [10–15]. The external forcing current is often used to change the excitability of neuron and different modes of electrical activities such as quiescent, spiking, bursting and even chaotic states can be observed in the sampled time series for membrane potentials by applying appropriate forcing currents. For example, the three-variable Hindmarsh–Rose neuron [11] is often appreciated due to its simple form and used for bifurcation analysis, synchronization setting, pattern selection in network and dynamical control [16–18], and further description can be found in the review survey [19,20] and references therein.

The realistic neuron holds complex anatomical structure; besides the ion channel, some interneurones have been found to form autapse, a specific synapse connected to its body via certain loop with time delay, and this type of signal modulation and propagation can be described by time-delayed feedback on the membrane potential [21]. The autapse is classified into two types such as electric autapse and chemical autapse according to the action mechanism [22], and the electrical activities of neuron can be controlled by autapse [23–25]; for example, mode transition between quiescent, spiking and bursting can be selected by applying appropriate feedback gain and time delay in the loop for autapse. The author [26] of this paper ever designed a class of neuronal circuit driven by autapse; it is found that autapse connection make neuron becomes self-adaptive to external forcing and gave appropriate response. Extensive results found that appropriate distribution of autapse in network can enhance synchronization of network [27], developing ordered wave and pulse [28], and thus, the collective behaviors of network can be regulated by a pacemaker associated with autapse, and defects [29] also can be induced to block the wave propagation by negative feedback in autapse. On the other hand, the electrical activities of neuron and neuronal network can be changed due to polarization and magnetization; for example, Refs. [30–32] discussed the effect of polarization on neurons exposed to magnetic field and the transition in rhythm of electrical activities and synchronization approach as well. Wu et al. [33] suggested that the effect of radiation can be described by adding loop current on the neuron, and the response of electrical activities is discussed.

In fact, most of the presented neuron models focus on the contribution of transmembrane currents, while the effect of electromagnetic induction induced by complex fluctuation of ion concentrations is missed. According to the law of electromagnetic induction, time-varying electromagnetic field can induce fluctuation of magnetic flux covered by a loop, and electromagnetic induction occurs. The exchange of intracellular and extracellular charged ions and the time-varying distribution of charged ions can induce complex electromagnetic induction in cell. As a result, induced current and field can be induced to change the membrane potential of neuron and it can be different from the transmembrane current generated by exchange of charged ions across the membrane. As a result, Ma et al. [34,35] suggested that magnetic flux can be used to describe the fluctuation of electromagnetic field, and memristor [36,37] is used to realize feedback coupling between magnetic flux and membrane potential of neuron. It is interesting to find emergence of multiple modes of electrical activities in the sampled time series for membrane potentials by applying external forcing at fixed parameters, and it is believed that the memory mechanism can be associated with magnetic storage. Memristor is a new type of electric device and often used as nonlinear device in chaotic circuit, the memductance is dependent on the input current, and its memductance is fixed after removal of external forcing; as a result, memory effect is verified and the memristor-coupled circuit is much dependent on the initial selection [38–42]. By now, the dynamical analysis and control has been investigated on these resonator coupled with memristor extensively, and it is also interesting to explore this problem on memristor-coupled circuits. In this paper, operational amplifier and multiplier are used to set a reliable neuronal circuit based on a neuron model [34,35] with electromagnetic induction being considered, and the synchronization problem will be carried out on this circuit.

2 Model description, circuit setting and experimental results

As mentioned above, the fluctuation of intracellular and extracellular ions of cell can induce electromagnetic induction and the membrane potential can be modulated; therefore, magnetic flux is proposed to describe

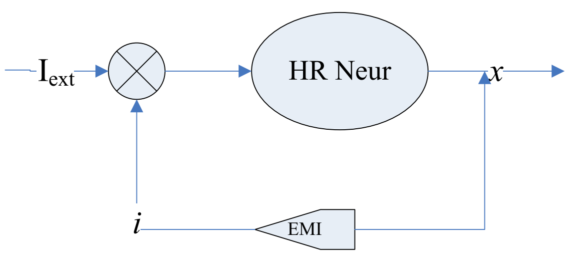

Fig. 1 Schematic diagram of the neuronal circuit. I_{ext} is the external forcing current, i is the induced current defined by $k_1\rho(\varphi)x$, and EMI is the abbreviation for electromagnetic induction

the effect of electromagnetic induction. According to the electromagnetic law, magnetic flux can change the membrane, vice versa. Memristor is proposed to realize coupling and interaction between magnetic flux and membrane potential to be consistent with physical units. The memductance of memristor is described by

$$q(\varphi) = \alpha\varphi + \beta\varphi^3 \text{ or } \frac{\mathrm{d}q(\varphi)}{\mathrm{d}\varphi} = \rho(\varphi) = \alpha + 3\beta\varphi^2 \tag{1}$$

where φ is magnetic flux, $\rho(\varphi)$ is the memductance of memristor controlled by magnetic flux, and α and β are parameters for memristor. The dynamical equations for Hindmarsh–Rose neuron [34,35] composed of electromagnetic induction are described by

$$\begin{cases} \frac{\mathrm{d}x}{\mathrm{d}t} = y - ax^3 + bx^2 - z + I_{\text{ext}} - k_1(\alpha + 3\beta\varphi^2)x \\ \frac{\mathrm{d}y}{\mathrm{d}t} = c - dx^2 - y \\ \frac{\mathrm{d}z}{\mathrm{d}t} = r[s(x + 1.6) - z] \\ \frac{\mathrm{d}\varphi}{\mathrm{d}t} = x - k_2\varphi \end{cases} \tag{2}$$

where $x, y, zand\varphi$ denote the membrane potential, recovery variable for slow current, adaption current and magnetic flux across the membrane respectively. I_{ext} is the external forcing current. *a, b, c, d, r* and *s* are parameters with similar description in the previous Hindmarsh–Rose model. The parameter k_1 defines the modulation gain on membrane potential resulting from induced current. The forth equation is Faraday's law of induction, while the parameter k_2 is associated with the media and suppresses the increase in magnetic flux infinitely. The parameters are set as $a = 1$, $b = 3$, $c = 1$, $d = 5$, $r = 0.006$, $s = 4$, $\alpha = 0.1$, $\beta = 0.01$,

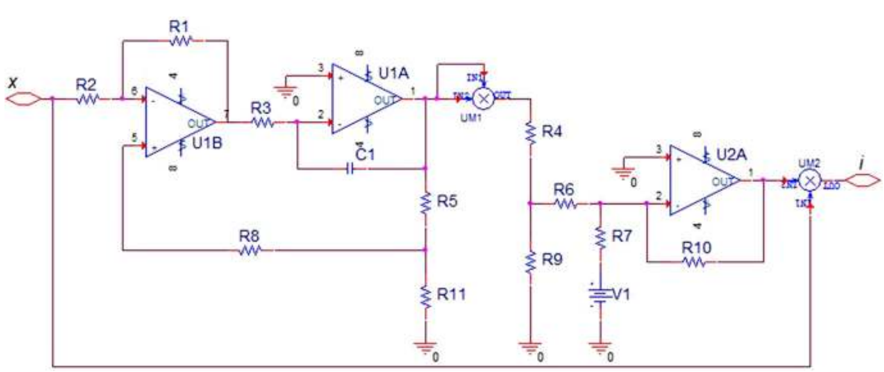

Fig. 2 Schematic diagram for the induced current $k_1\rho(\varphi)x$. The output for operational amplifier U1A is the nonlinear variable φ, which can be realized by using the resistance R1, R2, R3, R5, R8, R11, capacitor C1 and operational amplifier U1B. The output for operational amplifier U2A is the nonlinear variable $k_1\rho(\varphi)$, which can be realized by using the resistance R4 R6 R7 R9 R10 and multiplier Um1

$k_1 = 1, k_2 = 1$, and the schematic diagram for the HR circuit coupled with memristor is plotted in Fig. 1.

As a result, analog circuit can be designed for the induced current $k_1\rho(\varphi)x$; thus, the effect of electromagnetic induction can be considered. It reads in Fig. 2 as follows

According to Fig. 2, the output from multiplier Um2 produces the induced current for the neuron. The analog circuit for the variable x, y, z can be set up as well. It is known that two identical neurons can reach phase synchronization, generalized synchronization, lag synchronization and even complete synchronization by selecting appropriate coupling intensity. For non-identical neurons, phase synchronization or rhythm synchronization can be realized under appropriate coupling intensity between neurons. Furthermore, it is interesting to investigate the synchronization behavior between the improved neuronal circuits when the effect of electromagnetic induction is considered, and the schematic diagram for the coupled neuronal circuits is plotted in Fig. 3.

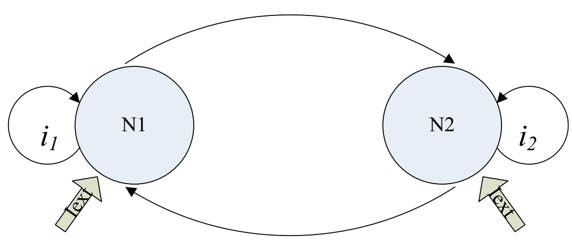

Fig. 3 Schematic diagram for two coupled neurons under electromagnetic induction

It is shown in Fig. 3 that two same neuronal circuits are coupled bidirectionally and driven by the same external forcing currents I_{ext}, while the electromagnetic induction currents $i_1 = k_1\left(\alpha + 3\beta\varphi_1^2\right)x_1$ and $i_2 = k_1\left(\alpha + 3\beta\varphi_2^2\right)x_2$ are considered as well, and the dynamical equations for the coupled neurons are described by

$$\begin{cases} \dot{x}_1 = y_1 - ax_1^3 + bx_1^2 - z_1 + I_{\text{ext}} - k_1(\alpha + 3\beta\varphi_1^2)x_1 \\ \qquad + D(x_2 - x_1) \\ \dot{y}_1 = c - dx_1^2 - y_1 \\ \dot{z}_1 = r[s(x_1 + 1.6) - z_1] \\ \dot{\varphi}_1 = x_1 - k_2\varphi_1 \\ \dot{x}_2 = y_2 - ax_2^3 + bx_2^2 - z_2 + I_{\text{ext}} - k_1(\alpha + 3\beta\varphi_2^2)x_2 \\ \qquad + D(x_1 - x_2) \\ \dot{y}_2 = c - dx_2^2 - y_2 \\ \dot{z}_2 = r[s(x_2 + 1.6) - z_2] \\ \dot{\varphi}_2 = x_2 - k_2\varphi_2 \end{cases} \tag{3}$$

where the subscript 1, 2 marks the neuron N1 and N2, D is the coupling intensity between neurons, a positive selection for D generates negative feedback and stabilization for each neuron, while negative selection for D can induce positive feedback for neurons, and it is interesting to detect the synchronization behaviors in this case.

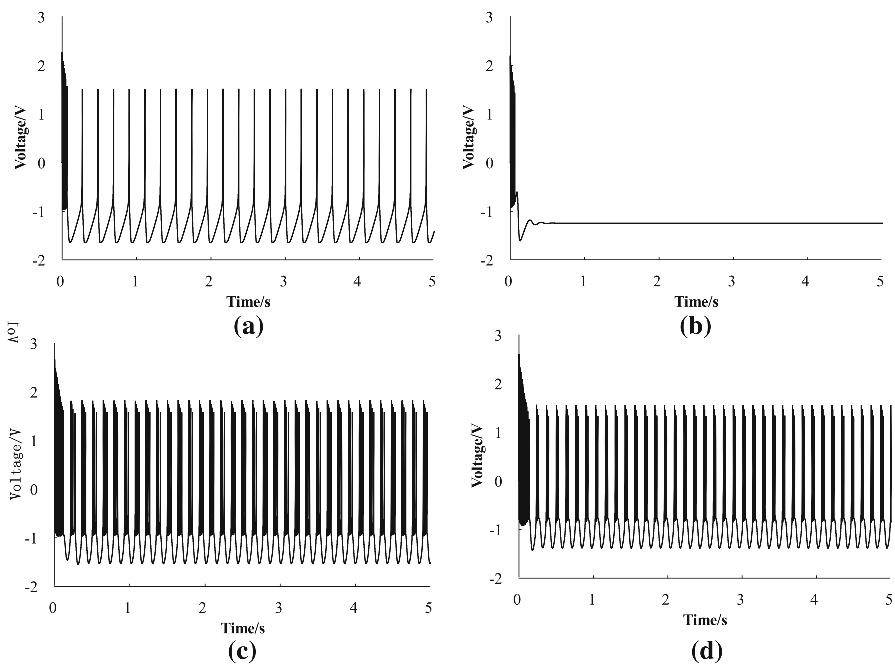

Fig. 4 Outputs for membrane potentials under external forcing with Spice. For **a** $I_{\text{ext}} = 130\mu A$, no electromagnetic induction is considered; **b** $I_{\text{ext}} = 130\mu A$ the effect of electromagnetic induction is considered; **c** $I_{\text{ext}} = 250\mu A$ no electromagnetic induction is considered; **d** $I_{\text{ext}} = 250\mu A$ the effect of electromagnetic induction is considered

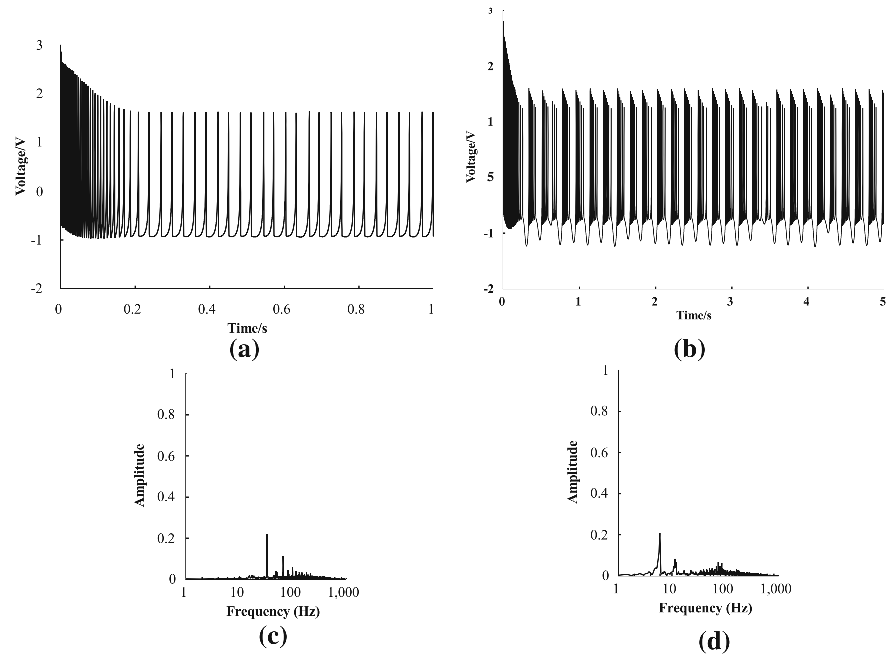

Fig. 5 Sampled time series for membrane potential and power spectrum at external forcing current $I_{\text{ext}} = 350\mu A$: for (**a**), (**c**), no electromagnetic induction is considered; for (**b**), (**d**), electromagnetic induction is considered

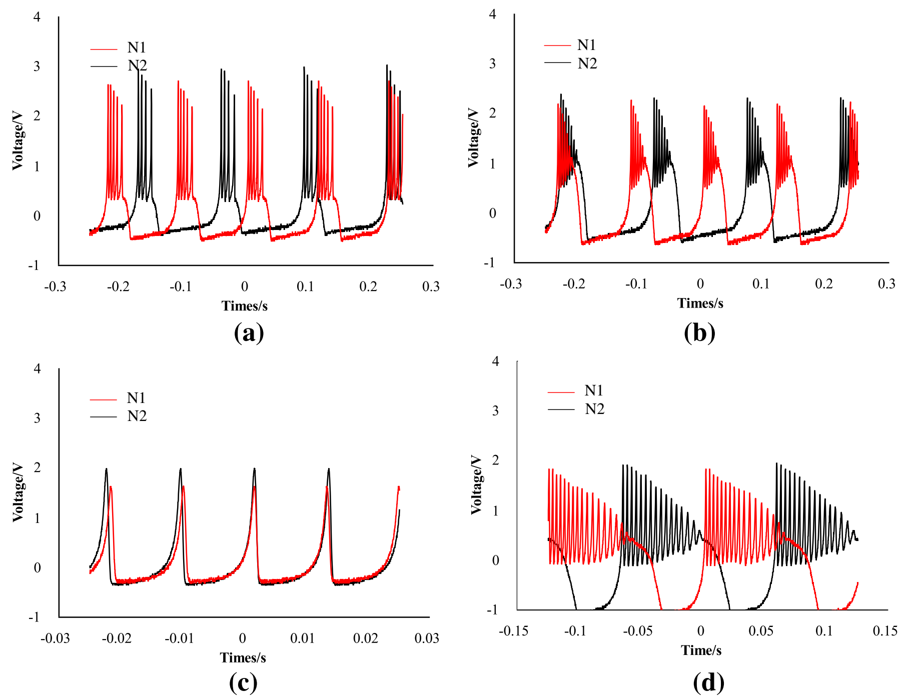

Fig. 6 Outputs for membrane potentials under external forcing with Spice. For **a** $I_{\text{ext}} = 250\mu A$ no electromagnetic induction is considered; **b** $I_{\text{ext}} = 250\mu A$ the effect of electromagnetic induction is considered; **c** $I_{\text{ext}} = 500\mu A$ no electromagnetic induction is considered; **d** $I_{\text{ext}} = 500\mu A$ the effect of electromagnetic induction is considered. The results are produced from two isolate PCBs

3 Experimental verification and discussion

At first, the Spice tool is used to check the outputs response and power spectrum by applying different external forcing currents. For example, the external forcing current is set as $I_{\text{ext}} = 130\mu A$, $I_{\text{ext}} = 250\mu A$, and the outputs for membrane potential are recorded in Fig. 4.

The results in Fig. 4a confirmed that distinct spiking can be induced by applying weak external forc-

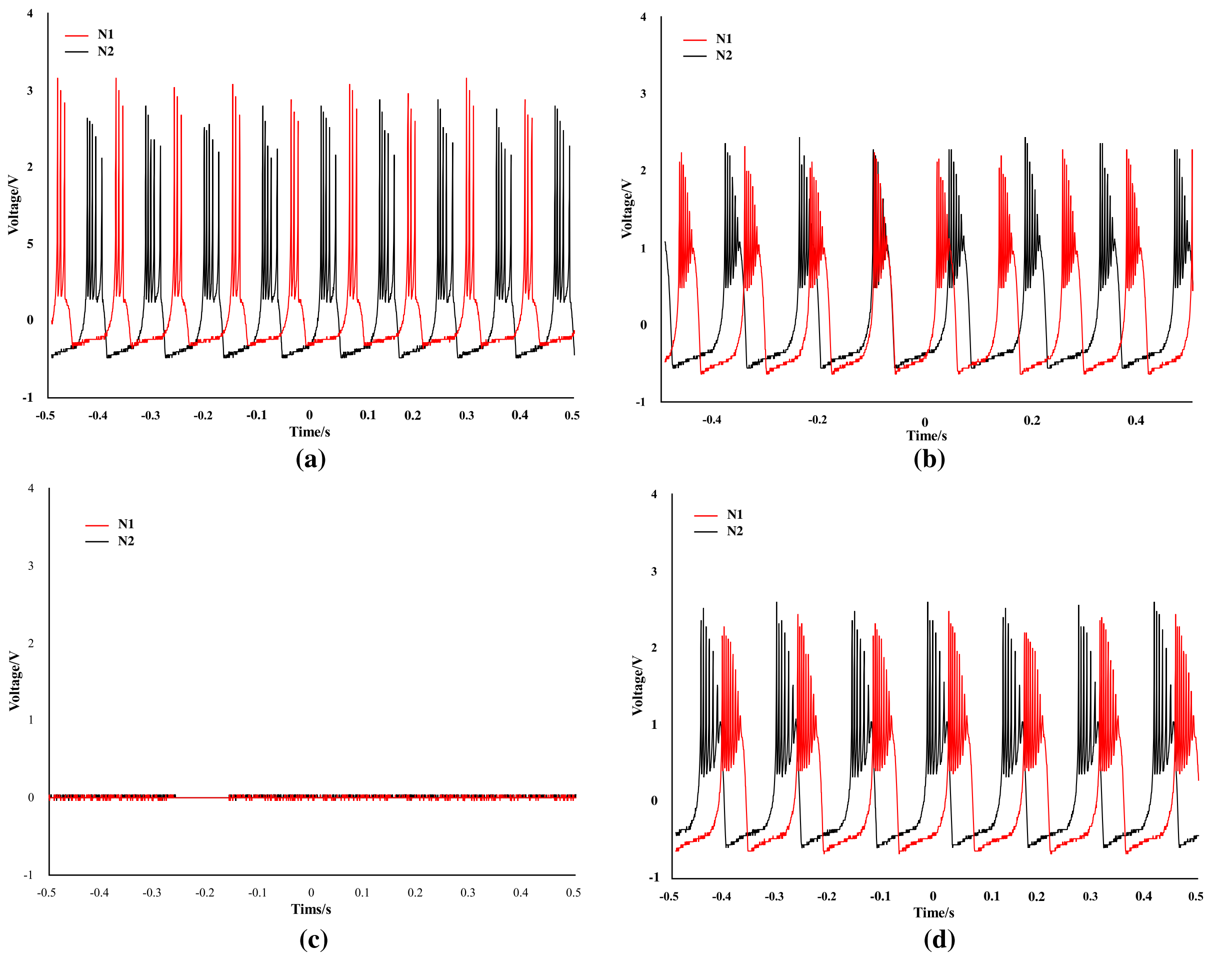

Fig. 7 Response of electrical activities is verified on PCBs for two neurons under bidirectional coupling. For **a** $D = -0.25$, without electromagnetic induction; **b** $D = -0.25$, electromagnetic induction is considered; **c** $D = -1.17$, without electromagnetic induction; **d** $D = -1.17$, electromagnetic induction is considered. The external forcing current is set as $I_{\text{ext}} = 250\mu A$

ing current without electromagnetic induction being considered, and the effect of electromagnetic induction in Fig. 4b can calm down the neuron to become quiescent state. Extensive investigation on Spice found that the quiescent potential and the excited threshold will be increased when the effect of electromagnetic induction is considered, the potential mechanism that more energy is required because some energy has to be stored for further release. The results in Fig. 4c, d show that the effect of electromagnetic induction can be suppressed by increasing the external forcing current to intermediate level, which can supply more energy for the neuronal circuit. Then, the external forcing current is increased, and the results are illustrated in Fig. 5.

It is found in Fig. 5 that the electromagnetic induction shows great impact on the mode of electrical activities by applying stronger external forcing current and bursting states can be enhanced; otherwise, its electrical activities keep the previous spiking state. Figure 5c confirms that the spectrum of discharge frequency can mainly be detected close to γ waveband (30–60 Hz) without electromagnetic induction being considered in the presence of strong external forcing current. However, the spectrum of discharge frequency is decreased to close the α waveband (8–10 Hz) and power distribution in continuous waveband (40–120 Hz). That is to say, the dynamical response becomes complex, and the power spectrum in electrical activities of neuron is extended with larger waveband that memory effect is

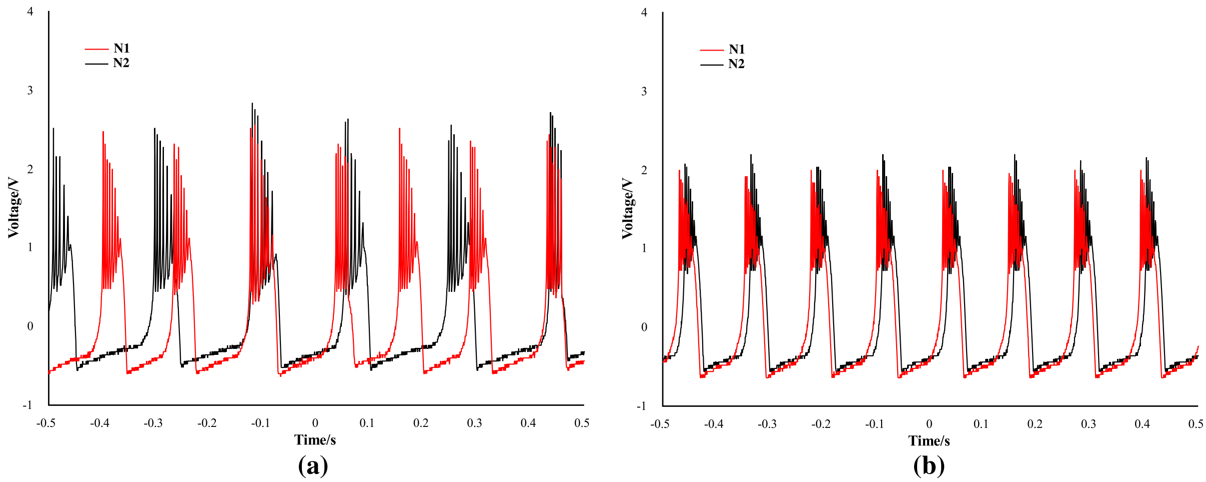

Fig. 8 Response of electrical activities is verified on PCBs for two neurons under bidirectional coupling. For **a** $D = .17$, without electromagnetic induction; **b** $D = .17$, electromagnetic induction is considered, and the external forcing current is set as $I_{\text{ext}} = 250\mu A$

enhanced. Furthermore, this scheme is investigated on printed circuit board (PCB) to analyze the outputs and dynamical response in the membrane potentials of two isolate neurons (without coupling), initial values are triggered in random wave, and the results are shown in Fig. 6.

The results presented the experimental outputs from PCB for two isolate neurons according to Eq. (1), operational amplifier TL08X and multiplier AD633 are used to set the neuronal circuit with and without electromagnetic induction being considered, and the external forcing current is selected by $I_{\text{ext}} = 250\mu\text{A}$ and $I_{\text{ext}} = 500\mu\text{A}$, respectively. The sampled signals from two different channels of oscilloscopes are recorded and confirmed the consistence with the results from Spice. That is to say, intermediate forcing current in intensity seems to disable the electromagnetic induction that shows weak impact on electrical activities. However, the electrical activities can be greatly modulated and changed by electromagnetic induction by applying stronger external forcing current on the neuronal circuit.

It is also interesting to discuss the synchronization between two neuronal circuits under bidirectional coupling. The PCB circuits are set according to Eq. (2) to detect the synchronization degree under different coupling intensities, and results are verified for $I_{\text{ext}} = 250\mu\text{A}$ in Fig. 7, the coupling intensity D is selected by negative value, and the response of electrical activities is recorded.

It is known that positive feedback can excite and enhance the oscillating behavior of neurons and oscillators. For example, positive feedback can make quiescent neuron become spiking and even bursting. Figure 7a, b confirms that positive feedback coupling cannot realize bursting synchronization when the coupling intensity is small. Then, the coupling intensity is increased, Fig. 7c confirms that the electrical activities are suppressed, and these phenomena can be much different from the previous works that amplitude of membrane potential is increased. Within Fig. 7d, it is important to find that the two neuronal circuits can reach phase synchronization with certain time delay that could be associated with the initial selection in diversity. Furthermore, negative feedback coupling is also investigated on PCBs, and the results are shown in Fig. 8.

It is found that the synchronization degree and behavior show much difference even the same coupling intensity is applied. Negative feedback coupling is effective to stabilize the synchronization, and the effect of electromagnetic induction seems to play important role in enhancing the synchronization between neuronal circuits due to the memory of magnetic field and flux.

4 Conclusions

The intracellular and extracellular ion concentrations of cells are time-varying, it can induce the complex electromagnetic field, and the membrane potential of neurons can be greatly changed. As a result, it is important to detect the effect of electromagnetic induction on neuronal activities and even the synchronization behaviors of coupled neurons. In most of the previous works, the model setting for neuronal activities used to focus on the transmembrane current generated by exchanges of charged ions, while the effect of electromagnetic induction is left out. As a result, magnetic flux [19,34,35] is suggested to describe the effect of electromagnetic induction, and further, electromagnetic radiation on mode transition of electrical activities (even the relevant disease induced by electromagnetic radiation) can be measured and understood. In dynamical view, mode selection and physical mechanism can be discussed on the model proposed in Refs. [34] even the effect of ion channel can be further discussed. It is more important to discuss this problem on realistic neuronal circuits, such as dynamical response and synchronization approaching in neuronal circuits when the effect of electromagnetic induction is considered completely. The results on Spice and PCBs in this paper confirmed that the electrical activities and synchronization between coupled neurons and circuits can be modulated by electromagnetic flux and the field, and the numerical results can be easily carried out by using the proposed model. Therefore, the scheme and discussion could be helpful for further investigation on synchronization problems and emergence of neuronal disease in physical mechanism, and researchers can further investigate this problem on network of neuron and large- scale integrated circuit.

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