Synchronization in time-varying networks

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We study the stability of the synchronized state in time-varying complex networks using the concept of basin stability, which is a nonlocal and nonlinear measure of stability that can be easily applied to high-dimensional systems [P. J. Menck, J. Heitzig, N. Marwan, and J. Kurths, Nature Phys. **9**, 89 (2013)]. The time-varying character is included by stochastically rewiring each link with the average frequency f. We find that the time taken to reach synchronization is lowered and the stability range of the synchronized state increases considerably in dynamic networks. Further we uncover that small-world networks are much more sensitive to link changes than random ones, with the time-varying character of the network having a significant effect at much lower rewiring frequencies. At very high rewiring frequencies, random networks perform better than small-world networks and the synchronized state is stable over a much wider window of coupling strengths. Lastly we show that the stability range of the synchronized state may be quite different for small and large perturbations, and so the linear stability analysis and the basin stability criterion provide complementary indicators of stability.

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I. INTRODUCTION

Complex networks have been extensively studied over the last few decades as they provide us a framework for modeling many physical, biological, social, and engineering systems [1–4]. Collective phenomena emerging on a network are primarily determined by the dynamics of the nodes and the interactions among them. Synchronization of dynamical units at nodes has particularly attracted researchers from diverse fields such as biology, ecology, sociology, power grids, climatology, etc. [5–9]. Initially many of them assumed the interactions among the nodes to be invariant over time, though lately there have been efforts to incorporate a time-varying nature of the interactions leading to evolving networks. In one way such time variations represent the evolution of interactions over time. In another way they can be helpful in representing discontinuities in interactions, i.e., when the nodes interact only for limited time. Such time-varying interactions are commonly found in social networks, communication, biological systems, spread of epidemics, computer networks, world wide web, etc., and have been shown to result in significantly different emergent phenomena [10–27].

Major advances have been made in the analysis of such time-varying networks and it has been shown that if connections change quite rapidly, then the network can be essentially modeled as the aggregate of the interactions over time [10,28]. It was also shown that if the Laplacian matrices at different times do not commute, the spread of transverse Lyapunov exponents decreases and for coupled Rössler oscillators the stability range of the time-varying case was found to be larger than that of the time-averaged case [11]. When the time period of variation of links is close to the time period of nodal dynamics, then new stable synchronized states may appear [17]. Very recently, similar results have been found in temporal networks, i.e., where only a single edge exists at one particular instant of time [29]. Long lasting interactions slowed down diffusion in such networks and the slow eigenmodes of the effective Laplacian matrix were shown to be affected more as compared to fast eigenmodes [25].

Many earlier approaches have studied the stability of the synchronized state by linearizing the dynamical equations [30,31]. Such approaches have enabled the analysis of stability of large class of synchronized oscillators. However, there have been studies [32] where local stability predictions do not corroborate with the actual dynamical response of the system. Jost *et al.* [33] have proved that linear stability provides conditions for stability of a synchronized solution that are necessary but not sufficient. Detailed studies of these cases reveal that local stability results can only be valid for small perturbations, and here small could actually be infinitesimal in some cases. Thus to correctly predict the dynamical response to any kind of perturbation, one should have a clear idea about the complete landscape of the coupled system. By complete landscape we mean that one should know the size of basin of attraction [34,35] for all local minimas present in the system. In this regard, Menck *et al.* [36,37] propounded the concept of basin stability (BS) based on the volume of basin of attraction and showed that linear stability and BS may be quite different and both approaches should be considered to evaluate the stability of the synchronized state.

In this work, we study the stability of the synchronized state when the underlying connection network evolves in time. The BS paradigm is particularly useful in case of time-varying networks as it can be applied to a very large class of systems, whereas the linear stability analysis can be done exactly only in some specific cases. For this reason, most of the previous studies have considered some special switching schemes such as very fast rewiring [10,28], on-off coupling [17,24], temporal networks [29], or a particular class of local dynamics

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[15]. Furthermore, linear stability and basin stability can be different in case of time-varying networks. Here, we consider Watts-Strogatz (WS) networks and vary the fraction of random links p to cover a broad range of networks varying from a regular ring topology for $p = 0$ to random networks for $p = 1$. For intermediate values of p, such networks are characterized by small path length and high clustering coefficient, typically referred to as small-world networks (SW). The time-varying character is considered by assuming that each link rewires with a rewiring frequency f. We discuss our model in the next section and present BS results in Sec. III. The linear stability analysis for fast rewiring case is shown in Sec. IV. In Sec. V, we study stability of the synchronized state in the on-off coupling model and show how results from linear stability analysis and BS may differ when the system is subjected to large perturbations. We present our conclusions in the last section.

II. MODEL

We begin by describing our link rewiring method. In our model each link in the network rewires stochastically and independently of the other links with an average frequency f. Specifically we consider ensembles of WS networks consisting of N -coupled Rössler oscillators, in which the dynamics at a node i is given by:

$$\begin{aligned} \dot{x_i} &= -y_i - z_i - c\sum_{j=1}^{N} L_{ij}x_j \\ \dot{y_i} &= x_i + a_1 y_i \\ \dot{z_i} &= a_2 + z_i(x_i - a_3), \end{aligned} \tag{1}$$

where c is a coupling constant, L the Laplacian matrix, and the parameters $a_1 = b_1 = 0.2$, and $a_3 = 7.0$. The Laplacian matrix is obtained from the adjacency matrix A whose elements $A_{ij} = 1$ if nodes i and j are connected and 0 otherwise. The diagonal elements of Laplacian matrix are sum of the corresponding rows of the adjacency matrix, i.e., if $i = j$, $L_{ij} = \sum_{j=1}^{N} A_{ij}$. For $i \neq j$, $L_{ij} = -A_{ij}$ and hence L_{ij} is zero row sum matrix. For the given set of parameter values, each uncoupled Rössler oscillator has a chaotic trajectory and the synchronous state corresponds to the case when all oscillators follow the same trajectory. To construct a WS network, we start with a regular ring in which each node is connected to its $2k$ nearest neighbors, k on either side. Then we rewire each link with probability p by cutting it from any one side and joining to some randomly selected distant node. Next at any time t, the link is rewired with probability $f\,dt$ where dt is the integration time step. If the edge is between two distant neighbors, it is rewired to a nearest neighbor of one of the nodes with a probability $1 - p$. Similarly if the edge is between two nearest neighbors, then with probability p, it is broken from one of the nodes and connected to a random distant node, and not rewired otherwise. Thus on average, pkN links couple distant neighbors and $(1 - p)kN$ links couple nearest neighbors and the rewiring represents the rate at which nearest neighbor links become random or vice versa. Whenever an edge is selected for rewiring, the node from which it will be cut is chosen at random. The nodes are initialized with random values, $x \in [-15 : 15], y \in [-15 : 15], z \in [-5 : 35]$. These values roughly correspond to the size of the chaotic attractor. We simulate the system for 10^6 time steps with an integration time step of 0.01, i.e., for a total time of 10 000 and check the final state of the system. We have checked the results for consistency for integration time steps up to 10^{-5}. There are three possibilities: (i) the system synchronizes, (ii) it does not synchronize, and (iii) it diverges. To determine whether a system is synchronized or not, we consider the differences $x_i - x_j, y_i - y_j, z_i - z_j$ for all pairs $i, j \in [1 : N]$, $(i \neq j)$ and call the final state as synchronized if each of these quantities is less than some threshold (taken to be 0.0001 in our representative results). If at some instant, any of $|x|$, $|y|$, or $|z|$ exceeds 1000, the system is assumed to be diverging. In all other cases, if till the end of 10^6 time steps, the system neither diverges nor synchronizes, then it is taken to be unsynchronized. BS is given as the average of the number of synchronized initial conditions for various values of coupling strengths chosen from a sufficiently large interval.

III. RESULTS

We simulate the above system for various fractions of random links p and coupling strengths c and further average out the results over a number of initial conditions. For every set of parameter values, we simulate our system for 500 different initial conditions. A new network is generated for every initial condition and the nodes are initialized with random values in the interval $x \in [-15 : 15], y \in [-15 : 15], z \in [-5 : 35]$. We vary the rewiring frequency f from low to high and calculate the percentage of initial conditions that arrive at the synchronized state. Here large f means that the edges are rewiring very quickly whereas a low value of f implies that the network is almost static.

Figure 1 shows the fraction of initial conditions arriving at the synchronous state for different f, c, p, and k. It can be seen that the range of coupling strength over which the synchronous state is stable increases considerably as the rewiring frequency f increases. Also, it is easily inferred that when the network approaches the global limit ($k \to N$), the change in connections will not affect the dynamics, while

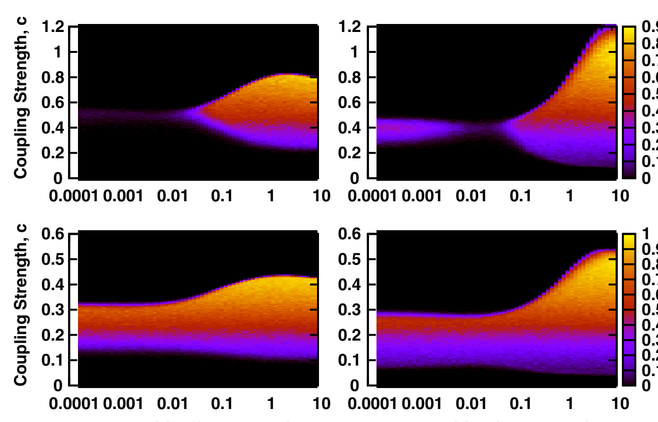

FIG. 1. (Color online) Color (grayscale) indicates the fraction of initial conditions arriving at the synchronized state for various rewiring frequencies f and coupling strengths c. Fraction of random links p is 0.2 (left) and 0.8 (right) and number of nearest neighbors on each side k is 2 (top) and 4 (bottom).

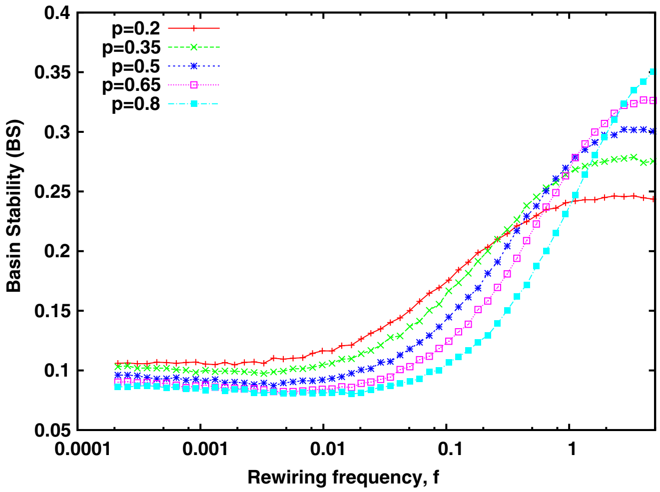

FIG. 2. (Color online) The fraction of initial conditions which arrive at the synchronized state for different rewiring frequencies averaged over different coupling strengths ($0 < c < 0.6$ for $k = 4$). The corresponding values of BS for the averaged networks are 0.24, 0.28, 0.31, 0.34, 0.37 for $p = 0.2, 0.35, 0.5, 0.65, 0.8$, respectively.

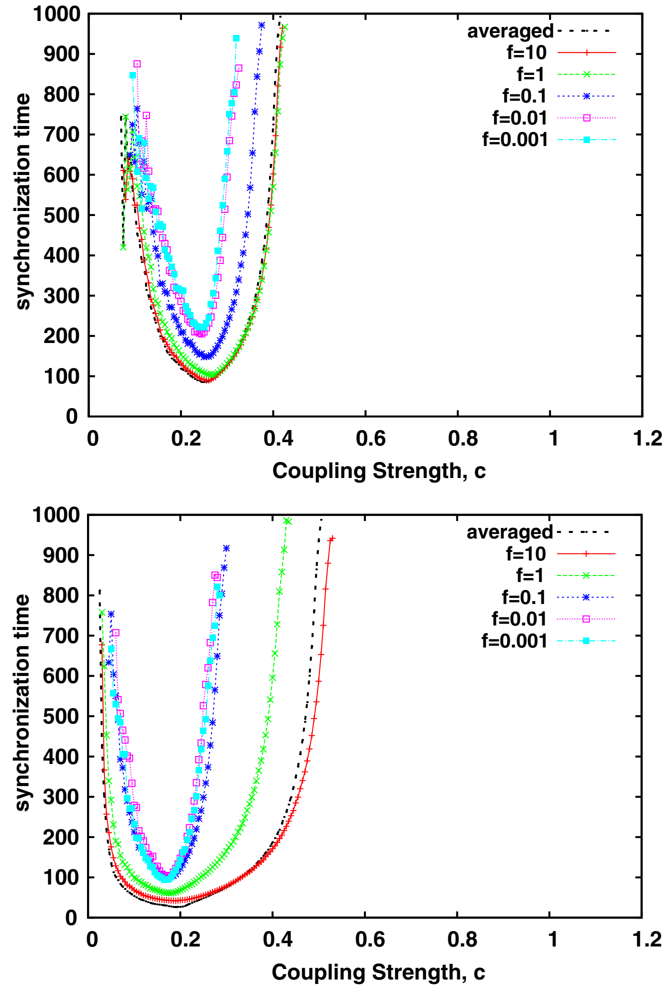

FIG. 3. (Color online) Time taken to reach the synchronized state for various coupling strengths, and rewiring frequencies at $p = 0.2$ (top) and $p = 0.8$ (bottom) for $k = 4$. It can be seen that fast rewiring networks take much lower time to reach the synchronized state.

the effects of time-varying links will be most pronounced in networks with lower number of neighbors k.

Further, notice that while the range of synchronization is largest when the fraction of random links p is high, the time-varying nature of the links starts to affect networks with low p at lower rewiring frequencies. In this sense networks with low p are more sensitive to dynamic connections. To further illustrate this point, we calculate the BS of the network for fixed p and f (see Fig. 2) [38]. When the rewiring frequency is close to 0, i.e., the network is almost static, BS is higher for low values of p as reported in Ref. [36]. As the rewiring frequency increases, BS for low p values rises rapidly, whereas the rise in BS for high p values is slower, indicating that BS for SW networks (low p) approaches to that of fast rewiring networks even for slowly rewiring networks, whereas a much higher rewiring frequency is needed for random (high p) networks to reach the level of fast rewiring networks. Notice also that at very fast rewiring times the basin stability is larger for networks with more spatial randomness, i.e., high p. So for networks where the connections change infrequently SW networks are more significantly affected than completely random networks, while for networks that change rapidly, random networks yield larger synchronization regions than SW networks.

The time taken to reach the synchronized state is plotted in Fig. 3 and it can be seen that more randomness (i.e., higher p) and higher rewiring frequency result in lower synchronization times. In Fig. 1 we observe that the fraction of initial conditions arriving at the synchronized state decreases for intermediate values of rewiring frequencies in the $k = 2$ case. This is so because in this range of rewiring frequencies, the time taken to arrive at the synchronized state increases (see Fig. 4) and many initial conditions neither synchronize nor go to infinity, but continue to stay in the unsynchronized state for long times. These time scales correspond to the transition from static to dynamic behavior. For frequencies lower than these, the time-varying character of the networks is lost and the dynamics is essentially that of static networks. Note that in the on-off coupling model discussed in Ref. [17], the dynamics can be determined by Lyapunov exponents for similar time scales (relating $f \sim 1/T$).

In Fig. 4 we also notice that the synchronous state in fast rewiring networks is stable for coupling strengths larger than the averaged case. This is more pronounced for large p (0.8 here). When a network rewires with a large f, the Laplacian matrix changes rapidly and many of these Laplacian matrices do not commute and hence the stability range is larger than the averaged case [11]. Further note that as the number of neighbors increases, this effect is reduced and the averaged case and fast rewiring case converge. This can be explained on the grounds that the Laplacian matrix is sparse in case of very few neighbors and so chances of consecutive matrices being noncommutative are higher.

To get further insights, we plot the fraction of initial conditions that synchronize as a function of the coupling strength and rewiring frequency in Fig. 5. We see that this fraction primarily depends on the coupling strength, with randomness p or rewiring frequency having little effect.

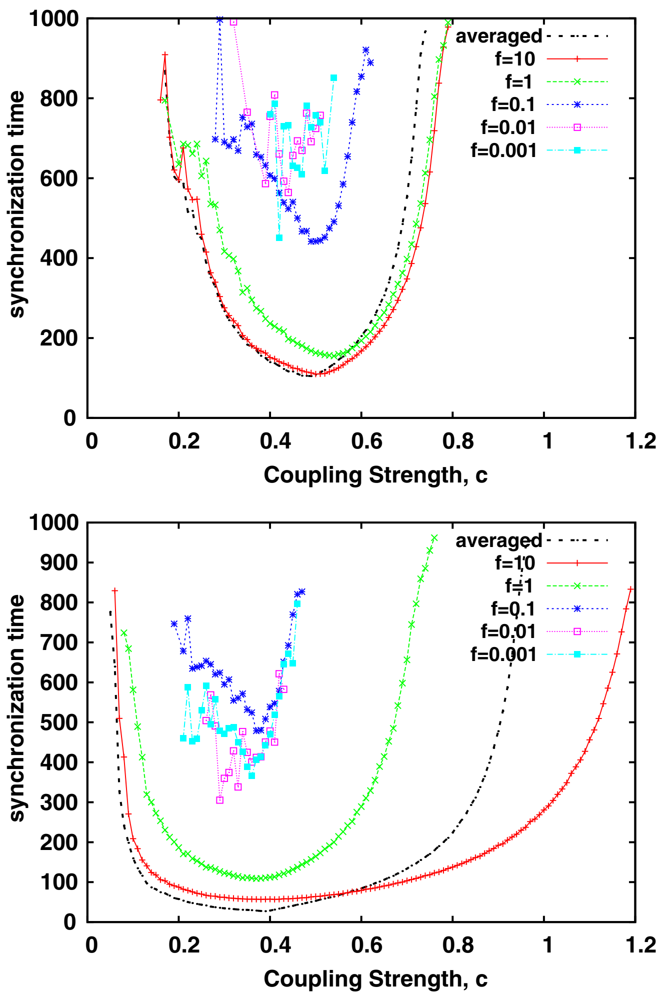

FIG. 4. (Color online) Time taken to reach the synchronized state for various coupling strengths, and rewiring frequencies at $p = 0.2$ (top) and $p = 0.8$ (bottom) for $k = 2$. It can be seen that synchronization time increases for intermediate values of rewiring frequencies $f \sim 0.1$ for $p = 0.8$.

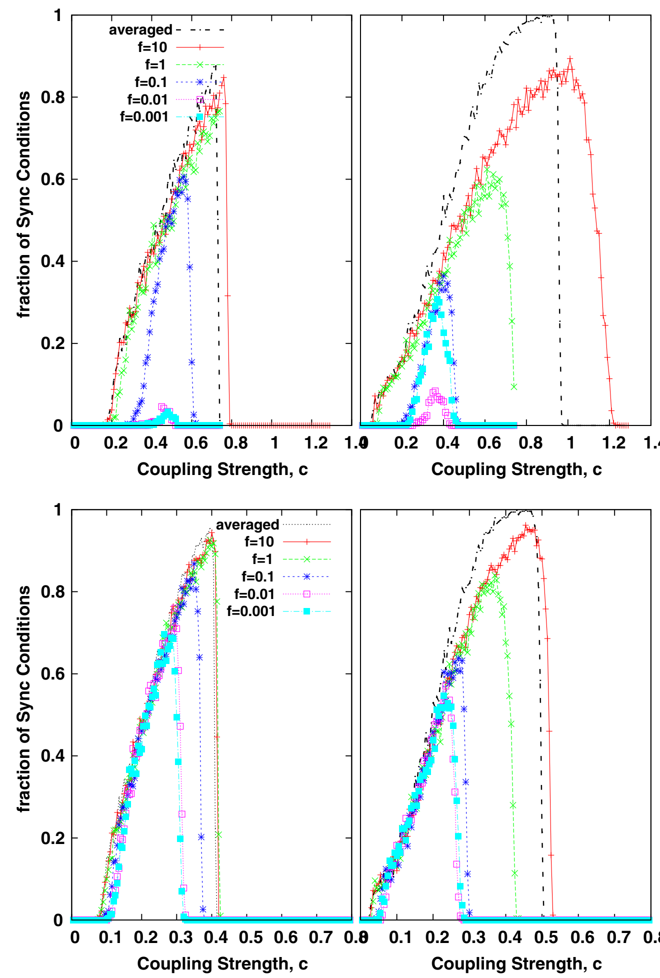

FIG. 5. (Color online) Fraction of initial conditions arriving at the synchronized state for $k = 2$ (top) and $k = 4$ (bottom) at various rewiring frequencies and $p = 0.2$ (left) and $p = 0.8$ (right). One can see that while the percentage of initial conditions arriving at synchronized state for a fixed coupling strength is larger for averaged network, the fast rewiring networks are stable for a larger range of coupling strengths.

IV. LINEAR STABILITY ANALYSIS

It was reported in Refs. [10,17,28] that a time-varying network can be approximated by the time-averaged network for sufficiently fast rewirings. In our case, for the time-averaged network the entries in the Laplacian matrix will be $-(1-p)$ for nearest neighbors and $-2kp/(N-2k-1)$ for distant nodes. Now we follow the master stability function (MSF) approach [30,31] and find the coupling range for which the synchronous state is locally stable. Our system can be written as:

$$\dot{X}_i = F(X_i) + c\sum_{j=1}^{N} A_{ij}[H(X_j) - H(X_i)] \tag{2}$$

$$= F(X_i) - c\sum_{j=1}^{N} L_{ij}H(X_j), \tag{3}$$

where $\boldsymbol{X_i} = (x_i, y_i, z_i)$ is the three-dimensional state vector of dynamical variables of the ith node. $F(X)$ is the functional form describing the dynamics of an isolated node. c is the coupling strength and A is the adjacency matrix such that $A_{ij} = 1$ if nodes i and j are connected and 0 otherwise. H defines the functional form of coupling. In our case, the oscillators are linearly coupled through their x component, so H is simply a 3×3 matrix with $H_{11} = x$ and $H_{ij} = 0$ otherwise. L_{ij} is the Laplacian matrix. The synchronization manifold is defined by $N-1$ constraints $X_1 = X_2 = \cdots = X_N$. The variational equation of this system is given by

$$\dot{\xi} = [1_N \otimes D\mathbf{F} + cL \otimes D\mathbf{H}]\xi, \tag{4}$$

where $\xi = (\xi_1, \xi_2 \ldots \xi_N)$ with ξ_i being the perturbation of the i-th node and $D\mathbf{F}$ and $D\mathbf{H}$ are the Jacobian functions. Upon block diagonalization this equation reduces to

$$\dot{\xi}_k = [D\mathbf{F} + c\gamma_k D\mathbf{H}]\xi_k, \tag{5}$$

where γ_k are the eigenvalues of L. $k = 0$ gives the variational equation of the synchronization manifold. For the synchronous state to be stable, perturbations along all the transverse modes must die out or the Lyapunov exponents of Eq. (5) must

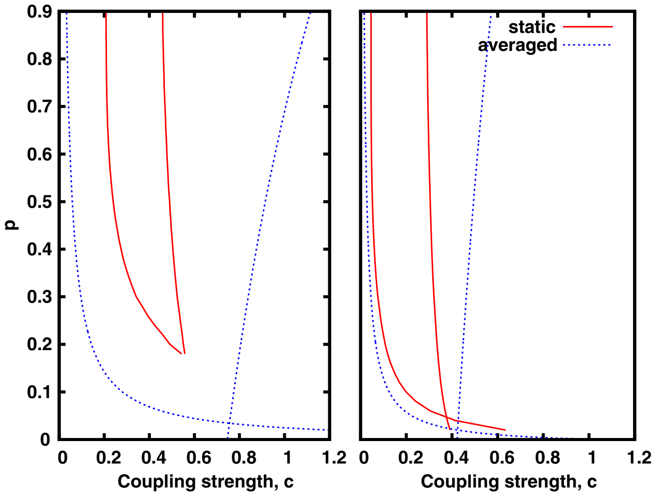

FIG. 6. (Color online) The region between solid red (dotted blue) corresponds to the range of coupling strengths for which the synchronized state is stable in the static (averaged) network for that particular value of SW rewiring probability p calculated by following the MSF approach for $k = 2$ (left) and $k = 4$ (right). For the static case, the values have been obtained by averaging over 10 000 different networks.

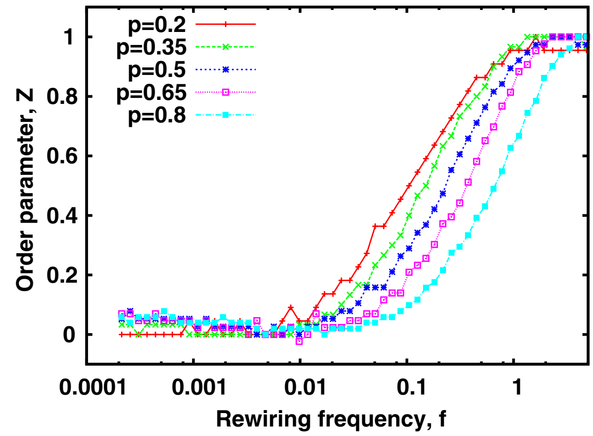

FIG. 7. (Color online) Order parameter Z as a function of rewiring frequency f. One can see that order parameter for low p values increases at lower values of rewiring frequencies.

be negative for $k > 0$. In our example this is true if $\alpha_1 < c\gamma_k < \alpha_2$ where $\alpha_1 = 0.1232$ and $\alpha_2 = 4.663$. In other words, the synchronous state is stable against local perturbations if the coupling strength is chosen from the interval $c \in (\alpha_1/\gamma_{\min},\alpha_2/\gamma_{\max})$ where $\gamma_{\min}$ and $\gamma_{\max}$ are the minimum and maximum non-zero eigenvalues of the Laplacian matrix.

In Fig. 6, we plot the range of the coupling strength for which the synchronized state is stable according to the MSF approach. We see that the coupling range for which the synchronous state is stable widens considerably for the averaged network as compared to the static one. Further, the difference between static and averaged cases is maximum when the number of neighbors is smallest. All these results are consistent with those obtained using BS (see Fig. 1).

To understand how the transition from the static case to time-averaged case depends on the rewiring frequency, we define an order parameter Z as,

$$Z = \left(c_{tv}^{\max} - c_s^{\max}\right)/\left(c^{\max} - c_s^{\max}\right), \tag{6}$$

where $c_{tv}^{\max}$ is the maximum coupling strength for which even a single initial condition settles at the synchronized state for that particular rewiring frequency f and $c_s^{\max}$ is the coupling strength for which the synchronized state is stable in case of a static network. $c^{\max}$ is the maximum value of $c_{tv}^{\max}$. For static networks, its value is 0 and for fast varying networks it is close to 1. The variation of this order parameter with the rewiring frequency shows that the transition from static case to time-averaged case is approximately linear with the log of the rewiring frequency (Fig. 7). Further, as the SW parameter p increases, the critical value of the rewiring frequency at which Z starts increasing, also increases. Note that the maximum value of the coupling strength for which the synchronized state is stable is $\alpha_2/\lambda_{\min}$ from the MSF approach. So the maximum coupling strength up to which the synchronous state is stable is directly related to the largest nonzero eigenvalue of the effective Laplacian matrix. Recently it was shown that in case of temporal networks [25], the effect of time scale of interactions is least on fast eigenmodes. In this case, as the maximum eigenvalue of the time-averaged SW networks is more than the maximum eigenvalue of time-averaged random networks, we can say that the eigenmodes of the time-averaged SW networks are faster as compared to eigenmodes of the time-averaged random networks. So the effect of rewiring is less on eigenmodes of time averaged SW networks as compared to those of random networks. Thus the time-averaged character can be retained even at slower rewirings for the case of SW networks.

V. STABILITY IN THE ON-OFF MODEL

In this section we investigate another model of time-varying networks in which time-varying character is implemented by switching the interactions on or off. Specifically, we study the on-off coupling model [17], in order to gain further insights into the power of the basin stability approach in time-varying networks. In this model, the network is switched on if $nT < t < (n+\theta)T$ and off if $(n+\theta)T < t < (n+1)T$ for $n = 0,1,2,\ldots$ and $0 \leqslant \theta \leqslant 1$. $\theta = 0$ implies that the network is always off and all nodes are isolated, whereas if $\theta = 1$, the nodes are always connected. For other values of θ the connections become on and off with time period T. It was shown in Ref. [17] that the time scale T is of critical importance for the network dynamics. If T is very small, the stability of the synchronized state can be predicted by a static time-averaged coupling. For very large T, stability can be explained by Lyapunov exponents. When T is of the order of the time scale of nodal dynamics, not only the region of stability increases but the time taken to reach the synchronized state also decreases. It was shown that for intermediate values of T, the traditional bound for synchronization due to short-wavelength bifurcations (SWBs) disappears and more stable regions emerge. This analysis, based on linear stability of the synchronous state is valid only for small perturbations. To understand whether the linear stability analysis can accurately predict the stability for large perturbations also, we calculate the BS (calculated as the number of initial conditions that

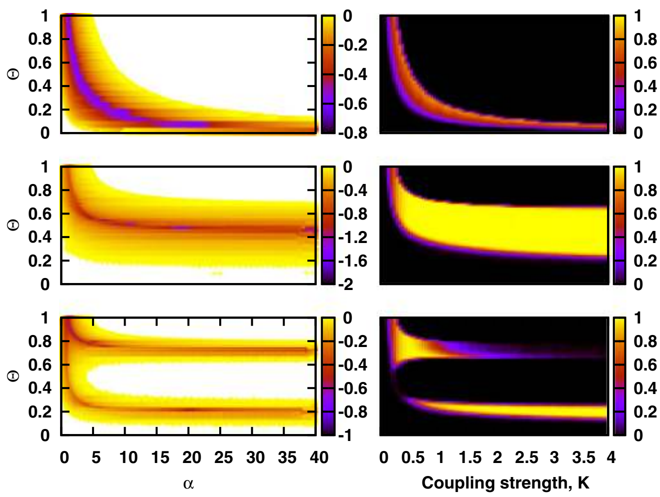

FIG. 8. (Color online) Maximum Lyapunov exponent (left) for the on-off coupling model obtained from linear stability analysis for $T = 0.1$ (top), $T = 3$ (middle), and $T = 6$ (bottom). Colored regions correspond to cases when maximum Lyapunov exponent is negative, i.e. the synchronized state is stable. The panels on right show the BS of WS networks for $p = 0.2$ and $k = 4$ for the same values of T as in the left panels. The results are qualitatively same for other values of p also.

arrive at the synchronous state) for an on-off coupling model by taking Rössler oscillators on WS networks. The results are shown in Fig. 8. As can be seen in the left panels, the linear stability indicated by largest Lyapunov exponent rises and falls gradually with increasing coupling strength, whereas the BS (right panels) rises smoothly for low values of coupling and then drops sharply. Further, in the bottom panel we see that whereas linear stability analysis predicts that the upper bound for synchronous state disappears, from the BS we find that it disappears only for the synchronous state corresponding to low θ values. For the synchronous state corresponding to high θ values, the BS approaches zero for high coupling strengths. Furthermore, the BS is much higher for intermediate time scales. These results indicate that for time-varying networks linear stability analysis alone is not sufficient and we need to calculate the BS also to accurately predict the stability of the synchronized state.

VI. CONCLUSION

We have studied the stability of a synchronous state in dynamic WS networks. In line with previous results [10,28], we have found that for sufficiently fast rewirings, time-varying networks can be approximated by static time-averaged networks. To the best of our knowledge, till now there is no method of finding out how fast the rewiring should be in order to consider it sufficiently fast. Using the BS framework, we have been able to estimate the rewiring frequency at which the network can be approximated by the static time average. Further we have gotten insights into how the transition from a static to a time-averaged case takes place. We have shown how the stability range changes at different rewiring time scales. Our central result from the extensive numerical simulations is that in case of SW networks the transition to the time-averaged case occurs at a much slower link rewiring frequency compared to random networks. We have found that not only the BS of SW networks is highest in static cases as reported earlier, but they approach the time-averaged coupling case fastest. That is, their BS approaches that of the time-averaged case even for very slow rewiring time periods. It has been observed that if the links are rewiring rapidly, random networks yield larger synchronization regions than SW networks. Further, we have found that the impact of rewiring is maximum when the number of neighbors is less. Lastly, faster rewiring networks have been uncovered to synchronize quickly.

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