System of Systems Based Security-Constrained Unit Commitment Incorporating Active Distribution Grids

Amin Kargarian, *Student Member, IEEE*, and Yong Fu, *Senior Member, IEEE*

***Abstract*—In restructured power systems, the transmission and distribution grids are autonomously utilized by independent system operator (ISO) and distribution companies (DISCOs), respectively. As the operating condition of one grid influences the decisions made by operators of other grids, the ISO and DISCOs should collaborate and cooperate with each other in order to operate the entire power system in a secure and economic manner. According to the concept of system of systems (SoS) engineering, this paper presents a decentralized decision-making framework to determine a secure and economical hourly generation schedule for a transmission system encompassing numbers of active distribution grids. Taking into consideration the physical connections and shared information between ISO and DISCOs, an SoS-based SCUC framework is designed and a hierarchical optimization algorithm is presented to find optimal operating points of all independent systems in the SoS-based power system architecture. The numerical results show the effectiveness of the proposed SoS framework and solution methodology.**

***Index Terms*—Active distribution grid, decentralized optimization, security-constrained unit commitment, system of systems.**

NOMENCLATURE

i	Index for generating unit.
j	Index for active distribution grids.
k	Index for outer loop iteration.
t	Index for time period.
w	Index for inner loop iteration.
NA	Number of active distribution grids.
NG	Number of generating units.
NT	Number of studied period.
$F_i(\cdot)$	Generation cost curve of unit i.
I_{it}	Commitment state of unit i at time t.
P_{it}	Generation of generating unit i at time t.
$PD_{S,jt}$	Power generated by ISO at period t and supplied to DISCO j in ISO's optimization problem.
$PG_{D,jt}$	Power demanded by DISCO j at time t and supplied by ISO in the $\mathrm{DISCO_j}$'s optimization problem.
SUD_{it}	Startup and shutdown cost of unit i at time t.
$\mathbf{x}$	Design variables of the ISO.
$\tilde{\mathbf{x}}$	Local variables for ISO, subvector of $\mathbf{x}$.
$\mathbf{y}$	Design variables of the DISCO.
$\tilde{\mathbf{y}}$	Local variables for DISCO, subvector of $\mathbf{y}$.
$\mathbf{z}$	Shared variables between independent systems, subvector of both $\mathbf{x}$ and $\mathbf{y}$.
$\boldsymbol{\mu}, \boldsymbol{\eta}$	Vector of response and target variables.
$\boldsymbol{\alpha}_{jt}^k, \boldsymbol{\beta}_{jt}^k$	Multipliers of penalty function in iteration k.

I. INTRODUCTION

DISTRIBUTED generations (DGs) are widely being installed near the load centers to locally supply power for the end-users. Connection of the DGs at different voltage and power levels has led to change in the characteristics of distribution grids, and consequently introduces active distribution grids (ADGs) in the restructured power systems. These ADGs are making power systems more reliable and cost-effective, and are improving the power quality required by local loads [1]–[3].

Security-constrained unit commitment (SCUC), which refers to a scheduling of generation resources to satisfy load demand at the least cost while considering system security, is an important decision-making tool in power system operation [4]–[10]. In restructured power systems without DGs and ADGs, the independent system operator (ISO) runs the SCUC module to schedule the hourly generation of conventional generating units connected to the transmission network. As a result, there is the lack of collaboration and cooperation between transmission and distribution grids. However, with the increasing deploy of DGs in present power systems, a decision-making model is required in order to facilitate the operation of DGs and ADGs, and enhance the collaboration and cooperation between transmission and distribution grids.

Incorporating DGs and ADGs into the SCUC market clearing process, and collaborating conventional generating units with DGs will improve the performance and the social welfare of the entire power system. For example, committing economic DGs in distribution network for producing energy close to the load centers might make the expensive conventional generators in transmission network stay off, and it might also impact power dispatch of the generators. Participation of the ADGs in the markets not only benefits the ISO but also provides the opportunity

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The authors are with the Department of Electrical and Computer Engineering, Mississippi State University, Mississippi State, MS 39762 USA (e-mail: ak836@msstate.edu; fu@ece.msstate.edu).

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for the ADGs to contribute into the market and make benefit out of their own DG units.

Indeed, day-ahead generation scheduling in the ADGs is an important issue providing the hourly plan for the operation of the system. Researchers have been working on the consideration of DGs and ADGs into the SCUC problem. Reference [11] presented a centralized optimization problem taking into account the impact of distributed energy resources in reliability constrained unit commitment. In [12], a unit commitment was formulated for distribution systems consisting of distributed generations, controllable loads, and storages. A two-stage scheduling algorithm for distribution grids was addressed in [13]. This algorithm finds the hourly power generations of distributed energy resources from a day-ahead energy market. Reference [14] presented an SCUC problem for day-ahead generation scheduling in active distribution grids considering the production cost of DG units and CO_2 emission penalty cost. In [15], a unit commitment algorithm was addressed for the coordination between mid-term maintenance outage decisions and short-term security-constrained scheduling in active distribution grids. A bilevel optimization problem was presented in [16] to solve a multi-period energy acquisition model for a DISCO with DGs and interruptible loads in a day-ahead electricity market.

Note that in conventional power systems, the distribution systems are passive grids which could not work separately from the upstream transmission network. However, as an important issue in restructured electric power industries, the ISO and active distribution grid operators (DISCOs) can function independently with their own operation and control regulations [4]. As these entities are independent systems being able to be utilized separately, the competition and collaboration relationship among them can be represented by the concept of system of systems (SoS). System of systems refers to a group of systems which are heterogeneous and independently operable with their own objectives, while they are linked together for realizing a secure and reliable operation of the entire SoS [17]. Although there are similarities between systems engineering and system of systems engineering, they are different fields of study. The traditional systems engineering intends to find the optimal operation point of an individual system. And the SoS tries to find the optimal operating point of the networks including interacting systems that work together to satisfy various objectives when guaranteeing constraints of the systems. In such an SoS-based power system, the dispatching and operational independence of each system should be respected, and meanwhile, the collaboration among systems should be encouraged. Using centralized optimization algorithms which need all the information of the autonomous systems (ISO/DISCO), might not be the appropriate way to find the optimal operating point of such SoS-based power system since generators, loads and network information of each autonomous system, are usually considered commercially sensitive. On the other hand, determining the entire power system operation including huge numbers of design and control variables in one centralized optimization model might be challenging.

In this paper, an SoS-based decentralized decision-making framework is presented in order to solve the SCUC problem of power systems encompassing active distribution grids. In this framework, the transmission network and ADGs are modeled as independent systems which are respectively utilized by ISO and DISCOs. A decentralized optimization model is formulated to minimize the operating cost of each individual and autonomous system while satisfying the security constraints associated with ISO's and DISCOs' operations. Considering the physical connections and shared variables between the systems, and the issue that the operating point of a system in the SoS may influence the operating point of the other independent systems, a hierarchical optimization algorithm is presented to determine an optimal operating point of all the independent systems in the SoS-based SCUC. This algorithm has a two-loop iterative solution procedure which guarantees a better convergence performance of the proposed decentralized optimization approach. The presented SoS-based decentralized SCUC is a procedure in which each independent utility or operator only deals with its own information and schedules for its own internal area and crossing borders with other systems. Thus, only a limited amount of information is exchanged among the operators of different systems, and the operators do not need to exchange all the information, which might be commercially sensitive, with each other.

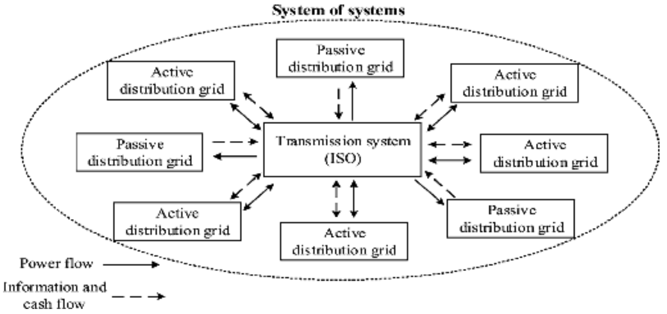

Fig. 1. Power system as a system of systems.

The rest of paper is organized as follows. The restructured power system is defined as a system of systems in Section II. The proposed decentralized decision-making framework and solution process of the SoS-based SCUC are carried out in Section III. Numerical testing results are presented in Section IV. The concluding remarks are provided in Section V.

II. SYSTEM OF SYSTEMS BASED SCUC FRAMEWORK

A. Power System as a System of Systems

Compared with that in conventional power systems, the distribution grid in the future power systems becomes active due to the inclusion of distributed generation units. As shown in Fig. 1, in such a power system, the electricity transportation, information and cash flow are bidirectional, from transmission system to active distribution grids or vice versa, and it complicates system analysis.

In the restructured power systems, defining the transmission system operator (ISO) and ADG operator (DISCO) as the independent system regulators, the operation and control schemes of the power systems can be designed and implemented in accordance with an SoS framework. An SoS is described as incorporation of task-oriented or dedicated systems in a unique system in which its components: 1) collect their resources and capabilities to construct a more complex system that has more capability and performance than simply the sum of the basic systems, and 2) are able to independently perform valid functions in their own right and continue to work to fulfill those purposes when are separated from the overall system [18]. Formulating the optimization problem, condition assessment and decision making under different circumstances, and modeling the economic issues and competitive behavior of the independent systems are among the challenges facing the implementation of the SoS framework [18]. In addition, the required data to model the behaviors of the independent systems and the data flow process between the systems should be determined and guaranteed.

In this paper, an SoS-based SCUC problem in the restructured power system is studied. In this model, the ISO is an autonomous system being responsible for the transmission system operation, and the DISCOs are other independent systems utilizing the active distribution grids. The ISO and each DISCO individually formulate and solve their own SCUC problem according to their own available generation sources, network topologies and forecasted loads.

B. SCUC for Autonomous Systems

In general, the SCUC is a constrained optimization problem intending to minimize the operating cost of the system including units' generation and startup/shutdown costs. The equality and inequality constraints consist of unit commitment constraints such as power balance, system spinning and operating reserve requirements, generating unit capacity, and units' ramping up/down and minimum on/off time limits, as well as power flow equations and transmission network security constraints.

The regular SCUC problem can be applied for each autonomous system, ISO and DISCOs. However, there are a few differences in characteristics of the generating units and network of the ISO and DISCOs, which result in differences in their individual SCUC formulations. For example, there might be many fast DG units in the ADGs, and the ramping constraints and the minimum on/off time requirements of such units may be negligible in its hourly SCUC problem. The transmission network is meshed while the distribution network is usually radial. The ISO and DISCOs solve their own individual SCUC problem in order to find their own optimal operating point. Note that in an ADG, there might be several DG units which have different owners, the DISCO or customers. Beside its own DG units, the DISCO can play the role of aggregator and handle the DGs owned by the customers in the proposed commitment and dispatch problems. Different approaches have been presented to solve the SCUC problem, like Lagrangian relaxation and mixed-integer programming (MIP) methods [4]–[9]. In this paper, the MIP model presented in [7] is applied to solve the regular SCUC of each independent system.

III. DECENTRALIZED DECISION-MAKING SOLUTION

If there is no connection between the transmission network and ADGs, the ISO and DISCOs can apply the existing SCUC algorithms to find the hourly generation schedule of their own generating units. However, when the transmission network and ADGs are indeed linked together, the optimal operating point of one of them impacts the operating point of others. In order to model this interaction between the systems in the SoS-based SCUC problem, and to find the optimal operating point of the ISO and DISCOs, a decentralized decision-making solution is presented in this section, which will better accommodate distributed technologies and active distribution grids participation into the market. A hierarchical two-level optimization algorithm can be applied to implement the decentralized decision-making solution to the SoS-based SCUC problem.

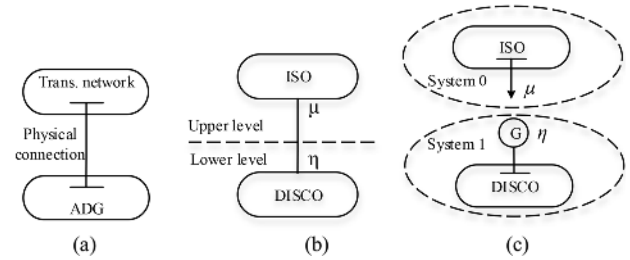

Fig. 2. (a) An ADG physically connected to ISO, (b) hierarchical two-level SoS structure, and (c) modeling target and response variables.

A. Hierarchical Two-Level Optimization Problem

Fig. 2(a) shows an ADG physically connected to the transmission system. Assume that the formula (1) expresses the general SCUC problem for the ISO:

$$\begin{aligned} Min \quad & f(\mathbf{x}) \\ s.t. \quad & \boldsymbol{g}(\mathbf{x}) \le 0 \\ & \boldsymbol{h}(\mathbf{x}) = 0 \end{aligned} \tag{1}$$

where $\mathbf{x}$ represents the design variables for the ISO, f is overall objective function, and $\boldsymbol{g}$ and $\boldsymbol{h}$ are all inequality and equality constraints for the ISO. The same general optimization problem is written for the DISCO as an independent system:

$$\begin{aligned} Min \quad & f(\mathbf{y}) \\ s.t. \quad & \boldsymbol{g}(\mathbf{y}) \le 0 \\ & \boldsymbol{h}(\mathbf{y}) = 0. \end{aligned} \tag{2}$$

In (2), $\mathbf{y}$ represents the design variables for the DISCO, f is the objective function, and $\boldsymbol{g}$ and $\boldsymbol{h}$ are constraints for the DISCO. The transmission system and ADG are linked through the substation system, and have shared variables with each other. Introduce $\tilde{\mathbf{x}}$ and $\tilde{\mathbf{y}}$ as the local design variables which are exclusively for ISO and DISCO, respectively. Also, introduce the set of design variables $\mathbf{z}$ which represents the shared variables between these two independent systems. Then, (1) and (2) can be rewritten as (3) and (4), respectively:

$$\begin{aligned} Min \quad & f(\tilde{\mathbf{x}}, \mathbf{z}) \\ s.t. \quad & \boldsymbol{g}(\tilde{\mathbf{x}}, \mathbf{z}) \le 0 \\ & \boldsymbol{h}(\tilde{\mathbf{x}}, \mathbf{z}) = 0 \end{aligned} \tag{3}$$

$$\begin{aligned} Min \quad & f(\tilde{\mathbf{y}}, \mathbf{z}) \\ s.t. \quad & \boldsymbol{g}(\tilde{\mathbf{y}}, \mathbf{z}) \le 0 \\ & \boldsymbol{h}(\tilde{\mathbf{y}}, \mathbf{z}) = 0. \end{aligned} \tag{4}$$

Because of the shared variable $\mathbf{z}$, (3) and (4) cannot be solved separately. In order to decompose the above optimization problems and make them independently solvable, a hierarchical two-level SoS structure is presented in Fig. 2(b) in which the ISO is located in upper level and DISCO is in lower level. Two different sets of variables are introduced to model the shared variables and formulate the self-governing objective functions and constraints related to each independent system. The first variable, $\boldsymbol{\eta}$, is called target variable which is vector of the shared variables between two systems sending from ISO to DISCO. In fact, $\boldsymbol{\eta}$ is transmitted from the system in upper level toward the system in lower level. Response variable, $\boldsymbol{\mu}$, is the second variable which is vector of the shared variables sending from DISCO in the lower level toward the ISO in the upper level. According to the target and response variables, the consistency constraint expressed by (5) is introduced [19]:

$$\boldsymbol{c} = \boldsymbol{\eta} - \boldsymbol{\mu} = 0. \tag{5}$$

Constraint (5) should be regarded in the optimization problems of ISO and DISCO. Using the penalty function, the consistency constraints can be relaxed. Then, the separated ISO and DISCO's local optimization problems are (6) and (7), respectively:

$$\begin{aligned} Min \quad & f(\tilde{\mathbf{x}}, \mathbf{z}) + \pi(\mathbf{c}) && (6)\\ s.t. \quad & \boldsymbol{g}(\tilde{\mathbf{x}}, \mathbf{z}) \leq 0 \\ & \boldsymbol{h}(\tilde{\mathbf{x}}, \mathbf{z}) = 0 \\ & \forall \mathbf{z} \in \{\boldsymbol{\eta}, \boldsymbol{\mu}\} \\ Min \quad & f(\tilde{\mathbf{y}}, \mathbf{z}) + \pi(\mathbf{c}) && (7)\\ s.t. \quad & \boldsymbol{g}(\tilde{\mathbf{y}}, \mathbf{z}) \leq 0 \\ & \boldsymbol{h}(\tilde{\mathbf{y}}, \mathbf{z}) = 0 \\ & \forall \mathbf{z} \in \{\boldsymbol{\eta}, \boldsymbol{\mu}\}. \end{aligned}$$

B. Modeling Target and Response Variables

In this subsection, both target and response variables, as shared variables between the systems, are identified based on the physical connection between the transmission system and ADGs. As shown in Fig. 2, the power exchange through the physical connection is the shared variable between these two independent systems. This variable links the SCUC problems of ISO and DISCO together. Assume that the power is transferred from the ISO toward DISCO. The target and response variables can be modeled as shown in Fig. 2(c) where ISO is the independent system 0 and DISCO is system 1. From the DISCO's perspective, the line flow is modeled as a pseudo generator supplying to DISCO; from the ISO's perspective, the line flow is modeled as a pseudo load supplied by ISO. Therefore, η is the pseudo generation for the DISCO, and $\mu$is the pseudo load for ISO. It should be noted that the pseudo generation might be negative which means the power is delivered to the ISO by DISCO, and the pseudo load of ISO, μ, may also be negative.

Here, the power demanded by DISCO and supplied by ISO in the DISCO's optimization problem is defined in (8); the power generated by ISO and supplied to DISCO in ISO's optimization problem is (9). In addition, both variables PG_D and PD_S should be between minimum and maximum capacity of the line connecting the transmission network to $\mathrm{ADG_j}$:

$$\begin{aligned} \mu &= PG_D && (8)\\ \eta &= PD_S. && (9) \end{aligned}$$

C. Multi-ADGs and Multi-Period Model

When there are many ADGs connected to the transmission network, Fig. 2 can be extended to Fig. 3. The only system in upper level is ISO and all DISCOs are located in lower level. The ISO has shared variables with many DISCOs, and its optimization problem (6) is modified by (10) which includes the penalty function modeling consistency constraints between ISO and all DISCOs:

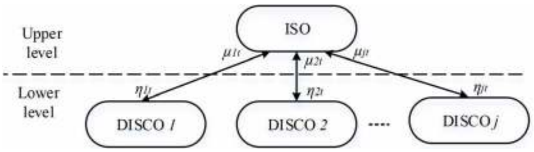

Fig. 3. Power system in the form of a hierarchical two-level SoS.

$$\begin{aligned} Min \quad & f(\tilde{\mathbf{x}}, \mathbf{z}_1, \mathbf{z}_2, \ldots, \mathbf{z}_j) + \pi(\mathbf{c}_1, \mathbf{c}_2, \ldots, \mathbf{c}_j) && (10)\\ s.t. \quad & \boldsymbol{g}(\tilde{\mathbf{x}}, \mathbf{z}_1, \mathbf{z}_2, \ldots, \mathbf{z}_j) \leq 0 \\ & \boldsymbol{h}(\tilde{\mathbf{x}}, \mathbf{z}_1, \mathbf{z}_2, \ldots, \mathbf{z}_j) = 0 \\ & \forall \mathbf{z}_j \in \{\boldsymbol{\eta}_j, \boldsymbol{\mu}_j\} \quad \forall j = 1, 2, \ldots, NA. \end{aligned}$$

Considering multi time intervals and using a second-order function to model the penalty function π [19], the ISO's optimization problem (10) is further modified by (11) where the symbol $\circ$ represents the Hadamard product: an entry-wise multiplication of two vectors:

$$\begin{aligned} Min \quad & \sum_{t=1}^{NT} f(\tilde{\mathbf{x}}, \mathbf{z}_{1t}, \mathbf{z}_{2t}, \ldots, \mathbf{z}_{jt}) && (11)\\ & + \sum_{t=1}^{NT} \sum_{j=1}^{NA} \left(\boldsymbol{\alpha}_{jt}(\boldsymbol{\eta}_{jt} - \boldsymbol{\mu}_{jt}) + \left\| \boldsymbol{\beta}_{jt} \circ (\boldsymbol{\eta}_{jt} - \boldsymbol{\mu}_{jt}) \right\|_2^2 \right) \\ s.t. \quad & \boldsymbol{g}(\tilde{\mathbf{x}}, \mathbf{z}_{1t}, \mathbf{z}_{2t}, \ldots, \mathbf{z}_{jt}) \leq 0 \\ & \boldsymbol{h}(\tilde{\mathbf{x}}, \mathbf{z}_{1t}, \mathbf{z}_{2t}, \ldots, \mathbf{z}_{jt}) = 0 \\ & \forall \mathbf{z}_{jt} \in \{\boldsymbol{\eta}_{jt}, \boldsymbol{\mu}_{jt}\} \quad \forall j = 1, 2, \ldots, NA, \ \forall t. \end{aligned}$$

Similarly, the optimization problem of $\mathrm{DISCO_j}$ can be rewritten in (12):

$$\begin{aligned} Min \quad & \sum_{t=1}^{NT} f(\tilde{\mathbf{y}}, \mathbf{z}_{jt}) && (12)\\ & + \sum_{t=1}^{NT} \left(\boldsymbol{\alpha}_{jt}(\boldsymbol{\eta}_{jt} - \boldsymbol{\mu}_{jt}) + \left\| \boldsymbol{\beta}_{jt} \circ (\boldsymbol{\eta}_{jt} - \boldsymbol{\mu}_{jt}) \right\|_2^2 \right) \\ s.t. \quad & \boldsymbol{g}(\tilde{\mathbf{y}}, \mathbf{z}_{jt}) \leq 0 \\ & \boldsymbol{h}(\tilde{\mathbf{y}}, \boldsymbol{z}_{jt}) = 0 \\ & \forall \mathbf{z}_{jt} \in \{\boldsymbol{\eta}_{jt}, \ \boldsymbol{\mu}_{jt}\} \quad \forall t. \end{aligned}$$

In (11) and (12), $\mathbf{z}_{jt}, \boldsymbol{\eta}_{jt}$, and $\boldsymbol{\mu}_{jt}$ are respectively shared, target and response variables between ISO and $\mathrm{DISCO_j}$ at time t. The penalty function consists of two terms, linear and quadratic. $\boldsymbol{\alpha}_{jt}$ and $\boldsymbol{\beta}_{jt}$ are multipliers associated with linear and quadratic terms, respectively, and they will be updated during the iterative solving process. An important feature of the second-order penalty function is that it is a convex quadratic curve. Thus, the problem can be easily solved using the quadratic optimization solvers. However, this quadratic penalty function can be piecewise linearized as presented in Appendix A.

D. Coupling Constraints Handling in SCUC Problems

SCUC problems of independent systems are connected together using the penalty function and target and response variables in order to find the results for the entire power system. Therefore, the following SCUC problem (13) is formulated for $\mathrm{DISCO_j}$:

$$\begin{aligned} Min \quad & \sum_{t=1}^{NT}\sum_{i=1}^{NG_j} F_i(P_{it})I_{it} + SUD_{it} \\ & + \sum_{t=1}^{NT}\left(\alpha_{jt}\left(PD^*_{S,jt} - PG_{D,jt}\right)\right. \\ & \left. + \left\|\beta_{jt}\circ\left(PD^*_{S,jt} - PG_{D,jt}\right)\right\|_2^2\right). \end{aligned} \tag{13}$$

The first term of (13) is for the production cost, startup and shutdown costs of $\mathrm{DISCO_j}$'s generating units. The second term is the penalty function related to the shared variables with ISO. Notice that in the penalty function, the response variables $PG_{D,jt}$ need to be determined, but the values of target variables $PD^*_{S,jt}$ are received from the ISO. Meanwhile, the regular SCUC constraints should be satisfied.

The SCUC problem (14) is for the ISO. The response variables received from the DISCOs are used to model the penalty function. In this problem, $PD_{S,jt}$ is treated as the vector of design variables while $PG^*_{D,jt}$ is treated as a constant term:

$$\begin{aligned} Min \quad & \sum_{t=1}^{NT}\sum_{i=1}^{\mathrm{NG}} F_i(P_{it})I_{it} + SUD_{it} \\ & + \sum_{t=1}^{NT}\sum_{j=1}^{NA}\left(\alpha_{jt}\left(PD_{S,jt} - PG^*_{D,jt}\right)\right. \\ & \left. + \left\|\beta_{jt}\circ\left(PD_{S,jt} - PG^*_{D,jt}\right)\right\|_2^2\right). \end{aligned} \tag{14}$$

Similarly, in (14), the first term represents the production cost, startup and shutdown costs of ISO's generating units, the second term is penalty function related to the shared variables with DISCOs, and the regular SCUC constraints should be satisfied.

In the SoS-based SCUC problem, the ISO and $\mathrm{DISCO_j}$, as the autonomous systems, may have different restrictions for amount of power exchange between them. Thus, in addition to regular SCUC constraints, the following constraint is regarded in the above SCUC problems of $\mathrm{DISCO_j}$ and ISO:

$$\begin{aligned} \max\{\underline{PT_{S,jt}}, \underline{PT_{D,jt}}\} \le & \{PD_{S,jt},\ PG_{D,jt}\} \\ & \le \min\{\overline{PT_{S,jt}}, \overline{PT_{D,jt}}\} \end{aligned} \tag{15}$$

where $\underline{PT_{S,it}}$ and $\overline{PT_{S,jt}}$ are minimum and maximum allowable values for the power exchange between ISO and $\mathrm{DISCO_j}$ at period t from the ISO's perspective; and $\underline{PT_{D,jt}}$ and $\overline{PT_{D,jt}}$ are minimum and maximum acceptable values from the $\mathrm{DISCO_j}$'s perspective.

E. Solution Procedure

Fig. 4 illustrates the solution procedure of the proposed hierarchical two-level optimization algorithm which determines the optimal SCUC results for the ISO and DISCOs. This algorithm has two iteration loops, inner and outer, which are explained as follows.

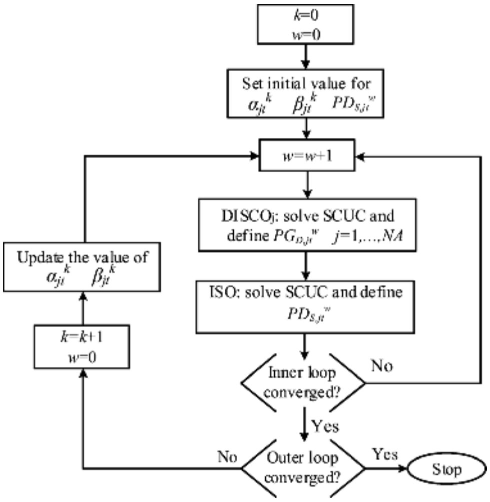

Fig. 4. Flowchart of the solving process.

Step 1: Set the iteration index $w = 0$ for the inner loop and $k = 0$ for the outer loop, and choose initial values for $PD^{*w}_{S,jt}$, α^k_{jt} and β^k_{jt}.
Step 2: Set $w = w + 1$. Solve the SCUC problem for each DISCO with $PG^w_{D,jt}$ as the design variables and the values of $PD^{*w-1}_{S,jt}$ from the previous iteration.
Step 3: Solve the SCUC problem for ISO with $PD^w_{S,jt}$ as design variables and the values of $PG^{*w}_{D,jt}$ obtained in Step 2.
Step 4: Use (16) and (17) to check the inner loop convergence. If they are not satisfied, return to Step 2 for the next iteration; otherwise, go to Step 5:

$$PD^w_{S,jt} - PD^{w-1}_{S,jt} \le \varepsilon_1 \quad \forall j,\ \forall t \tag{16}$$

$$PG^w_{D,jt} - PG^{w-1}_{D,jt} \le \varepsilon_1 \quad \forall j,\ \forall t. \tag{17}$$

Step 5: Check the following necessary-consistency (18) and sufficient (19) stopping criteria for the outer loop. If they are not satisfied, go to Step 6; otherwise, the converged optimal result is obtained and the solution procedure stops.
Necessary-consistency condition:

$$PD^w_{S,jt} - PG^w_{D,jt} \le \varepsilon_2 \quad \forall j,\ \forall t \tag{18}$$

Sufficient condition:

$$\left|\frac{f_s\left(\mathbf{x}^{(w)}\right) - f_s\left(\mathbf{x}^{(w-1)}\right)}{f_s\left(\mathbf{x}^{(w)}\right)}\right| \le \varepsilon_3 \tag{19}$$

where f_s is the objective function of the independent system S.
Step 6: Set $k = k + 1$ and update the values of multipliers α^k_{jt} and β^k_{jt} using (20) and (21):

$$\alpha^{(k+1)}_{jt} = \alpha^{(k)}_{jt} + 2\left(\beta^{(k)}_{jt}\right)^2\left(PD^w_{S,jt} - PG^w_{D,jt}\right) \tag{20}$$

$$\beta^{(k+1)}_{jt} = \lambda\beta^{(k)}_{jt} \tag{21}$$

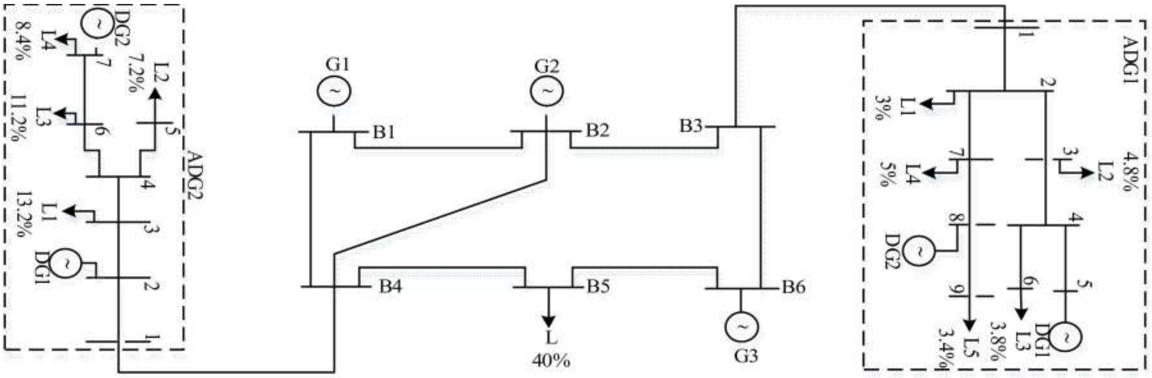

Fig. 5. Six-bus test system with two active distribution grids.

where the coefficient λ is necessary to be equal or larger than one in order to get the converged optimal results. This method for updating Lagrangian multipliers is proven to converge to the optimal solution in [20].

Step 7: Set $PD_{S,jt}^{*0} = PD_{S,jt}^{w}, \forall j, \forall t, w = 0$, and return to Step 2.

Note that in the inner loop process of this algorithm, the penalty multipliers are fixed, and only $PD_{S,jt}$ and $PG_{D,jt}$ need to be updated. Such process helps to improve the accuracy of the final results, especially when we do not have a good initial guess for the shared variables between the systems. Also, in practice, the following stopping criteria may be added to the inner and outer loops in order to avoid facing the dead loop:

$$w \geq \bar{W} \tag{22}$$

$$k \geq \bar{K} \tag{23}$$

where $\bar{W}$ and $\bar{K}$ are the maximum allowable number of inner and outer loops iterations, respectively. Suppose in a case, the inner loop stops because of the number of maximum iterations and before reaching the pre-specified threshold. The process of updating penalty multipliers in the outer loop and going back to the inner loop will control the entire optimization algorithm to converge to the optimal results [21].

In the proposed decentralized decision-making framework, as there are many DISCOs collaborating and communicating with one ISO, the ISO can be committed as the entity that is in charge of updating the penalty multipliers and send them to the DISCOs. It is assumed that the systems are working in a fair and clear market, and they accept the penalty multipliers defined by the ISO. However, an active distribution system can have its own right to refuse the penalty multipliers and can work in islanded mode without power exchange with the ISO. In this condition, there is no need to consider this system in the decentralized optimization process.

IV. CASE STUDIES

A six-bus, the modified IEEE RTS 24-bus and IEEE 118-bus test systems are applied to illustrate the performance of the proposed hierarchical two-level optimization algorithm for the SoS-based SCUC. All cases utilize ILOG CPLEX 12.4's MIQP solver on a 2.8-GHZ personal computer. Note that we can also use ILOG CPLEX 12.4's MIP solver once the quadratic penalty functions in the problem objectives are piecewise linearized.

TABLE I
GENERATOR DATA

Unit	P_{min} (MW)	P_{max} (MW)	a (MBtu)	b (MBtu/MWh)	c (MBtu/MW^{2}h)	Min OFF (hr)	Min ON (hr)
1	40	220	100	7	0.03	4	4
2	10	100	104	10	0.07	3	2
3	0	25	110	8	0.05	1	1

TABLE II
NETWORK INFORMATION

From Bus	To Bus	X (pu)	Flow Limit (MW)
1	2	0.170	200
1	4	0.258	200
2	3	0.037	190
2	4	0.197	200
3	6	0.018	180
4	5	0.037	190
5	6	0.140	180

TABLE III
HOURLY LOAD OVER 24-h HORIZON

Hour	Pd(MW)	Hour	Pd(MW)	Hour	Pd(MW)	Hour	Pd(MW)
1	175	7	173	13	242	19	246
2	169	8	174	14	244	20	237
3	165	9	185	15	249	21	237
4	155	10	202	16	256	2	233
5	155	11	228	17	256	23	210
6	165	12	236	18	247	24	210

A. Six-Bus System

The system topology is shown in Fig. 5. The six-bus test system has 3 generating units, 7 branches, and 3 demand sides in the transmission system. The characteristics of generating units, network information, and the hourly load distribution over 24-h horizon are given in Tables I–III. Two active distribution grids are connected to the transmission system through buses 3 and 4, and one passive distribution grid is connected to bus 5. ADG1 consists of 9 buses, 8 distribution lines, 5 loads and 2 DGs. Also ADG2 includes 7 buses, 6 distribution lines, 4 loads and 2 DGs. DG unit and network characteristics of the ADGs are shown in Tables IV and V. The load distribution percent for each bus is indicated in Fig. 5.

In order to analyze the effectiveness of the proposed algorithm, we consider the following three case studies:

Case 1: Without active distribution grids

Case 2: With active distribution grids, but no network security

TABLE IV
DISTRIBUTED GENERATOR DATA

ADG No.	DG	P_{min} (MW)	P_{max} (MW)	a (MBtu)	b (MBtu/MWh)	c (MBtu/MW^{2}h)
ADG1	1	0	15	100	7	0.08
	2	0	18	65	3	0.03
ADG2	1	5	25	140	5	0.04
	2	0	19	50	25	0.00

TABLE V
DISTRIBUTION LINE DATA FOR ADG 1 AND 2

ADG1				ADG2			
From	To	X (pu)	Flow Limit (MW)	From	To	X (pu)	Flow Limit (MW)
B3	1	0.2	60	B4	1	0.2	70
1	2	0.19	60	1	2	0.15	70
2	3	0.21	30	2	3	0.20	90
2	7	0.21	30	3	4	0.16	70
3	4	0.20	40	4	5	0.18	40
4	5	0.18	20	4	6	0.18	50
4	6	0.18	30	6	7	0.16	40
7	8	0.19	20	-	-	-	-
8	9	0.19	20	-	-	-	-

TABLE VI
UC SOLUTION IN CASE 1

Units	Hours (1-24)
1	1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1
2	0 0 0 0 0 0 0 0 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1
3	1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1

TABLE VII
GENERATION DISPATCH (MW) IN CASE 1

Hour	Unit1	Unit2	Unit3	Hour	Unit1	Unit2	Unit3
1	150.0	0	25	13	166.9	50.1	25
2	144.0	0	25	14	168.3	50.7	25
3	140.0	0	25	15	171.8	52.2	25
4	130.0	0	25	16	176.7	54.3	25
5	130.0	0	25	17	176.7	54.3	25
6	140.0	0	25	18	170.4	51.6	25
7	148.0	0	25	19	169.7	51.3	25
8	149.0	0	25	20	163.4	48.6	25
9	127.0	33.0	25	21	163.4	48.6	25
10	138.9	38.1	25	22	160.6	47.4	25
11	157.1	45.9	25	23	144.5	40.5	25
12	162.7	48.3	25	24	144.5	40.5	25

Case 3: With active distribution grids and network security

Case 1: In this case, there is no DG in the system and all distribution grids are passive, which are modeled as the constant (forecasted) loads connected to buses 3, 4 and 5. The conventional centralized SCUC problem is solved to find the optimal operating point of the system. Table VI shows hourly ON/OFF states of the units. The generation dispatch is listed in Table VII. During the off-peak load, unit 2 which is an expensive unit, is not committed, and when it is within the peak hours, this unit is scheduled to be ON. The total operating cost is \$65 414.44.

Case 2: As shown in Fig. 5, two ADGs are connected to the transmission system through buses 3 and 4, respectively. According to the SoS concept, the ISO and each DISCO are modeled as the independent systems. The power transferred between the ISO and DISCOs are limited by the capacity of the line connecting the systems together. The initial value for $\boldsymbol{\alpha}_{jt}^{0} = \boldsymbol{\beta}_{jt}^{0} = 1$ (j $= 1, 2,$ and t $= 1:24$); and convergence thresholds ε_1, ε_2, and ε_3 are set to 0.01, 0.001, and 0.001, respectively. The decentralized SoS-based SCUC is applied to find the optimal operating point of the transmission system and ADGs. Notice that in this case, the transmission and distribution network security is not taken into account. In other words, it is a UC problem incorporating ADGs.

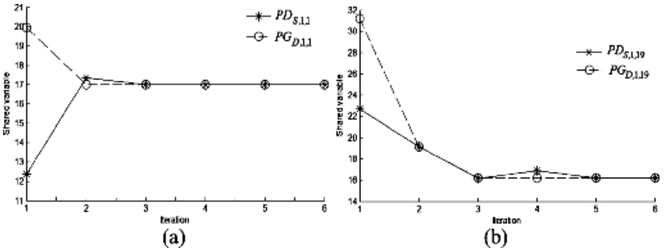

Fig. 6. Power exchange between ISO and ADG1 at hours (a) 1, (b) and 19.

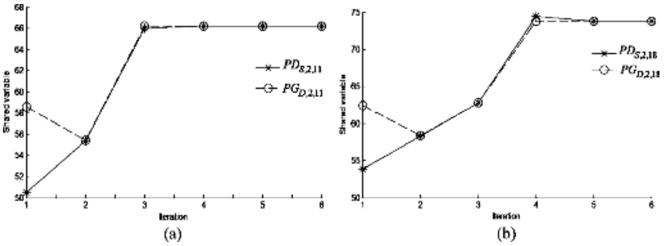

Fig. 7. Power exchange between ISO and ADG2 at hours (a) 11, (b) and 18.

TABLE VIII
UC SOLUTION IN CASE 2

Ind. Syst.	Units	Hours (1-24)
ISO	1	1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1
	2	0 0 0 0 0 0 0 0 **0 0 0 0** 1 1 1 1 1 1 **1 0 0 0 0 0**
	3	1 1 **0 0 0 0** 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1
DISCO1	1	0 0 0 0 0 0 0 0 0 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1
	2	1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1
DISCO2	1	1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1
	2	0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0

After 6 outer loop iterations, the converged optimal power exchange is obtained. As the examples, Fig. 6 shows the shared variable (amount of power exchange (MW)) between ISO and DISCO1 at hours 1 and 19; and Fig. 7 depicts the shared variable (amount of power exchange (MW)) between ISO and DISCO2 at hours 11 and 18 in each outer loop iteration. Table VIII shows the ON/OFF states of each generating unit, and the generation dispatch is depicted in Table IX. The highlighted values in these tables show differences between the UC and generation dispatch of Case 2 and that of Case 1. In the off-peak hours, the expensive generating units, unit 2 of transmission system and DG1 of ADG1 and DG2 of ADG2, are OFF. When it is around peak load hours, unit 2 of transmission system and DG1 of ADG1 is scheduled to be ON but DG2 of ADG2 which is very expensive should stay OFF. The operating costs of ISO, DISCO1, and DISCO2 are \$46 113.62, \$6434.28, and \$6960, respectively; and the total system operating cost is \$59 507.90. Compared with the cost (\$65 414.44) of Case 1, the total operating cost is reduced due to incorporation of cheap DG units in the SCUC problem. As the network capacity is not considered in Case 2, the lines 1–2 and 4–6 in ADG2, and the tie-line connecting the transmission grid to ADG2 are overloaded according to the generation dispatches obtained in this case.

Case 3: In order to meet the network security for each independent system, the capacity limit of lines is considered in this case. Using the initial values $\boldsymbol{\alpha}_{jt}^{0} = \boldsymbol{\beta}_{jt}^{0} = 1$ (j $= 1, 2,$

TABLE IX
GENERATION DISPATCH (MW) IN CASE 2

Hour	ISO			DISCO1		DISCO2	
	Unit1	Unit2	Unit3	DG1	DG2	DG1	DG2
1	107	0	25	0	18	25	0
2	101	0	25	0	18	25	0
3	122	0	0	0	18	25	0
4	112	0	0	0	18	25	0
5	112	0	0	0	18	25	0
6	122	0	0	0	18	25	0
7	105	0	25	0	18	25	0
8	106	0	25	0	18	25	0
9	117	0	25	0	18	25	0
10	119	0	25	15	18	25	0
11	145	0	25	15	18	25	0
12	153	0	25	15	18	25	0
13	126.30	32.7	25	15	18	25	0
14	127.70	33.30	25	15	18	25	0
15	131.19	34.78	25	15	18	25	0
16	136.10	36.90	25	15	18	25	0
17	136.10	36.90	25	15	18	25	0
18	129.80	34.20	25	15	18	25	0
19	129.1	33.90	25	15	18	25	0
20	154	0	25	15	18	25	0
21	154	0	25	15	18	25	0
22	150	0	25	15	18	25	0
23	127	0	25	15	18	25	0
24	127	0	25	15	18	25	0

TABLE X
UC SOLUTION IN CASE 3

Ind. Syst.	Units	Hours (1-24)
ISO	1	1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1
	2	0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 1 0 0 0 0 0 0 0
	3	1 1 0 0 0 0 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1
DISCO1	1	0 0 0 0 0 0 0 0 0 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1
	2	1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1
DISCO2	1	1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1
	2	0 0 0 0 0 0 0 0 0 0 1 1 1 1 1 1 1 1 1 1 1 1 0 0

and $\mathrm{t} = 1 : 24$); and setting the values 0.01, 0.001, and 0.001 for the convergence thresholds ε_1, ε_2, and ε_3, respectively, the proposed SoS-based SCUC is implemented. The algorithm converges after 5 outer loop iterations. The UC schedule for the generating units of each independent system is shown in Table X. The generation dispatch is listed in Table XI. The highlighted values in Tables X and XI show differences between these tables and Tables VIII and IX. Compared with case 2, to remove the congestion on the lines 1–2 and 4–6 in ADG2, and the tie-line connecting the transmission grid to ADG2, unit 2 is scheduled to be OFF in hours 13–15 and 18–19, and DG2 of ADG2, which is an expensive generation source, is committed ON from hour 11 to 22. The operating costs of ISO, DISCO1, and DISCO2 are \$44 817.94, \$6434.28, and \$9670, respectively; and the total system operating cost is \$60 922.22 which is \$1414.5 more than that in Case 2, and \$4492.2 less than that in Case 1.

Notice that in order to check the validity of the results of the proposed decentralized optimization algorithm, both Cases 2 and 3 are also solved considering the entire power system as a single system and applying centralized optimization algorithm. Its results are same as those shown in Tables VIII–XI, which are obtained based on the decentralized optimization algorithm.

B. Modified IEEE-RTS 24-Bus System

A modified IEEE-RTS 24-bus test system is used to study the SOS-based SCUC. The transmission network has 10 generators and 34 lines, and it encompasses 8 passive distribution grids, and 9 independent active distribution grids connected to buses 1, 2, 3, 5, 6, 7, 10, 13, and 19. The total 25 distributed generators are installed near to the load centers in ADGs. The peak load of 1,869 MW occurs at hour 11. The system input data is given in Appendix B. Set the initial values $PD^0_{S,jt} = \boldsymbol{\alpha}^0_{jt} = \boldsymbol{\beta}^0_{jt} = 1$ ($\mathrm{j} = 1 : 9$ and $\mathrm{t} = 1 : 24$), and pick the values 0.01, 0.001, and 0.001 for the convergence thresholds ε_1, ε_2, and ε_3, respectively. The algorithm takes 5 s to obtain the optimal results after 5 outer loop iterations. Table XII shows the ON/OFF states of the generating units. Generating unit 3 of the ISO is a very expensive unit and is committed to be OFF all over the operating time horizon. Unit 4 which is an expensive unit is only committed to be ON when the load is near the peak. Also, DGs 4 of DISCO2, 3 of DISCO3, 2 of DISCO6, 4 of DISCO7, and 2 of DISCO9 are very expensive and are scheduled to be OFF over 24 h in this case. The operating cost of each independent system, ISO and DISCOs, is depicted in Table XIII. The total operating cost of the SoS-based power system is \$463 729.80.

TABLE XI
GENERATION DISPATCH (MW) IN CASE 3

Hour	ISO			DISCO1		DISCO2	
	Unit1	Unit2	Unit3	DG1	DG2	DG1	DG2
1	107	0	25	0	18	25	0
2	101	0	25	0	18	25	0
3	122	0	0	0	18	25	0
4	112	0	0	0	18	25	0
5	112	0	0	0	18	25	0
6	122	0	0	0	18	25	0
7	105	0	25	0	18	25	0
8	106	0	25	0	18	25	0
9	117	0	25	0	18	25	0
10	119	0	25	15	18	25	0
11	143.79	0	25	15	18	25	1.20
12	148.59	0	25	15	18	25	4.40
13	152.19	0	25	15	18	25	6.80
14	153.39	0	25	15	18	25	7.60
15	156.39	0	25	15	18	25	9.60
16	124.42	33.18	25	15	18	25	12.40
17	127.42	33.18	25	15	18	25	12.40
18	155.19	0	25	15	18	25	8.80
19	154.59	0	25	15	18	25	8.40
20	149.19	0	25	15	18	25	4.80
21	149.19	0	25	15	18	25	4.80
22	146.76	0	25	15	18	25	3.20
23	127	0	25	15	18	25	0
24	127	0	25	15	18	25	0

C. Modified IEEE 118-Bus System

A modified IEEE 118-bus test system, as a relatively large power system, is used to study the proposed SoS-based SCUC algorithm. The system has 30 independent active distribution grids each of which is operated by a DISCO. Also 61 inactive distribution grids are also connected to the transmission network. The initial values of $PD^0_{S,jt}$, $\boldsymbol{\alpha}^0_{jt}$, and $\boldsymbol{\beta}^0_{jt}$ ($\mathrm{j} = 1 : 30$ and $\mathrm{t} = 1 : 24$) are set to be 1. The decentralized decision-making algorithm takes 2 min to converge to an optimal solution after 7 outer loop iterations. Table XIV depicts the total operating cost of the entire SoS-based power system which is \$1 257 170. To check the validity of the results, the centralized algorithm is also implemented and the total operating cost is \$1 254 586. The difference between the operating costs obtained by these two algorithms is 0.2% which is acceptable. Although the SoS-based decentralized algorithm could result in a slight increase in operating cost, it needs a limited information to be exchanged between the independent systems which can meet the privacy of the systems.

TABLE XII
UC SOLUTION FOR IEEE-RTS 24-BUS SYSTEM

Ind. Syst.	Units	Hours (1-24)																							
ISO	1	0	0	0	0	0	0	0	1	1	0	1	0	0	0	0	0	0	1	1	0	0	1	0	0
	2	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
	3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0
	4	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	0	0	1	1	0	0	0
	5	0	1	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
	6-10	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
DISCO1	1-2	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
DISCO2	1-3	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
	4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
DISCO3	1-2	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
	3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
DISCO4	1	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0
	2	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
DISCO5	1-3	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
DISCO6	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
DISCO7	1	0	0	0	0	0	0	0	1	1	0	0	0	0	0	0	1	0	1	1	0	0	1	0	0
	2-3	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
	4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
DISCO8	1-3	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
DISCO9	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

TABLE XIII
OPERATING COST OF ISO AND DISCOS

Independent System	Operating Cost ($)
ISO	383,660.20
DISCO1	8,441.28
DISCO2	8,945.52
DISCO3	6,546.00
DISCO4	8,351.31
DISCO5	11,424.00
DISCO6	6,960.00
DISCO7	11,059.50
DISCO8	10,692.00
DISCO9	7,650.00

TABLE XIV
OPERATING COST OF SOS-BASED POWER SYSTEM

Solution Algorithm	Operating Cost ($)	# of outer loop
Centralized	1254586	-
Decentralized	1257170	7

V. CONCLUSION

Incorporation of generation sources of active distribution grids in power systems operation enhances the economic and security in restructured power systems. As the transmission and distribution grids are utilized by different system operators in the electricity market, making a collaborative and optimal operation among these systems is an important problem. In this paper, the power system was modeled as a system of systems, in which the ISO and each DISCO were autonomous and independent systems. And, a decentralized decision-making framework was proposed to find the optimal SoS based SCUC schedule for ISO and DISCOs. In order to solve the problem, a hierarchical optimization algorithm was presented taking into account the shared variables/information between the independent systems. The numerical tests on a six-bus and an IEEE-RTS 24-bus test systems showed the accuracy and convergence performance of the proposed algorithm. The hourly results of day-ahead market verified that considering the DG units in SCUC problem improves the market efficiency in terms of economic and security issues. Consequently, this decentralized decision-making framework can facilitate the participation of active distribution grids into the market.

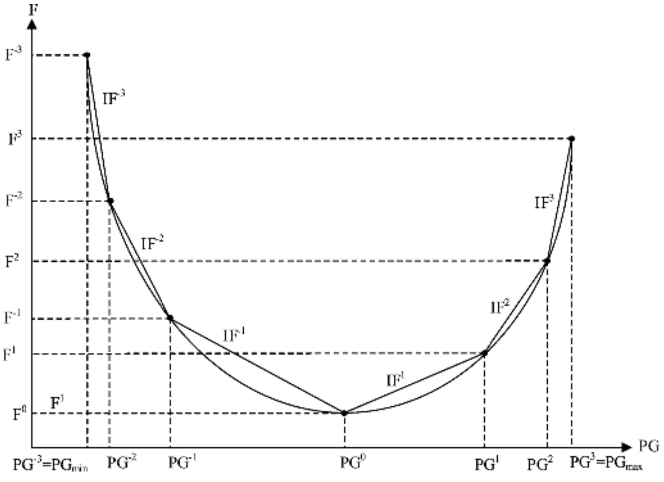

Fig. 8. Piecewise linear approximation of the quadratic penalty function.

In this paper, the dispatchable DGs like gas turbines and diesel generators are studied, and we have not discussed the generation uncertainty of renewable energy sources like wind turbines and solar PVs. Our future work will incorporate the uncertainty of non-dispatchable renewable energy sources into this SoS-based decentralized SCUC problem.

APPENDIX A

Assume the quadratic penalty function used in (13). In each iteration of the solution procedure, the multipliers α and β, and the value of *PD* are known, and *PG* is the only variable in this penalty function. Thus, this convex quadratic penalty function can be piecewise linearized in each iteration using the following steps:

Step 1: Find the minimum point of the curve by $PG^0 = (2\beta * PD + \alpha)/(2\beta)$.

Step 2: The convex penalty function is approximated by a set of piecewise linear functions. Fig. 8 shows an example of the piecewise linear approximation in which the convex penalty function between $PG^{-3} = PG_{\min}$ and $PG^3 = PG_{\max}$ is divided into six segments represented by straight lines. The negative and positive superscripts are respectively used for the left and right hand sides of the minimum point on the curve. The pseudo power generated by DISCO at the dth breakpoint is represented by PG^d. And the power dispatched at segment d is PGx^d which is between zero and $[PG^{d+1} - PG^d]$. The incremental cost at segment d is $IF^d = (F^{d+1} - F^d)/(P^{d+1} - P^d)$. Thus, the penalty function can be replaced as $\pi = \sum_d (IF^d * PGx^d)$.

The above linearization process needs to be implemented at each time interval in each iteration. A similar procedure can be applied to linearize the penalty function of (14).

TABLE XV
HOURLY LOAD OVER 24-h HORIZON

Hour	Pd(MW)	Hour	Pd(MW)	Hour	Pd(MW)	Hour	Pd(MW)
1	1190	7	1400	13	1813	19	1750
2	1211	8	1701	14	1785	20	1785
3	1183	9	1715	15	1834	21	18200
4	1190	10	1820	16	1855	2	1736
5	1225	11	1869	17	1785	23	1540
6	1295	12	1813	18	1771	24	1288

TABLE XVI
ISO GENERATOR DATA

Unit	P_{min} (MW)	P_{max} (MW)	a (MBtu)	b (MBtu/MWh)	c (MBtu/MW^{2}h)
1	30	192	155	10	0.09
2	30	192	130	6	0.03
3	50	00	240	10	0.07
4	200	591	115	5	0.06
5	50	215	150	8	0.05
6	40	155	165	6	0.04
7	80	400	130	7	0.06
8	80	400	110	5	0.07
9	60	300	120	6	0.05
10	200	660	105	4	0.01

TABLE XVII
DISTRIBUTED GENERATOR DATA

ADG No.	DG	P_{min} (MW)	P_{max} (MW)	a (MBtu)	b (MBtu/MWh)	c (MBtu/MW^{2}h)
ADG1	1	0	15	100	7	0.08
	2	0	18	65	3	0.03
ADG2	1	0	8	110	6	0.07
	2	0	10	80	4	0.04
	3	0	5	60	5	0.05
	4	0	5	100	7	0.03
ADG3	1	0	10	100	5	0.06
	2	0	15	65	3	0.03
	3	0	5	100	7	0.08
ADG4	1	0	15	120	7	0.07
	2	0	18	50	6	0.06
ADG5	1	0	10	100	4	0.05
	2	0	10	65	8	0.08
	3	5	10	100	7	0.08
ADG6	1	5	25	140	5	0.04
	2	0	19	50	25	0
ADG7	1	0	5	80	6	0.05
	2	5	20	140	5	0.04
	3	0	10	100	7	0.07
	4	0	15	50	25	0
ADG8	1	5	15	110	4	0.05
	2	0	10	60	6	0.04
	3	0	15	90	2	0.09
ADG9	1	5	25	150	6	0.03
	2	0	20	60	24	0

APPENDIX B

The general information about IEEE 24-bus transmission system is given in [22]. The other input data used in the case studies is listed in Tables XV–XVII.

REFERENCES

[1] M. H. Bollen and F. Hassan, *Integration of Distributed Generation in the Power System*. New York, NY, USA: Wiley, 2011.

[2] S. P. Chowdhury, P. Crossley, and S. Chowdhury, Microgrids and Active Distribution Networks, Institution of Engineering and Technology, Jul. 2009.

[3] A. Khodaei and M. Shahidehpour, “Microgrid-based co-optimization of generation and transmission planning in power systems,” *IEEE Trans. Power Syst.*, vol. 28, no. 2, pp. 1582–1590, May 2013.

[4] M. Shahidehpour, H. Yamin, and Z. Li, *Market Operations in Electric Power Systems*. New York, NY, USA: Wiley, 2002.

[5] X. Guan, P. B. Luh, H. Yan, and J. A. Amalfi, “An optimization-based method for unit commitment,” *Electr. Power Energy Syst.*, vol. 14, no. 1, pp. 9–17, Feb. 1992.

[6] J. M. Arroyo and A. J. Conejo, “Optimal response of a thermal unit to an electricity spot market,” *IEEE Trans. Power Syst.*, vol. 15, no. 3, pp. 1098–2000, Aug. 2000.

[7] M. Carrion and J. M. Arroyo, “A computationally efficient mixed-integer linear formulation for the thermal unit commitment problem,” *IEEE Trans. Power Syst.*, vol. 21, no. 3, pp. 1371–1378, Aug. 2006.

[8] Y. Fu, M. Shahidehpour, and Z. Li, “Security-constrained unit commitment with AC constraints,” *IEEE Trans. Power Syst.*, vol. 20, no. 3, pp. 1538–1550, Aug. 2005.

[9] Y. Fu and M. Shahidehpour, “Fast SCUC for the large-scale power systems,” *IEEE Trans. Power Syst.*, vol. 22, no. 4, pp. 2144–2151, Nov. 2007.

[10] A. Kalantari, J. F. Restrepo, and F. D. Galiana, “Security-constrained unit commitment with uncertain wind generation: The loadability set approach,” *IEEE Trans. Power Syst.*, vol. 28, no. 2, pp. 1787–1796, May 2013.

[11] S. Goleijani, T. Ghanbarzadeh, F. Sadeghi Nikoo, and M. Parsa Moghaddam, “Reliability constrained unit commitment in smart grid environment,” *Electr. Power Syst. Res.*, vol. 97, pp. 100–108, Jan. 2013.

[12] M. E. Khodayar, M. Barati, and M. Shahidehpour, “Integration of high reliability distribution system in microgrid operation,” *IEEE Trans. Smart Grid*, vol. 3, no. 4, pp. 1997–2006, Dec. 2012.

[13] T. Logenthiran, D. Srinivasan, A. Khambadkone, and H. Aung, “Multiagent system for real-time operation of a microgrid in real-time digital simulator,” *IEEE Trans. Smart Grid*, vol. 3, no. 2, pp. 925–933, Jun. 2012.

[14] H. Moghimi Ghadikolaei, E. Tajika, J. Aghaei, and M. Charwand, “Integrated day-ahead and hour-ahead operation model of discos in retail electricity markets considering DGs and CO2 emission penalty cost,” *Appl. Energy*, pp. 174–185, 2012, vol..

[15] M. A. Fotouhi Ghazvini, H. Morais, and Z. Vale, “Coordination between mid-term maintenance outage decisions and short-term security-constrained scheduling in smart distribution systems,” *Appl. Energy*, vol. 96, pp. 281–291, 2012.

[16] H. Li, Y. Li, and Z. Li, “A multiperiod energy acquisition model for a distribution company with distributed generation and interruptible load,” *IEEE Trans. Power Syst.*, vol. 22, no. 2, pp. 588–596, May 2007.

[17] J. M. Held, “The modeling of systems of systems,” Ph.D. dissertation, University of Sydney, Sydney, Australia, 2008.

[18] M. Jamshidi, *System of Systems Engineering: Innovations for the 21th Century*, 1st ed. New York, NY, USA: Wiley, 2008.

[19] S. Tosserams, L. F. P. Etman, P. Y. Papalambros, and J. E. Rooda, “An augmented Lagrangian relaxation for analytical target cascading using the alternating directions method of multipliers,” *Struct. Multidisc. Optim.*, vol. 31, no. 3, pp. 176–189, 2006.

[20] D. P. Bertsekas, *Nonlinear Programming*, 2nd ed. Belmont, MA, USA: Athena Scientific, 2003.

[21] N. F. Michelena, H. Park, and P. Y. Papalambros, “Convergence properties of analytical target cascading,” *AIAA J.*, vol. 41, no. 5, pp. 897–905, May 2003.

[22] C. Grigg, P. Wong, P. Albrecht, Q. Allann, M. Bhavaraju, R. Billinton, Q. Chen, C. Fong, S. Haddad, S. Kuruganty, W. Li, R. Mukerji, D. Patton, N. Rau, D. Reppen, A. Schneider, A. Shahidehpour, and C. Singh, “The IEEE reliability test system—1996,” *IEEE Trans. Power Syst.*, vol. 14, no. 3, pp. 1010–120, Aug. 1999.

Amin Kargarian (S’10) received the B.S. and M.S. degrees in electrical engineering from the University of Isfahan and Shiraz University, Iran, in 2007 and 2010, respectively. He is currently pursuing the Ph.D. degree in electrical engineering at Mississippi State University, Mississippi State, MS, USA.

His research interests include power system optimization and economics.

Yong Fu (M’05–SM’13) received his B.S. and M.S. in E.E. from Shanghai Jiaotong University, China, in 1997 and 2002, respectively and Ph.D. degree in E.E. from Illinois Institute of Technology, USA, in 2006. Presently, he is an assistant professor in the Department of Electrical and Computer Engineering at Mississippi State University. His research interests include power system optimization and economics, power system reliability and renewable energy integration.