

Received April 9, 2019, accepted May 21, 2019, date of publication June 14, 2019, date of current version July 2, 2019.

Digital Object Identifier 10.1109/ACCESS.2019.2923166

Synthesis and Optimization of Fractional-Order Elements Using a Genetic Algorithm

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Research described in this paper was supported by the King Abdullah University of Science and Technology, Saudi Arabia, the National Sustainability Program under grant LO1401, the Ministry of Education, and the Czech Science Foundation under grant no. 19-24585S. For the research, infrastructure of the SIX Center was used.

ABSTRACT This study proposes a new approach for the optimization of phase and magnitude responses of fractional-order capacitive and inductive elements based on the mixed integer-order genetic algorithm (GA), over a bandwidth of four-decade, and operating up to 1 GHz with a low phase error of approximately $\pm 1^\circ$. It provides a phase optimization in the desired bandwidth with minimal branch number and avoids the use of negative component values, and any complex mathematical analysis. Standardized, IEC 60063 compliant commercially available passive component values are used; hence, no correction on passive elements is required. To the best knowledge of the authors, this approach is proposed for the first time in the literature. As validation, we present numerical simulations using MATLAB[®] and experimental measurement results, in particular, the Foster-II and Valsa structures with five branches for precise and/or high-frequency applications. Indeed, the results demonstrate excellent performance and significant improvements over the Oustaloup approximation, the Valsa recursive algorithm, and the continued fraction expansion and the adaptability of the GA-based design with five different types of distributed RC/RL network.

INDEX TERMS Cauer network, constant phase element, continued fraction expansion, distributed RC network, distributed RL network, Foster network, fractional-order capacitor, fractional-order element, fractional-order inductor, genetic algorithm, impedance optimization, phase optimization, RC network, RL network, recursive algorithm, Valsa network.

I. INTRODUCTION

Recently, tremendous efforts have been made to design fractional-order elements (FOEs). Indeed they offer additional degrees of freedom and versatility in electrical circuits [1]–[4], such as filters [5]–[7], oscillators [8]–[11], controllers [12]–[15], bioimpedance modeling [16], lithium-ion battery modeling [17], transmission line design [18], reluctance inductive transducer realization [19], dc–dc boost converters [20] and references cited therein. The versatility of fractional-order circuits leads researchers to believe that the future of discrete element circuit design will undergo a

paradigm shift in favor of FOEs [21]. Fig. 1 shows the fundamental components and possible FOEs in four quadrants [22], [23]. Their impedance is described as $Z(s) = Ks^\alpha$, where ω is the angular frequency in $s = j\omega$, and obviously has a real part dependent on the non-zero frequency. In particular, the impedance of Type IV FOEs, i.e. fractional-order capacitors (FOCs), is provided with an order of $-1 < \alpha < 0$ and pseudocapacitance of $C_\alpha = 1/K$, whereas fractional-order inductors (FOIs) in quadrant I (Type I) have an order of $0 < \alpha < 1$ and pseudoinductance of $L_\alpha = K$. These two FOEs are the key components in fractional-order circuit design and our main object of investigation in this work. Their characteristics such as pseudocapacitance, pseudoinductance, constant phase zone (CPZ), constant phase angle (CPA – defined phase

The associate editor coordinating the review of this manuscript and approving it for publication was Hisao Ishibuchi.

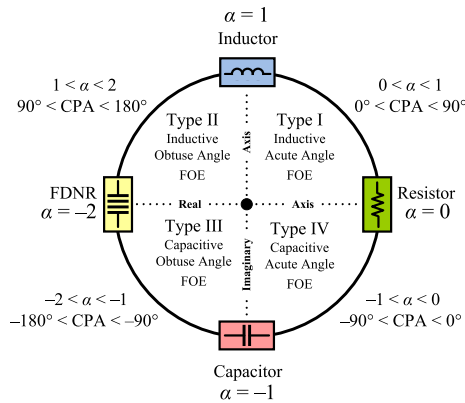


FIGURE 1. Description of fractional-order elements in four quadrants [22], [23].

angle in CPZ), and phase angle deviation (PAD – maximum difference between a designed/measured phase and a target phase) profoundly impact the transfer function of those systems [1], [2], [4], [22].

Different approaches to the realization of an FOC using dielectric materials [24]–[27], a liquid electrochemical capacitor [28], [29], and other active emulators [30] exist in the open literature. However, they are in the conceptual stage and still require much work before they meet the expectations of circuit designers. One method of implementing an FOC is to approximate the rational function of s in a desired bandwidth using the methods of Carlson [31], Matsuda [32], El-Khazali [33], Maione [34], Oustaloup [35], Continued Fraction Expansion (CFE) [36], and Recursive Algorithms (RAs) [37], [38] among others [39]–[43]. Once a rational function is obtained, it can be synthesized by the use of a weighted sum of first-order filter sections [44], active building blocks [45], controllers [46], bilinear sections [47], tree structures [48], RC ladder networks [36], or a Valsa structure [37]. Subsequently, by defining the C_α , their overall transfer function mimics the impedance of FOCs at a specific frequency range. On the other hand, there are few approaches that mimic the FOI response such as RL-networks [20], equiripple and El-Khazali approximations [33], using generalized impedance converters (GICs) [19], [23], [23], [49]–[52], or other active emulators [45]–[47]. However, the trade-off between an operating frequency range and the number of circuit elements required in the FOC or FOI design limits the use of passive or active emulators in many applications. In addition, all above mentioned approximation methods are analytical and require not standard IEC 60063 compliant values of resistors, capacitors, and inductors to have better results. If the used values are replaced by the closest standardized values, the accuracy of the approximation decreases, which leads to an increase of PAD and overall degradation of the performance of the FOE (for instance [15], [20], [23]).

Up until now, evolutionary computing algorithms have been used to reduce the drawbacks in traditional optimization

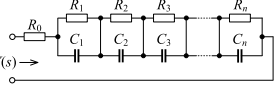
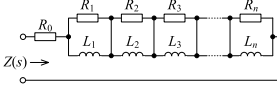
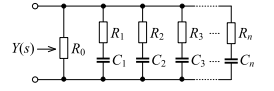
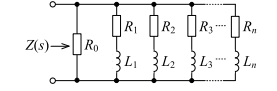
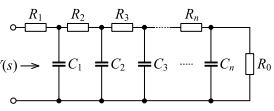
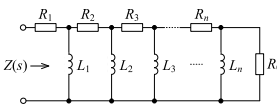
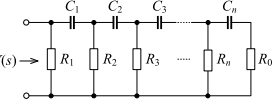
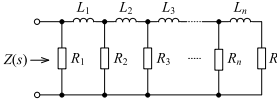
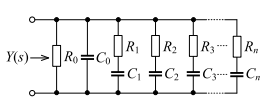
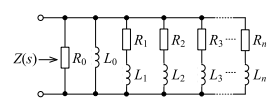
methods and to solve complex problems where conventional techniques fail in many areas of the fractional-order domain such as chaos [53], control [54], or extracting the design parameters of filters [55]. In this regard, flower pollination algorithms [56], particle swarm optimization [57], or genetic algorithms (GA) [58] are used. In this work, a mixed integer-order GA in MATLAB® is used. Instead of approximating s^α using the above mentioned approximations at a certain frequency (or bandwidth), we optimize the phase and/or magnitude responses of RC/RL networks in the whole desired frequency range. In brief, the GA is a powerful computational technique, which mimics the process of natural selection theory. It consists of a population of representations of candidate solutions to an optimization problem, which evolve toward enhanced solutions. It is important to mention that the GA uses the objective function itself, not derivatives or other auxiliary knowledge based on probabilistic/deterministic characterization. These features make this optimization method the most suitable technique to optimize the CPA in distributed RC/RL networks. Furthermore, the required values are obtained with GA, even if the passive component values are restricted to commercially available kit values defined by standard IEC 60063, and still maintain superb results. Hence, the paper aims to introduce an FOE optimization method that achieves a broad operating frequency range with CPA deviation of approximately $\pm 1^\circ$ using commercially available passive component values in RC and RL structures with five branches of Foster-I, Foster-II, Cauer-I, Cauer-II, and Valsa networks. Most crucially, the presented approach avoids the use of negative component values, GICs, or random passive element values. Thus, this article deals with the optimal emulation of an FOE currently available in the literature. In particular, Foster-II and Valsa networks are selected as our main objective, because the former offers a minimum total capacitance value and the latter provides a minimum CPA deviation. Here it is also worth noting that, to the best knowledge of authors, an FOI design using the listed five RL networks is studied for the first time in the literature.

The rest of this work is organized as follows. Section II describes the proposed optimization approach using GA as well as design considerations. In Section III, five types of RC networks for FOC design with optimized passive component values are analyzed numerically and the particular cases experimentally. Furthermore, the FOC results of comparisons with other algorithms (CFE, Oustaloup, and RA) are presented. A numerical study of FOI with five types of RL networks using GA is presented for the first time in the literature in the Section IV. As an example, the behavior of the optimized Valsa structure is measured. Section V discusses the results and performance characteristics of all examples, whereas the Section VI concludes the study.

II. DESCRIPTION OF THE GA APPROACH USED IN OPTIMIZATION OF FOE

Table 1 summarizes the FOC and FOI approximation methods used in this work with their synthesized RC and RL net-

TABLE 1. FOC and FOI approximation methods used in this study (note: all below networks are optimized using GA).

FOC Approximation			FOI Approximation		
Method	Admittance, $Y(s)$	RC Network	Method	Impedance, $Z(s)$	RL Network
Oustaloup	$Ck_f \sum_{i=1}^n \frac{1 + (s/\omega_i')}{1 + (s/\omega_i)}$ [35]		GA	$R_0 + \sum_{i=1}^n \frac{sL_i R_i}{R_i + sL_i}$	
CFE	$\frac{1}{R_0} + \sum_{i=1}^n \frac{sC_i}{sC_i R_i + 1}$		GA	$\frac{1}{\frac{1}{R_0} + \sum_{i=1}^n \frac{1}{R_i + sL_i}}$	
CFE	$R_1 + \frac{1}{sC_1 + \frac{1}{R_2 + \frac{1}{sC_2 + \dots}}}$ $\dots \frac{1}{sC_n + R_0}$		GA	$R_1 + \frac{1}{\frac{1}{sL_1} + \frac{1}{R_2 + \frac{1}{sL_2 + \dots}}}$ $\dots \frac{1}{sL_n + R_0}$	
CFE	$\frac{1}{\frac{1}{R_1} + \frac{1}{sC_1} + \frac{1}{R_2} + \frac{1}{sC_2} + \dots}$ $\dots \frac{1}{\frac{1}{sC_n} + R_0}$		GA	$\frac{1}{\frac{1}{R_1} + \frac{1}{sL_1} + \frac{1}{R_2} + \frac{1}{sL_2} + \dots}$ $\dots \frac{1}{sL_n + R_0}$	
RA	$\frac{1}{R_0} + sC_0 + \sum_{i=1}^n \frac{sC_i \varepsilon^i}{sCR + (\eta \varepsilon)^i}$ [37], [38] Note: η is capacitance and ε is resistance scaling factors		GA	$\frac{1}{\frac{1}{R_0} + \frac{1}{sL_0} + \sum_{i=1}^n \frac{1}{R_i + sL_i}}$	

Algorithm 1 Genetic Algorithm Pseudocode

```

1: for  $i = 1$  to  $NumOfGenerations$  (or until an
   acceptable solution is found) do
2:   if first generation then
3:     Generate the initial population with primitives
       (CPZ, CPA, pseudocapacitance, pseudoinductance,
       number of branches, resistor, capacitor, and
       inductor set)
4:   else
5:     With current population, generate a new one using
       crossover and mutation operators
6:   end if
7:   Calculate fitness of population members
8:   if fitness  $\neq 0$  then
9:     Return to generate new population
10:  else
11:    Break the loop
12:  end if
13: end for
    Return best individual in last population

```

TABLE 2. Genetic algorithm parameters.

Run Parameter	Value
Population size	100
Max. stall generation	125
Max. generation	10 000
Input variables	6
Crossover probability	0.5
Random range of resistor values (Ω)	1 – 10 M (in total, 1 000 points divided into n logarithmically equal sections are used)
Random range of capacitor values (F)	1 f – 1 μ (in total, 1 000 points divided into n logarithmically equal sections are used)
Commercially available resistor kits	YAGEO RC-0603-FR-07 E96 Kit [59] and Vishay 0402 [60]
Commercially available capacitor kits	muRata C0603 [61] and Kemet 0402 [62]
Commercially available inductor kit	Coilcraft 0603 ceramic inductors [63]

works and equivalent admittances or impedances. The admittances of some of the RC networks can be found in [4], [36]. The impedance and phase optimization of all structures using

the GA is obtained with predefined R and C values. The desired constant phase and/or pseudocapacitance, pseudoinductance, number of branches, and frequency range (i.e. CPZ) are defined as design parameters.

To provide more detail regarding the exact steps that were performed by the GA approach, we present its pseudocode in

Algorithm 1 based on [58]. Table 2 presents the parameters employed during the training phase of the GA approach. Fitness, also known as the cost function of the solution set, is determined using the following equation:

$$F = |\varphi_{\text{sim}} - \varphi_{\text{targ}}|, \quad (1)$$

where φ_{sim} and φ_{targ} express the simulated and target phase, respectively.

III. OPTIMIZATION AND VERIFICATION OF FOC

As an exemplary study, primarily the Foster-II [36] and Valsa [37] networks are optimized within this section. For the reason that; Foster-II network offers a minimum total capacitance value and Valsa network provides a minimum PAD.

A. OPTIMIZATION OF FOSTER-II STRUCTURE FOR FOC DESIGN

For a Foster-II realization, the component values are given by the partial fraction expansion and its admittance is expressed in Table 1. Here, n is the number of branches, R_0 is the initial resistor, and R_i and C_i are the resistances and capacitances of i -th branch. Firstly, the performance of the network obtained using the GA with the Oustaloup and CFE methods is compared to show the advantage of the GA. The desired bandwidth, number of branches which is equivalent of a fifth-order admittance function ($n = 5$), and CPA are respectively set as 100 Hz–1 MHz, 5, and -45° with a pseudocapacitance of $C_\alpha = 100 \text{ nF}\cdot\text{s}^{-0.5}$. As a population, the random and commercially available passive elements defined in Table 2 are used. The central frequency in case of CFE is set to 10 kHz. It can be observed from Fig. 2(a) that all three approximations provide a constant phase response with target CPA near a central frequency, specifically between 1 kHz and 100 kHz. However, errors in phase for the approximation models increase significantly when the frequency is 2 decades above and below the central frequency, whereas the phase response obtained using the GA is satisfied in the whole frequency range of interest. Furthermore, Fig. 2(b) shows relative phase errors and corresponding normalized histograms (%) of phase angle deviation from CPA as an inset. It can be seen that the maximum deviation in the GA is limited to only $\pm 2^\circ$, whereas in both CFE and Oustaloup, $\pm 25^\circ$ errors occur. Because no direct control exists over the R and C values obtained from the last two approximations, a correction to use the commercially available RC kit values is obligatory to build the FOCs. However, this correction is not needed for the results obtained by the GA since it directly provides the standard IEC 60063 compliant RC values as the results. Indeed, it is possible to include in the population, i.e. available R and C values to MATLAB® and the GA performs the optimization using only given values. Fig. 2(c) shows the simulated phase response of corrected RC network values using the Oustaloup, CFE, and optimized network using the GA, while the commercially available 0603 size R and C kit values defined in Table 2 are used. The rest of

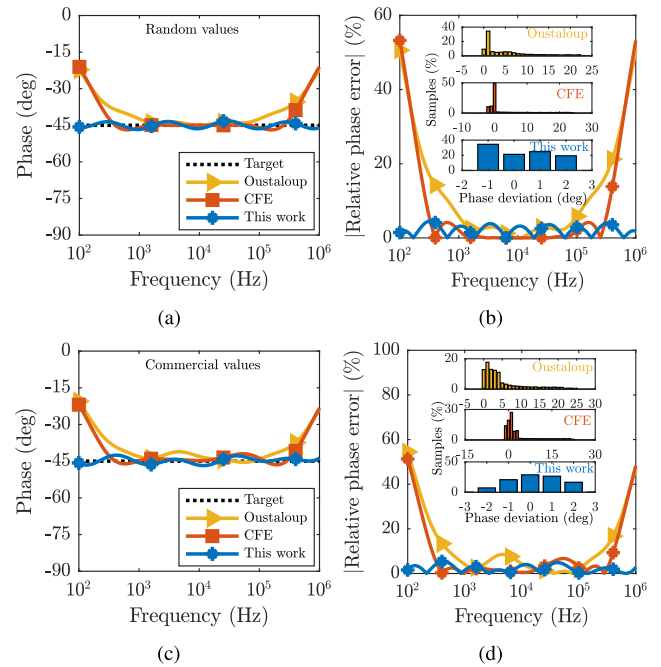


FIGURE 2. (a) Numerical phase response plots of the Foster-II RC network using the Oustaloup, CFE, and GA methods with random values, (b) relative phase errors and corresponding normalized histograms (%) of phase angle deviation from CPA as inset, (c) phase angle response of the RC network using the Oustaloup and CFE methods after RC value correction, and the GA optimized for commercially available RC kit values, (d) relative phase errors and corresponding normalized histograms (%) of phase angle deviation from CPA as inset. Phase responses are optimized in the frequency range of 100 Hz–1 MHz.

the simulation setup is identical to the simulation setup for Fig. 2(a). Fig. 2(d) plots the relative phase errors and corresponding normalized histograms (%) of phase angle deviation from CPA as an inset. As it can be observed, the maximum deviation in the GA is limited to $\pm 2.8^\circ$, whereas $\pm 30^\circ$ error is obtained in both Oustaloup and CFE approximation results. Notably, the maximum error in the phase obtained from both approximation methods are further increased compared with the results in Fig. 2(b) with no RC value correction. However, no significant change is observed in the phase of the circuit obtained using the GA.

Figs. 3(a) and (c) show the target, simulated, and measured phase angle and pseudocapacitance responses of the RC network optimized using the GA. The same passive element values are used from the commercially available RC kits as depicted in Fig. 2(c) (see “This work”) with the setup listed in Appendix A. The experimental verification uses the Agilent 4294A Precision Impedance Analyzer. Standard calibration tests (open and short circuits) of the Keysight 16048G Test Leads are performed to calibrate the instrument. From the results in Figs. 3(b) and (d), it can be seen that the maximum CPA deviation between target (ideal) and simulated as well as measured values is only $\pm 2.8^\circ$ and $\pm 3.2^\circ$, respectively, whereas the pseudocapacitance is $\pm 6.6 \text{ nF}\cdot\text{s}^{-0.5}$ and $\pm 7.3 \text{ nF}\cdot\text{s}^{-0.5}$.

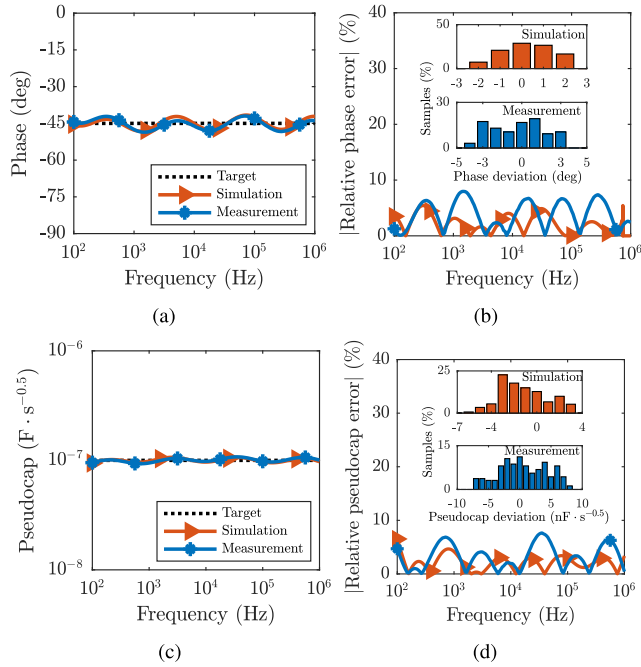


FIGURE 3. Target (ideal), simulated, and measured (a) phase responses, (b) relative phase errors and corresponding normalized histograms (%) of phase angle deviation from CPA as an inset, (c) pseudocapacitance responses, and (d) relative pseudocapacitance errors and corresponding normalized histograms (%) of pseudocapacitance deviation from CPA as an inset, respectively, of the Foster-II network optimized using GA. Impedance and phase responses are optimized in the frequency range of 100 Hz–1 MHz.

Statistical analysis of Monte Carlo (MC) was performed in OrCAD PSpice® simulation software with passive element tolerances based on their datasheets [59], [61] and 200 runs to observe effects due to manufacturing processes. The histogram shown in Fig. 4 demonstrates the variation of the phase at 100 kHz of the Foster-II network optimized using GA. The mean value with standard deviation 0.555 is -44.8109° , which is very close to the theoretical value -45° confirming that the proposed network has low sensitivity characteristic on passive components. The analysis results of MC for all studied networks at their middle frequency are listed in Appendix A.

B. OPTIMIZATION OF VALSA STRUCTURE FOR FOC DESIGN

The Valsa network in Table 1 [37] is proposed to emulate FOC behavior and realized using RA. The possibility of designing this network using commercially available R and C values was claimed by the authors [37]. However, the RA allows us to set only initial values and the remaining branch values must be adjusted according to commercially available passive element values. With this in mind, similar to with the Foster-II structure, the GA is applied to the Valsa network in this subsection. The admittance function is given in Table 1, where compared to Foster-II network the additional C_0 denotes an initial capacitor. In our study, to provide a fair comparison with [37], the phase responses of RA and the GA of an

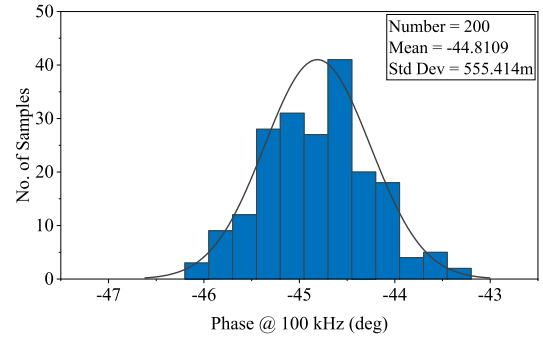


FIGURE 4. Monte Carlo analysis: Phase variation at 100 kHz of the Foster-II network optimized using GA (values used in Fig. 3(a)).

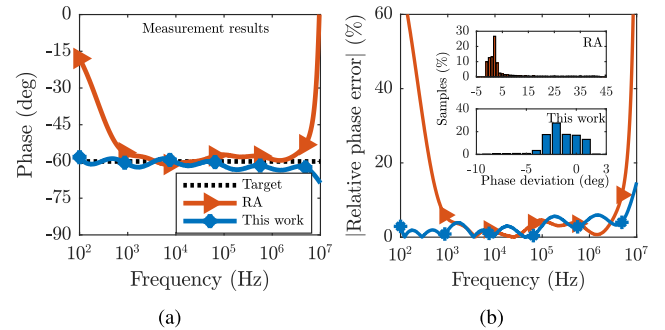


FIGURE 5. (a) Measured phase responses of the Valsa structure using the RA and GA methods (commercially available kits are used), and (b) relative phase errors and corresponding normalized histograms (%) of phase angle deviation from CPA as an inset. Impedances are measured in the frequency range of 100 Hz–10 MHz.

order of $\alpha = -0.67$ using commercially available 0603 size RC kit values are experimentally evaluated. The used passive element values are listed in Appendix B (see “Fig. 5” columns). During the experimental verification, the same instruments listed in Section III(A) are used. With the phase error equal to $\pm 2.1^\circ$, the approximation with the GA experimentally reaches a wider bandwidth of 100 Hz – 5 MHz, as shown in Fig. 5(a). Considering the full frequency band up to 10 MHz, the error is still only $\pm 3.2^\circ$ (see Fig. 5(b)).

Furthermore, the measurement results of $\alpha = -0.5$ order FOCs using an ENA Series Network Analyzer E5071C (300 kHz – 20 GHz) in three different frequency ranges [case study (a) in 1 MHz – 100 MHz, (b) 5 MHz – 500 MHz, and (c) 50 MHz – 1 GHz] are shown in Fig. 6. Two variants of the FOC device with dimensions of 20 mm \times 20 mm were designed (for 0402 and 0603 size passive components) employing a subminiature version A (SMA) coaxial RF connector. The fabricated printed circuit board for 0402 size kit values is shown in Fig. 6(c) as an inset. Considering an input impedance 50 Ω of the connector, the phase is measured by defining the equation of impedance as $Z = 50 \cdot [(1 + S_{11}) / (1 - S_{11})]$. As passive elements, RF-type resistors from Vishay [60] and capacitors from Kemet [62] are used. Because of the producers fabrication boundaries, used passive components having CPA in limited frequency

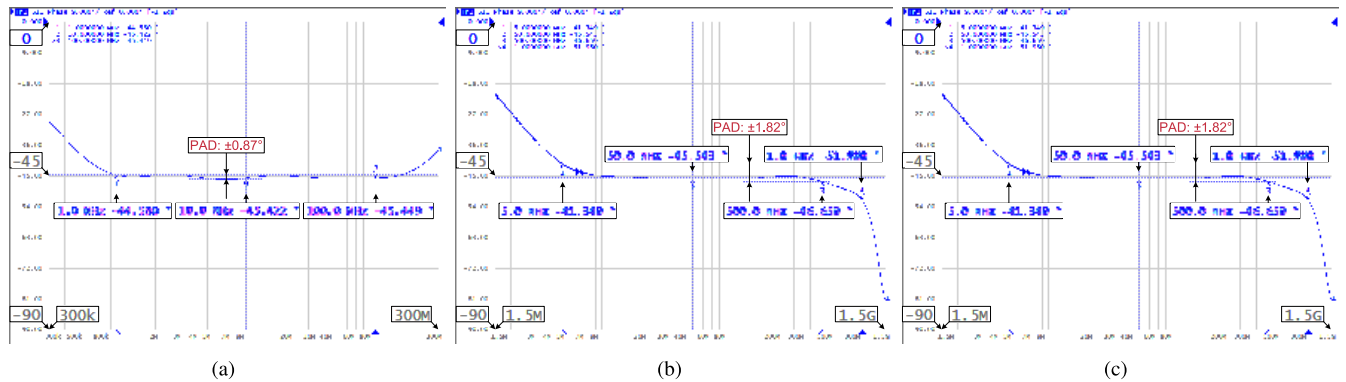


FIGURE 6. Measurement results of an $\alpha = -0.5$ order FOC implemented using the Valsa network optimized using GA for two decades in different frequency ranges: (a) 1 MHz–100 MHz, (b) 5 MHz–500 MHz, and (c) 50 MHz–1 GHz.

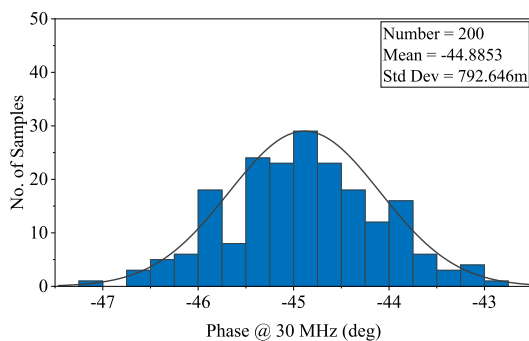


FIGURE 7. Monte Carlo analysis: Phase variation at 30 MHz of the Valsa network optimized using GA (values used in Fig. 6(a)).

range, operate up to a maximum of 5 GHz. In addition, this frequency range is inversely proportional to the resistance values. For instance, a 100 Ω resistor works until 8 GHz, whereas a 1 k Ω resistor has a constant zero-degree phase response up to 800 MHz and so forth. At high frequencies, the transmission line effect becomes dominant; therefore, we maintain the distance between passive elements the least. Despite the above mentioned limitations, we obtained the results until 1 GHz with low phase angle deviations as shown in Figs. 6(a)–(c).

MC analysis was performed in OrCAD PSpice® simulation software with 0402 kit resistors [60] and capacitors [62] with tolerance according to their datasheets, and 200 runs. The histogram shown in Fig. 7 demonstrates the variation of the phase at 30 MHz with values used in Fig. 6(a). The mean value with standard deviation 0.793 is -44.8853° , which is again very close to the theoretical value -45° .

One of the advantage of the proposed GA to design FOC is its suitability for any RC ladder topology, such as Cauer-I, Cauer-II, or Foster-I. In general, by replacing the admittance function of the desired topology, it is possible to determine the required resistance and capacitance values to build an FOC with desirable electrical properties. Notably, the list of admittances of listed networks can be found in Table 1. Fig. 8 shows the phase and pseudocapacitance responses

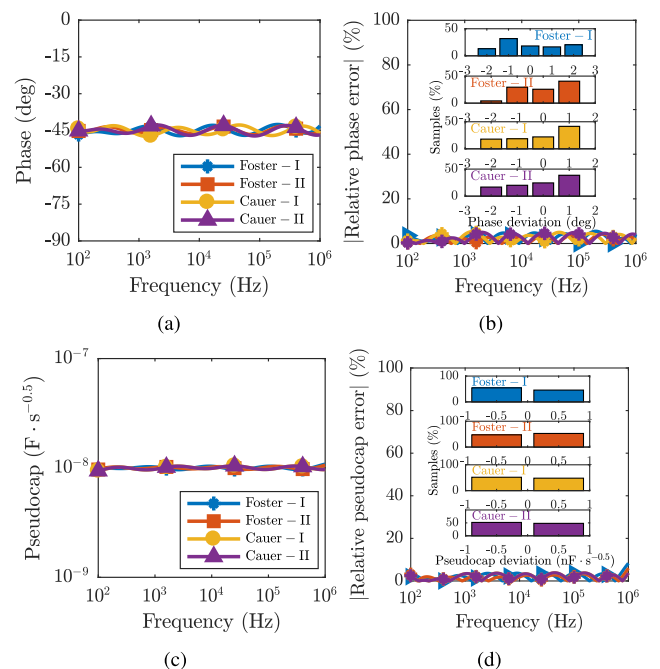


FIGURE 8. (a) Simulated phase and (b) relative phase errors and corresponding normalized histograms (%) of phase angle deviation from CPA as an inset, (c) pseudocapacitance responses, (d) relative pseudocapacitance errors and corresponding normalized histograms (%) of pseudocapacitance deviation from CPA as an inset, respectively, of four RC networks optimized using GA. Responses are optimized in the frequency range of 100 Hz–1 MHz.

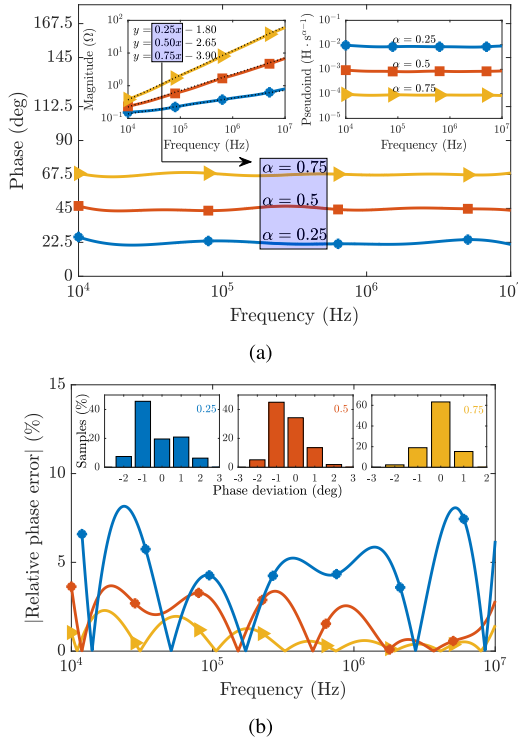
with corresponding relative errors and normalized histograms (%) of deviations of four RC topologies while the target phase, pseudocapacitance, and frequency bandwidth are set to -45° , $10 \text{ nF}\cdot\text{s}^{-0.5}$, and 4 decades in the frequency range of $100 \text{ Hz} - 1 \text{ MHz}$, respectively. The largest deviation between the desired and simulated phase values in all topologies is up to $\pm 2.5^\circ$ with low pseudocapacitance deviation.

IV. OPTIMIZATION AND VERIFICATION OF FOI

The most popular technique to mimic an inductor is using a GIC employing Op-Amps, resistors, and capacitors [19],

TABLE 3. Comparison of simulation and measurement results of used methods for FOI design.

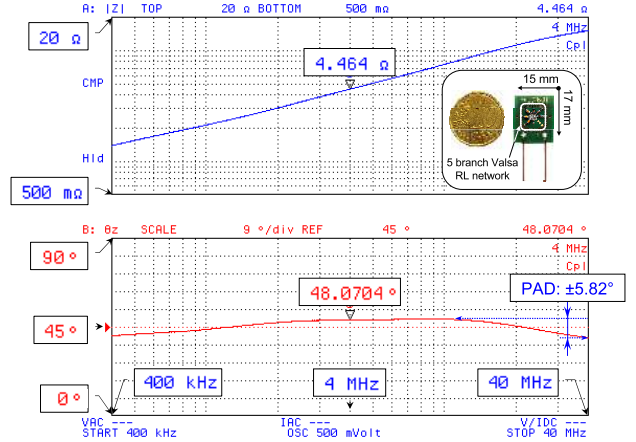
Topology	Method	No. of Branches	No. of Capacitors	No. of Resistors	ICPA Deviationl (°)	BW (decades)	FoM (°)
Foster-II	Oustaloup (Fig. 2(c)) (simul.)	5	5	6	5	2.1	7.64×10^{-3}
	CFE (Fig. 2(c)) (simul.)	5	5	6	5	2.35	8.55×10^{-3}
	This work (Fig. 2(c)) (simul.)	5	5	6	2.4	4	30.3×10^{-3}
Valsa	RA (Fig. 5) (meas.)	5	6	6	5	3.8	12.67×10^{-3}
	This work (Fig. 5) (meas.)	5	6	6	2.1	4.7	37.3×10^{-3}
	This work (Fig. 6(a)) (meas.)	5	6	6	0.87	3	57.47×10^{-3}

**FIGURE 9.** Numerical simulation results of five-branches Valsa RL network using 0603 kit R and L values for FOI design: (a) phase, pseudoinductance, and magnitude responses, (b) relative phase errors and corresponding normalized histograms (%) of three different orders in the frequency range of 10 kHz–10 MHz.

[20], [33], [45]–[47], [49]–[52]. However, the performances of these GIC-based active inductance simulators often suffer from the non-idealities of Op-Amps. Therefore, this section deals with the optimal emulation of an FOI for the first time in the literature. The FOI design using the GA is studied numerically and experimentally verified.

A. OPTIMIZATION OF VALSA STRUCTURE FOR FOI DESIGN

The Valsa RC network in Table 1 is modified to an RL-type structure by replacing all capacitors with inductors as shown in corresponding figure. Its equivalent impedance function is given in Table 1, where n is the number of branches, R_0 is the initial resistor, L_0 is the initial inductor, R_i and L_i are the resistances and inductances of the i -th branch, respectively, while the fitness function is described as (1). The frequency response of five-branch FOIs with three different angles using

**FIGURE 10.** Measurement results of an $\alpha = 0.5$ order FOI from Fig. 9 and the fabricated device with dimensions of 15 mm × 17 mm as in inset (blue line - impedance response; red line - phase response).

0603 kit R [59] and L [63] values is studied numerically and shown in Fig. 9. The pseudoinductances of orders $\alpha = \{0.25, 0.5, 0.75\}$ are $8.52 \text{ mH} \cdot \text{s}^{-0.75}$, $834.62 \text{ } \mu\text{H} \cdot \text{s}^{-0.5}$, and $89.62 \text{ } \mu\text{H} \cdot \text{s}^{-0.25}$, which are constant with small deviations in the whole frequency range. Furthermore, the slope of magnitude in the inset of Fig. 9(a) shows that the inductive reactance (impedance) of the FOI increases as the supply frequency across it increases. To estimate the equivalent order α , the simulated magnitude responses are fitted to the function $\log_{10}|Z| = \alpha \log f + \log_{10}(2\pi)^{\alpha} L_{\alpha}$ using the linear least squares method. The equivalent equations from fitting the magnitude are provided inside Fig. 9(a). The maximum PAD and relative phase errors of the related orders are $\{\pm 1.84^{\circ}, \pm 1.66^{\circ}, \pm 1.55^{\circ}\}$ and $\{\pm 8.16\%, \pm 3.68\%, \pm 2.29\%\}$, respectively, as depicted in Fig. 9(b). The operating frequency range is chosen between 10 kHz and 10 MHz because of the working frequency range of the 0603 kit ceramic chip inductors [63]. Considering that the maximum PAD is around $\pm 2^{\circ}$, order of 0.25 is limited from 12 kHz. The used passive element values are listed in Appendix C.

Moreover, the behavior of an $\alpha = 0.5$ order FOI, numerically simulated in Fig. 9, was verified using the Agilent 4294A precision Impedance Analyzer. Standard calibration tests (open and short circuits) of the 16047E Test Fixture were performed to calibrate the instrument. During the experimental validation in the frequency range 400 kHz–40 MHz (801 logarithmically spaced points in two decades),

TABLE 4. Passive element values for FOC using the Foster-II network and their performance characteristics.

Element	Fig. 2(a)			Fig. 2(c)		
	$\alpha = -0.5; C_\alpha = 100 \text{ nF}\cdot\text{s}^{-0.5}$					
	This work	CFE	Oustaloup	This work	CFE	Oustaloup
$R_0 \text{ (}\Omega\text{)}$	979.5 k	438.84 k	3.99 k	910 k	430 k	3.9 k
$R_1 \text{ (}\Omega\text{)}$	42.1 k	4.44 k	4.12 k	36 k	4.3 k	4.3 k
$R_2 \text{ (}\Omega\text{)}$	13.6 k	37.87 k	11.52 k	15 k	39 k	12 k
$R_3 \text{ (}\Omega\text{)}$	2.1 k	94.10 k	28.58 k	330 k	91 k	30 k
$R_4 \text{ (}\Omega\text{)}$	409.8 k	155.28 k	77.49 k	120 k	150 k	75 k
$R_5 \text{ (}\Omega\text{)}$	132.5 k	202 k	272.24 k	2.4 k	200 k	270 k
$C_1 \text{ (F)}$	229.3 p	74.04 p	153.90 p	120 p	68 p	150 p
$C_2 \text{ (F)}$	70.8 p	87.66 p	346.88 p	390 p	100 p	330 p
$C_3 \text{ (F)}$	33.5 p	127 p	852.84 p	10 p	120 p	680 p
$C_4 \text{ (F)}$	2.3 n	248.16 p	2.05n	0.47 p	270 p	2.2 n
$C_5 \text{ (F)}$	722.5 p	913.85 p	3.69 n	120 p	1 n	3.3 n
	Operating frequency range					
	100 Hz – 1 MHz					
	Total resistance (Ω) / Total capacitance (F)					
	467.7 k / 3.17 n	932.53 k / 1.45 n	397.94 k / 7.09 n	1.41 M / 3.05 n	914.3 k / 1.56 n	395.2 k / 6.66 n
	Spread of resistance / capacitance					
	198.75 / 69.79	98.84 / 12.34	68.23 / 23.98	379.17 / 66.67	100 / 14.71	69.23 / 22
	Max. phase angle deviation ($^\circ$) / relative phase error (%)					
	± 1.8 / ± 4.09	± 25 / ± 53.23	± 25 / ± 50.61	± 2.4 / ± 5.43	± 40 / ± 79.44	± 30 / ± 54.5
	Monte Carlo analysis: Variation of phase @ 10 kHz (mean / min / max ($^\circ$))					
	-46.35 /	-45.06 /	-45 /	-46.74 /	-44.77 /	-43.02 /
	-48 /	-46.45 /	-46.25 /	-47.7 /	-46.1 /	-44.41 /
	-44.83	-43.68	-43.51	-45.61	-43.33	-41.41

TABLE 5. Passive element values for FOC using the Valsa network and their performance characteristics.

Element	Fig. 5 ($\alpha = -0.67$)		Fig. 6 ($\alpha = -0.5$)		
	This work	RA	This work (a)	This work (b)	This work (c)
$R_0 (\Omega)$	10 M	24 k	1 k	330	250
$R_1 (\Omega)$	24 k	10 k	1 k	250	10
$R_2 (\Omega)$	4.7 k	3 k	221	330	100
$R_3 (\Omega)$	110 k	820	221	50	25
$R_4 (\Omega)$	2.4 M	270	250	500	200
$R_5 (\Omega)$	470 k	75	20	221	221
$R_6 (\Omega)$	–	–	1 k	20	330
$C_0 (\text{F})$	7 p	1 n	0.1 p	4.7 p	1.5 p
$C_1 (\text{F})$	18 p	10 n	100 p	68 p	2.7 p
$C_2 (\text{F})$	10 p	4.7 n	68 p	8 p	8 p
$C_3 (\text{F})$	39 p	3.3 n	12 p	15 p	6 p
$C_4 (\text{F})$	200 p	1.5 n	12 p	12 p	12 p
$C_5 (\text{F})$	91 p	0.82 n	22 p	15 p	6.8 p
$C_6 (\text{F})$	–	–	100 p	7 p	100 p
Operating frequency range					
100 Hz – 10 MHz					
1 MHz – 100 MHz					
5 MHz – 500 MHz					
50 MHz – 1 GHz					
Total resistance (Ω) / Total capacitance (F)					
13.01 M / 365 p					
38.17 k / 21.32 n					
3.71 k / 314.1 p					
1.7 k / 129.7 p					
1.14 k / 137 p					
Spread of resistance / capacitance					
416.67 / 28.57					
320 / 12.2					
50 / 1000					
25 / 14.47					
33 / 66.67					
Max. phase angle deviation ($^\circ$) / relative phase error (%) Note: *100 Hz – 5 MHz					
$\pm 2.1^* / \pm 4.04^*$					
$\pm 74.7 / \pm 124.51$					
$\pm 0.87 / \pm 1.94$					
$\pm 1.82 / \pm 4.63$					
$\pm 1.94 / \pm 4.58$					
Monte Carlo analysis: Variation of phase (mean / min / max ($^\circ$))					
@ 30 kHz:					
-60.08 /					
-60.29 /					
-62.93 /					
-57.39					
-60.29 /					
-62.13 /					
-57.78					
@ 30 MHz:					
-44.98 /					
-46.67 /					
-46.23					
@ 60 MHz:					
-45.54 /					
-46.84 /					
-43.89					
@ 400 MHz:					
-45.24 /					
-46.88 /					
-43.88					

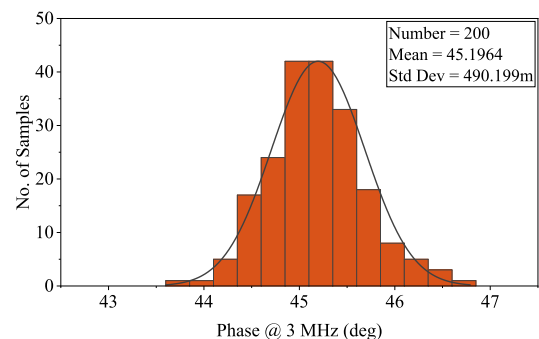
TABLE 6. Passive element values for FOI using the RL network and their performance characteristics.

Element	Fig. 9 (This work – Valsa network)			Fig. 13 ($\alpha = 0.5$)			
	$\alpha = 0.25$ $L_\alpha = 8.52$ $\text{mH}\cdot\text{s}^{-0.75}$	$\alpha = 0.5$ $L_\alpha = 834.62$ $\mu\text{H}\cdot\text{s}^{-0.5}$	$\alpha = 0.75$ $L_\alpha = 896.23$ $\mu\text{H}\cdot\text{s}^{-0.25}$	Foster-I $L_\alpha = 8$ $\text{mH}\cdot\text{s}^{-0.5}$	Foster-II $L_\alpha = 1$ $\text{mH}\cdot\text{s}^{-0.5}$	Cauer-I $L_\alpha = 4.78$ $\text{mH}\cdot\text{s}^{-0.5}$	Cauer-II $L_\alpha = 4.84$ $\text{mH}\cdot\text{s}^{-0.5}$
R_0 (Ω)	1	15	6.80 k	1	16	1	1 k
R_1 (Ω)	1	1	1	18	2.2	5.6	24
R_2 (Ω)	1	10	30 k	6.8	1	13	8.2
R_3 (Ω)	1	1	7.5	120	6.8	33	3
R_4 (Ω)	1	1.2	36	1.8	1	120	470 k
R_5 (Ω)	1	3.9	110	1	1	180 k	1
L_0 (H)	6.80 μ	6.80 μ	7.80 μ	–	–	–	–
L_1 (H)	270 n	6.80 μ	7.80 μ	1.5 μ	820 n	7.8 μ	360 n
L_2 (H)	3.30 μ	220 n	27 n	3.9 μ	3.3 μ	2.7 μ	1.8 μ
L_3 (H)	36 n	6.8 μ	5.60 μ	820 n	270 n	1.2 μ	4.7 μ
L_4 (H)	1.50 μ	1.2 μ	3.90 μ	7.8 μ	7.8 μ	470 n	6.8 μ
L_5 (H)	3.90 μ	470 n	1.50 μ	7.8 μ	7.8 μ	33 n	7.8 μ
Operating frequency range							
10 kHz – 10 MHz							
Total resistance (Ω) / Total inductance (H)							
6 / 15.81 μ	32.1 / 22.29 μ	36.96 k / 26.63 μ	149 / 21.82 μ	27 / 20 μ	180.17 k / 12.2 μ	471.04 k / 21.46 μ	
Spread of resistance / inductance							
1 / 188.9	15 / 30.91	30 k / 288.89	120 / 9.51	16 / 2.89	180 k / 236	470 k / 21.67	
Max. phase angle deviation ($^\circ$) / relative phase error (%) Note: *12 kHz – 10 MHz							
$\pm 1.84^*$ / $\pm 8.16^*$	± 1.66 / ± 3.68	± 1.55 / ± 2.29	± 1.9 / ± 4.22	± 2.44 / ± 4.26	± 20.04 / ± 44.54	± 3.91 / ± 8.69	
Monte Carlo analysis: Variation of phase (mean / min / max ($^\circ$))							
@ 300 kHz: 21.45 / 20.58 / 22.37	@ 3 MHz: 45.2 / 43.79 / 46.71	@ 300 kHz: 67.26 / 64.55 / 69.4	@ 100 kHz: 45.16 / 43.99 / 46.42	@ 100 kHz: 45.7 / 44.78 / 46.92	@ 300 kHz: 44.81 / 42.47 / 46.97	@ 300 kHz: 45.53 / 42.97 / 47.73	

a sinusoidal input signal with a default AC voltage of 500 mV and a frequency of 1 MHz was applied, while one of terminals was grounded. The measurement results and a photograph of the fabricated device with dimensions of 15 mm \times 17 mm are depicted in Fig. 10. The measured PAD in two decades of the frequency range of our interest is $\pm 5.82^\circ$.

In addition to an $\alpha = 0.5$ order FOI, a MC statistical analysis was also performed in the OrCAD PSpice® simulation software. The passive element tolerances according to 0603 kit datasheets [59], [63] and 200 runs were set to observe affects due to manufacturing processes. The histogram shown in Fig. 11 demonstrates the variation of the phase at 3 MHz. The mean value with standard deviation 0.49 is 45.1964° , which is very close to the theoretical value 45° confirming that the proposed network has low sensitivity characteristic on passive components.

In addition, for the first time in the literature, the Foster-I, Foster-II, Cauer-I, and Cauer-II type of RL networks are also studied. The impedance function of all networks optimized using GA are given in Table 1. Fig. 12 shows the phase and pseudoinductance responses with corresponding relative errors and normalized histograms (%) of deviations of four RL topologies. The target phase and frequency bandwidth are set to 45° and 3 decades in the frequency range of 10 kHz – 10 MHz, respectively, with no pseudoinductance

**FIGURE 11.** Monte Carlo analysis: Phase variation at 3 MHz of the Valsa RL network optimized using GA ($\alpha = 0.5$ order FOI with values used in Figs. 9 and 10).

specification to obtain the best result. In summary, the minimal error is obtained with Foster-I structure while the least spread of passive element values are observed with the Foster-II. The detailed analysis including MC results for all studied networks is presented in Appendix C.

V. BRIEF DISCUSSION OF RESULTS

Table 3 compares the performance of RC networks built using Oustaloup, CFE, RA, and the GA. For instance, for the Foster-II network composed of the same number of

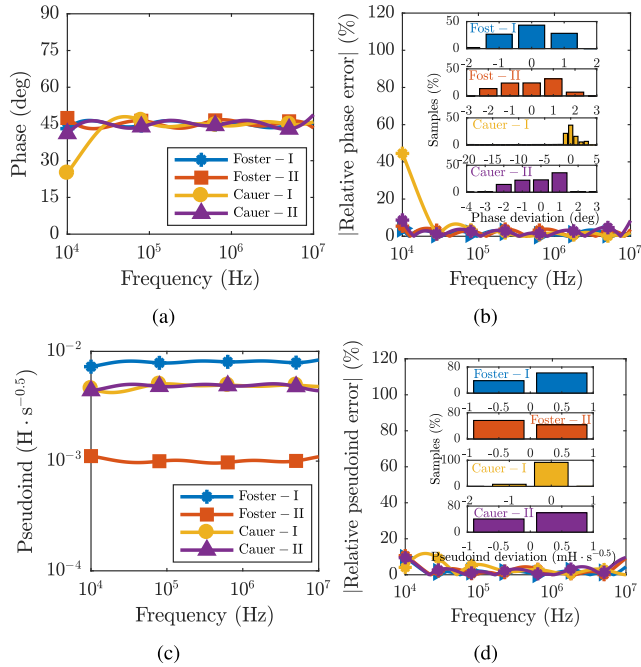


FIGURE 12. (a) Simulated phase responses, (b) relative phase errors and corresponding normalized histograms (%) of phase angle deviation from CPA as an inset, (c) pseudoinductances responses, and (d) relative pseudoinductances errors and corresponding normalized histograms (%) of pseudoinductances deviation from CPA as an inset, respectively, of different RL networks optimized using GA for FOI design. Impedance and phase responses are optimized in the frequency range of 10 kHz–10 MHz.

branches, the performance of the GA is compared with that of Oustaloup and CFE. The results obtained with the GA have the lowest PAD in a wider frequency range than Oustaloup and CFE. To provide the overall performance evaluation, a numeric Figure of Merit (FoM) value is calculated as:

$$\text{FoM} = \frac{\text{BW}}{\text{No. of Branches} \times \text{No. of Cap.} \times \text{No. of Res.} \times |\text{CPA Dev.}|} \quad (2)$$

Notably, the FoM in our study for Foster-II network using the GA was 30.3×10^{-3} , which is the largest value. An improvement of approximately 396% over the Oustaloup and 354% over the CFE was achieved using the same number of elements and least CPA deviation in a wider bandwidth. In the same manner, the Valsa structure is compared between the RA and GA. Evidently, the GA provides a wider bandwidth than RA with lower CPA error. Moreover, the FoM shows significant improvements to the Valsa network with the GA (294% in case of Fig. 5 results). To comprehensively evaluate the performance of the Valsa structure using the RA and GA in Table 3, a radar chart is depicted in Fig. 13, which shows that a smaller area of pentagon provides superior performance (for instance, the GA in Fig. 6(a)). On the other hand, the error increases by increasing the bandwidth while maintaining the five branches as default (see numerical study in depicted in Fig. 14). It is clear that the phase angle deviation is less

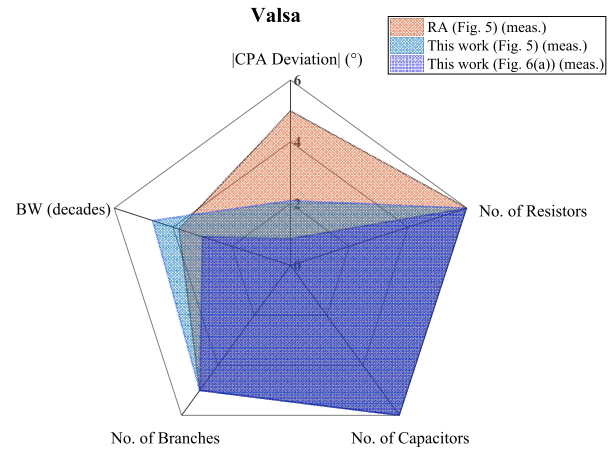


FIGURE 13. Radar chart showing an evaluation of Valsa RC structure results from Table 3.

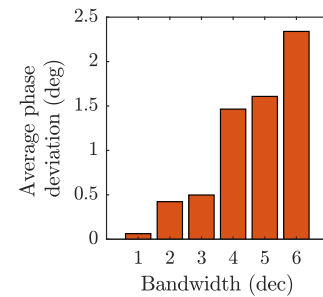


FIGURE 14. Numerical study of five-branches RC networks using random R and C values and plot of average phase angle deviation of an order of $\alpha = -0.5$ by increasing the operation bandwidth from 100 Hz up to 100 MHz.

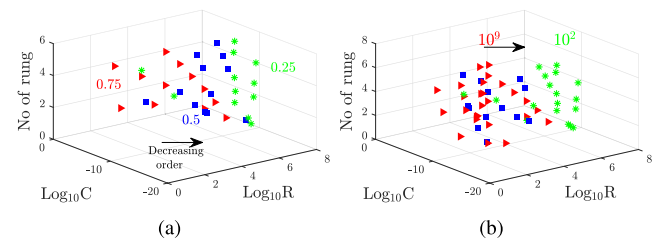


FIGURE 15. (a) Order and (b) frequency effect on R and C values on each rung of the Foster-II and Valsa structures for FOC design.

than $\pm 2.3^\circ$ even with 6 decades of operation (from 100 Hz up to 100 MHz).

The performance of RC network optimizations is primarily compared through the utilization of the results in Appendix A and Appendix B. Here, total and spread of resistances and capacitances, phase angle deviation, relative phase error, and MC analysis for all networks are given. According to these, the GA generally provides the minimum total capacitance value and can be limited in any range of the designer's choice. Furthermore, as the order increases, the total capacitance increases and the resistance decreases, as shown in Fig. 15(a). Maintaining the order constant and increasing the capacitance value provides the same results as in the previous case. The

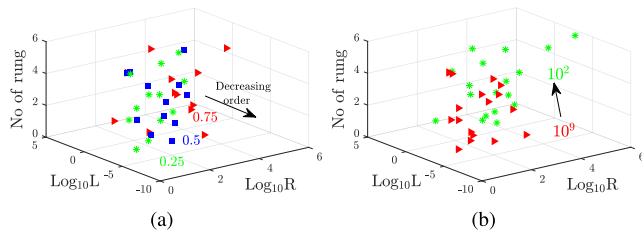


FIGURE 16. (a) Order and (b) frequency effect on R and L values on each rung of the Valsa structures for FOI design.

frequency effect on the values is shown in Fig. 15(b). At high frequencies, small R and C values are used (as also seen in Appendix B), whereas larger passive values are used at low frequencies. This fact can be explained by the dissipation factor (DF) expressed as $DF = ESR/X_C$, where ESR and X_C denote the equivalent series resistance and capacitors reactance, respectively, or as a tangent of the loss angle [64].

Fig. 16 shows the distribution of R and L values depending on an order and the frequency range for FOI design. Different to the FOC evaluation, resistance and inductance vary linearly with an order. It is also clear from Appendix C that an increasing FOI order has the effect of increasing passive values. This result can be explained by the quality factor (Q) definition $Q = X_L/R$, where X_L is the inductive reactance and R is the DC resistance [65]. Maintaining the Q constant, increasing an order (effecting X_L) has the effect of increasing the R . At low frequencies and within limits, both passive values become much greater than their equivalents at high frequencies.

VI. CONCLUSION

In this study, a new approach for the design of an FOE, mainly from the Foster-II and Valsa structures, with desired properties was introduced. The mixed integer-order GA was used to determine the optimal phase response with minimum phase angle deviation in a defined frequency range. The values of the passive elements have been optimized in accordance with the commercially available IEC 60063 compliant kits. Therefore, the introduced approach offers enormous freedom to design RC/RL networks without making any value adjustments, which could lead to degeneration of FOE performance during measurements. Furthermore, designers can obtain the optimal phase and impedance response at low-, mid-, and high frequencies with wide bandwidths and low phase errors with minimum total passive element. The results demonstrated excellent performance as well as adaptability for application to various types of structure, such as Cauer-I, Cauer-II, and Foster-I either for FOC or FOI design, which was carried out for the first time in the literature. All these features make this approach strong and beneficial analogue designer. It is also important to note that the proposed approach outperformed other approximations or algorithms such as the recursive algorithm [15], Oustaloup's approximations [35], graphical method [20], meta-heuristic algorithms [44], and the design using bilinear sections [47]. In other words, a fifth-order approximation of FOE using

GA shows better performance than for example fourteenth- [15] or ninth-order [20] approximations. In addition, during our research, we found that there is a connection between the FOE and the equivalent circuit model of the inductors and electrolytic capacitors. Thus, more accurate models of these components can be developed in the future.

APPENDIX A

See Table 4.

APPENDIX B

See Table 5.

APPENDIX C

See Table 6.

ACKNOWLEDGMENT

For the research, infrastructure of the SIX Center was used. (Aslihan Kartci and Agamyrat Agambayev contributed equally to this work.)

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