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Synchronization of an uncertain unified chaotic system via adaptive control

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Abstract

A new adaptive control method is proposed for adaptive synchronization of two uncertain chaotic systems, using a specific uncertain unified chaotic model as an example for illustration.

1. Introduction

Since the study of chaos synchronization by Pecora and Carroll in 1990, this topic has received increasing attention [1–3]. Chaos synchronization can be applied to many areas such as secure communication, chemical reactions, biological systems, and information processing [1].

Chaos synchronization can be classified into two types – mutual synchronization and master–slave synchronization – according to their coupling configuration. Over the last decade, a large variety of approaches have been proposed for chaos synchronization such as the master–slave method [4], backstepping design method [5,6], impulsive control method [7], and invariant manifold method [3], among many others [1,8].

In this paper, a new approach combining both parameters identification and chaos synchronization is proposed, which works for a large class of uncertain chaotic systems. This method provides a detailed design process for chaos synchronization, even for uncertain systems. An uncertain unified chaotic system is used as an example for easier and detailed description of the method, on which simulation will also be performed.

2. Adaptive synchronization of the uncertain unified chaotic system

Given two identical chaotic systems, one is used as the master system and another the slave system, the task of the master–slave synchronization is to force the response of the slave system to synchronize with the master system, where the slave system received driving signals from the master system [3]. This paper studies this problem, but for uncertain chaotic systems.

2.1. The problem formulation

More precisely, we consider the drive chaotic system in the form of

$$\dot{x} = f(x) + F(x)\theta, \tag{1}$$

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where $x \in R^n$ is the state vector of the system, $\theta \in R^m$ is the parameter vector of the system, $f \in C^1(R^n, R^n)$ and $F \in C^1(R^n, R^{n\times m})$ are nonlinear functions. On the other hand, the response system is given by

$$\dot{u} = f(u) + F(u)\alpha, \tag{2}$$

which has the same structure as the drive system but the parameter vector $\alpha \in R^m$ is completely unknown, or uncertain.

In practice, the output signals of the drive system (1) can be received by the response system (2), but the parameter vector of the drive system (2) may not be known to the receiver. Therefore, the goal of control is to design and implement an appropriate controller U for the slave system, such that the controlled response system

$$\dot{u} = f(u) + F(u)\alpha + U, \tag{3}$$

can synchronize with the drive system (1).

The methodology to be described below works for a large class of chaotic systems, including the Duffing oscillator, Van der Pol oscillator, Rössler system, and several variants of Chua's circuits. Since a description of the general case is rather messy and uneasy, at least notationally, to facilitate the description and discussion, we use a specific chaotic system as an example. This system is still not too special, in the sense that it unifies both the Lorenz [2] and the Chen systems [9,10], where the latter is the *dual system* of the former in a sense defined in [11]: The Lorenz system satisfies the condition $a_{12}a_{21} > 0$ while the Chen system satisfies $a_{12}a_{21} < 0$, where $A = [a_{ij}]_{3\times 3}$ are the matrix of the linear part of the chaotic systems. Recently, Lü et al. [12–14] found a unified chaotic system that not only includes the case of $a_{12}a_{21} = 0$ [12] but also the Lorenz and Chen systems as two extreme cases. This unified system will be used in this paper for the study of synchronization, as further discussed in the following.

2.2. Synchronization of uncertain unified system

Consider the unified chaotic systems described by [14]

$$\begin{aligned} \dot{x} &= (25\theta + 10)(y - x), \\ \dot{y} &= (28 - 35\theta)x - xz + (29\theta - 1)y, \\ \dot{z} &= xy - \frac{8+\theta}{3}z, \end{aligned} \tag{4}$$

where $\theta \in [0,1]$. Obviously, when $\theta = 0$ it is the Lorenz system, while when $\theta = 1$ it is the Chen system. What is interesting is that, as θ changes continuously from 0 to 1, the resulting system remains continuously to be chaotic.

In this study, we first rewrite it in the form of system (1), where

$$f(\mathbf{x}) = \begin{pmatrix} 10(y-x) \\ 28x - xz - y \\ xy - \frac{8}{3}z \end{pmatrix}.$$

$$F(\mathbf{x}) = \begin{pmatrix} 25(y-x) \\ -35x + 29y \\ -\frac{z}{3} \end{pmatrix}\theta.$$

Then, we design an adaptive controller for the response system in the form of (4) where, however, all the system parameters are assumed to be unknown or uncertain therefore need to be identified.

Assume that the response states are $(u, v, w)^{\mathrm{T}}$, and let the state tracking error be $e = (u - x, v - y, w - z)^{\mathrm{T}}$. Then, if the system parameter θ is known, the error dynamics is

$$\begin{aligned} \dot{e}_1 &= (25\theta + 10)(e_2 - e_1) + u_1, \\ \dot{e}_2 &= (28 - 35\theta)e_1 + (29\theta - 1)e_2 - we_1 - ue_3 + e_1e_3 + u_2, \\ \dot{e}_3 &= -\tfrac{\theta+8}{3}e_3 + ue_2 + ve_1 - e_1e_2 + u_3, \end{aligned} \tag{5}$$

where $U = (u_1, u_2, u_3)$ is the controller to be designed for the response system.

Construct a Lyapunov function of the form

$$V(e) = \frac{1}{2}(e_1^2 + e_2^2 + e_3^2).$$

Then its derivative along the solution of (5) is

$$\frac{\mathrm{d}V}{\mathrm{d}t} = e_1((25\theta+10)(e_2+e_1)+u_1) + e_2((28-35\theta)e_1 + (29\theta-1)e_2 - we_1 - ue_3 + u_2) + e_3\left(-\tfrac{8+\theta}{3}e_3 + ue_2 + ve_1 + u_3\right).$$

Select

$$u_1 = -ye_3 + ze_2 + 25\theta e_1, \quad u_2 = -29\theta e_2 + (10\theta - 38)e_1, \quad u_3 = \frac{\theta}{3}e_3.$$

Then

$$\frac{\mathrm{d}V}{\mathrm{d}t} = -10e_1^2 - e_2^2 - \frac{8}{3}e_3^2 \leqslant -2V(e).$$

Therefore, $V(e) \leqslant V(0)\mathrm{e}^{-2t}$, implying synchronization of the derive–response systems.

When the parameter θ of the system (2) is unknown, and in this case it will be denoted by α, we do not have the above synchronization result. In this case, we have to design an adaptive controller instead.

Here, we propose to design the following adaptive controller for the intended synchronization:

$$\begin{aligned} \dot{u} &= (25\alpha(t)+10)(v-u) + u_1, \\ \dot{v} &= (28 - 35\alpha(t))u - uw + (29\alpha(t)-1)v + u_2, \\ \dot{w} &= uv - \frac{8+\alpha(t)}{3}w + u_3 \end{aligned} \tag{6}$$

in which

$$\dot{\alpha}(t) = -F^{\mathrm{T}}(x)(\mathrm{grad}V(e))^{\mathrm{T}} = 25(x-y)e_1 + (35x - 29y)e_2 + \frac{ze_3}{3}, \tag{7}$$

where

$$u_1 = -ye_3 + ze_2 + 25\alpha(t)e_1, \quad u_2 = -29\alpha(t)e_2 + (10\alpha(t) - 38)e_1, \quad u_3 = \frac{\alpha(t)}{3}e_3.$$

This adaptive controller works. Indeed, according to the drive system (1) and the controlled response system (3), we have the following error dynamical system:

$$\dot{e} = f(u) - f(x) + F(u)\alpha - F(x)\theta + U. \tag{8}$$

For this system, we can use the Lyapunov function

$$V_1(e,\alpha) = V(e) + \frac{1}{2}(\alpha-\theta)^{\mathrm{T}}(\alpha-\theta).$$

Its derivative along the solution of system (3) satisfies

$$\begin{aligned} \frac{\mathrm{d}V_1}{\mathrm{d}t} &= (\mathrm{grad}V(e), f(u) - f(x) + F(u)\alpha - F(x)\theta + U) + \dot{\alpha}^{\mathrm{T}}(\alpha-\theta) \\ &= (\mathrm{grad}V(e), f(u) - f(x) + F(u)\alpha - F(x)\alpha + U) + (\mathrm{grad}V(e), F(x)(\alpha-\theta)) + \dot{\alpha}^{\mathrm{T}}(\alpha-\theta) \\ &\leqslant -2V(e) = W(e). \end{aligned}$$

Since $V_1(e,\alpha)$ is a positive definite functional and $\mathrm{d}V_1/\mathrm{d}t$ is a negative semi-definite functional, it follows that the equilibrium points $e=0$ and $\alpha=0$ of the systems (7) and (8) are uniformly asymptotically stable. Hence, $e(t)$ and $\alpha(t)$ are bounded on the interval $(0,+\infty)$.

Furthermore, let

$$\overline{\lim_{t\to\infty}} e(t) = \bar{e}, \ \overline{\lim_{t\to\infty}} \alpha(t) = \bar{a}.$$

If $\bar{e} \neq 0$, then there exist two positive constants, δ, ε, such that $\|e(t) - \bar{e}\| < \delta$, which implies that $W(e) > \varepsilon$. By the definition of upper limit, there exists a sequence $\{t_n\} \subset R^+$ such that $(e(t_n), \alpha(t_n)) \to (\bar{e}, \bar{\alpha})$ as $n \to \infty$. Let n^* be the integer such that $\|e(t_n) - \bar{e}\| < \frac{1}{2}\delta$ for any $n > n^*$. Then, by the continuity of $V_1(e,\alpha)$, for sufficiently large n, we have

$$V_1(e(t_n), \alpha(t_n)) - V_1(\bar{e}, \bar{\alpha}) < \frac{\varepsilon\delta}{4r}, \tag{9}$$

where $\varepsilon\delta/4r$ is a constant, which is chosen such that on $(t_n, t_n + \frac{\delta}{3r})$ we have both $\|e(t) - \bar{e}\| < \delta$ and $W(e) > \varepsilon$. Therefore,

$$V_1(e(t_n), \alpha(t_n)) - V_1(\bar{e}, \bar{\alpha}) \geqslant \int_{t_n}^{t_n+\frac{\delta}{3r}} W(e)\mathrm{d}t > \frac{\varepsilon\delta}{3r}. \tag{10}$$

This is a contradiction, which implies that $\overline{\lim}_{t\to\infty} e(t) = 0$. Similarly, we have $\underline{\lim}_{t\to\infty} e(t) = 0$. Thus, $\lim_{t\to\infty} e(t) = 0$. Therefore, the controlled response system (6) is synchronizing with the drive system (4), and satisfies that $\lim_{t\to+\infty} \|\alpha(t) - \theta\| = 0$.

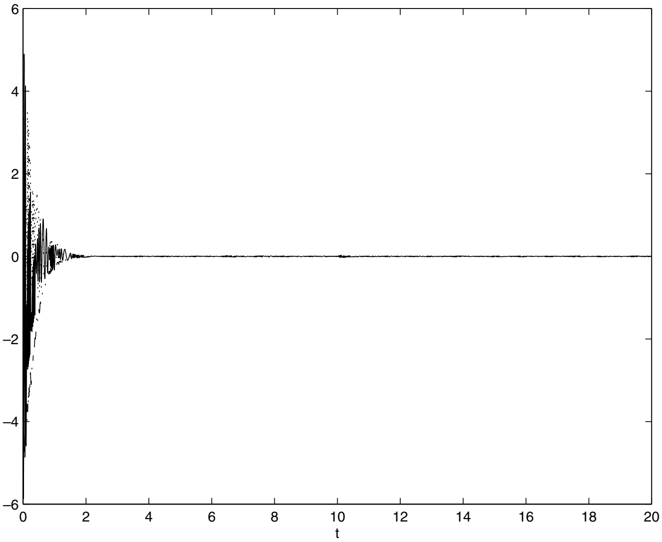

Fig. 1. Graph of synchronization errors versus time t. Solid line: $e_1(t) = u(t) - x(t)$; dotted line: $e_2(t) = v(t) - y(t)$; dashdotted line: $e_3(t) = w(t) - z(t)$.

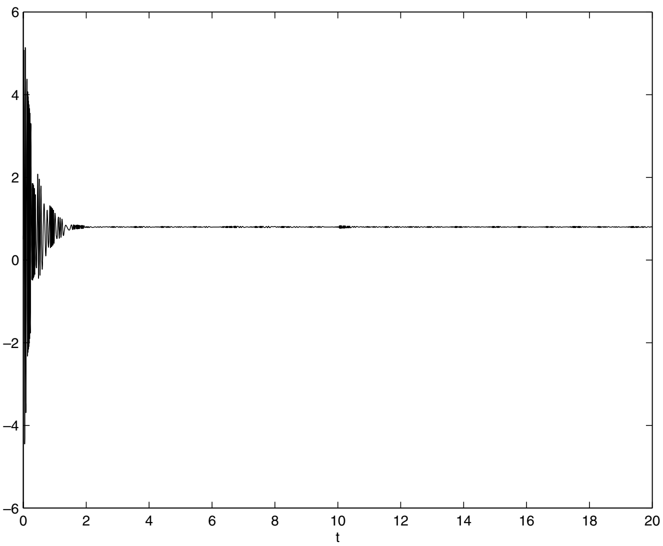

Fig. 2. Graph of the parameter identification result.

2.3. Numerical simulation

In this simulation, the parameter $\theta = 0.8$, for which the drive system (4) is chaotic [14]. The initial conditions of the drive system and the controlled system are set to be $(8.0, 9.0, 10.0)^{\mathrm{T}}$ and $(3.0, 4.0, 5.0)^{\mathrm{T}}$, respectively, and the unknown parameter $\alpha(t)$ has zero initial condition.

The results of parameters identification and adaptive synchronization are shown in Figs. 1 and 2, respectively. Obviously, the numerical simulations verify the theoretical analysis.

3. Conclusion

An effective method for adaptive synchronization of uncertain chaotic systems has been provided in this paper. This approach can be used for a large class of chaotic systems, including the Duffing oscillator, Van der Pol oscillator, Rössler system, and several variants of Chua's circuits, to name just a few.

Using a unified chaotic system as an example, both theoretical proof and numerical simulations of the proposed method have been carried out, which demonstrates the effectiveness and feasibility of the proposed adaptive synchronization method. It should be noted that the parameter's identification method proposed in this paper is an online estimation process, therefore it should have a wide spectrum of practical applications to uncertain and complex systems synchronization.

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