Synthesis of Arbitrary Quantum States via Adiabatic Transfer of Zeeman Coherence

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A scheme for the preparation of general coherent superpositions of photon-number states is proposed. By strongly coupling an atom to a cavity field, atomic ground-state Zeeman coherence can be transferred by (coherent) adiabatic passage to the cavity mode and a general field state can be generated without atomic projection noise.

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The significance of a fully quantized theory of light has been punctuated in recent years by the experimental realization of highly nonclassical states of the radiation field. In particular, squeezed light and sub-Poissonian light, which exhibit statistics that cannot be associated with any classical stochastic process, have been generated by a number of groups in a variety of configurations [1–3]. Beyond this, exciting challenges still exist for experimentalists in the form of Fock states of the field (which have no intensity fluctuations), and coherent superposition, or "Schrödinger cat," states. Several proposals for the preparation of such states have been put forward [4,5]. These proposals make use of the fascinating and unique possibilities offered by cavity quantum electrodynamics, in which beams of atoms interact strongly with a single quantized field mode of a cavity. Experimental progress in this field over the past few years has been dramatic both in the microwave [2,6] and optical [3,7] regimes, with atom-cavity–mode coupling strengths now exceeding dissipative rates (from spontaneous emission and cavity losses) and making the above-mentioned proposals of very direct relevance.

We would like to propose a new and novel scheme for the preparation of Fock states and *general* superposition states of a cavity field mode which should also be of considerable relevance to existing and future experiments. Our scheme involves the passage of an atom (or atoms) with Zeeman substructure through overlapping cavity and laser fields, and is based on the adiabatic transfer of atomic ground-state Zeeman coherence to the cavity mode.

The preparation of Fock states in a cavity using an adiabatic transfer technique has been proposed previously by Raimond *et al.* [4]. Their scheme utilizes the passage of a two-level atom through a microwave cavity, the resonant frequency of which is continuously tuned (during the atomic transit time) so as to produce an adiabatic transformation of the initial eigenstate describing the atom-cavity system into a final eigenstate corresponding to the cavity mode being in a Fock state. This eigenstate contains a contribution from the excited atomic state, but spontaneous emission (which destroys the adiabatic evolution) is not a major problem in the microwave regime.

In a similar fashion, our scheme makes use of the adiabatic transformation of an eigenstate of an atom-cavity system subject to appropriate time-dependent coupling and laser excitation. However, a new and important feature of our scheme is that the adiabatic transformation is applied to a "dark" eigenstate of the system; i.e., the relevant eigenstate contains no contribution from the excited atomic state, and hence the technique is immune to the effects of atomic spontaneous emission, regardless of the spectral regime one may be considering. This has very significant practical consequences, as it raises the possibility of experimental tests in the *optical regime*, where coherence-destroying spontaneous emission is usually a limiting factor.

The absence of spontaneous emission from the dynamics means that coherent superpositions of atomic ground-state Zeeman sublevels can be mapped directly onto coherent superpositions of cavity photon-number states. Noteworthy features of our scheme are (i) the interactions can be resonant throughout the transfer, (ii) the generality of the superpositions that can be produced in the cavity is limited only by the extent to which one can prepare general superpositions of atomic Zeeman sublevels (e.g., using optical or radio frequency pumping), and (iii) following the transfer, the atomic population is in a *single* atomic state, thus avoiding the introduction of atomic-state "measurement noise."

The particular adiabatic passage procedure that we are proposing has been studied previously in the context of coherent population transfer [8] and coherent atomic-beam deflection [9], in which atomic population is coherently shifted via Raman transitions induced by a pair of (overlapping) time-delayed laser pulses. These previous studies of adiabatic passage have dealt with situations in which photons associated with the Raman transitions are absorbed from, and emitted into, light fields that can be regarded as classical fields (i.e., coherent laser

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fields). Here, we consider the field into which photons are emitted to be the field mode of a cavity, and that the requisite time-dependent coupling to this field (i.e., the "pulse") is provided by the atom's motion through the cavity. Hence, we now consider one of the fields to be quantized.

To illustrate the basic principle and demonstrate the preparation of Fock states of the cavity mode, we consider first the simplest possible configuration. This consists of a three-level atom with two ground states $|g_1\rangle$ and $|g_2\rangle$ (for simplicity we consider these states to have the same energy, but they need not be degenerate) coupled to an excited state $|e\rangle$ via, respectively, a classical laser field $\Omega(t)$ of frequency ω_L and a cavity mode field of frequency ω. The Hamiltonian for this system can be written as (for the moment we shall omit dissipation)

$$H(t) = \hbar\omega a^\dagger a + \hbar\omega_{eg}|e\rangle\langle e| - i\hbar g(t)\left(|e\rangle\langle g_2|a - \text{H.c.}\right) + \frac{1}{2}i\hbar\Omega(t)\left(|e\rangle\langle g_1|e^{-i\omega_L t} - \text{H.c.}\right), \quad (1)$$

where a is the annihilation operator for the cavity mode and $g(t)$ gives the atom-cavity–mode coupling strength. The time dependence of $\Omega(t)$ and $g(t)$ denotes the motion of the atom across the laser- and cavity-field profiles.

The Hamiltonian $H(t)$ has the property that it couples only states within the family, or manifold, $\{|g_1,n\rangle, |e,n\rangle, |g_2,n+1\rangle\}$, where $|g,n\rangle \equiv |g\rangle|n\rangle$, $|e,n\rangle \equiv |e\rangle|n\rangle$, and $|n\rangle$ represents an n-photon Fock state of the cavity mode. Such a family is depicted in Fig. 1(a). The adiabatic energy eigenvalues of the Hamiltonian associated with a particular family of states are

$$E_n = n\hbar\omega,$$
$$E_n^{\pm} = n\hbar\omega + \hbar\{\Delta \pm \left[\Delta^2 + 4g(t)^2(n+1) + \Omega(t)^2\right]^{1/2}\}/2,$$

where we have assumed that $\omega = \omega_L$, and $\Delta = \omega_{eg} - \omega$ is the detuning. Of particular interest to us is the eigenstate corresponding to $E_n = n\hbar\omega$, which is given by

$$|E_n\rangle = \frac{2g(t)\sqrt{n+1}\,|g_1,n\rangle + \Omega(t)\,|g_2,n+1\rangle}{\sqrt{\Omega(t)^2 + 4g(t)^2(n+1)}}. \quad (2)$$

This eigenstate does not contain any contribution from the excited state (hence the term "dark state"), and is independent of the detuning Δ. The possibility for adiabatic passage arises from the following behavior of $|E_n\rangle$:

$$|E_n\rangle \to \begin{cases} |g_1,n\rangle \text{ for } \Omega(t)/g(t) \to 0, \\ |g_2,n+1\rangle \text{ for } g(t)/\Omega(t) \to 0. \end{cases} \quad (3)$$

That is, for the pulse sequence in which the $\Omega(t)$ pulse is time delayed with respect to $g(t)$, the state $|g_1,n\rangle$ may be adiabatically transformed into the state $|g_2,n+1\rangle$. A possible experimental configuration providing such a pulse sequence is shown schematically in Fig. 1(b). Assuming simple Gaussian pulse profiles for $\Omega(t)$ and $g(t)$ of width T (FWHM) and peak intensities Ω_{max} and g_{max}, the necessary condition for adiabatic following is [8]

$$\Omega_{max}T, \;\; 2g_{max}\sqrt{n+1}\,T \gg 1. \quad (4)$$

This condition results from the requirement that the probability for transitions from $|E_n\rangle$ to other states be very small. Given that the pulses have a significant overlap in time, it ensures that $|E_n\rangle$ is well separated from $|E_n^{\pm}\rangle$ throughout the interaction, and that nonadiabatic coupling between these eigenstates is not significant.

An immediate consequence of adiabatic passage in this system is the generation of Fock states of the cavity mode, an intriguing point of note being that single-photon Fock states are produced from adiabatic passage "out of the vacuum." In particular, if the atom enters the interaction region in state $|g_1\rangle$ and the cavity mode is initially in the *vacuum* state $|0\rangle$, then by adiabatic passage the initial state $|g_1,0\rangle$ is transformed into the state $|g_2,1\rangle$ and a single-photon Fock state is realized. Sequences of atoms can be used to generate Fock states of higher photon number, each atom providing a "shift" of one photon through the transformation $|g_1,n\rangle \to |g_2,n+1\rangle$. More generally, any initial distribution of Fock states will experience such a "shift" as a result of the passage of an atom through the cavity and laser fields.

Dissipation in the form of spontaneous emission and cavity damping is, of course, an important practical issue. The adiabatic passage technique is robust against the effects of spontaneous emission, as, in principle, the excited atomic state is never appreciably populated. Cavity damping is certainly a problem as its effects come into play as soon as the cavity mode is excited, leading to degradation of the adiabatic transfer (a simple picture of the effect of dissipation is that it couples manifolds $\{|g_1,n\rangle, |e,n\rangle, |g_2,n+1\rangle\}$ of different n: ideal adiabatic transfer occurs when the passage is across a single manifold). Hence, the technique will be optimized when

$$\Omega_{max}, g_{max} \gg \Gamma, n_{max}\kappa \;\text{ and }\; T \ll (n_{max}\kappa)^{-1}, \quad (5)$$

where Γ is the rate of spontaneous emission, κ is the cavity decay rate, and n_{max} is the maximum photon number attained by the cavity mode. That is, we require conditions under which "vacuum Rabi splitting" would be observable in the coupled atom-cavity system [2,4,6,7].

The adiabatic passage procedure is readily generalized to more complicated atomic-level configurations, as, for

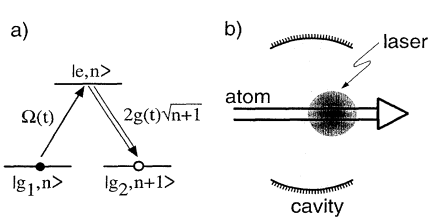

FIG. 1. (a) Λ three-level atom. (b) Proposed configuration for the preparation of Fock states using three-level atoms. The propagation direction of the pump laser is perpendicular to the page.

instance, in an atom possessing Zeeman substructure. The associated increase in the possible number of Raman transitions means that a single atom may be used to generate a multiphoton Fock state of the cavity. As an example, we consider an atomic $J_g = N \rightarrow J_e = N-1$ transition in the adiabatic passage configuration, with a π-polarized cavity mode field and a σ^+-polarized laser field. Including dissipation, we turn to a master equation description which takes the form

$$\frac{\partial \rho}{\partial t} = -i(H_{eff}\rho - \text{H.c.}) + \Gamma \sum_{\sigma=0,\pm1} A_\sigma \rho A_\sigma^\dagger + 2\kappa a \rho a^\dagger, \tag{6}$$

where $\rho(t)$ is the reduced density operator of the system,

$$H_{eff} = (\Delta - i\Gamma/2)\sum_{m_e} |J_e m_e\rangle\langle J_e m_e| - i\kappa a^\dagger a$$
$$-i\Omega(t)(A_{+1} - A_{+1}^\dagger) + ig(t)(a^\dagger A_0 - A_0^\dagger a) \tag{7}$$

and the atomic lowering operators A_σ are given by

$$A_\sigma = \sum_{m_e,m_g} |J_g m_g\rangle\langle J_g m_g; 1\sigma | J_e m_e\rangle\langle J_e m_e|, \tag{8}$$

where $\langle J_g m_g; 1\sigma | J_e m_e\rangle$ is a Clebsch-Gordan coefficient for the dipole transition $|e\rangle \rightarrow |g\rangle$ with polarization $\sigma = 0, \pm1$. Provided that the conditions for adiabatic transfer are satisfied, i.e., that we have a pulse sequence in which $g(t)$ precedes $\Omega(t)$, and that the conditions (4) and (5) are satisfied, then passage may occur along a dark state, which, for the particular transformation $|g_{-N}, n\rangle \rightarrow |g_{N-1}, n+2N-1\rangle$ takes the form

$$|E_n, g_{-N}\rangle = \mathcal{N}\{\,|g_{-N}, n\rangle G_{-N+1}^{(n)} G_{-N+2}^{(n)} \cdots G_{N-1}^{(n)} + |g_{-N+1}, n+1\rangle \Omega_{-N+1} G_{-N+2}^{(n)} \cdots G_{N-1}^{(n)}$$
$$+ \cdots + |g_{N-1}, n+2N-1\rangle \Omega_{-N+1}\Omega_{-N+2}\cdots\Omega_{N-1}\}, \tag{9}$$

where $G_k^{(n)} = g(t)\sqrt{n+N+k}\langle J_g(m_g = k); 10 | J_e(m_e = k)\rangle$ $(k < N)$, $\Omega_k = \Omega(t)\langle J_g(m_g = k-1); 11 | J_e(m_e = k)\rangle$ $(k > -N)$, and $\mathcal{N}$ is a normalization factor. Hence, for an initial vacuum state of the cavity mode, passage of a single atom yields a $(2N-1)$-photon Fock state in the cavity, and each subsequent atom (entering in the state $|g_{-N}\rangle$) increases the photon number by $2N-1$.

For a specific example, we consider a $J_g = 2 \rightarrow J_e = 1$ transition ($N = 2$), as depicted in Fig. 2(a), where we have drawn the (seven) states in the manifold of which the initial state $|g_{-2}, 0\rangle$ is a member. Adiabatic passage in this system corresponds to the transformation $|g_{-2}, 0\rangle \rightarrow |g_{+1}, 3\rangle$ and hence to the preparation of a three-photon Fock state. This preparation is shown in more detail in Fig. 3, where we display the time variation of (a) the exciting pulses, (b) the (seven) energy eigenvalues associated with the particular manifold involved, (c) the populations of the atomic ground-state sublevels, and (d) the mean cavity photon number, $\langle a^\dagger a\rangle$, and the Mandel Q parameter. These results were obtained by numerical integration of the master equation. The Q parameter is a measure of intensity fluctuations in the cavity field ($Q = -1 + [\langle (a^\dagger a)^2\rangle - \langle a^\dagger a\rangle^2]/\langle a^\dagger a\rangle$) and is equal to -1 for a pure Fock state. In Fig. 3(d), we have also included results obtained with nonzero cavity damping. This damping clearly limits the maximum obtainable cavity photon number and the maximum reduction

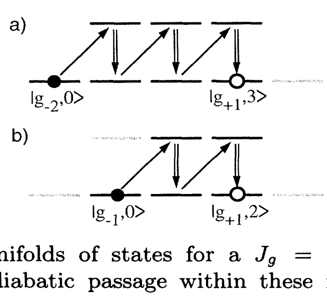

FIG. 2. Manifolds of states for a $J_g = 2 \rightarrow J_e = 1$ transition. Adiabatic passage within these manifolds produces the transformations (a) $|g_{-2}, 0\rangle \rightarrow |g_{+1}, 3\rangle$, and (b) $|g_{-1}, 0\rangle \rightarrow |g_{+1}, 2\rangle$. (The family of states $|g_0, 0\rangle \rightarrow |g_{+1}, 1\rangle$ and $|g_{+1}, 0\rangle \rightarrow |g_{+1}, 0\rangle$ are not shown.)

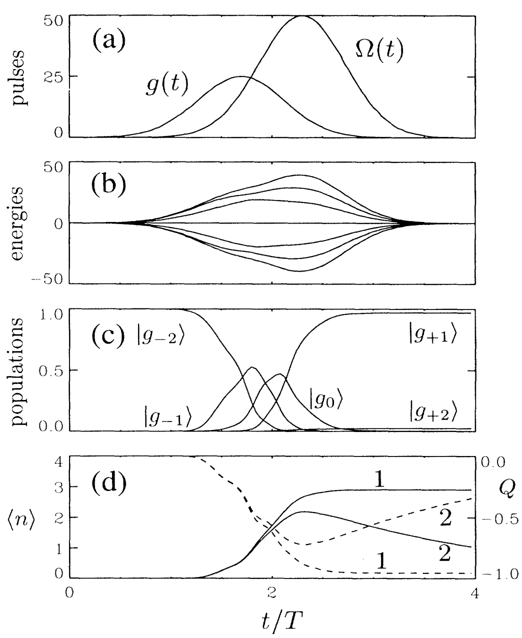

FIG. 3. Preparation of a three-photon Fock state via the adiabatic transformation $|g_{-2}, 0\rangle \rightarrow |g_{+1}, 3\rangle$. Time variation of (a) exciting pulses (units of T^{-1}), (b) energy eigenvalues for the manifold shown in Fig. 2(a) (units of $\hbar T^{-1}$), (c) populations of the atomic ground-state sublevels, (d) mean cavity photon number (solid) and Mandel Q parameter (dashed) for $2\kappa T = 0$ (1) and $2\kappa T = 0.5$ (2). Other parameters for this figure are $g_{max}T = 25$, $\Omega_{max}T = 50$, $\Gamma T = 5$, $\Delta = 0$.

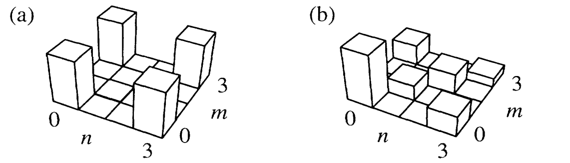

FIG. 4. Field superposition state $(|0\rangle + |3\rangle)\sqrt{2}$ generated for the parameters and pulse sequence of Fig. 3, with the atom initially prepared in the state $(|g_{-2}\rangle + |g_{+1}\rangle)/\sqrt{2}$. The figures show the moduli of the density matrix elements $\langle n|\rho_f|m\rangle$ of the cavity mode at time $t/T = 3.0$, with (a) $2\kappa T = 0$, (b) $2\kappa T = 0.5$.

in intensity fluctuations. A more comprehensive study of the effects of dissipation on our scheme will be presented elsewhere.

The situation in which the cavity mode is initially in the vacuum state is of special interest because, in this case, the system states $|g_m, 0\rangle$, with $m = -N, -N + 1, \ldots, N-1$, each belong to different (orthogonal) manifolds of the system Hamiltonian, and hence they evolve (under ideal conditions) independently of each other. Furthermore, each of these manifolds possesses a dark state along which adiabatic passage may proceed, yielding the general transformation $|g_m, 0\rangle \rightarrow |g_{N-1}, N-m-1\rangle$. For the $J_g = 2 \rightarrow J_e = 1$ transition, this means that we can also consider adiabatic passage in, for example, the manifold of states beginning with $|g_{-1}, 0\rangle$, as shown in Fig. 2(b), which yields a two-photon Fock state.

This possibility of independent adiabatic passage along dark states belonging to different manifolds suggests a means of preparing superposition states of the cavity mode. In particular, given that the atom is prepared in a coherent superposition of ground-state Zeeman sublevels, and that the cavity is initially in the vacuum state, adiabatic passage will coherently "map" this superposition onto the cavity mode Fock states. This is summarized, for a $J_g = N \rightarrow J_e = N-1$ atomic transition, by the transformation equation

$$\sum_{m=-N}^{N-1} c_m |g_m\rangle\, |0\rangle \rightarrow |g_{N-1}\rangle \sum_{m=-N}^{N-1} c_m |N-m-1\rangle. \quad (10)$$

This is the central result of the paper, and illustrates a means by which we can, in principle, generate arbitrary superpositions of Fock states, limited only by the Zeeman degeneracy of the atom, and by the extent to which one can prepare arbitrary superpositions of ground-state Zeeman sublevels.

As an example, in Fig. 4 we illustrate the superposition state $(|0\rangle + |3\rangle)\sqrt{2}$ generated in the cavity as a result of the passage of an atom ($J_g = 2 \rightarrow J_e = 1$ transition) initially prepared in the state $(|g_{-2}\rangle + |g_{+1}\rangle)/\sqrt{2}$. The off-diagonal elements (coherences) of the field density matrix which characterize coherent superpositions are still found to persist for a significant time when cavity damping is present [Fig. 4(b)].

From an experimental point of view (in the optical regime), the realization of parameters satisfying the conditions (4) and (5) appears feasible with realistic improvements to present experiments [3,7]. Photon counting measurements (with existing quantum efficiencies), in conjunction with interrogation of the state of the atom after it leaves the cavity, should suffice to record nonclassical values of the Mandel Q parameter ($Q < 0$) in the case of intracavity Fock states. For a more comprehensive characterization of the state of the field, the Wigner function of the cavity field could be determined from a sequence of tomographic measurements which record the probability distributions for photocurrent fluctuations in homodyne detection for various phase offsets between the local oscillator and signal fields [10]. Such measurements could be made in a conditional fashion (e.g., homodyne current conditioned upon transit of an atom through the cavity).

Finally, we note that the adiabatic passage scheme is reversible; i.e., by reversing the order of $g(t)$ and $\Omega(t)$, states of the cavity field can be mapped onto atomic ground-state sublevels. Probing these sublevels following the passage of an atom would thus provide detailed information on the initial cavity field. Hence, adiabatic transfer could itself serve as a powerful tool for the *measurement* of cavity fields.

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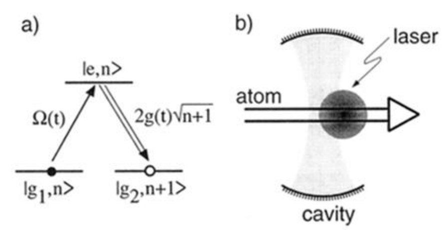

FIG. 1. (a) Λ three-level atom. (b) Proposed configuration for the preparation of Fock states using three-level atoms. The propagation direction of the pump laser is perpendicular to the page.