Synthesis Techniques for a Class of SSB-AM-PM Signals

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Abstract

This paper develops synthesis techniques for a particular type of single-sideband sinusoidal carrier which is phase modulated by a subcarrier. Mathematical expressions for signal efficiency, sensitivity of design to parameter variation, and ratio of peak to average power are derived and incorporated in a computer program. Given the desired power ratios for modulated signal components, the program solves for the corresponding modulation parameters and evaluates signal efficiency, design sensitivity, and peak to average power ratio. A sample signal design is presented for clarity.

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Introduction

The use of angle-modulated sinusoidal subcarriers on low-deviation phase-modulated (PM) sinusoidal carriers is widely accepted in space communications. Such signals are used in NASA manned space programs and deep space scientific programs, and in USAF programs. In particular, use of such signals is common to the supporting communication systems for the Manned Space Flight Network, Deep Space Network, and Space-Ground-Link System. So-called "subcarrier-PM" signals are almost a standard in space communications and are referred to as such in this paper.

Given the number of data channels to be transmitted and the signal-to-noise ratio (SNR) requirements for each channel, the synthesis procedure for a standard subcarrier-PM signal consists of just three parts. First, subcarrier frequencies are selected. Second, the phase or frequency deviation of each data channel on its respective subcarrier is selected. Third, the phase deviations of each subcarrier onto the carrier are determined. Selection of subcarrier frequencies is generally controlled by minimization of interchannel interference, and becomes an increasingly difficult problem with increasing numbers of data channels. Selection of angle deviations for data on each subcarrier is straightforward. Determination of phase deviations for each subcarrier onto the carrier is reasonably straightforward, and is treated in the literature [1], [2]. Essentially, the phase deviations are chosen so as to satisfy constraints on ratios of powers of the modulation and residual carrier components.

It is possible to generate a single-sideband (SSB) signal by simultaneously amplitude-and-phase modulating a sinusoidal carrier with angle-modulated subcarriers. Such a signal is compatible with carrier demodulators used for the standard subcarrier-PM signal. It is the purpose of this paper to develop a general synthesis procedure for such signals.

Signal Modeling

The Basic Signal

There exist eight unique mathematical models which give SSB-AM-PM signals for a sinusoidal (subcarrier) modulating function [3]. Four of these models give signals having upper sidebands. The other four give signals having lower sidebands. Of the four upper sideband signals, two actually have the modulating function in the carrier phase term. For a single subcarrier of the form

$$f(t) = \beta \cos [\omega_m t + \theta(t)] \tag{1}$$

where β, ω_m are constants and $\theta(t)$ is subcarrier angle modulation, the two SSB-AM-PM signals are

$$s_1(t) = A \exp [-f(t)] \cos [\omega_c t + f(t)] \tag{2}$$

$$s_2(t) = A \exp [f(t)] \cos [\omega_c t - f(t)] \tag{3}$$

where A, ω_c are constants and the caret denotes Hilbert transform.

In the standard subcarrier-PM signal, the total modulated carrier power is constant, being shared between modulation products and residual carrier power. For modulation by a single subcarrier with relatively low phase deviation, the resulting power spectrum is localized in frequency with a delta function at ω_c representing the residual carrier, and other components at $\omega_c + n\omega_m$ where n is a member of the set of integers. The amount of power in any distinct spectral component, say that one at $\omega_c \pm \omega_m$, is independent of the subcarrier angle modulation function $\theta(t)$. That is, the total power in the first-order spectral component (or any other component) is the same whether or not the subcarrier is modulated. This property also holds for the SSB-AM-PM signal. This may be verified by expanding the spectral densities of the modulated carriers in infinite series for the cases where the subcarrier is and is not modulated, and then applying Parseval's relation. Thus, a simplification may be made in that it is only necessary to consider modulating functions of the form

$$f(t) = \beta \cos \omega_m t. \tag{4}$$

In the synthesis of the standard subcarrier-PM signal it is necessary to treat an arbitrary number k of subcarriers, since the power in each spectral component is dependent on the phase deviation of all subcarriers. However, in the SSB-AM-PM case, the power in any first-order subcarrier spectral component depends only on the phase deviation of that particular subcarrier. This may be shown for an arbitrary number k of subcarriers by expanding the analytic form [4] of the modulated carrier to obtain a k-fold product of infinite series. Application of the Cauchy rule for multiplying convergent series yields a series expression for the modulated signal from which the jth subcarrier term (of frequency $\omega_c + \omega_j$) may be isolated. Thus, attention may now be restricted to the case of modulation by a single sinusoid of the form of (4).

By expanding the analytic versions of $s_1(t)$ and $s_2(t)$ in MacLaurin series, (2) and (3) may be written as

$$\begin{aligned} s_1(t) &= A \exp\left[-\beta \sin \omega_m t\right] \cos\left[\omega_c t + \beta \cos \omega_m t\right] \\ &= \sum_{n=0}^{\infty} A \frac{\beta^n}{n!} \cos\left[(\omega_c + n\omega_m)t + \frac{n\pi}{2}\right] \end{aligned} \tag{5}$$

$$\begin{aligned} s_2(t) &= A \exp\left[\beta \sin \omega_m t\right] \cos\left[\omega_c t - \beta \cos \omega_m t\right] \\ &= \sum_{n=0}^{\infty} A(-1)^n \frac{\beta^n}{n!} \cos\left[(\omega_c + n\omega_m)t + \frac{n\pi}{2}\right]. \end{aligned} \tag{6}$$

As shown in Fig. 1, the signals $s_1(t)$ and $s_2(t)$ differ only in the relative phasing of the modulation components. From (5) and (6) it is immediately obvious that the power in the residual carrier P_0 and the power in the first-order component P_1 are given for both signals by

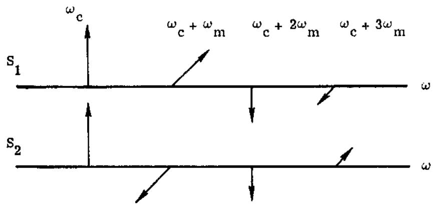

Fig. 1. Relative phases of signals $s_1(t)$ and $s_2(t)$.

$$P_0 = \frac{A^2}{2} \tag{7}$$

$$P_1 = \beta^2 \frac{A^2}{2}. \tag{8}$$

It is readily determined that the total modulated signal power is given by

$$\begin{aligned} P_T &= \frac{A^2}{2} \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} \exp\left[-2\beta \sin \omega_m t\right] dt \quad \text{for } s_1(t) \\ P_T &= \frac{A^2}{2} \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} \exp\left[2\beta \sin \omega_m t\right] dt \quad \text{for } s_2(t). \end{aligned} \tag{9}$$

Noting that both integrands are periodic in 2π for

$$\theta = \omega_m t, \tag{10}$$

it follows that for both signals

$$P_T = \frac{A^2}{2\pi} \int_0^{\pi} \exp\left[2\beta \cos \theta\right] d\theta \triangleq \frac{A^2}{2} I_0(2\beta) \tag{11}$$

where I_0 denotes the modified Bessel function.

Now, if an efficiency factor E is defined as the ratio of first-order component power to total power, (8) and (11) yield

$$E \triangleq \frac{P_1}{P_T} \equiv \frac{\beta^2}{I_0(2\beta)}. \tag{12}$$

The function E has an absolute maximum of

$$E \max = 0.475; \qquad \beta = 1.29. \tag{13}$$

Thus it is seen that less than half the available power can be concentrated in the first-order component. Also, to vary the ratio of P_1 to P_0, as is required in synthesis, means that control over efficiency is lost. Hence, it appears that using a signal model of the form $s_1(t)$ or $s_2(t)$, alone, is undesirable.

The Combined Signal

What is needed is a signal structure where a high ratio of subcarrier power to total power is obtained while retaining control over the ratio of subcarrier to residual carrier power. Such a signal is obtained by subtracting a signal of the form of $s_1(t)$ from a signal of the form of $s_2(t)$. The combined signal is given by

$$\begin{aligned} s(t) &= A_2 \exp\left[\beta_2 \sin \omega_m t\right] \\ &\quad \cdot \cos\left[\omega_c t - \beta_2 \cos \omega_m t\right] \\ &\quad - A_1 \exp\left[-\beta_1 \sin \omega_m t\right] \\ &\quad \cdot \cos\left[\omega_c t + \beta_1 \cos \omega_m t\right] \\ &= \sum_{n=0}^{\infty} \frac{A_2(-1)^n \beta_2{}^n - A_1 \beta_1{}^n}{n!} \\ &\quad \cdot \cos\left[(\omega_c + n\omega_m)t + n\frac{\pi}{2}\right]. \end{aligned} \tag{14}$$

From (14) it may be seen that the powers residing in the residual carrier P_0, first-order subcarrier component P_1, and second-order subcarrier component P_2, are, respectively,

$$\begin{aligned} P_0 &= \tfrac{1}{2}(A_2 - A_1)^2 \\ P_1 &= \tfrac{1}{2}(A_2\beta_2 + A_1\beta_1)^2 \\ P_2 &= \tfrac{1}{8}(A_2\beta_2{}^2 - A_1\beta_1{}^2)^2. \end{aligned} \tag{15}$$

Now, let four parameters be defined as

$$\begin{aligned} C_\alpha &= \frac{A_2}{A_1} \quad \text{(carrier factor)} \\ C_\beta &= \frac{\beta_2}{\beta_1} \quad \text{(subcarrier factor)} \\ k_1{}^2 &= \frac{P_1}{P_0} \quad \text{(modulation to residual carrier power ratio)} \\ k_2{}^2 &= \frac{P_2}{P_0} \quad \text{(harmonic to residual carrier power ratio).} \end{aligned} \tag{16}$$

Substituting (15) into (16) gives

$$\begin{aligned} k_1(C_\alpha - 1) &= \beta_1(C_\alpha C_\beta + 1) \\ 2k_2(C_\alpha - 1) &= \beta_1{}^2(C_\alpha C_\beta{}^2 - 1). \end{aligned} \tag{17}$$

The basic synthesis problem is the selection of phase deviations of the subcarrier onto the carrier. For standard subcarrier-PM signals, the phase deviations are chosen so as to satisfy a constraint on the ratio of first-order subcarrier power to carrier power while maximizing efficiency. In the standard signal the only adjustable parameter is the phase deviation, itself. In the combined SSB-AM-PM signal, constructed as in (14), additional adjustable parameters are the ratios of the two carrier amplitudes and of the two carrier deviations. Hence, additional constraints may be satisfied.

Equations (17) relate the ratios of first- and second-order subcarrier component powers to residual carrier power with the adjustable parameters C_α, C_β, and β_1. Obviously, it is justifiable to impose a third constraint before attempting to obtain solutions for C_α, C_β, and β_1. In synthesis of the standard subcarrier-PM signal the solution for carrier deviation by the subcarrier is actually obtained by maximizing an efficiency expression. It will be seen later that this approach cannot be followed for the SSB-AM-PM signal. Thus, the question is, "What is a sensible constraint to impose on the signal design?"

There are three general areas in which to seek a constraint. One area is that of signal efficiency, or amount of unusable power. The second area is that of sensitivity or stability of design to small changes in parameters C_α, C_β, β_1. The third area is that of peak to average power ratio.

To investigate signal efficiency requires an expression for total signal power, which is derived as follows. By use of the analytic form of $s(t)$, noting periodicity, an expression for total signal power P_T is obtained as

$$\begin{aligned} P_T = \frac{1}{2\pi}\int_0^{\pi} &\{A_2{}^2 \exp\left[2\beta_2 \cos\phi\right] \\ &+ A_1{}^2 \exp\left[-2\beta_1 \cos\phi\right] \\ &- 2A_1A_2 \exp\left[(\beta_2 - \beta_1)\cos\phi\right] \\ &\cdot \cos\left[(\beta_2 + \beta_1)\sin\phi\right]\} d\phi. \end{aligned} \tag{18}$$

Applying several Bessel function identities and the definitions of (16) gives

$$\begin{aligned} P_T = \frac{A_1{}^2}{2}\Big\{&(C_\alpha{}^2 I_0(2C_\beta\beta_1) + I_0(2\beta_1) \\ &- 2C_\alpha\{J_0[(C_\beta + 1)\beta_1]I_0[(C_\beta - 1)\beta_1] \\ &+ 2\sum_{k=1}^{\infty} J_{2k}[(C_\beta + 1)\beta_1]I_{2k}[(C_\beta - 1)\beta_1]\}\Big\}. \end{aligned} \tag{19}$$

Equation (19) converges very rapidly for low deviation modulation. Hence few terms of the series need be used in practical calculations.

There are two different efficiency factors which may be considered. The first is the ratio of first-order subcarrier component power to total power, P_1/P_T. Since, in some system applications, the second-order subcarrier component is utilized, along with the residual carrier, a second efficiency factor is the ratio $(P_0+P_1+P_2)/P_T$. Substituting (16) into (15) gives

$$P_0 = \frac{A_1{}^2}{2}(C_\alpha - 1)^2$$

$$P_1 = \frac{A_1{}^2}{2}(\beta_1{}^2)(C_\alpha C_\beta + 1)^2$$

$$P_2 = \frac{A_1^2}{2}\left(\frac{\beta_1^4}{4}\right)(C_\alpha C_\beta^2 - 1)^2. \tag{20}$$

Denoting

$$\begin{aligned} G_0 &= (C_\alpha - 1)^2 \\ G_1 &= (\beta_1)^2(C_\alpha C_\beta + 1)^2 \\ G_2 &= \left(\frac{\beta_1^4}{4}\right)(C_\alpha C_\beta^2 - 1)^2 \end{aligned} \tag{21}$$

and

$$\begin{aligned} F_1 = \Big\{ & C_\alpha^2 I_0(2C_\beta\beta_1) + I_0(2\beta_1) \\ & - 2C_\alpha\{J_0[(C_\beta+1)\beta_1]I_0[(C_\beta-1)\beta_1] \\ & + 2\sum_{k=1}^{\infty} J_{2k}[(C_\beta+1)\beta_1]I_{2k}[(C_\beta-1)\beta_1]\}\Big\}, \end{aligned} \tag{22}$$

define

$$\begin{aligned} F_2 &\triangleq \frac{P_0 + P_1 + P_2}{P_T} = \frac{G_0 + G_1 + G_2}{F_1} \\ F_3 &\triangleq \frac{P_1}{P_T} = \frac{G_1}{F_1}. \end{aligned} \tag{23}$$

F_2 and F_3 are the efficiency factors described above.

Considered next is the sensitivity of the signal design to variation in C_α, C_β, β_1. In a physical SSB-AM-PM system, C_α, C_β, β_1 can only be realized within certain tolerances which are a function of the hardware. Hence, it is certainly desirable to minimize the sensitivity of the design to changes in the parameters. What is meant by sensitivity of the design is the sensitivity of the ratios k_1^2 and k_2^2. Stability factors may be defined very conveniently as partial derivatives. Define

$$\begin{aligned} S_{k1}(C_\alpha) &\triangleq \frac{\partial k_1}{\partial C_\alpha} = -\frac{\beta_1(C_\beta+1)}{(C_\alpha-1)^2} \\ S_{k1}(C_\beta) &\triangleq \frac{\partial k_1}{\partial C_\beta} = \frac{\beta_1 C_\alpha}{C_\alpha - 1} \\ S_{k1}(\beta_1) &\triangleq \frac{\partial k_1}{\partial \beta_1} = \frac{(C_\alpha C_\beta + 1)}{(C_\alpha - 1)} \\ S_{k2}(C_\alpha) &\triangleq \frac{\partial k_2}{\partial C_\alpha} = \left(\frac{\beta_1^2}{2}\right)\frac{(1 - C_\beta^2)}{(C_\alpha-1)^2} \\ S_{k2}(C_\beta) &\triangleq \frac{\partial k_2}{\partial C_\beta} = \beta_1^2\frac{C_\alpha C_\beta}{(C_\alpha - 1)} \\ S_{k2}(\beta_1) &\triangleq \frac{\partial k_2}{\partial \beta_1} = \beta_1\frac{(C_\alpha C_\beta^2 - 1)}{(C_\alpha - 1)}. \end{aligned} \tag{24}$$

A last possible area in which to look for a constraint on the signal design is that of peak signal power. From (14) it may be determined that the absolute peak value squared of $s(t)$ is

$$\begin{aligned} |spk|^2 &= |A_2\exp(\beta_2) - A_1\exp(-\beta_1)|^2 \\ &= \left(\frac{A_1^2}{2}\right)2\,|C_\alpha\exp(C_\beta\beta_1) - \exp(-\beta_1)|^2. \end{aligned} \tag{25}$$

A ratio F_p of peak power to average power may then be defined as

$$F_p \triangleq \frac{|spk|^2}{P_T} = \frac{H}{F_1} \tag{26}$$

where

$$H = 2\,|C_\alpha\exp(C_\beta\beta_1) - \exp(-\beta_1)|^2. \tag{27}$$

Note that the peak to average power factor of (26) has value $F_p = 2$ for no modulation, which is just the square of the peak-to-rms value for a sinusoid.

Synthesis

General Technique

A digital computer is used in synthesis of standard subcarrier-PM signals in the interest of efficiency. Although the design equations are solvable graphically, obtaining the solution which maximizes efficiency requires iteration or "cut-and-try." Likewise, in the synthesis of the SSB-AM-PM signal a computer is used. A printout of the program used in obtaining results for this paper is appended.

The initial approach, here, is to set up a diagnostic program which solves (17) for C_α, C_β, β_1, given a set of pairs of values for k_1^2, k_2^2. Since, for a given k_1^2, k_2^2, there are infinitely many solutions to (17), bounds are placed on C_α, C_β, β_1, such that solutions are obtained in the ranges

$$\begin{aligned} 1 < {} & C_\alpha \le 10 \\ 0 \le {} & C_\beta \le 10 \\ 0.1 \le {} & \beta_1 \le 2.00 \end{aligned} \tag{28}$$

by incrementing β_1 in increments of 0.1. The first bound, reached, then terminates the search for solutions for the given pair of values for k_1^2, k_2^2. For each solution pair, C_α, C_β, the values are printed out for the factors defined in (22)–(24), (26), and (27).

It was mentioned earlier that maximization of an efficiency expression can not be used as a constraint on the solution for the SSB-AM-PM case. Observation of the printout of the diagnostic program described above shows why. Solutions are examined for k_1^2 varying from 0 to +15 dB and k_2^2 from 0 to −20 dB.

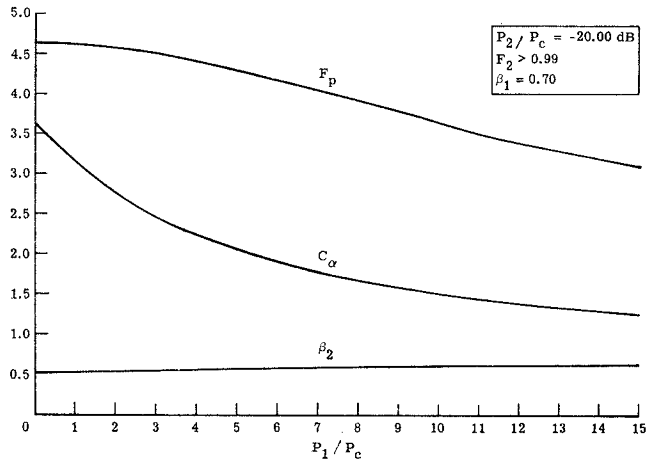

Fig. 2. Variation of F_p, C_α, β_2 for 0 dB $\leq P_1/P_c \leq +15$ dB.

TABLE I

$20 \times \log_{10}$ (K1) = 15.0000, $20 \times \log_{10}$ (K2) = −15.0000

C ALPHA	C BETA	B1	CB×B1	F1	F2	F3	H	FP	B1	SK1CA	SK1CB	SK1B1	SK2CA	SK2CB	SK2B1
1.05	1.86	0.10	0.19	0.09	1.00	0.97	0.26	2.91	0.10	−103.36	2.00	56.23	−4.45	0.37	5.02
1.09	1.40	0.20	0.28	0.26	1.00	0.97	0.77	2.94	0.20	−59.57	2.43	28.12	−2.36	0.68	2.51
1.13	1.23	0.30	0.37	0.53	1.00	0.97	1.59	2.99	0.30	−41.22	2.65	18.74	−1.43	0.98	1.67
1.17	1.14	0.40	0.46	0.90	1.00	0.97	2.74	3.05	0.40	−31.15	2.81	14.06	−0.89	1.28	1.26
1.20	1.08	0.50	0.54	1.37	1.00	0.97	4.28	3.12	0.50	−24.81	2.94	11.25	−0.51	1.59	1.00
1.24	1.04	0.60	0.62	1.96	1.00	0.96	6.28	3.20	0.60	−20.45	3.05	9.37	−0.23	1.90	0.84
1.28	1.00	0.70	0.70	2.67	0.99	0.96	8.79	3.29	0.70	−17.28	3.16	8.03	−0.02	2.22	0.72
1.33	0.97	0.80	0.78	3.51	0.99	0.96	11.90	3.39	0.80	−14.87	3.26	7.03	0.16	2.53	0.63
1.37	0.95	0.90	0.85	4.48	0.98	0.95	15.70	3.50	0.90	−12.98	3.35	6.25	0.31	2.86	0.56
1.41	0.92	1.00	0.92	5.61	0.98	0.95	20.31	3.62	1.00	−11.47	3.44	5.62	0.43	3.18	0.50
1.45	0.90	1.10	0.99	6.91	0.97	0.94	25.85	3.74	1.10	−10.23	3.53	5.11	0.54	3.51	0.46
1.50	0.89	1.20	1.06	8.40	0.96	0.93	32.45	3.87	1.20	−9.19	3.62	4.69	0.63	3.85	0.42
1.54	0.87	1.30	1.13	10.09	0.95	0.91	40.29	3.99	1.30	−8.32	3.71	4.33	0.71	4.18	0.39
1.59	0.85	1.40	1.19	12.03	0.93	0.90	49.53	4.12	1.40	−7.57	3.79	4.02	0.79	4.52	0.36
1.63	0.84	1.50	1.25	14.24	0.91	0.88	60.38	4.24	1.50	−6.93	3.88	3.75	0.85	4.87	0.33
1.68	0.82	1.60	1.32	16.76	0.89	0.86	73.06	4.36	1.60	−6.37	3.96	3.51	0.91	5.21	0.31
1.72	0.81	1.70	1.37	19.64	0.87	0.84	87.82	4.47	1.70	−5.87	4.05	3.31	0.96	5.56	0.30
1.77	0.80	1.80	1.43	22.93	0.85	0.82	104.93	4.58	1.80	−5.44	4.14	3.12	1.00	5.92	0.28
1.82	0.78	1.90	1.49	26.70	0.82	0.79	124.69	4.67	1.90	−5.05	4.22	2.96	1.04	6.28	0.26
1.87	0.77	2.00	1.54	31.02	0.79	0.77	147.44	4.75	2.00	−4.71	4.31	2.81	1.08	6.64	0.25

Over the bounded ranges given in (28), both F_2 and F_3, the efficiency expressions defined in (23), are monotonically decreasing functions of β_1. F_p is monotonically increasing with β_1. Observation also shows that k_1^2 is much more sensitive to β_1 and C_α than is k_2^2. The ratio of peak to average power F_p is reasonably stable, varying from about 3 to about 6. Fig. 2 shows the variation in peak to average power ratio F_p, carrier factor C_α, and phase deviation β_2, as a function of changing ratio of modulation to residual carrier power. The functions are plotted for signals where the second harmonic of the modulation is 20 dB below the residual carrier power and where total signal efficiency is greater than 99 percent.

Design of a signal is naturally tailored to a particular communication system. In particular systems some design constraints may be more heavily weighted than others. For instance, in a particular spacecraft transmitter, there may be an absolute upper limit for peak to average power ratio. On the other hand, a particular ground system may have no limit for peak to average power ratio, but may require a high efficiency factor F_2 to restrict higher order modulation components. It may happen, in a particular system implementation, that drifts in C_α, C_β, due to hardware temperature and aging effects, are correlated. Then it may be desirable to choose a design which realizes a particular ratio of $Sk_1(C_\alpha)$ to $Sk_1(C_\beta)$.

Consideration of the above observations on design constraints suggests that a general approach to synthesis of a SSB-AM·PM signal of the type dealt with here is the use of the diagnostic program. For fixed k_1^2, k_2^2, the program gives as wide a range of solutions for C_α, C_β, β_1 as desired, and presents the various efficiency, sensitivity, and peak to average power factors. A unique solution is then chosen according to the constraints peculiar to the system at hand. On the other hand, a specialized synthesis program can easily be obtained by setting suitable boundaries on any or all of the various factors in the diagnostic program.

Sample Design

Table I is a computer printout of a design for ratios of first-order subcarrier component power to residual carrier power of +15.0 dB and second-order subcarrier power to residual carrier power of −15.0 dB. Of course, the second-order subcarrier component may be eliminated exactly by choosing

$$C_\alpha = 1/C_\beta^2, \tag{29}$$

as may be seen from (17). However, there are occasions where a controlled amount of subcarrier second harmonic may be desirable. An example is a system wherein the subcarrier modulation is binary phase shift keying with a deviation on the subcarrier of $\pm\pi/2$ radians. In this case the subcarrier second harmonic is a ready-made reference for demodulation of the subcarrier.

Observation of Table I shows that the search for solutions is terminated by β_1 reaching the boundary of 2.00. Total efficiency, defined for the sum of residual carrier, first- and second-order subcarrier components, is seen to vary from greater than 99 percent to 79 percent. The efficiency for subcarrier components varies from 97 percent (ideal limiting value) to 77 percent. The various sensitivities are seen to vary considerably. The peak to average power ratio varies only from 2.9 to 4.75. The choice of the particular solution from Table I depends, of course, on the constraints of the system which gives rise to the choices of k_1^2, k_2^2. With no prior constraints, and based on the sensitivity and efficiency factors, one would probably choose a solution with β_1 between 1.00 and 2.00.

Conclusion

A synthesis technique has been given for selecting modulation parameters for a single-sideband carrier which is phase modulated by a sinusoidal subcarrier. The modulated carrier is a composite of two carriers which are each modulated in both amplitude and phase. The composite carrier is used in order to increase signal efficiency. An appended computer program, useful for signal synthesis, is applied to the solution of a sample problem.

Appendix

Diagnostic Program and Subroutines

```
      PROGRAM PAINTER(INPUT,OUTPUT,TAPE5=INPUT,TAPE6=OUTPUT)
      DIMENSION ANSJN(60)
      REAL K1,K2,K1LOW,K1HIGH,K1INC,K2LOW,K2HIGH,K2INC
      EXTERNAL FI02N
      COMMON AN,ARG4
  101 CONTINUE
      READ(5,100)CALOW,CAHIGH,CAINC
      IF(EOF,5)123,456
  456 CONTINUE
      READ(5,100)CBLOW,CBHIGH,CBINC
      READ(5,100)B1LOW,B1HIGH,B1INC
  100 FORMAT(3F10.2)
   32 FORMAT(1H02X*CALPHA*2X*CBETA*7X*B1*4X*CBXB1*3X*F1*7X*F2*5X*F3*4X*S
     1K1CA*3X*SK1CB*3X*SK1B1*3X*SK2CA*3X*SK2CB*3X*SK2B1*4X*H*7X*FP*///)
      EPS=.00001
      K1LOW=1.
      K1HIGH=10.**(.75)
      K2LOW=10.**(-1.)
      K1INC=0.
      K2HIGH=10.**(-.15)
      EPF=.001
      EPG=.00001
      K1=K1LOW
   27 K2=K2LOW
      K2INC=-20.
      TK1=20.*ALOG10(K1)
      TK2=20.*ALOG10(K2)
      WRITE(6,666)TK1,TK2
      I=0
   26 B1=B1LOW
   25 CALPHA=(CAHIGH-CALOW)/2.
      CBETA=(CBHIGH-CBLOW)/2.
      GO TO 8
  250 CALPHA=CALPHA+.1
      CBETA=CBETA-.1
    8 FAB=B1*(CALPHA*CBETA+1.)-K1*(CALPHA-1.)
      GAB=B1*B1*(CALPHA*CBETA**2-1.)-2.*K2*(CALPHA-1.)
      IF(ABS(FAB).GT.EPF)GO TO 4
      IF(ABS(GAB).GT.EPG)GO TO 4


      ARG1=2.*CBETA*B1
      ARG2=2.*B1
      ARG3=(CBETA+1.)*B1
      ARG4=(CBETA-1.)*B1
      CAI0=CALPHA**2*AI0(ARG1)
      CI0=AI0(ARG2)
      C3=AI0(ARG4)
      CALL BSSLS(ARG3,ANSJN,0,IERR)
      IF(IERR.EQ.0)GO TO 23
      WRITE(6,14)IERR
   14 FORMAT(1H *ERROR=*I10)
   23 CONTINUE
      SUMIJ=0.
      DO 22 NO=3,11,2
      NO=NO-1
      AN=NO
      PI=3.1415926535
      CALL GAUS1(0.,PI,10,AISUM,FI02N)
      CALL BSSLS(ARG3,ANSJN,NO,IERR)
      IF(IERR.GT.0)WRITE(6,28)
   28 FORMAT(1H *ERROR IN BSSLS*)
   22 SUMIJ=AISUM*ANSJN(NO)*1./PI+SUMIJ
      ASM=2.*SUMIJ
      FL=CAI0+CI0-2.*CALPHA*(ANSJN(1)*C3+ASM)
      FFL=FL
      CBX=CBETA*B1
      GGG=(CALPHA-1.)**2+B1**2*(CALPHA*CBETA+1.)**2+.25*B1**4*(CALPHA*CB
     1ETA**2-1.)**2
      FL=GGG/FFL
      F3L=(B1**2*(CALPHA*CBETA+1.)**2)/FFL
      F4=B1*(CBETA+1.)/(CALPHA-1.)**2
      F5=(B1*CALPHA)/(CALPHA-1.)
      DENC=CALPHA-1.
      SK1CA=-B1*(CBETA+1.)/DENC**2
      SK1CB=B1*CALPHA/DENC
      SK1B1=(CALPHA*CBETA+1.)/DENC
      SK2CA=B1**2*(1.-CBETA**2)/(2.*(DENC)**2)
      SK2CB=B1**2*CALPHA*CBETA/DENC
      SK2B1=B1*(CALPHA*CBETA**2-1.)/DENC


      H=2.*(ABS(CALPHA*EXP(CBETA*B1)-EXP(-B1)))**2
      FP=H/FFL
  666 FORMAT(1H0*20XLOG10(K1)=*F14.4,10X*20XLOG10(K2)=*F14.4///
     1             2X*CALPHA*2X*CBETA*7X*B1*4X*CBXB1*5X*F1*6X*F2*7X*F3*4X*S
     2K1CA*3X*SK1CB*3X*SK1B1*3X*SK2CA*3X*SK2CB*3X*SK2B1*4X*H*7X*FP*///)
      WRITE(6,20)CALPHA,CBETA,B1,CBX,FFL,FL,F3L       ,SK1CA,SK1CB,SK1B1,
     1SK2CA,SK2CB,SK2B1,H,FP
   20 FORMAT(1H 15F8.2)
   16 B1=B1+B1INC
      I=0
      IF(B1.LE.B1HIGH)GO TO 250
      K2INC=K2INC+1.
      K2=10.**(K2INC/20.)
      TK1=20.*ALOG10(K1)
      TK2=20.*ALOG10(K2)
      WRITE(6,666)TK1,TK2
      IF(K2.LE.K2HIGH)GO TO 26
      K1INC=K1INC+1.
      K1=10.**(K1INC/20.)
      IF(K1.LE.K1HIGH)GO TO 27
      GO TO 30
    4 I=I+1
      IF(I.LT.1000)GO TO 181
      I=0
      GO TO 16
```

```
181 CONTINUE
    DFCA=B1*CBETA-K1
    DFCB=B1*CALPHA
    DGCA=B1**2*CBETA**2-2.*K2
    DGCB=2.*B1**2*CALPHA*CBETA
    DETR=DFCA*DGCB-DGCA*DFCB
    IF(DETR.EQ.0.)GO TO 99
    HH=(-FAB*DGCB-(-GAB)*DFCB)/DETR
    AK=(DFCA*(-GAB)-DGCA*(-FAB))/DETR
    CALPHA=CALPHA+HH
    CBETA=CBETA+AK
    IF(CALPHA.GT.CAHIGH)CALPHA=CALPHA/2.
    IF(CALPHA.LT.CALOW)CALPHA=CALPHA+1.
    IF(CBETA.GT.CBHIGH)CBETA=CBETA/2.
    IF(CBETA.LT.CBLOW)CBETA=CBETA+1.

    GO TO 8
 99 CALPHA=CALPHA+.1
    CBETA=CBETA-.1
    WRITE(6,411)
411 FORMAT(1H *DETERMINANT IS 0*)
    GO TO 181
 30 CONTINUE
    GO TO 101
123 STOP
    END

    FUNCTION FI02N(THETA)
    COMMON AN,ARG4
    POW=ARG4*COS(THETA)
    EP=EXP(POW)
    CTH=COS(AN*THETA)
    FI02N=EP*CTH
    RETURN
    END
    FUNCTION AI0(X)
    DIMENSION A(6),B(8)
    T=X/3.75
    IF(T.GT.1.)GO TO 1
    DATA(A(M),M=1,6)/3.5156229,3.0899424,1.2067492,.2659732,.0360768,
   1.0045813/
    ALINK=T*T
    CHAIN=ALINK
    AI0=1.
    DO 2 K=1,6
    AI0=AI0+CHAIN*A(K)
    CHAIN=ALINK*CHAIN
  2 CONTINUE
    RETURN
  1 ALINK=T
    DATA(B(M),M=1,8)/1.328592E-2,2.25319E-3,-1.57565E-3,9.16281E-3,
   1-2.057706E-2,2.635537E-2,-1.647633E-2,3.92377E-3/
    CHAIN=ALINK
    AI0=.39894228
    DO 3 K=1,8
    AI0=AI0+B(K)/CHAIN
    CHAIN=ALINK*CHAIN
  3 CONTINUE
    AI0=AI0*EXP(X)/SQRT(X)
    RETURN
    END
```

```
    SUBROUTINE GAUS1(A,B,M,SUM,FOFZ)
    DIMENSION U(5),R(5)
    U(1)=.425562830509184
    U(2)=.283302302935376
    U(3)=.160295215850488
    U(4)=.067468316655508
    U(5)=.013046735741414
    R(1)=.147762112357376
    R(2)=.134633359654998
    R(3)=.109543181257991
    R(4)=.074725674575290
    R(5)=.033335672154344
    SUM=0.
    IF(A.EQ.B)RETURN
    FINE=M
    DELTA=FINE/(B-A)
    DO 3 K=1,M
    X1=K-1
    FINE=A+X1/DELTA
    DO 2 II=1,5
    UU=U(II)/DELTA+FINE
  2 SUM=SUM+R(II)*FOFZ(UU)
    DO 3 L=1,5
    UU=(1.-U(L))/DELTA+FINE
  3 SUM=SUM+R(L)*FOFZ(UU)
    SUM=SUM/DELTA
    RETURN
    END
```


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REFERENCES

[1] B. D. Martin, "The Pioneer IV lunar probe: a minimum-power FM/PM system design," Jet Propulsion Laboratory, Pasadena, Calif., Tech. Rept. 32-215, March 15, 1962.

[2] J. H. Painter, "Synthesis techniques for a class of phase-modulated signals," *Supplement to IEEE Trans. Aerospace*, vol. AS-3, pp. 264–269, June 1965.

[3] ——, "Single-sideband hybrid AM-PM signal models," *Proc. IEEE* (Letters), vol. 54, p. 440, March 1966.

[4] E. Bedrosian, "The analytic signal representation of modulated waveforms," *Proc. IRE*, vol. 50, pp. 2071–2076, October 1962

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