Synchronization of Mutually Coupled Self-Mixing Modulated Lasers

Kenju Otsuka and Ryoji Kawai
Department of Applied Physics, Tokai University, 1117 Kitakaname, Hiratsuka, Kanagawa 259-1292 Japan

Siao-Lung Hwong, Jing-Yuan Ko, and Jyh-Long Chern
Nonlinear Science Group, Department of Physics, National Cheng Kung University, Tainan, Taiwan 70101, Republic of China
(Received 20 October 1999)

Synchronization of mutually coupled chaotic lasers has been demonstrated in a microchip $LiNdP_4O_{12}$ laser array with self-mixing feedback modulation. An abrupt transition to synchronized chaos by way of "phase squeezing" was observed when coupling between the two lasers was increased. This phenomenon is well reproduced by numerical calculations using model equations. It is also shown that low energy variation as well as high disorder are concurrently established in synchronized chaos.

PACS numbers: 05.45.Xt, 42.50.Lc, 42.55.Sa, 42.65.Sf

A recent study in nonlinear dynamics has revealed that two, or more than two, chaotic systems can be synchronized when they are coupled appropriately [1,2]. This interesting phenomenon plays a key role in the chaotic dynamics of communication signals and may be applied to the real-time recovery of signals which have been masked in a chaotic background and thus to encoded communications [3]. In the context of interacting chaotic oscillators, there are different interpretations of the term "synchronization," such as master-slave synchronization [2] and the synchronization based on mutually coupled oscillators [4,5]. Synchronization based on generalized functional dependence between two different variables has also been included to reveal the dynamical correlations [6]. On the other hand, a related chaos synchronization phenomenon can be developed in terms of a suitably defined "phase" of a chaotic oscillator [7]. Chaotic amplitude fluctuation usually triggers a diffusion of phase. In the phase synchronization state, amplitudes of coupled oscillators remain chaotic but their phases are in step with a common timing characteristic. Phase synchronization may be an important consideration in schemes for communication using the natural symbolic dynamics of chaos [8]. The clock timing of information bits is typically a key factor in a communication system, so, a termination of phase diffusion is crucial in an application to encoded communication. Although chaos synchronization has been demonstrated in electronic systems [2,9], including single-chip systems [10] as well as laser systems [3,5,11,12], the role of phase in chaos synchronization has not yet been clarified and more experimental schemes are desired to find potential applications to signal processings.

In this Letter, experimental results on synchronized chaos in mutually coupled lasers subjected to delayed self-mixing laser-Doppler-velocimetry (SMLDV) modulations [13] are reported. An abrupt transition from asynchronous chaos to synchronous chaos via a "phase-squeezed state" has been observed. Coupled laser array equations with self-mixing feedback are proposed and observed behaviors have been reproduced successfully.

The experimental system, in which we used an Ar-laser-pumped $LiNdP_4O_{12}$ (LNP) laser, is shown in Fig. 1. The output light was divided into two beams by the beam splitter and was focused on the input surface of the LNP crystal by the common focusing lens. The intensity of pump beam 2 was about 3 times that of beam 1 and was controlled by the variable attenuator. The facets of the plane-parallel 1-mm-thick LNP crystal were directly coated by dielectric mirrors, M_1 (transmission at pump wavelength $\lambda_p = 514.5$ nm: 80%; reflection at lasing wavelength $\lambda_l = 1048$ nm: 99.9%) and M_2 (transmission at λ_l: 1%). The stoichiometric LNP crystal has a Nd concentration which is 30 times higher than Nd:YAG and the absorption length at 514.5 nm was only 400 μm. Consequently, the pumping of laser 2 by the oblique beam 2 resulted in only a slight increase in threshold pump power because of the pump-induced thermal lens effect. The threshold pump power P_{th} for both lasers was 120 mW, and linearly polarized TEM_{00} oscillations were obtained. The

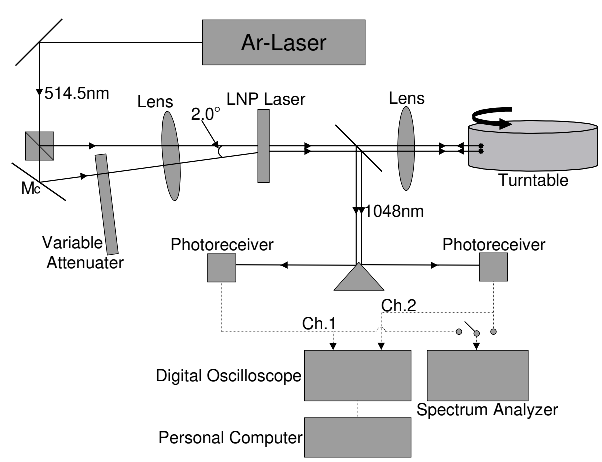

FIG. 1. Experimental setup of a LNP laser array subjected to Doppler-shifted light injections.

0031-9007/00/84(14)/3049(4)$15.00

spatial separation d of the pump positions of laser 1 and 2 was larger than their pump beam spot size ($\simeq 40\ \mu$m) averaged over the absorption length. Therefore, cross-saturation of population inversions did not take place between two lasers. The coupling between the two lasers occurred through the overlap of their lasing fields which each have spot size $w_l \simeq 200\ \mu$m. The coupling coefficient, $|\eta| = \exp(-d^2/2w_l^2)$, is normalized so that $|\eta| = 1$ for $d = 0$ [14]. At the separation of 0.8 mm used in this experiment, $|\eta|$ is estimated to be 3.35×10^{-4}. The separation d was varied precisely by tilting the mirror M_c in Fig. 1. π out-of-phase locking was observed in the present system, so η can be considered to have a negative real value [14]. The two parallel beams from the LNP laser were incident to and scattered from the turntable to modulate themselves by the SMLDV feedback, in which microcavity lasers are loss modulated because of the interference between the lasing and scattered feedback fields, which possess a narrow-band Gaussian frequency distribution, at the Doppler-shift frequency f_D, when f_D is higher than the injection lock-in range of the laser Δf_L [13]. The distance between the LNP laser and the turntable was 50 cm. *The distinct difference between the present system and the pump (or loss) modulation scheme* [5,12] *is that the self-mixing feedback modulation depends upon whether or not the two coupled lasers are locked and on each laser's own intensity. The perturbation to the laser system is unidirectional and no feedback action occurs in the usual pump (or loss) modulation scheme.* The two monitoring beams were converted to electrical signals by two InGaAs photoreceivers (New Focus 1811, bandwidth: 125 MHz) and observed by using a digital oscilloscope (Tektronix 420A, bandwidth: 200 MHz, 100 MS/s sampling). Time series consisting of 30 000 data points were analyzed by using the personal computer. The rf spectrum analyzer (Tektronix 2712) was used to monitor the power spectra of the two lasers. The far-field pattern and intensity profile were measured by using a PbS infrared television. When the distance between beams 1 and 2 was shortened, a two-lobed far-field pattern was observed, indicating π out-of-phase locking of two lasers similar to [14].

The following experiments were carried out in the single-longitudinal-mode oscillation regimes of both lasers. When the modulation frequency f_D was tuned to be near the relaxation oscillation frequency $f_R = (1/2\pi)\sqrt{(w-1)/\tau\tau_p}$, where $w = P/P_{\rm th}$ is the relative pump, τ is the fluorescence lifetime, and τ_p is the photon lifetime, chaotic relaxation oscillations were easily obtained. Synchronization was obtained when the pump power of beam 2 was set so that the relaxation oscillation frequencies of both coupled free-running lasers almost coincided, i.e., for $f_R \cong 600$ kHz. Otherwise, synchronization did not occur. Figure 2(a) is a plot of the correlation in amplitude between the two signals when $d \simeq 1.5$ mm. To examine the phase correlation of the chaotic pulsations, the time interval between the nth peak and the subsequent peak for laser 2, $T_2(n)$, was plotted against that for laser 1, $T_1(n)$, in Fig. 2(b). In this case, asynchronous chaotic fluctuations in both amplitude and phase are apparent, and the two lasers are found to be behaving independently. When the separation between the two lasers was decreased, a mutual interaction appeared and the phase fluctuations of two lasers were squeezed, while their amplitudes remained uncorrelated, just before the onset of chaos synchronization, as shown in Figs. 2(c) and 2(d). We refer to this hereafter as the phase-squeezed state. This phenomenon could be interpreted in terms of a slaving principle [15]: system dynamics is governed by the longer time scales involved in oscillations. Pulsation periods (i.e., phase) are governed by $(\tau\tau_p)^{1/2}$, while the amplitudes are determined by the much shorter time scale of τ_p. Detailed results will be published elsewhere.

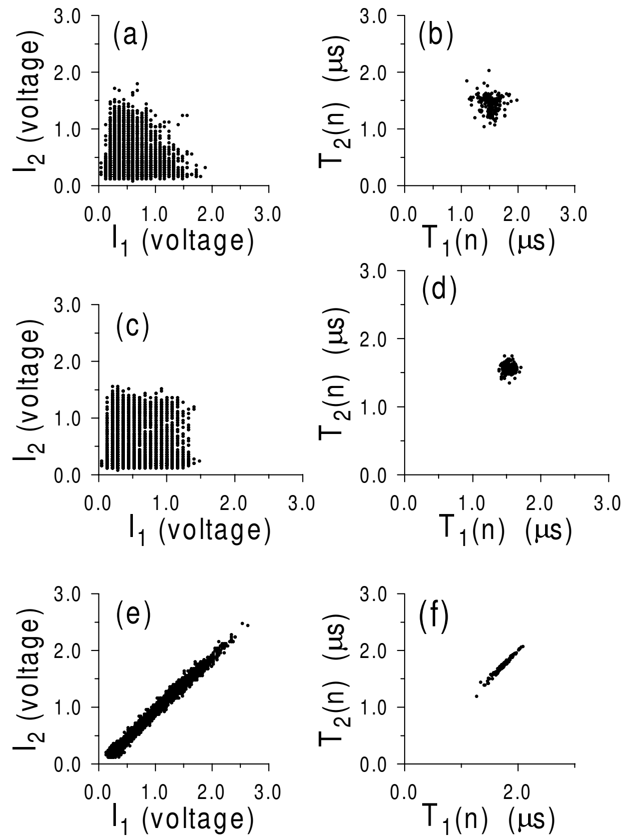

FIG. 2. Signal correlations of the LNP laser array: (a),(b) at weak coupling ($d \simeq 1.5$ mm); (c),(d) at $d \simeq 0.85$ mm; (e),(f) at stronger coupling ($d \simeq 0.8$ mm). (a),(c),(e) are for amplitude correlation, and (b),(d),(f) are for phase correlation.

When d was decreased further, synchronized chaotic states, in which the two lasers exhibited chaotic pulsations with both strong amplitude and phase correlation within allowable errors, were obtained. Results for $d \simeq 0.8$ mm are shown in Figs. 2(e) and 2(f). The nature of synchronized chaos has been found to be qualitatively different from that of asynchronized chaos before the transition by the singular-value-decomposition (SVD) analysis of time series [16].

The following model equations are used to explore the dynamics of coupled lasers subjected to Doppler-shifted light injections:

$$dE_i/dt = N_i E_i + \eta E_{i+1}\cos\theta + m_i E_i(t - t_d)\cos\varphi_i\,, \tag{1}$$

$$d\varphi_i/dt = \Omega_i - \Omega_{s,i} - m_i[E_i(t - t_d)/E_i(t)]\sin\varphi_i\,, \tag{2}$$

$$dN_i/dt = [w_i - 1 - N_i - (1 + 2N_i)E_i^2]/K\,, \tag{3}$$

$$d\theta/dt = \Omega_2 - \Omega_1 - \eta(E_1/E_2 + E_2/E_1)\sin\theta\,, \quad i = 1,2\,, \tag{4}$$

where E_i is the normalized field amplitude ($E_3 = E_1$), N_i is the normalized population inversion, $w_i = P_i/P_{i,\text{th}}$ is the relative pump power, $\theta = \phi_2 - \phi_1$ is the relative phase of two electric fields, $\Omega_i = \omega_i\tau_p$ is the normalized oscillation frequency, $\Omega_{s,i} = \omega_{s,i}\tau_p$ is the normalized frequency of the scattered field, φ_i is the phase difference between the lasing field and the scattered field fed back to the resonator, m_i is the feedback coefficient, $K = \tau_f/\tau_p$, and t and t_d are the time and delay time normalized by τ_p. The present system is more complicated than a simple pump [5,12] or loss modulation and in fact the experimental results are different from those in [5,12]. This may result from the self-mixing feedback effect which we have explained above. An example of correlation analyses of numerically generated synchronized chaos is shown in Fig. 3, assuming $w_{1,2} = 1.05$, frequency detuning of the two lasers $\Omega_2 - \Omega_1 = 10^{-5}$, $\eta = -4 \times 10^{-4}$, $m_{1,2} = 0.005, \Omega_1 - \Omega_{s,1} = \Omega_2 - \Omega_{s,2} = 2\pi \times 10^{-3}$, and $K = 2 \times 10^3$. The delay time of $t_d = 3$ ns was much smaller than the fluctuation time scale, so we assumed $t_d \ll 1$ (a short delay limit). The distinctive feature of synchronized chaos states in the present system is that oscillations of the two chaotic lasers are strongly localized in the vicinity of the "perfect" synchronous state even in the presence of frequency detuning and, as shown in Fig. 3, cannot escape from this state. That is, they are stable *stagnant motions*. It has also been confirmed numerically that phase squeezing occurs when the coupling is reduced, similar to the experimental result shown in Figs. 2(c) and 2(d), as shown below.

Let us show the global feature by varying the coupling coefficient numerically. The degree of synchronization can be characterized by the average errors

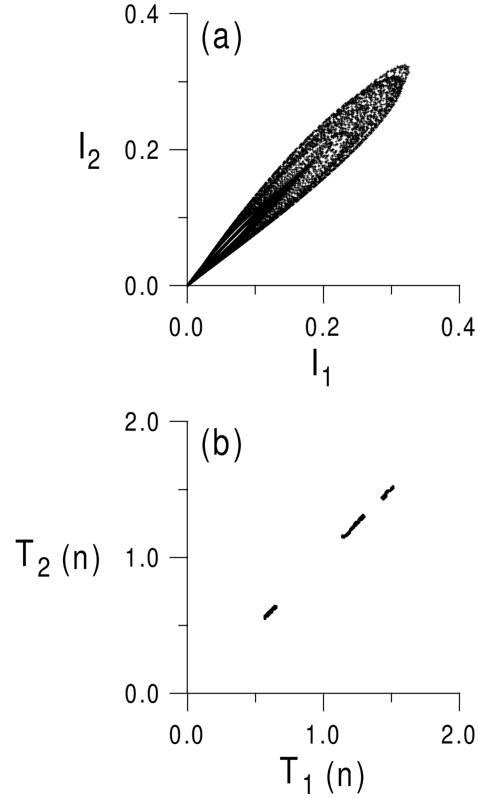

FIG. 3. Numerical example of synchronized chaos where (a) is for amplitude correlation and (b) is for phase correlation. Adopted parameter values are given in the text.

defined as $\langle\varepsilon_I\rangle = \frac{1}{\Delta T}\int_0^{\Delta T} |E_1(t)^2 - E_2(t)^2|\,dt$ for the intensity ($\Delta T \gg 1$) and as $\langle\varepsilon_T\rangle = \frac{1}{M}\sum_{i=1}^{M} |T_1(i) - T_2(i)|$ for the phase ($M \gg 1$). The degree of phase squeezing can be identified by the average standard deviation of phase. It is defined as $\delta_T = \frac{1}{2}\sum_{i=1}^{2}\langle(T_i - \langle T_i\rangle)^2\rangle$ where $\langle\cdots\rangle$ denotes the time average. As shown in Fig. 4, transition occurred at the coupling $|\eta| \sim 1.38 \times 10^{-4}$ and phase squeezing took place just before the onset of chaos synchronization. These numerical characterizations parallel the experimental observations. To explore more features around the transition, we calculated the average standard deviation of intensity (i.e., energy), defined as $\delta_I = \frac{1}{2}\sum_{i=1}^{2}\langle(E_i^2(t) - \langle E_i^2\rangle)^2\rangle$, as a function of the coupling. As shown in Fig. 4(b), an abrupt change can be seen. This means that synchronized chaos requires a low variation of energy. We also calculated the average variation of disorder based on the Shannon entropy. That is, we calculated $\delta_H = \frac{1}{2}\sum_{i=1}^{2} H_i$, where $H_i = -\sum_{l=1} P_i(l)\ln P_i(l)$, in which $P_i(l)$ is the probability of the intensity localized within the lth interval during time evolution for laser i ($i = 1,2$) [17]. The result, shown in Fig. 4(b), implies that when the synchronized chaos occurs a larger disorder is established while the mutual information between the two lasers is increased. From these results, it should be pointed out that a lower variation of energy as well as a higher disorder are

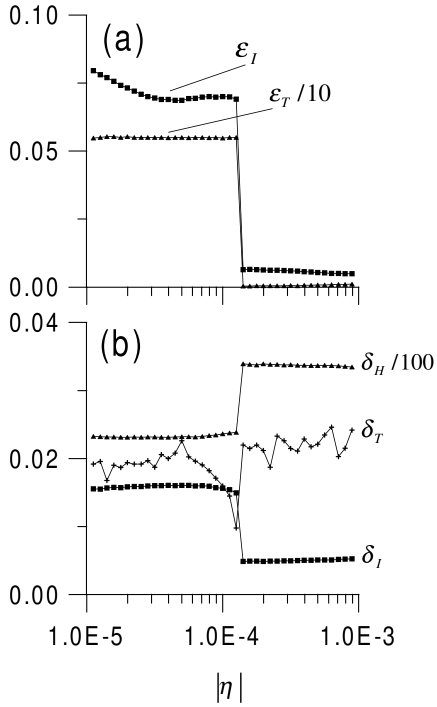

FIG. 4. Variations of dynamical states as a function of coupling $|\eta|$. Adopted parameter values are the same as in Fig. 3. (a) average errors for amplitude ε_I and phase ε_T; (b) average standard deviations of phase δ_T and intensity δ_I, and entropy δ_H.

required for synchronized chaos to be established. This is a significant feature and could be a general characteristic of the synchronization of mutually coupled chaotic oscillators in systems where energy can be defined. At any rate, the nature of synchronized chaos is qualitatively different from that of asynchronized chaos before the transition.

In summary, we experimentally examined synchronous chaos in two coupled microchip lasers subjected to self-mixing feedback. A transition from an asynchronous state to synchronized chaos via a phase-squeezing state has been found by amplitude and phase correlation analysis of long-term experimental time series. Theoretical model equations of coupled laser arrays with frequency-shifted feedback have been proposed and a key feature of the experimental results has been reproduced numerically.

This research was supported in part by the Monbusho International Scientific Research Program: Joint Research 10044175. The work of the NCKU group is partially supported by the National Science Council, Taiwan, under the Project No. NSC89-2112-M-006-023.

[1] A. V. Gaponov-Grekhov, M. I. Rabinovich, and I. M. Starobinets, Pis'ma v Zh. Eksp. Teor. Fiz. **39**, 561 (1984).

[2] L. M. Pecora and T. L. Carroll, Phys. Rev. Lett. **64**, 821 (1990); L. M. Pecora and T. L. Carroll, Phys. Rev. A **44**, 2374 (1991).

[3] G. D. VanWiggeren and R. Roy, Science **279**, 1198 (1998), and references therein.

[4] H. G. Winful and L. Rahman, Phys. Rev. Lett. **65**, 1575 (1990); J.-L. Chern and J. K. McIver, Phys. Lett. A **151**, 150 (1990).

[5] R. Roy and K. S. Thornburg, Jr., Phys. Rev. Lett. **72**, 2009 (1994).

[6] N. F. Rulkov, M. M. Sushchik, L. Sh. Tsimring, and H. D. I. Abarbanel, Phys. Rev. E **51**, 980 (1995).

[7] M. G. Rosenblum, A. S. Pikovsky, and J. Kurths, Phys. Rev. Lett. **76**, 1804 (1996).

[8] S. Hayes, C. Grebogi, E. Ott, and A. Mark, Phys. Rev. Lett. **73**, 1781 (1994); E. Rosa, Jr., S. Hayes, and C. Grebogi, Phys. Rev. Lett. **78**, 1247 (1997).

[9] K. M. Cuomo and A. V. Oppenheim, Phys. Rev. Lett. **71**, 65 (1993).

[10] J.-L. Chern, T.-C. Hsiao, J.-S. Lih, L.-E. Li, and K. Otsuka, Chin. J. Phys. **36**, 667 (1998).

[11] T. Sugawara, M. Tachikawa, T. Tsukamoto, and T. Shimizu, Phys. Rev. Lett. **72**, 3502 (1994).

[12] A. Uchida, M. Shinozuka, T. Ogawa, and F. Kannari, Opt. Lett. **24**, 890 (1999).

[13] K. Otsuka, Appl. Opt. **33**, 1111 (1994).

[14] L. Fabiny, P. Colet, R. Roy, and D. Lenstra, Phys. Rev. A **47**, 4287 (1993).

[15] H. Haken, in *Encyclopedia of Physics* edited by L. Genzel (Springer, Heidelberg, 1970).

[16] J.-S. Lih, J.-Y. Ko, J.-L. Chern, and I.-M. Jiang, Europhys. Lett. **40**, 7 (1997).

[17] H.-J. Li and J.-L. Chern, Phys. Rev. E **52**, 297 (1995), and references therein.