SYNTHESIS OF ACTIVE RC NETWORKS CONTAINING

DISTRIBUTED AND LUMPED ELEMENTS

W. J. KERWIN
Ames Research Center, NASA
Moffett Field, California 94035

N68-18790
CODE 1
Cat. 10
PAGE 11
TMX60856

JUL 1967

ABSTRACT

A practical procedure for synthesizing distributed, lumped active (DLA) networks is developed by determining a set of equivalent rational pole positions corresponding to the amplitude response of three specific DLA networks which together allow the realization of filters with left half-plane poles and $j\omega$ axis zeros. An example of the synthesis of a 5-pole, 4 $j\omega$ axis zero elliptic function low-pass filter is given.

INTRODUCTION

The increasing importance of linear integrated circuits makes it vitally necessary that a practical, economical means be found to realize transfer functions that normally require inductive components without the use of inductors, since it is not practical in most cases to integrate inductors (ref. 1, pg. 36). Additionally, the size and weight of inductive components is a severe disadvantage for many other applications (satellites and space probes, for example).

Many approaches to active RC synthesis have been used, each with a particular advantage in specific cases. The lumped element active RC networks using a voltage-controlled voltage source (VCVS), as introduced by Sallen and Key (ref. 2), use a relatively large number of passive elements whereas the active element is relatively simple. Further, those realizations using fewer passive elements tend to require a more complex active system (e.g., gyrator realization (ref. 3, pg. 140)). As will be shown, the use of distributed, lumped, active (DLA) systems in which the active element is a VCVS can greatly reduce the number of passive components while using a simple active element, and in addition, in many cases allow the use of a very simple VCVS.

A practical computational procedure has been developed for analyzing DLA networks, and from this a general synthesis method was evolved whereby the amplitude response corresponding to any combination of left half-plane (LHP) poles and $j\omega$ axis zeros can be realized by cascading a few simple DLA RC networks (ref. 4).

Code 10 cat.

TMX 60856 Code 1 page 11

TRANSFER FUNCTIONS NEEDED

If we now consider the synthesis problem in general, a linear system rational transfer function can be factored into a product of complex root quadratics and first-degree terms (for practical reasons, $m \leq n$ is assumed) as

$$T(p) = K\frac{p^m + a_1p^{m-1} \; . \; . \; . \; a_m}{p^n + b_1p^{n-1} \; . \; . \; . \; a_n} = K\frac{(p^2+\alpha_1p+\alpha_2)(p^2+\alpha_3p+\alpha_4) \; . \; . \; . \; (p+\alpha_5)}{(p^2+\beta_1p+\gamma_1)(p^2+\beta_2p+\gamma_2) \; . \; . \; . \; (p+\gamma_3)} \qquad (1)$$

Each quadratic factor needed can then be realized by passive RC elements combined with a VCVS. The VCVS is chosen for simplicity and availability in practical and integrated form, and to allow the overall transfer function to be obtained by simple cascading of the individual quadratic factor network realizations (ref. 5). A passive RC realization of the first-degree term (or terms) is then added in cascade. In this way, no additional active elements are needed. Active synthesis procedures need therefore only be applied to the quadratic terms, which, in the most general case, will be of the form shown in Eq. (1). However, we will consider only those cases in which the zeros lie on the $j\omega$ axis ($\alpha_1 = \alpha_3 = \alpha_5 \; . \; . \; . = 0$) since these are generally the most useful. The networks developed here are, however, easily modified to produce complex zeros anywhere in the p plane (ref. 6).

ACTIVE RC NETWORKS

Figure 1(a) shows a lumped active RC network previously used by the author to realize two complex poles and two $j\omega$ axis zeros (ref. 7). Figure 1(b) shows a DLA network of the type to be discussed in this paper which produces essentially the same amplitude response as the network of Fig. 1(a), but is considerably simpler and requires a VCVS gain about half as large (i.e., $A_1 \cong 1/2\ K_1$). Although we will be considering networks containing distributed elements that do not have a rational transfer function, the amplitude response produced by these networks is adequately represented by the amplitude response of a rational transfer function.

EQUIVALENT POLE POSITIONS

Since we are concerned with obtaining an amplitude response equivalent to that of either a 2-pole 2-zero, or a 2-pole function, we will consider equivalent pole positions for the DLA networks. It is this concept of an equivalent set of pole positions which allows the development of a simple and practical synthesis procedure for DLA networks. This is similar to the effective dominant pole idea for passive, distributed RC networks proposed by Ghausi and Kelly (ref. 8).

The first DLA network to be considered (Fig. 2) produces an amplitude response similar to that of the network of Fig. 1(a), which has two complex poles and two $j\omega$ axis zeros. The values have been chosen to produce $j\omega$ axis zeros at $\omega = \pm 1.0$ rps, that is, $R_f/R_o = 17.786$ and $R_oC_o = 11.192$ (ref. 6) for a uniform line. If we match the DLA network amplitude response for various values of gain, A_1, and capacity, C, to that of a rational 2-pole, 2 $j\omega$ axis zero function in a sufficient number of cases, the equivalent pole positions of the DLA network can be plotted as a function of the DLA network parameters A_1, and C; this allows synthesis directly from a given rational voltage transfer function. The zeros must be located at a greater distance from the origin than the poles since, for the DLA network of Fig. 2, ϵ_1 must be less than 1.

Figure 3 compares the response of 2-pole 2-zero functions and the calculated response of the DLA network of Fig. 2 for two different values of A_1 and C. The only significant deviation occurs well into the stop band, and is in a direction to improve the performance of the filter.

The result of matching this amplitude response in many cases is shown as a pole position ($\sigma + j\omega$) in Fig. 4 for values of A_1 from 0.5 to 1.7 and of C from 0 to 0.5 fd. The zeros of this network are, of course, located at $\omega = 1.0$ rps for all values of A_1 and C.

The DLA network suited to the production of an equivalent set of two complex poles and two $j\omega$ axis zeros with poles that lie above and to the left of the region given by the curve $C = 0$ in Fig. 4 is shown in Fig. 5. This network is normalized to unit feedback resistance and the line elements are chosen to give zeros on the $j\omega$ axis at $\omega = \pm 1.0$ rps when an untapered RC line is used as before. The only adjustable parameters are then A_2 and R. The calculation of the amplitude response of this DLA network for various values of A_2 and R allowed a match to be made to a 2-pole, 2-zero function. Typical comparisons are shown in Fig. 6 for two values of A_2 and R. The deviations shown are typical of the type to be expected. The positions of the equivalent poles (for $\sigma + j\omega$) are shown in Fig. 7 for values of A_2 from 0.6 to 1.35 and of R from 10 Ω to ∞. The zeros are located at $\omega = \pm 1.0$ rps. The synthesis procedure is thus to locate the desired pole position on Fig. 7 and read off the values of A_2 and R required in the network of Fig. 5 to produce that equivalent pole position.

The last DLA network has an amplitude response equivalent to that of a single-pole pair. In the past, the active RC network (Fig. 8(a)) has been used to produce a 2-pole function using a VCVS as the active element (ref. 2). A very simple network producing approximately a 2-pole transfer function is the DLA network shown in Fig. 8(b). In this case, the amplifier gain required (A_3) is always less than 0.9206! A typical comparison of the amplitude response of this network and a 2-pole function is shown in Fig. 9. The resulting equivalent pole positions for this network are shown in Fig. 10 for various values of line capacity C_o and amplifier gain A_3, for $R_o = 1\ \Omega$. Note that variations of C_o (or R_o) <u>do not change</u> the system Q, only the frequency is affected; Q is changed only by variations in the amplifier gain.

This circuit is particularly useful as an oscillator since only a single RC line and an emitter follower are required. The frequency of oscillation can be determined from Fig. 10 for various values of capacity C_o for $R_o = 1\ \Omega$ by noting the intersection of the C_o curves with the $j\omega$ axis.

DLA NETWORK SYNTHESIS EXAMPLE

As an example of the application of the procedures developed here, a 5-pole, 4 $j\omega$ axis zero, low-pass filter was synthesized. The transfer function was obtained from Skwirzynski (ref. 9, pp. 439-500). This is a convenient source of transfer functions in factored form which are directly applicable to synthesis with DLA RC networks.

The function chosen, Eq. (2), has an equal ripple pass band with a tolerance of 0.5 dB and an equal ripple stop band with a minimum attenuation of 40 dB:

$$T(p) = \frac{0.168(0.359p^2+1)(0.738p^2+1)}{(p^2+0.488p+0.501)(p^2+0.114p+0.808)(p+0.416)} \qquad (2)$$

This function has a cut-off frequency (-0.5 dB) of $\omega = 0.886$ and is 40 dB down at $\omega = 1.13$ (ref. 9).

We will now split this function into three parts (neglecting the constant multiplier) and realize each one separately with DLA networks. A multiplier of 0.416 is assumed for the third factor, $T_c(p)$, to make it realizable with a passive RC network. If the overall gain realized is not acceptable, either attenuation or additional gain can, of course, be added:

$$T_a(p) = \frac{0.359p^2 + 1}{p^2 + 0.488p + 0.501} \qquad (3)$$

$$T_b(p) = \frac{0.738p^2 + 1}{p^2 + 0.114p + 0.808} \qquad (4)$$

$$T_c(p) = \frac{0.416}{p + 0.416} \qquad (5)$$

If we now substitute $s^2 = 0.359p^2$ in $T_a(p)$, Eq. (3), to place the zeros at $\omega = \pm 1$ we obtain the normalized equation

$$T_a'(s) = \frac{s^2 + 1}{2.785s^2 + 0.814s + 0.501} \qquad (6)$$

This must be done in all cases since the design charts have their zeros at $\omega = \pm 1$. The poles of $T_a'(s)$ are located at $s = -0.146 \pm j0.398$. Since the zeros are beyond the poles, this function can be realized by the DLA network of Fig. 2. From Fig. 4 we find that a gain $A_1 = 1.11$ and a capacity $C = 0.14$ fd are required. The resulting network is shown in Fig. 11. Since we made the substitution $s^2 = 0.359p^2$ $(p = 1.67s)$, the transformation to the p plane is accomplished by dividing all capacitances of Fig. 11 by 1.67 to obtain the 1st section of the network of Fig. 13, which approximates $T_a(p)$.

The next factor to be realized, $T_b(p)$, Eq. (4) is normalized by substituting $s^2 = 0.738p^2$ which gives Eq. (7).

$$T_b'(s) = \frac{s^2 + 1}{1.355s^2 + 0.133s + 0.808} \tag{7}$$

The poles of $T_b'(s)$ are located at $s = -0.049 \pm j0.77$ (zeros beyond the poles as before), and we find from Fig. 4 that a gain $A_1 = 1.03$ and a capacity $C = 0.023$ fd are necessary. The network approximating $T_b'(s)$ is shown in Fig. 12. From the relation $s^2 = 0.738p^2$ $(p = 1.164s)$, we transform the network of Fig. 12 to the p plane by dividing all capacitances by 1.164 to obtain the 2nd section of the network shown in Fig. 13, which approximates $T_b(p)$. The realization of $T_c(p)$, Eq. (5), is shown as the 3rd section of the network in Fig. 13 and as indicated in Fig. 13 the cascade connection of the individual sections realizing $T_a(p)$, $T_b(p)$, and $T_c(p)$ completes the design. To bring the capacitor levels closer together, each section can be independently impedance-scaled (alternate values are shown in parentheses in Fig. 13).

The deviation of the DLA network responses from the theoretical amplitude responses are shown in Fig. 14 where the response of $T_a(p)$ is compared with that of DLA_1, $T_b(p)$ with DLA_2, and the overall amplitude response $T(p)$ with the DLA network response, DLA. The resultant filter performance is unchanged from the theoretical equal ripple response in both the pass and stop bands, but has a greater attenuation following the second stop band peak. For clarity in Fig. 14, the individual D.C. gains are not shown; that is, all functions are plotted normalized to a D.C. value of 0 dB.

CONCLUSIONS

In all of the DLA networks considered, the number of capacitors required is equal to or less than the minimum number of capacitors required by a lumped active RC realization, if we count the capacity of a distributed RC line as a single capacitor. This minimum number of capacitors is equal to the degree of the denominator of the transfer function for the lumped active RC network, although practical networks generally use more than the minimum number.

The DLA networks used have been chosen for their practicality and simplicity. The element spread is generally no more than 20 to 1 for the resistances and capacitances, and the element sizes required are similar to those required by other active RC techniques. The frequency limitations are generally determined by the amplifier used and the gain stability required in the overall circuit. At Q values from 10 to 20, the amplifier must be 20 to 40 times as stable in gain as the required system stability. If the system is to be stabilized to 1 or 2%, then this Q value is about as high as is practical.

The synthesis procedure developed for DLA networks has been concerned only with the amplitude response. No attempt has been made to obtain a particular phase or transient response; however, two maximally flat, low-pass filters (3-pole, 2-zero) with a cut-off frequency of 5.4 KHz and an infinite rejection frequency of 10.8 KHz were compared experimentally. The first was an active RC circuit containing only lumped elements and the second used DLA RC networks. There was no measurable difference in delay time, rise time, or overshoot to a step input.

BIBLIOGRAPHY

1. Holland, L., "Thin Film Microelectronics," John Wiley and Sons, Inc., New York, 1965.

2. Sallen, R. P., & Key, E. L., "A Practical Method of Designing RC Active Filters," IRE Transactions on Circuit Theory, Vol. CT-2, No. 1, March 1955, pp. 74-84.

3. Huelsman, L. P., "Circuits, Matrices and Linear Vector Spaces," McGraw-Hill Book Company, Inc., New York, 1963.

4. Kerwin, W. J., "Analysis and Synthesis of Active RC Networks Containing Distributed and Lumped Elements," Doctoral Dissertation, Stanford Univ., 1967.

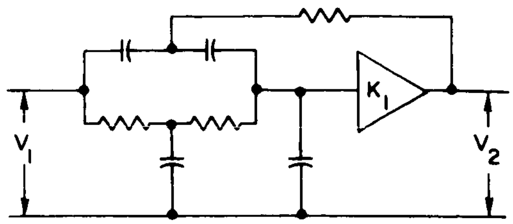
5. Thomas, R. E., "Polynomial Decomposition in Active Network Synthesis," IRE Transactions on Circuit Theory, Vol. CT-8, No. 3, September 1961, pp. 270-274.

6. Kaufman, W. M., & Garrett, S. J., "Tapered Distributed Filters," IRE Transactions on Circuit Theory, Vol. CT-9, No. 4, December 1962, pp. 329-336.

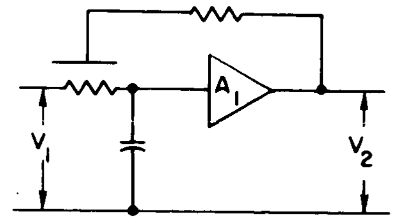
7. Kerwin, W. J., "An RC Active Elliptic Function Filter," 1966 IEEE Region Six Conference Record, Vol. 2, April 1966, pp. 648-654.

8. Kelly, J., & Ghausi, M. S., "On the Effective Dominant Pole of the Distributed RC Networks," Journal of the Franklin Institute, Vol. 279, No. 6, June 1965.

9. Skwirzynski, J. K., "Design Theory and Data for Electrical Filters," D. Van Nostrand and Company, London, 1965.



(a) Lumped.



(b) DLA

Fig. 1.- Active RC networks for $T(p) = K_1' \dfrac{p^2+\alpha_2}{p^2+\beta_1 p+\gamma_1}$

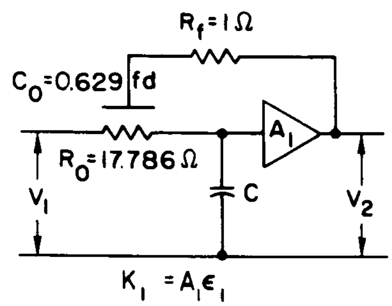


Fig. 2.- DLA network approximating $T_1(p) = \left.\dfrac{K_1(p^2+1)}{p^2+\delta_1 p+\epsilon_1}\right]_{\epsilon_1<1}$

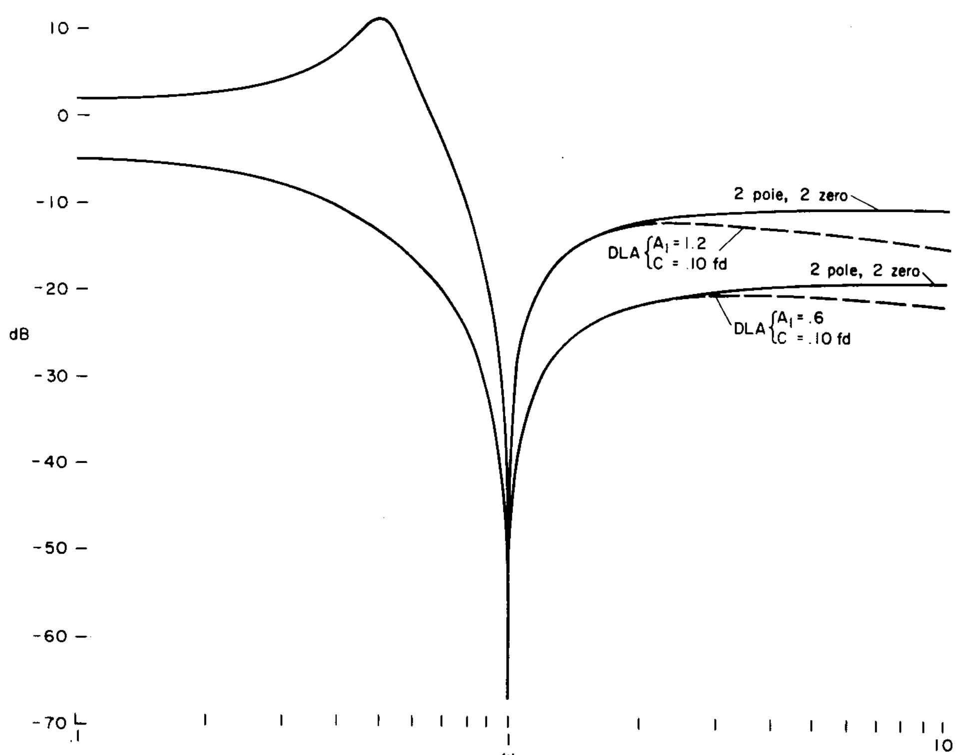


Fig. 3.- Amplitude responses for two choices of parameters of the distributed-lumped-active network of Fig. 2 and a 2-pole, 2 $j\omega$ axis zero function.

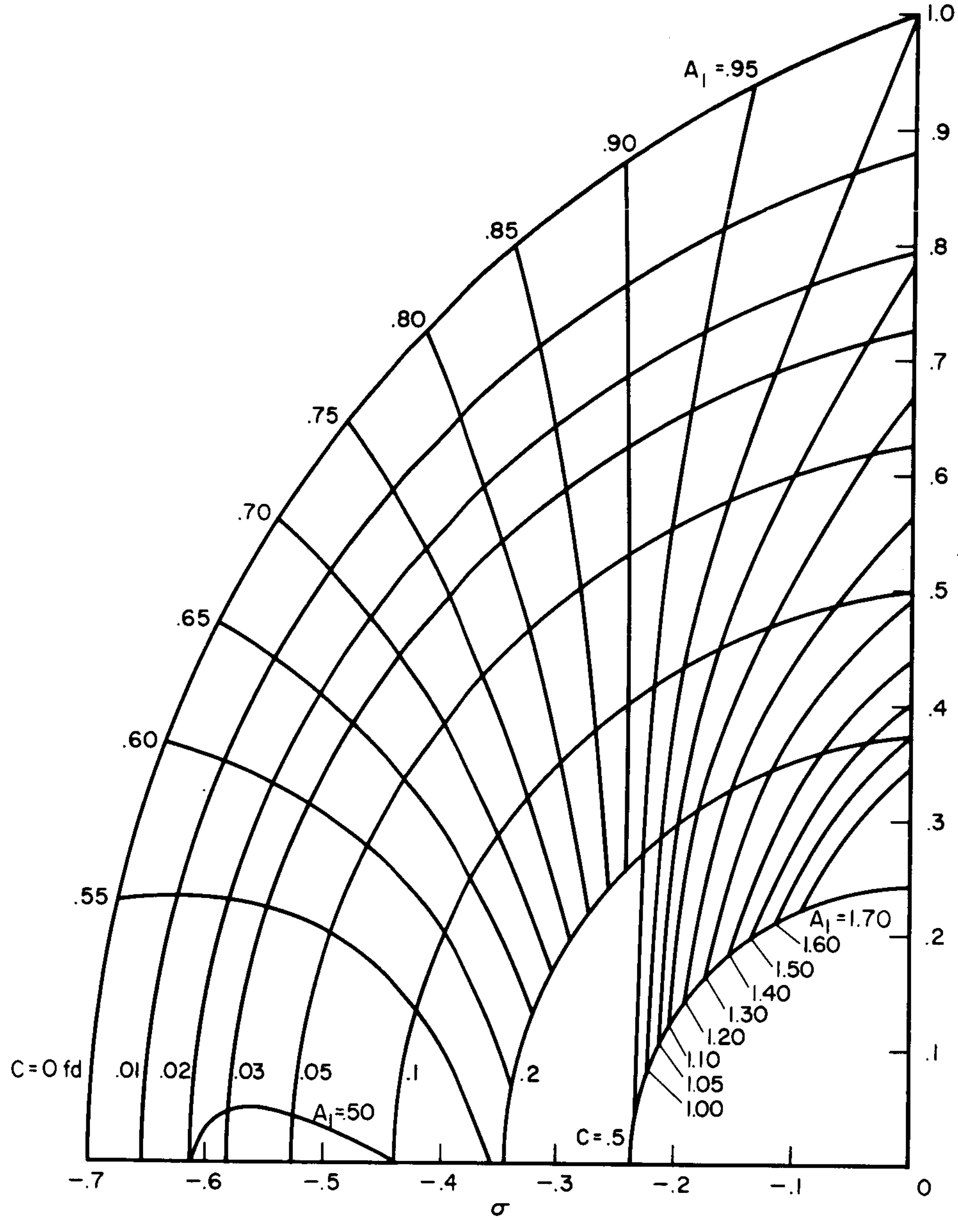


Fig. 4.- Equivalent pole positions for the distributed-lumped-active network of Fig. 2.

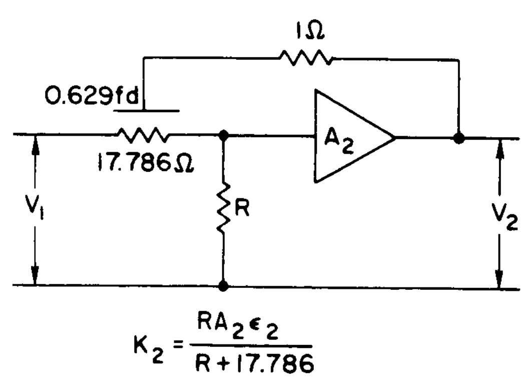


Fig. 5.- DLA network approximating $T_2(p) = \frac{K_2(p^2+1)}{p^2+\delta_2 p+\epsilon_2}$

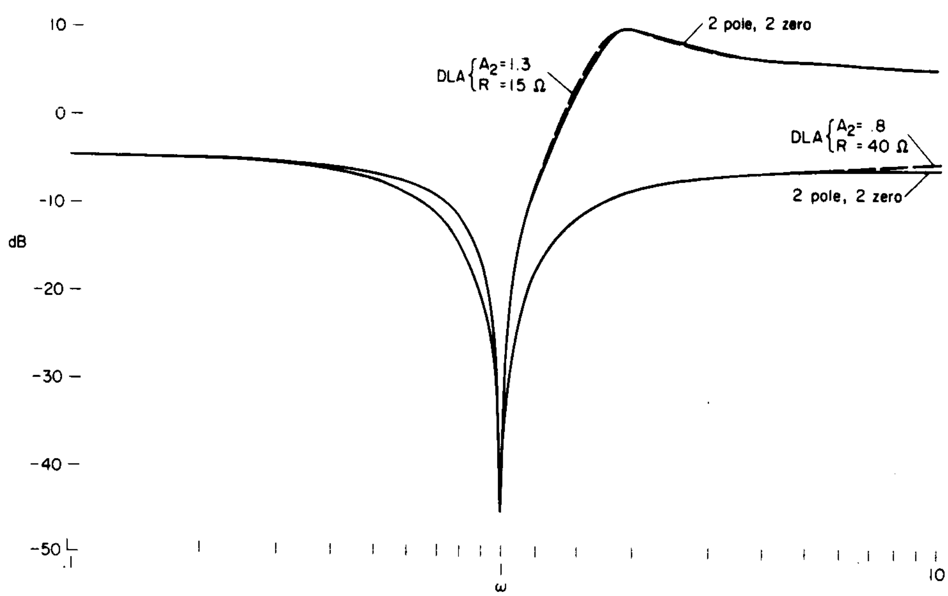


Fig. 6.- Amplitude responses for two choices of parameters of the distributed-lumped-active network of Fig. 5 and a 2-pole, 2 jω axis zero function.

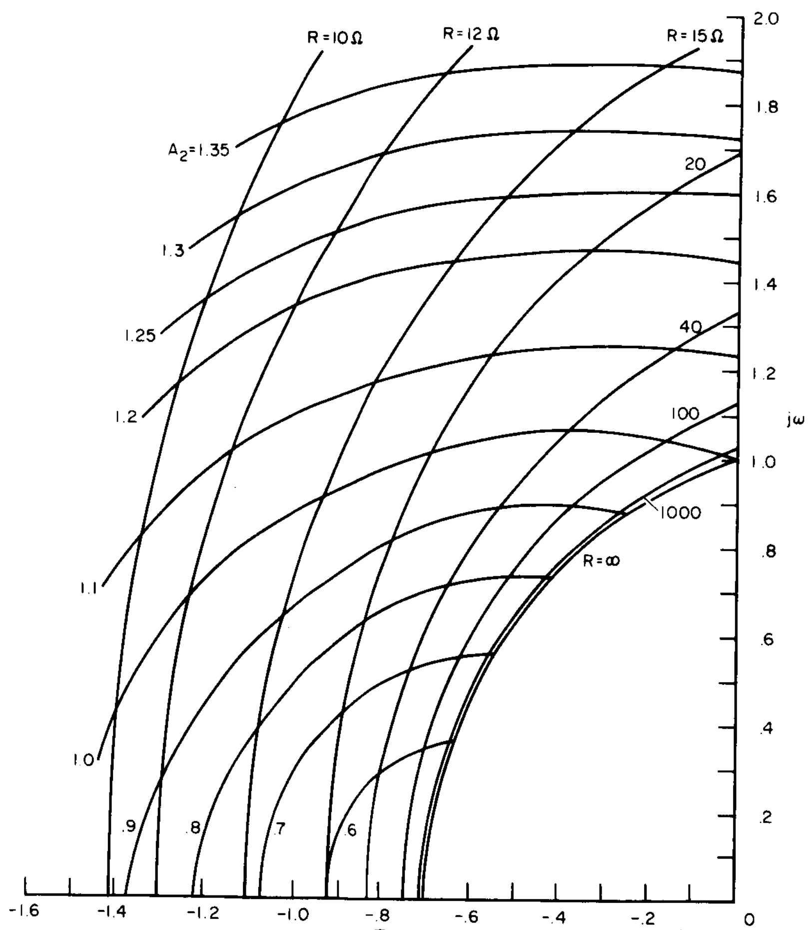
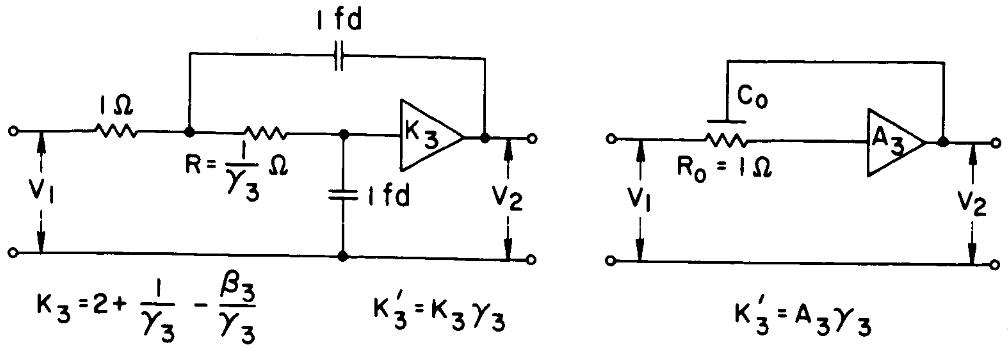


Fig. 7.- Equivalent pole positions for the distributed-lumped-active network of Fig. 5.



(a) Lumped. (b) DLA

Fig. 8.- Active RC networks for $T_3(p) = \dfrac{K_3'}{p^2+\beta_3 p+\gamma_3}$

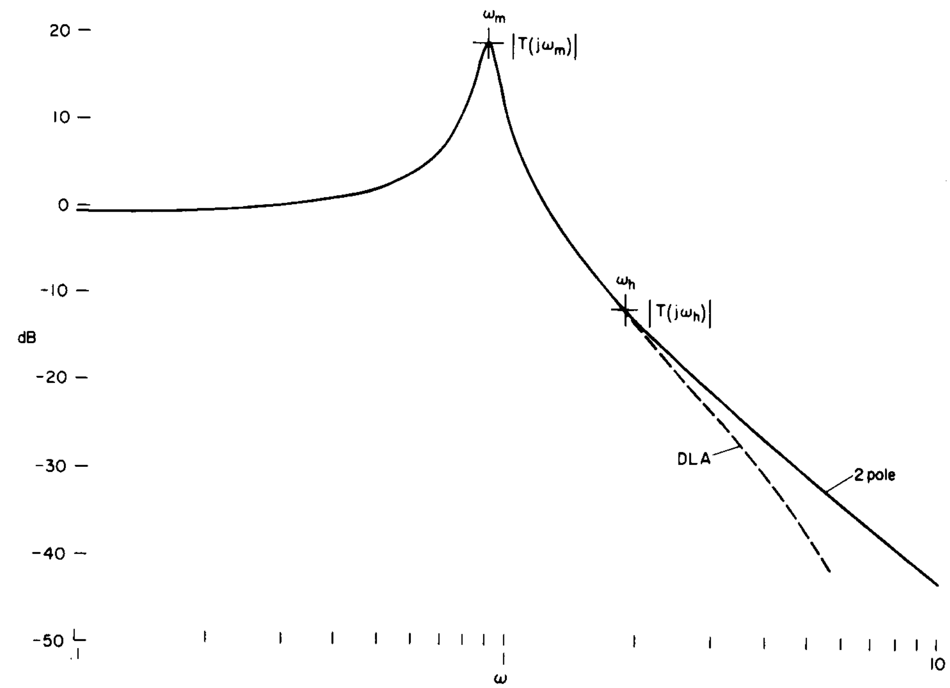


Fig. 9.- Typical amplitude response of the distributed-lumped-active network of Fig. 8(b) and a 2-pole function.

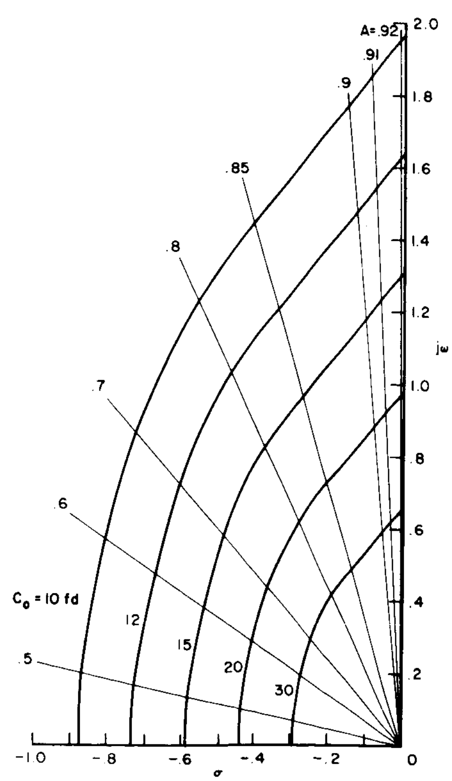


Fig. 10.- Equivalent pole positions for the distributed-lumped-active network of Fig. 8(b).

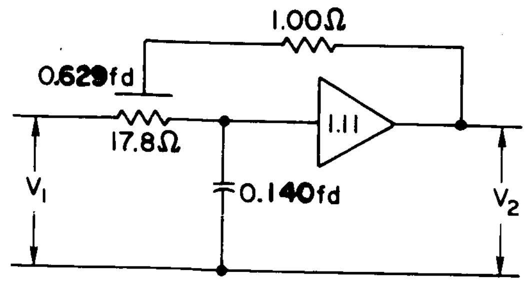


Fig. 11.- DLA network approximating $T_a'(s) = \dfrac{0.556(s^2+1)}{2.785s^2+0.814s+0.501}$

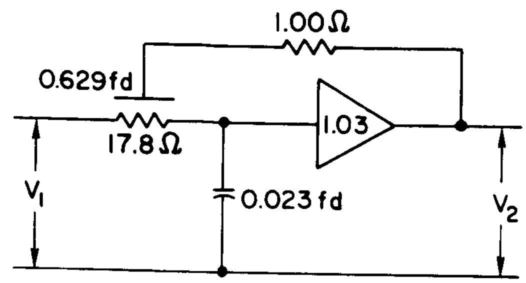


Fig. 12.- DLA network approximating $T_b'(s) = \dfrac{0.832(s^2+1)}{1.355s^2+0.133s+0.808}$

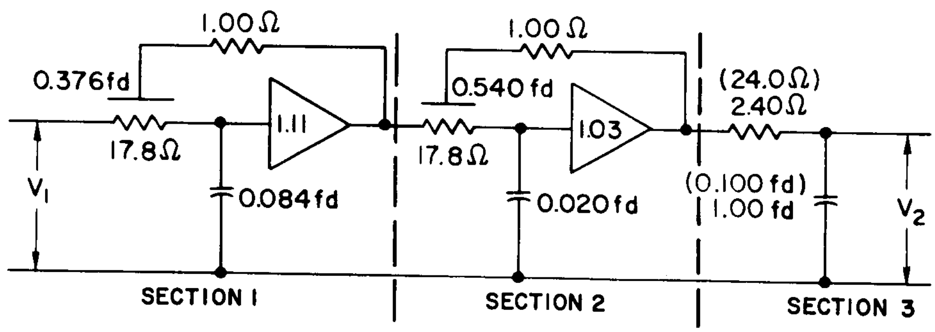


Fig. 13.- DLA network approximating Eq. (2).

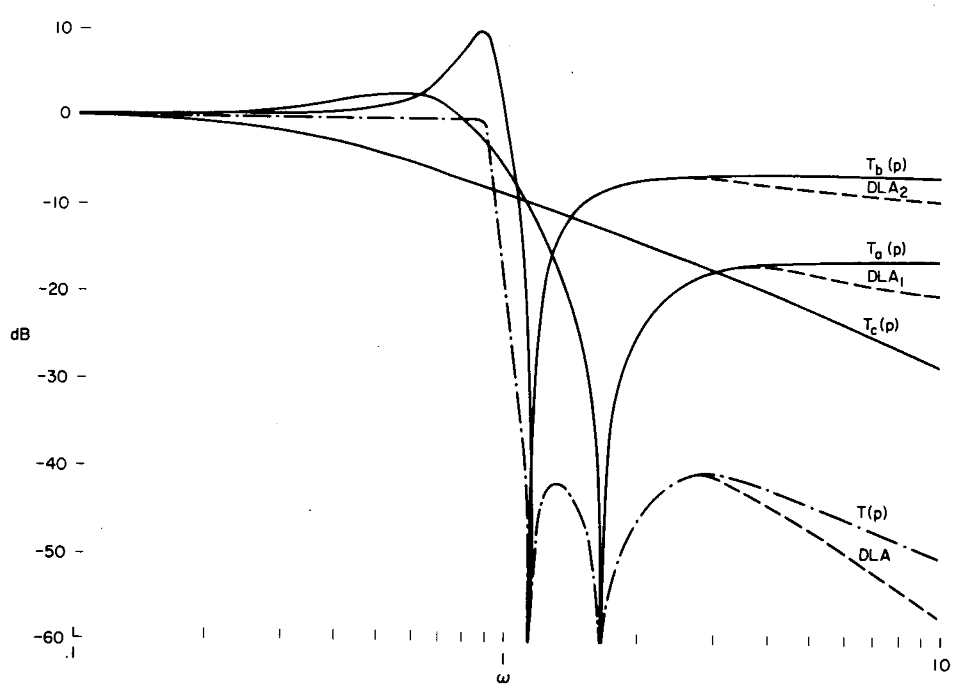


Fig. 14.- 5-pole, 4 jω axis zero low-pass elliptic function filter response.