Geophys. J. R. astr. Soc. (1985) **83**, 307–317

Synthetic *SH* seismograms in a laterally varying medium by the discrete wavenumber method

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Accepted 1985 March 6. Received 1985 February 8; in original form 1984 August 6

Summary. We present a new method to calculate the *SH* wavefield produced by a seismic source in a half-space with an irregular buried interface. The diffracting interface is represented by a distribution of body forces. The Green's functions needed to solve the boundary conditions are evaluated using the discrete wavenumber method. Our approach relies on the introduction of a periodicity in the source-medium configuration and on the discretization of the interface at regular spacing. The technique developed is applicable to boundaries of arbitrary shapes and is valid at all frequencies. Some examples of calculation in simple configurations are presented showing the capabilities of the method.

Key words: seismology, synthetic seismograms, diffraction, vertical seismic profiles.

Introduction

The discrete wavenumber method (Bouchon & Aki 1977; Bouchon 1981) is a powerful technique of simulation of wave propagation in a viscoelastic flat-layered medium. Our aim is to develop the generalization of this method to a laterally heterogeneous medium. This problem is of crucial interest for geophysical prospecting purposes as well as for the interpretation of earthquake data. In the last decade seismologists have devoted a lot of work to these studies. A large number of techniques have been used including finite differences (Boore 1972; Kelly *et al.* 1976; Virieux 1985), finite elements (Smith 1974) and frequency-wavenumber methods (Aki & Larner 1970). High-frequency asymptotic ray theory has also been applied following different formulations (Červený, Molotkov & Pšenčík 1977; Hong & Helmberger 1978, MacMechan & Mooney 1980; Haines 1983) including the Gaussian beam method (Červený 1983; Červený & Pšenčík 1984; Novak & Aki 1985). Finally, boundary integral equations have been used to compute the complete response of an irregular free surface (e.g. Sanchez-Sesma & Esquivel 1979; Sanchez-Sesma 1983) or of an irregular buried interface (Dravinski 1983).

In this paper we present a method of representation of the scattered wavefields by distributing sources along the diffracting interface (Huyghens's principle). We follow the approach proposed by Bouchon (1985) to calculate the response of a topographic profile to an incident plane wave. This method is valid at any frequency. We shall restrict the topic to the calculation of synthetic *SH* seismograms in a two-dimensional half-space with one irregular interface.

Description of the problem

We consider the two-dimensional problem of the response of an irregularly layered elastic medium to a seismic excitation. We assume the medium to consist of a layer and a half-space separated by an irregular interface C (Fig. 1). V_1, ρ_1 and V_2, ρ_2 represents respectively the wave velocity and the density of the layer (1) and of the half-space (2). For a source located in the layer (1), the displacement at a location $r(x, z)$ may be written as:

$$V^{(1)}(x,z) = V_C^{(1)}(x,z) + V_s(x,z); \qquad r \in (1)$$
$$V^{(2)}(x,z) = V_C^{(2)}(x,z); \qquad r \in (2) \tag{1}$$

where $V_C^{(i)}$ represents the displacement associated with the field diffracted at the interface in medium (i) and V_s denotes the displacement radiated directly by the source.

The diffracted field $V_C^{(1)}$ may be represented by the radiation of a specific distribution of seismic sources (forces) $F^{(1)}$ along C. This representation will be exact for the entire zone (1) if the radiation is calculated with an accurate Green's function and if the boundary conditions are complied with. For region (1) the free surface condition implies the use of the homogeneous half-space Green's function while the boundary conditions to be complied with are on C. The diffracted field $V_C^{(2)}$ will be represented by the source distribution $F^{(2)}$ on C. In this case only the infinite space Green's function is required.

This approach implies the consideration of two different virtual media to represent the 'real' medium: region (1) is part of the virtual half-space A and region (2) is part of the virtual homogeneous infinite space B.

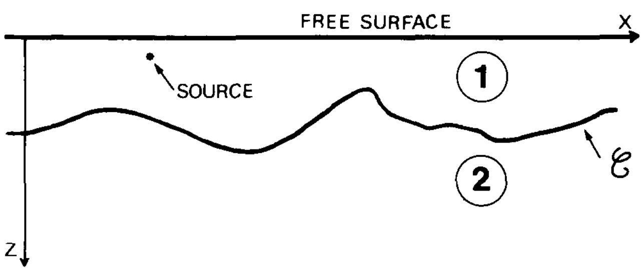


Figure 1. Geometry of the problem. The interface C has an arbitrary shape.

Boundary conditions

The source distributions $F^{(1)}$ and $F^{(2)}$ on C are determined by the boundary conditions at the interface itself. These distributions are consistent with a particular choice of the Green's functions (i.e. with a particular choice of the virtual regions in which these sources act). The boundary conditions are, for any (x, z) on C:

$$V^{(1)}(x,z) = V^{(2)}(x,z)$$

$$\mu_1 \left[nx \frac{\partial V^{(1)}(x,z)}{\partial x} + nz \frac{\partial V^{(1)}(x,z)}{\partial z} \right] = \mu_2 \left[nx \frac{\partial V^{(2)}(x,z)}{\partial x} + nz \frac{\partial V^{(2)}(x,z)}{\partial z} \right] \tag{2}$$

where μ_1 and μ_2 are the Lamé coefficients in mediums 1 and 2, and $\mathbf{n}(nx, nz)$ represents the normal vector to C at $r(x, z)$.

Let $G^{(1)}(x, z; x', z')$ and $G^{(2)}(x, z; x', z')$ be the Green's functions of virtual media A and B for the particular frequency $\omega/2\pi$. The displacements are given by:

$$V^{(1)}(x, z) = \int_C F^{(1)}(x', z') G^{(1)}(x, z; x', z') d\xi + V_s(x, z)$$

$$V^{(2)}(x, z) = \int_C F^{(2)}(x', z') G^{(2)}(x, z; x', z') d\xi \tag{3}$$

with $\xi = (x', z')$.

The use of these expressions in equations (2) yields two integral equations which give theoretically the distributions $F^{(1)}$ and $F^{(2)}$ through the Green's functions of the virtual media and their spatial derivatives. We have now to discretize the integrals over C and to obtain a suitable formulation for the Green's functions.

Evaluation of the Green's functions for a discretized interface

To this end we assume that the source medium configuration is periodic in the x-direction with a spacial periodicity L. We shall now use the discrete wavenumber method (Bouchon & Aki 1977) to evaluate the Green's functions. We assume that the problem may be resolved by representing the irregular interface by an array of $2M + 1$ discrete points regularly spaced in x (Fig. 2), where forces are applied and where the boundary conditions are matched. The abcissae of these points are given by: $x_m = (m - 1)L/(2M + 1)$. The value of M is chosen such that the following condition of sampling is satisfied: $L/(2M + 1) < \pi\beta_1/\omega$ where β_1 is the lowest wave velocity of the medium and ω the angular frequency.

The *SH* displacement V produced at $r(x, z)$ by a transverse line force F acting at (x', z') is given by (Lamb 1904):

$$V(x, z) = \frac{F}{4\pi\mu i} \int_{-\infty}^{+\infty} \frac{\exp\left[-ik(x - x') - i\gamma|z - z'|\right]}{\gamma} dk \tag{4}$$

with

$$\gamma = \left(\frac{\omega^2}{\beta^2} - k^2\right)^{1/2}, \ \mathrm{Im}(\gamma) \leqslant 0$$

where β denotes the shear wave velocity.

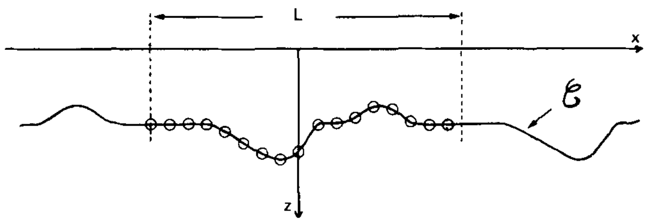


Figure 2. Introduction of a periodicity in the medium geometry and discretization of the interface at regular spacing.

For an infinite array of periodic sources equation (4) becomes:

$$V(x, z) = \frac{F}{2Li\mu} \sum_{n=-\infty} \frac{\exp\left[-ik_n(x - x') - i\gamma_n |z - z'|\right]}{\gamma_n} \tag{5}$$

with

$$k_n = n\,\frac{2\pi}{L}, \qquad \gamma_n = \left(\frac{\omega^2}{\beta^2} - k_n^2\right)^{1/2}, \qquad \mathrm{Im}\,(\gamma_n) \leqslant 0.$$

Using equation (3) we can evaluate the displacement produced by the entire distribution $F^{(i)}$ in medium i. It comes:

$$V_C^{(i)}(x, z) = \sum_{m=-M}^{M} F_m^{(i)} \frac{1}{2iL\mu_i} \sum_{n=-\infty}^{+\infty} \frac{\exp\left[-ik_n(x - x_m) - i\gamma_n^i |z - z_m|\right]}{\gamma_n^i} \tag{6}$$

or

$$V_C^{(i)}(x, z) = \frac{1}{2iL\mu_i} \sum_{n=-\infty}^{+\infty} \sum_{m=-M}^{M} F_m^{(i)} \frac{\exp\left[-ik_n(x - x_m) - i\gamma_n^i |z - z_m|\right]}{\gamma_n^i}. \tag{7}$$

This expression defines $V_C^{(i)}$ for all the positions $r(x_m, z_m)$ on C.

Its Fourier transform over x_m is given by:

$$\tilde{V}_C^{(i)}(k_p, z) = \frac{1}{2M+1} \sum_{j=-M}^{M} V_C^{(i)}(x_j, z) \exp(ik_p x_j)$$

with

$$k_p = p\,\frac{2\pi}{L}.$$

Using equation (7) this expression becomes:

$$\tilde{V}_C^{(i)}(k_p, z) = \frac{1}{2iL\mu_i(2M+1)} \sum_{n=-\infty}^{\infty} \left(\sum_{m=-M}^{M} F_m^{(i)} \frac{\exp\left[-i\gamma_n^i |z - z_m| + (2i\pi n x_m/L)\right]}{\gamma_n^i} \times \right.$$

$$\left. \times \sum_{j=-M}^{M} \exp[\![i(2\pi/L)(p - n) j\,[1/(2M+1)]]\!] \right) \tag{8}$$

where we have:

$$\sum_{j=-M}^{M} \exp\left[2i\pi(p - n)j/(2M+1)\right] = \begin{cases} 2M+1 & \text{if } n = p \\ 0 & \text{if } n \neq p. \end{cases}$$

Therefore equation (8) is simply:

$$\tilde{V}_C^{(i)}(k_p, z) = \frac{1}{2i\mu_i L} \sum_{m=-M}^{M} F_m^{(i)} \frac{\exp\left(-i\gamma_p^i |z - z_m| + ik_p x_m\right)}{\gamma_p^i}. \tag{9}$$

With this expression it appears that the field radiated by the entire set of forces is not simply decomposable into plane waves. As we have assumed the interface to be correctly described by a discrete set of regularly spaced points, we can assume the system of boundary conditions to be matched with a corresponding spacial resolution. This suggests that the expression of V_C needed to resolve the discrete boundary conditions system is obtained by truncated Fourier series in the form:

$$V_C^{(i)}(x_j, z) = \frac{1}{2i\mu_i L} \sum_{m=-M}^{M} F_m^{(i)} \sum_{p=-M}^{M} \frac{\exp\left[-i\gamma_p^i |z - z_m| - ik_p(x_j - x_m)\right]}{\gamma_p^i}. \tag{10}$$

The hypothesis of discretization of the interface has led to the use of a truncated series in place of the actual Green's function. Inhomogeneous waves, however, are included in the solution as we have seen with equation (9). Using equatation (2) and writing the boundary conditions at each points of the contour C leads to a system of $4M + 2$ linear equations in $F_m^{(i)}$. After resolution of this system we are able to evaluate the elastic field anywhere in the medium. The calculation is made for each frequency and the results are synthetized in the time domain. The unwanted effect of source-medium periodicity is avoided by giving the frequency a small constant imaginary part (Bouchon & Aki 1977) and removing its attenuation effect from the time domain solution.

A test of the method: a flat layer over a half-space

As a way to test the accuracy of our approach we compute synthetic seismograms in the case where the interface C is flat and we compare them with the results obtained by the flat-layer discrete wavenumber method.

The source considered is a transverse horizontal line force F_0 and its radiated displacement is given by:

$$V_s(x, z) = \frac{F_0}{2iL\mu} \sum_{n=-\infty}^{\infty} \frac{\exp\left[-ik_n(x - x_0) - i\gamma_n |z - z_0|\right]}{\gamma_n}$$

where x_0 and z_0 denote the source coordinates. The source problem configuration and the synthetic seismograms obtained by the two methods are displayed in Fig. 3. The calculations are done for frequencies between 0 (static) and 16 Hz. The periodicity length is 4 km and the interface is represented by an array of 119 points. The source function used is

$$\left(S(\omega) = \frac{\omega^2 t_0}{\sinh\left[(\omega \pi t_0/4)\right]}\right)$$

with $t_0 = 0.06$ s.

The results obtained by both techniques are in good agreement. The waveshapes are exactly superposable while the amplitudes show differences which do not exceed 3 per cent. This discrepancy may be attributed to the representation of the continuous interface by a set of discrete points. This example shows the validity of our approach.

Simulation of a vertical seismic profile near a dipping interface

In this section we first present a calculation of synthetic seismograms along a vertical seismic profile located on the flank of a sine-shaped dome. The geometry and the parameters of the medium are shown in Fig. 4. We have investigated the two following cases:

Case I: the source is at position A, 10 m below the surface. The receivers are located along a line which passes through B and are distributed at regular intervals between the depths of 150 and 850 m.

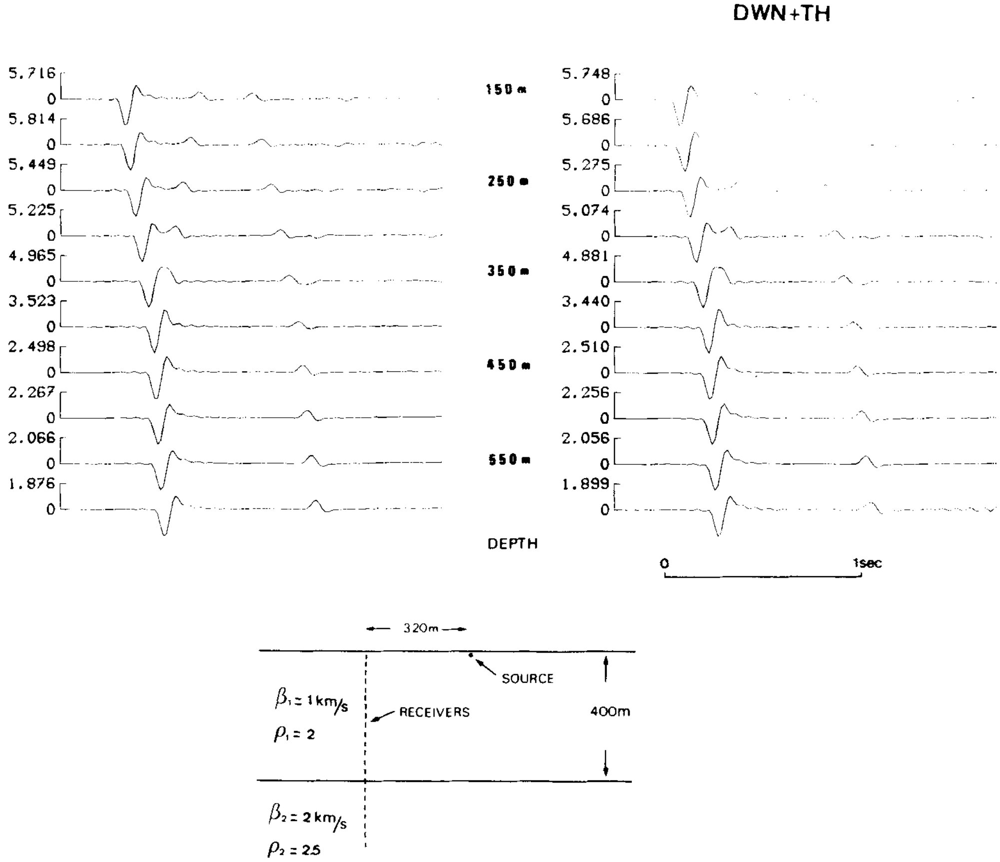


Figure 3. Source-receiver geometry used and comparison between our results (left part) and synthetic seismograms obtained by the discrete wavenumber method using Thomson–Haskell propagators (DWN + TH). Each seismogram is normalized independently. Numbers on the left of each trace indicate the maximum amplitude. The source depth is 10 m.

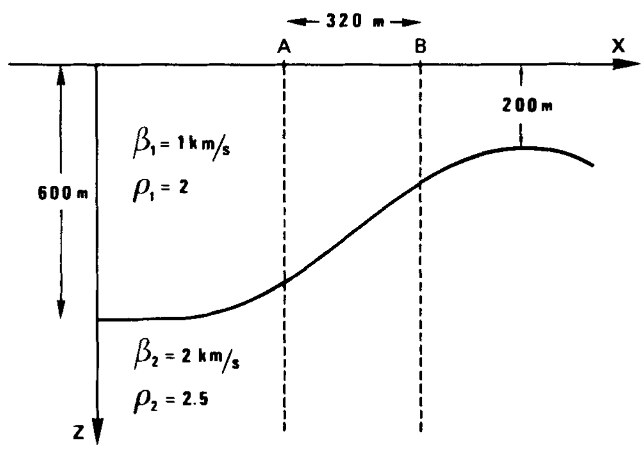


Figure 4. Source-receiver positions and geometry of the medium used to investigate the effect of a dipping interface on a vertical seismic profile (see text).

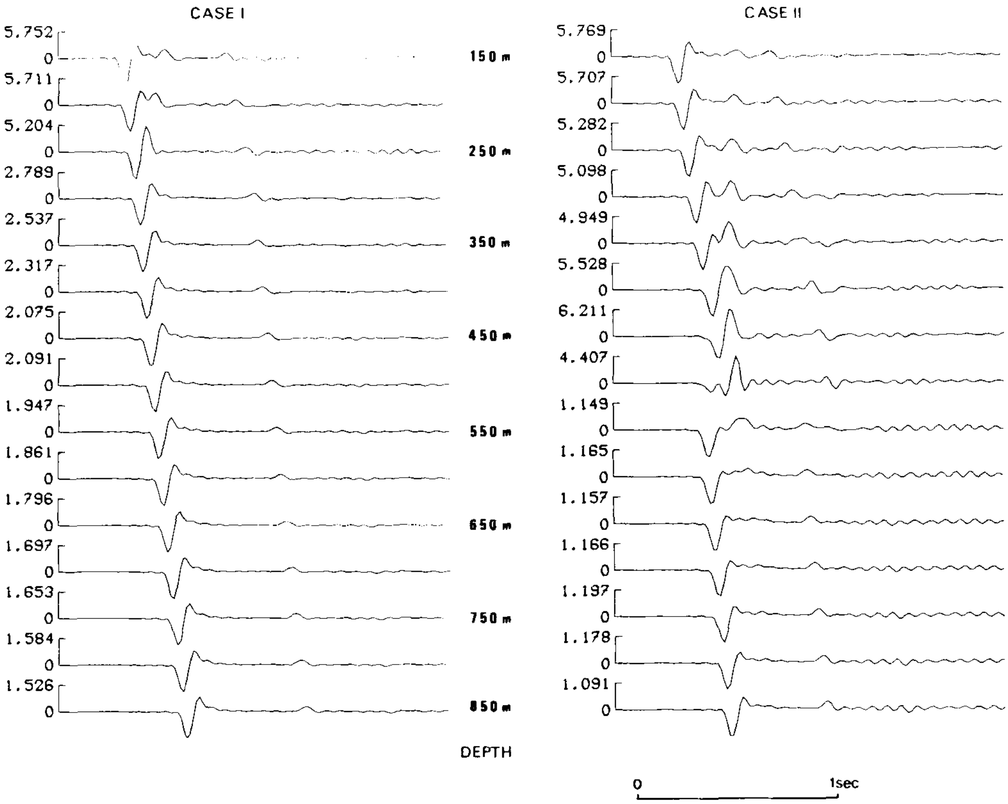


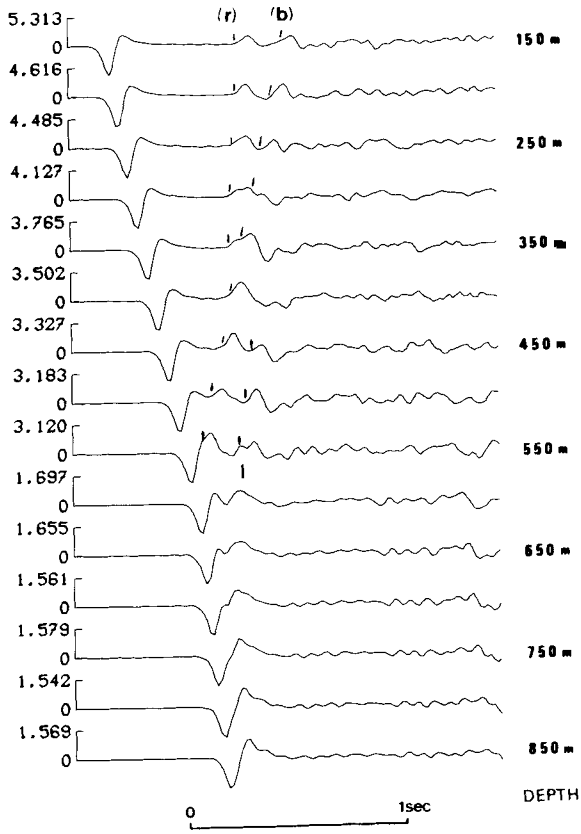
Figure 5. Synthetic seismograms produced in the two cases described in the text. Each seismogram is normalized to its maximum amplitude given at the left of each trace.

Case II: the x-coordinates of the source and the receiver-line are interchanged.

The seismograms are displayed in Fig. 5. The parameters of the calculation are identical to the ones used in the last section. In case I, we observe reflected and refracted waves. As the incidence angle is small the pulse shape of the transmitted wave in medium 2 is similar to the one of the direct wave. In the method, the transmitted wavefield is represented by the radiation of the sources distributed along the curved interface. The validity of the approach is pointed out by the quality of the pulse shape of the transmitted wave.

The profile obtained in case II shows the large distortion of the refracted and reflected pulses which occurs when the direction of propagation of the wave is close to the dip of the interface. For a receiver location far below the interface the incidence angle decreases and the source radiated pulse shape reappears. In these two cases up-going and down-going multi-reflected waves are present.

We consider now a simple model of graben (Fig. 6). In this case the horizontal distance between source and receivers is now 160 m and the value of t_0 (which defines the source function) is now 0.08 s. The other parameters remain identical to the ones used in the previous calculations. The interface presents a very rough shape with angular points. We can identify numerous arrivals on the synthetics but we will focus our attention on the waves reflected on the different parts of the interface. The wave reflected at the bottom of the graben is denoted by 'b', the reflection on the left side by 'l' and the one on the right side by 'r'. The last two reflected waves have large apparent velocities. In this case, the lateral



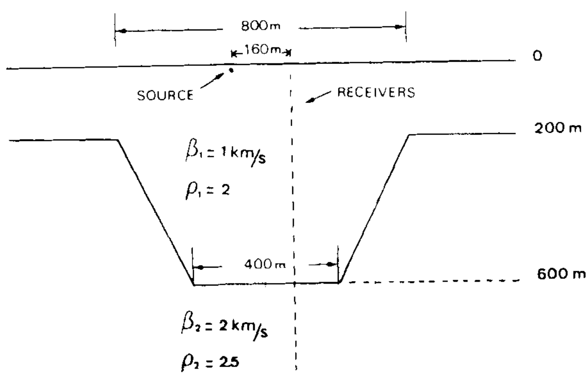


Figure 6. Geometry of the simple model of graben used and synthetic vertical seismic profile.

propagation plays an important role on the vertical seismic profile obtained. Multiple reflections at the sides of the graben and at the free surface are responsible for later arrivals. In the lower medium, the pulse shape presents an important distortion due to interference between the pulse transmitted through the bottom of the graben and a pulse refracted along its right flank.

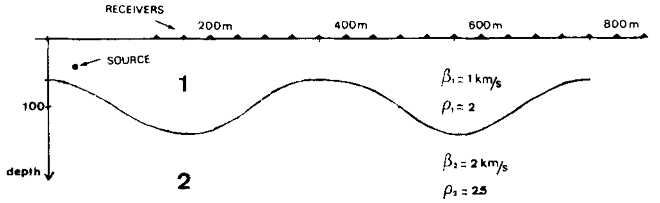


Figure 7. Geometry of a surface layer of periodically varying thickness. The location of the source and of the receivers is indicated.

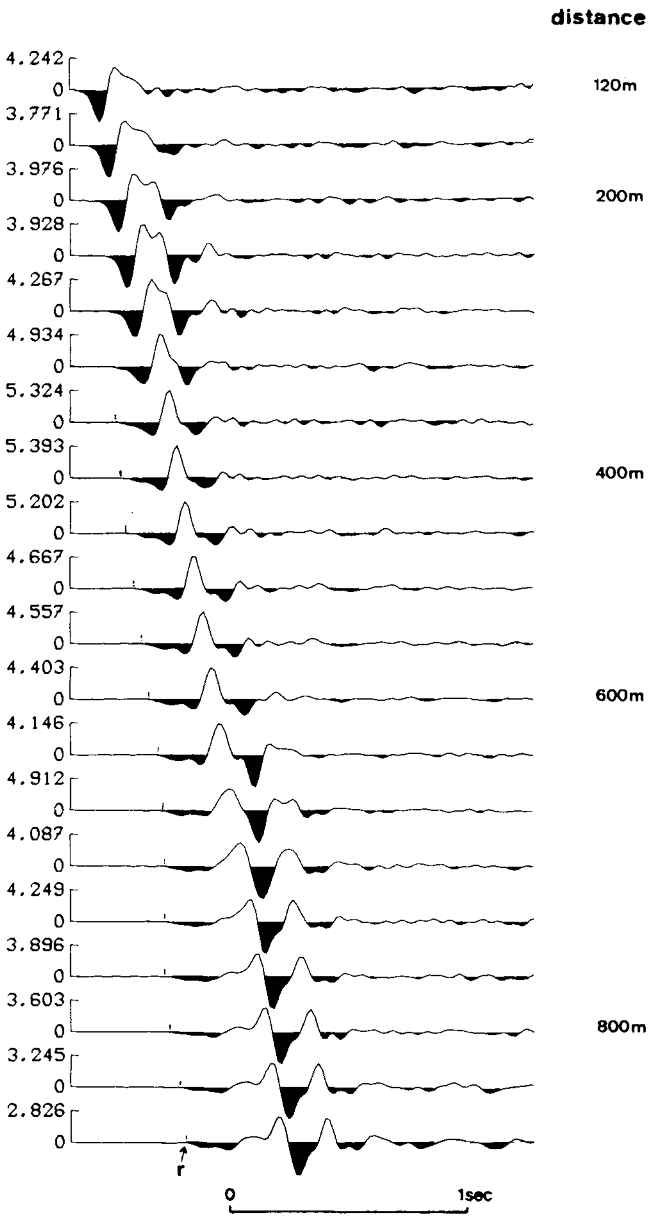


Figure 8. Synthetic refraction profile obtained for distances from the source between 120 and 880 m. The medium considered is represented in Fig. 7.

Simulation of a refraction profile

As another example of application we consider the case of a surface refraction profile. The configuration is chosen to investigate the effect of a weathered zone of varying thickness. The interface is centred on a depth of 100 m and its shape is a sine function with period of 400 m and amplitude of 40 m (Fig. 7). The source is 40 m below the surface and the receivers span the horizontal distance range between 120 and 880 m. The synthetics are depicted in Fig. 8. The other calculations parameters remain unchanged. They show complex and varying wave-shapes due to the interaction of direct, reflected and refracted waves. The refracted first arrival displays a very weak amplitude. Its apparent velocity changes with the position of the receiver and reflects the variations of the basement depth. The progressive decrease of the thickness of the weathered layer produces a strong amplification effect at a distance of around 400 m.

Conclusion

We have presented a new method to compute the complete *SH* wavefield produced in a two-dimensional half-space with an irregular interface. This method may be considered as a propagation technique associated with the discrete wavenumber representation of the seismic source radiation. The comparison of our results with calculations done using the flat-layer discrete wavenumber method shows, in the case of a plane interface, the validity of our technique. We have presented examples of synthetic seismograms computed for simple configurations in presence of an irregular interface. The formulation of this method does not rely on the shape of the diffracting interface or on the frequency range considered.

Acknowledgments

Most of the calculations presented in this paper were performed at the 'Centre de Calcul Vectoriel pour la Recherche'. This work was supported by CNRS through contract 'ATP Géophysique Appliquée'.

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