

Systematic approach for the design of reconstruction algorithm in digital holography

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ABSTRACT

In digital holography, holograms are recorded by a CCD-array, and the complex amplitude of the object wave is numerically reconstructed via computer. For different recording conditions and different properties of objects, different reconstruction algorithms are required. The conventional reconstruction algorithms were conceived directly by replacing the diffraction integral with summation. Each method has its limitation in the valid range for correctly calculating the diffraction integral. The Single Fourier Transform method is valid for far Fresnel zone hologram, whereas the convolution method is appropriate for near Fresnel holograms. Here, we present a general reconstruction model from the perspective of "Generalized sampling theory". Given that the function space in which the unknown complex amplitude lies, an approximation of the continuous complex amplitude at the CCD can be synthesized from a set of basis functions with the recorded samples as weights. Back-propagation of the approximated complex amplitude to the original object plane yields an expression relating the continuous complex amplitude of the object with the recorded samples. By adopting different basis functions and different formulas for describing the diffraction process, an optimal reconstruction algorithm can be developed for various recording conditions and different diffraction characteristics of the object. Contrary to the conventional algorithms where values are available only at specific grid, complex amplitude at any position of the object can be obtained using this model. In addition, the effect due to the non-zero fill factor of the CCD can also be incorporated into the reconstruction algorithm to be further compensated by over-weighting the high frequency components. Two basis functions: Dirac delta- and Sinc-, are studied in detail.

Keywords: Digital holography, Numerical reconstruction, Sampling theory

1. INTRODUCTION

In digital holography, the reconstruction algorithm plays a core role. After the first demonstration of the reconstruction of hologram via computer, many works on the design of reconstruction algorithm have been reported [1, 2]. Although these algorithms are useful, they have limitation in valid range for correctly reconstructing image. The Single Fourier transform algorithm [1] is valid for far Fresnel zone holograms, whereas, the convolution method [2] is appropriate for near Fresnel zone holograms. In principle, the performance of reconstruction algorithm depends on the specifications of image acquisition device, the diffraction characteristics of the object under investigation, and the geometry of recording setup (esp. the distance of object to the CCD). When the object is far away from the CCD, then what is recorded is more or less the spectrum of the complex amplitude of the object. In contrast, if the object is close to the CCD, then the acquisition is nearly done on the complex amplitude of object itself. If digital holography is thought of as an imaging technique, it exhibits, with different recording distance, the evolution process from transformed domain imaging to direct imaging. In the recording process, the continuous optical signal is digitalized by the CCD. According to the sampling theory, the reconstruction of the original signal from the recorded samples is accompanied by a reconstruction basis function. In the case of band-limited object complex amplitude, the reconstruction basis function can be a sinc function. Generally, in order to give the best representation of the optical signal, different basis function is required. However, the conventional reconstruction algorithms are developed metaphysically whose reconstruction basis function implicitly adopted is a Delta function as shown in Section 4. The constant Delta reconstruction basis function is not adaptable to practical situations and this can partially account for the limited performance of the existing algorithms. In this paper, we aim at presenting a general model for the design of reconstruction algorithm.

2. MODEL FOR NUMERICAL RECONSTRUCTION

As a prerequisite of accurate reconstruction of digital holograms, we need a model that faithfully describes the recording process. For the sake of brevity, we limit ourselves to the 1-D case. The extension to 2-D is straightforward.

Let $U_o(x_o)$ be the complex amplitude of an object wavefront, $h(x, x_o)$ be the impulse response of free space propagation from the object plane to the hologram plane, then the complex amplitude in the hologram plane $U(x)$ can be written as

$$U(x) = \int_{-\infty}^{\infty} U_o(x_o) h(x, x_o) dx_o. \quad (2.1)$$

Denote the complex amplitude of reference beam by $U_R(x)$, and then the incident wavefront on the CCD is

$$U_{total}(x) = U(x) + U_R(x). \quad (2.2)$$

Here we assume that the difference of light path between the object and the reference is within the coherence length of the laser used, and the reference and the object beams have the same polarization direction. The readouts of CCD are the integrated intensity of light over each pixel. Denote the pixel aperture of CCD as $\phi(x)$. The integrated intensities are given by

$$I(x) = |U_{total}(x)|^2 \otimes \phi(x) = [|U(x)|^2 + |U_R(x)|^2] \otimes \phi(x) + U_{AV}(x) + U_{AV}^*(x), \quad (2.3)$$

with

$$U_{AV}(x) = U(x)U_R^*(x) \otimes \phi(x). \quad (2.4)$$

Symbols \otimes and $*$ stand for convolution and complex conjugation operations. The convolution leads to a high signal to noise ratio for low spatial frequency fringes since the phase of diffracted object field are nearly the same over one pixel. However, the phase change within a pixel aperture cannot be ignored for high spatial frequency fringes. The integration will average out the detail information of the object diffracted field. This kind of low-pass filtering effect will be discussed in detail in Section 3.2. The hologram discretization process is described as

$$I[m] = I(m\Delta x), \text{ with } m=0, 1 \dots N-1. \quad (2.5)$$

where $I[m]$ is the recorded hologram samples, N is the number of pixels of CCD, and Δx is the CCD pixel pitch. By use of the phase-shifting algorithm, we can derive complex amplitude from intensities [3]. Given four holograms, for example, we have

$$U_{AV}[m] = \{I_0[m] - I_\pi[m] + i(I_{\pi/2}[m] - I_{3\pi/2}[m])\} / 4 \quad (2.6)$$

where

$$U_{AV}[m] = U_{AV}(m\Delta x) \quad (2.7)$$

are samples of the averaged wavefront. In order to reconstruct the wavefront of object, it is necessary to represent the continuous diffracted field with the derived N samples $U_{AV}[m]$. According to the generalized sampling theory, a reconstruction of the continuous wavefront can be expressed as

$$U_{AV}^r(x) = \sum_{m=0}^{N-1} U_{AV}[m] \psi(x/p_H - m), \quad (2.8)$$

where $\psi(x)$ is a reconstruction basis function. For band-limited complex amplitude, it should be the sinc function.

Generally, the selection of the basis function is based on some pre-knowledge about the object complex amplitude. It is important to notice that, for a given reconstruction basis function, the signal that can be described by the finite samplings are limited, many signals are undistinguishable. Denote

$$\hat{U}(x) = U(x)U_R^*(x) \quad (2.9)$$

then its reconstruction can be written as

$$\hat{U}'(x) = \int_{-\infty}^{\infty} U_{AV}'(u)\varphi^r(x-u)du = \sum_{m=0}^{N-1} U_{AV}[m] \int_{-\infty}^{\infty} \psi(u/p_H - m)\varphi^r(x-u)du, \quad (2.10)$$

where $\varphi^r(x)$ is a deconvolution kernel for compensating for the blurring due to $\varphi(x)$. If it fulfills the following equation

$$\int_{-\infty}^{\infty} \varphi(x-u)\varphi^r(u)du = \delta(x), \quad (2.11)$$

the effect due to the non-zero fill factor of CCD is completely compensated. Where $\delta(x)$ is the Dirac delta function. Finally, the reconstructed object wavefront is expressed as

$$\begin{aligned} U_I(X) &= \int_{-\infty}^{\infty} \hat{U}'(x)g(X,x)/U_R^*(x)dx \\ &= \sum_{m=0}^{N-1} U_{AV}[m] \int_{-\infty}^{\infty} g(X,x)/U_R^*(x) \int_{-\infty}^{\infty} \psi(u/\Delta x - m)\varphi^r(x-u)dudx \end{aligned} \quad (2.12)$$

where $g(X,x)$ is the impulse response of back propagation from the hologram plane to the image plane. In order to get a focused image, it should fulfill the relationship

$$\int_{-\infty}^{\infty} g(X,x) \int_{-\infty}^{\infty} U_o(x_o)h(x,x_o)dx_o dx = U_o(X). \quad (2.13)$$

Equation (2.12) is the general formula for the design of reconstruction algorithm. Based on the generalized development, we study some practical cases.

3. FOURIER HOLOGRAM

In digital holography, Fourier holograms are usually recorded in a lensless setup [4]. In this setup, a reference point light source is put at a location with the same distance to the CCD as the object. Therefore, we have

$$h(x,x_o) = \exp\left(-i\pi \frac{(x-x_o)^2}{\lambda z_o}\right), \quad (3.1)$$

and

$$U_R(x) = \exp\left(-i\pi \frac{x^2}{\lambda z_o}\right) \quad (3.2)$$

where z_o is the recording distance. Let

$$g(X,x) = \exp\left(i\pi \frac{(X-x)^2}{\lambda z_r}\right). \quad (3.3)$$

to fulfill (2.13).

3.1 Neglecting the effect due to CCD pixel aperture

Assume that the CCD has a negligible fill factor. Then, the pixel aperture can be represented as a Dirac impulse function mathematically. Accordingly, the deconvolution kernel is also a Dirac impulse function.

$$\varphi(x) = \varphi^r(x) = \delta(x) \quad (3.4)$$

A. $\psi(x)$ takes the form of $\delta(x)$

First, we assume that the reconstruction basis function is a Dirac function.

$$\psi(x) = \delta(x) \quad (3.5)$$

Substitution of equations (3.1- 3.5) into (2.12) yields

$$\begin{aligned} U_I(X) &= \sum_{m=0}^{N-1} U_{AV}[m] \int_{-\infty}^{\infty} \exp\left(i\pi \frac{(X-x)^2}{\lambda z_r}\right) \exp\left(-i\frac{\pi x^2}{\lambda z_o}\right) \int_{-\infty}^{\infty} \delta(u/\Delta x - m) \delta(x-u) du dx \\ &= \exp\left(i\pi \frac{X^2}{\lambda z_r}\right) \sum_{m=0}^{N-1} U_{AV}[m] \int_{-\infty}^{\infty} \delta(x/\Delta x - m) \exp\left(-i\frac{2\pi Xx}{\lambda z_r}\right) \exp\left(i\frac{\pi x^2(z_o - z_r)}{\lambda z_r z_o}\right) dx \\ &= \Delta x \exp\left(i\pi \frac{X^2}{\lambda z_r}\right) \sum_{m=0}^{N-1} U_{AV}[m] \exp\left(-i2\pi m \frac{X\Delta x}{\lambda z_r}\right) \exp\left(i\frac{\pi(m\Delta x)^2(z_o - z_r)}{\lambda z_r z_o}\right). \end{aligned} \quad (3.6)$$

If the reconstruction distance differs from the recording distance, the chirp term in the last expression of (3.6) will change the phase of diffracted object field. The formula can be used to evaluate the influence of defocusing on the reconstruction. When $z_r = z_o$, Eq. (3.6) reduces to

$$U_I(X) = \Delta x \exp\left(i\pi \frac{X^2}{\lambda z_r}\right) \sum_{m=0}^{N-1} U_{AV}[m] \exp\left(-i2\pi m \frac{X\Delta x}{\lambda z_r}\right). \quad (3.7)$$

Assume that the pixel spacing in the reconstruction is given by

$$\Delta X = \frac{\lambda z_r}{N\Delta x}, \quad (3.8)$$

we arrive at the conventional Fourier transform formula that is used in the literature for the reconstruction of Fourier Hologram [4].

$$U_I[n] = U_I(n\Delta X) = \exp\left(i\pi m^2 \frac{\lambda z_r}{(N\Delta x)^2}\right) \sum_{m=0}^{N-1} U_{AV}[m] \exp\left(-i\frac{2\pi mn}{N}\right), \quad n = 0, 1, \dots, N-1. \quad (3.9)$$

Generally, if values at a different grid are of interest, one can implement the summation in (3.7) through convolution. Denote the coordinate of reconstruction samples as $n\Delta X$, where ΔX is the desired reconstruction pixel spacing, and introduce another variable $\alpha = \Delta X \Delta x / (\lambda z_r)$ for conciseness, then Eq. (3.7) can be rewritten as

$$\begin{aligned} U_I[n] &= \exp\left(i\pi \frac{(n\Delta X)^2}{\lambda z_r}\right) \sum_{m=0}^{N-1} U_{AV}[m] \exp(-i2\pi m n \alpha) \\ &= \exp\left(i\pi \frac{(n\Delta X)^2}{\lambda z_r}\right) \sum_{m=0}^{N-1} U_{AV}[m] \exp\{i\pi [(m-n)^2 - m^2 - n^2]\} \\ &= \exp\left(i\pi \frac{(n\Delta X)^2}{\lambda z_r}\right) \exp(-i\pi n^2) \sum_{m=0}^{N-1} U_{AV}[m] \exp(-i\pi m^2) \exp[i\pi (n-m)^2] \end{aligned} \quad (3.10)$$

where a nonsignificant constant has been dropped. The last expression in (3.10) is a linear convolution of a chirp sequence with the product of the measured samples and another chirp sequence. The linear convolution can be calculated via FFT [5].

B. $\psi(x)$ takes the form of Sinc(x)

Under the previous assumptions, an explicit expression for $U_{AV}(x)$ is given by

$$\begin{aligned} U_{AV}(x) &= U(x)U_R^*(x) = \exp\left(i\pi \frac{x^2}{\lambda z_o}\right) \int_{-\infty}^{\infty} U_o(x_o) \exp\left(-i\pi \frac{(x-x_o)^2}{\lambda z_o}\right) dx_o \\ &= \int_{-\infty}^{\infty} U_o(x_o) \exp\left(-i\pi \frac{x_o^2}{\lambda z_o}\right) \exp\left(i2\pi \frac{xx_o}{\lambda z_o}\right) dx_o. \end{aligned} \quad (3.11)$$

hence $U_{AV}(x)$ and the product $U_o(x)\exp[-i\pi x^2/(\lambda z_o)]$ constitute a Fourier transform pair. In practice, the object has a finite extent. Therefore, wavefront $U_{AV}(x)$ is a band-limited signal. According to Shannon sampling theory, if we choose

$$\psi(x) = \text{sinc}(x) = \sin(\pi x) / \pi x, \quad (3.12)$$

the continuous wavefront $U_{AV}(x)$ can be reconstructed from its samples without aliasing distortion. Now, (2.12) becomes

$$\begin{aligned} U_I(X) &= \sum_{m=0}^{N-1} U_{AV}[m] \int_{-\infty}^{\infty} \exp\left(i\pi \frac{(X-x)^2}{\lambda z_r}\right) \exp\left(-i\pi \frac{x^2}{\lambda z_o}\right) \int_{-\infty}^{\infty} \text{sinc}(u/\Delta x - m) \delta(x-u) du dx \\ &= \exp\left(i\pi \frac{X^2}{\lambda z_r}\right) \sum_{m=0}^{N-1} U_{AV}[m] \int_{-\infty}^{\infty} \text{sinc}(x/\Delta x - m) \exp\left(i\pi \frac{x^2(z_o - z_r)}{\lambda z_r z_o}\right) \exp\left(-i\pi \frac{2\pi Xx}{\lambda z_r}\right) dx \end{aligned} \quad (3.13)$$

and when the reconstruction is in focus, we have

$$\begin{aligned} U_I(X) &= \exp\left(i\pi \frac{X^2}{\lambda z_r}\right) \sum_{m=0}^{N-1} U_{AV}[m] \int_{-\infty}^{\infty} \text{sinc}(x/\Delta x - m) \exp\left(-i\pi \frac{2\pi Xx}{\lambda z_r}\right) dx \\ &= \exp\left(i\pi \frac{X^2}{\lambda z_r}\right) \sum_{m=0}^{N-1} U_{AV}[m] \exp\left(-i2\pi \frac{mX\Delta x}{\lambda z_r}\right) \int_{-\infty}^{\infty} \text{sinc}(u/\Delta x) \exp\left(-i\pi \frac{2\pi Xu}{\lambda z_r}\right) du \\ &= \Delta x \exp\left(i\pi \frac{X^2}{\lambda z_r}\right) \text{rect}\left(\frac{X\Delta x}{\lambda z_r}\right) \sum_{m=0}^{N-1} U_{AV}[m] \exp\left(-i2\pi \frac{mX\Delta x}{\lambda z_r}\right) \end{aligned} \quad (3.14)$$

where $\text{rect}(\cdot)$ stands for a rectangle function defined as

$$\text{rect}(x) = \begin{cases} 1 & -\frac{1}{2} \leq x \leq \frac{1}{2} \\ 0 & \text{otherwise} \end{cases} \quad (3.15)$$

Equation (3.14) explicitly shows that the reconstructed image is confined within a closed range. When the pixel spacing of reconstruction is set as (3.8), the N reconstructed samples just fill this range. For other selections of the reconstruction pixel spacing, only samples within the range are valid. As far as we know, this point has not been clearly pointed out in literature.

3.2 Rectangular pixel aperture of CCD

In the previous discussion, we assume that the CCD has a negligible fill factor of its pixel aperture. In practice, however, most CCD has a large pixel size in order to achieve high sensitivity. For example, the CCD camera used in our experiment (Hamamatsu ORCA-ER) has a fill factor nearly equal to 100%. Here we assume a rectangular CCD pixel

aperture which agrees with typical specification of actual devices.

$$\varphi(x) = \text{rect}\left(\frac{x}{\tau\Delta x}\right) \quad (3.16)$$

where τ , $0 \leq \tau \leq 1$, is a fill factor. Then

$$U_{AV}(x) = \{U(x)U_R^*(x)\} \otimes \varphi(x) = \varphi(x) \otimes \int_{-\infty}^{\infty} U_o(x_o) \exp\left(-i\pi \frac{x_o^2}{\lambda z_o}\right) \exp\left(i2\pi \frac{xx_o}{\lambda z_o}\right) dx_o \quad (3.17)$$

As we have mentioned above, the integration term in (3.17) has band-limited spectrum. According to the convolution theorem, the wavefront $U_{AV}(x)$ has also a band-limited spectrum. Therefore, the optimal reconstruction basis function should be the sinc function. Thus, (2.12) becomes

$$\begin{aligned} U_I(X) &= \sum_{m=0}^{N-1} U_{AV}[m] \int_{-\infty}^{\infty} \exp\left(i\pi \frac{(X-x)^2}{\lambda z_r}\right) \exp\left(-i\pi \frac{x^2}{\lambda z_o}\right) \int_{-\infty}^{\infty} \text{sinc}\left(\frac{x-u}{\Delta x} - m\right) \varphi^r(u) du dx \\ &= \exp\left(i\pi \frac{X^2}{\lambda z_r}\right) \sum_{m=0}^{N-1} U_{AV}[m] \int_{-\infty}^{\infty} \varphi^r(u) \int_{-\infty}^{\infty} \exp\left(i\pi \frac{x^2(z_o - z_r)}{\lambda z_r z_o}\right) \exp\left(-i\pi \frac{2\pi Xx}{\lambda z_r}\right) \text{sinc}\left(\frac{x-u-m\Delta x}{\Delta x}\right) dx du \end{aligned} \quad (3.18)$$

At the focused plane, Eq. (3.18) reduces to

$$\begin{aligned} U_I(X) &= \exp\left(i\pi \frac{X^2}{\lambda z_r}\right) \sum_{m=0}^{N-1} U_{AV}[m] \int_{-\infty}^{\infty} \varphi^r(u) \int_{-\infty}^{\infty} \exp\left(-i\pi \frac{2\pi Xx}{\lambda z_r}\right) \text{sinc}\left(\frac{x-u-m\Delta x}{\Delta x}\right) dx du \\ &= \exp\left(i\pi \frac{X^2}{\lambda z_r}\right) \text{Rect}\left(\frac{X\Delta x}{\lambda z_r}\right) \Phi(X) \sum_{m=0}^{N-1} U_{AV}[m] \exp\left(-i\pi \frac{2\pi X m \Delta x}{\lambda z_r}\right) \end{aligned} \quad (3.19)$$

with

$$\Phi(X) = \int_{-\infty}^{\infty} \varphi^r(u) \exp\left(-i\pi \frac{2\pi Xu}{\lambda z_r}\right) du \quad (3.20)$$

Taking consideration of (2.11) and (3.16), (3.20) is expressed explicitly as

$$\Phi(X) = 1 / \int_{-\infty}^{\infty} \varphi(u) \exp\left(-i\pi \frac{2\pi Xu}{\lambda z_r}\right) du = \left[\tau \Delta x \text{sinc}\left(\frac{\tau \Delta x}{\lambda z_r} X\right) \right]^{-1} \quad (3.21)$$

Thus, the effect due to non-zero fill factor of CCD can be compensated by dividing a sinc function after reconstruction. The numerical calculation of (3.19) can follow the same procedure as used in the derivation obtaining (3.9) and (3.10). In comparison with conventional algorithm, Eq. (3.19) is a more accurate reconstruction formula for the Fourier digital hologram. For the typical values $\tau = 1$, $\Delta x = 6.45 \mu\text{m}$, $\lambda = 632.8 \text{ nm}$, and $z_r = 150 \text{ mm}$, function $\Phi(X)$ is shown in Fig. 1.

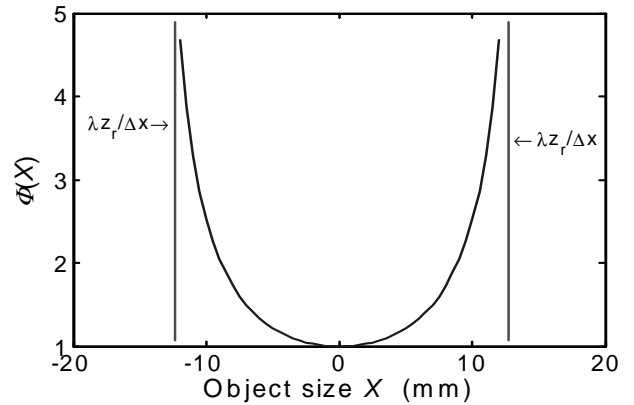


Fig.1. Plot of correction function $\Phi(X)$.

4. FRESNEL HOLOGRAM

In Section 3, we addressed the reconstruction of Fourier holograms. The derived formula (3.19) gives a more accurate reconstruction. In practice, however, more frequently, a plane wave normally incident onto the CCD is used as reference beam, which results in the so-called Fresnel hologram. In this section, we discuss the reconstruction of Fresnel hologram. The formulae for $h(x, x_o)$ and $g(X, x)$ are still the same as (3.1) and (3.3). Assume that the CCD performs ideal point sampling. Thus, the aperture of CCD may be represented as a Dirac delta function. According to (2.11), the deconvolution kernel also has the same form.

$$\varphi(x) = \varphi^r(x) = \delta(x) \quad (4.1)$$

4.1 $\psi(x)$ takes the form of $\delta(x)$

Assume that the reconstruction basis function is a Dirac delta function

$$\psi(x) = \delta(x) \quad (4.2)$$

Carrying out all substitutions and derivations as before, (2.12) reduces to

$$U_I(X) = \sum_{m=0}^{N-1} U_{AV}[m] \exp\left(i\pi \frac{(X - m\Delta x)^2}{\lambda z_r}\right) \quad (4.3)$$

Equation (4.3) is similar to the conventional discrete form of Fresnel integral except that the variable X can take continuous values. If the pixel spacing ΔX equals to the CCD pitch Δx , (4.3) becomes

$$U_I[n] = \sum_{m=0}^{N-1} U_{AV}[m] \exp\left(i\pi \frac{\Delta x^2}{\lambda z_r} (n - m)^2\right) = \sum_{m=0}^{N-1} U_{AV}[m] \exp\{i\pi \beta^2 (n - m)^2\}, \quad (4.4)$$

where $\beta = \Delta x / (\lambda z_r)^{1/2}$. Equation (4.4) is a linear convolution with respect to m and n , and can be calculated through circular convolution that is then implemented via FFT. Assume that the number of reconstructed samples is M , then the computational complexity is $2(M+N-1)\log(M+N-1)$. The artifacts arising from the truncation of hologram can be handled more flexibly. For example, mirror extension of holograms usually gives better results. Since M can be larger than N , we can reconstruct object even larger than CCDs. If the pixel spacing is equal to

$$\Delta X = r\Delta x, \quad (r \in \mathbb{Z}^+), \quad (4.5)$$

(4.3) becomes

$$U_I[n] = \sum_{m=0}^{N-1} U_{AV}[m] \exp\left(i\pi \frac{\Delta x^2}{\lambda z_r} (rn - m)^2\right) = \sum_{m=0}^{rN-1} U^s_{AV}[m] \exp\left(i\pi \frac{\Delta x^2}{\lambda z_r} (rn - m)^2\right), \quad (4.6)$$

where discrete sequence $U^s_{AV}[m]$ is defined as

$$U^s_{AV}[m] = \begin{cases} U_{AV}[n] & m = rn \\ 0 & \text{otherwise} \end{cases}, \quad (4.7)$$

By calculating the linear convolution of length rN by means of FFT, and then picking up elements per r samples from the results, we can obtain the complex amplitude of object at a lattice with spacing $r\Delta x$. When the pixel spacing has value

$$\Delta X = \frac{\lambda z_r}{M\Delta x}, \quad M \geq N, \quad (4.8)$$

Eq. (4.3) becomes

$$\begin{aligned} U_I[n] &= \sum_{m=0}^{N-1} U_{AV}[m] \exp\left(i\pi \frac{[n\lambda z_r / (M\Delta x) - m\Delta x]^2}{\lambda z_r}\right) \\ &= \exp\left(i\pi \frac{\lambda z_r}{(M\Delta x)^2} n^2\right) \sum_{m=0}^{N-1} U_{AV}[m] \exp\left(i\pi \frac{\Delta x^2}{\lambda z_r} m^2\right) \exp\left(-2i\pi \frac{mn}{M}\right). \end{aligned} \quad (4.9)$$

When $M=N$, equation (4.9) is exactly the Single Fourier transform algorithm. If $M > N$, in order to utilize the FFT algorithm, we need to pad $U_{AV}[m]$ to M elements with zeros. This operation will lead to an interpolated reconstruction.

4.2 $\psi(x)$ takes the form of sinc function.

For the same reason as pointed out in Section 3, the assumption $\psi(x)=\delta(x)$ is too ideal and impractical. Referring to the Fresnel integral, the Fourier spectrum of the diffracted field is the convolution between the object complex amplitude $U_o(x_o)$ and a chirp function (see Eq. (3.11)). In practice, the object has a finite extent. The convolution also has a concentrated power spectrum when the object is located at far Fresnel zone. Therefore, it is reasonable to assume that $U(x)$ is a band-limited signal although it may not be exactly accurate. Therefore, we have

$$\psi(x) = \text{sinc}(x) \quad (4.10)$$

and then Eq. (2.12) gives

$$U_I(X) = \sum_{m=0}^{N-1} U_{AV}[m] \int_{-\infty}^{\infty} \text{sinc}\left(\frac{u}{\Delta x} - m\right) \exp\left(i\pi \frac{(X-u)^2}{\lambda z_r}\right) du = \sum_{m=0}^{N-1} U_{AV}[m] DR(X - m\Delta x) \quad (4.11)$$

with a kernel $DR(x)$ that is represented by

$$\begin{aligned} DR(x) &= \int_{-\infty}^{\infty} \text{sinc}(u/\Delta x) \exp\left(i\pi \frac{(x-u)^2}{\lambda z_r}\right) du \\ &= C_1 \int_{-\infty}^{\infty} \text{rect}(f\Delta x) \exp(-i\pi \lambda z_r f^2) \exp(2i\pi x f) df \\ &= C_1 \frac{1}{\sqrt{2\lambda z_r}} \exp\left\{i\pi \frac{x^2}{\lambda z_r}\right\} \int_{(-\frac{1}{\Delta x} - \frac{x}{\lambda z_r})\sqrt{2\lambda z_r}}^{(\frac{1}{\Delta x} - \frac{x}{\lambda z_r})\sqrt{2\lambda z_r}} \exp\left(-i\frac{\pi}{2} u^2\right) du \\ &= C_1 \frac{1}{\sqrt{2\lambda z_r}} \exp\left\{i\pi \frac{x^2}{\lambda z_r}\right\} \left\{ C\left[\left(\frac{1}{\Delta x} - \frac{x}{\lambda z_r}\right)\sqrt{2\lambda z_r}\right] - C\left[\left(-\frac{1}{\Delta x} - \frac{x}{\lambda z_r}\right)\sqrt{2\lambda z_r}\right] \right\} \\ &\quad - iC_1 \frac{1}{\sqrt{2\lambda z_r}} \exp\left\{i\pi \frac{x^2}{\lambda z_r}\right\} \left\{ S\left[\left(\frac{1}{\Delta x} - \frac{x}{\lambda z_r}\right)\sqrt{2\lambda z_r}\right] - S\left[\left(-\frac{1}{\Delta x} - \frac{x}{\lambda z_r}\right)\sqrt{2\lambda z_r}\right] \right\} \end{aligned} \quad (4.12)$$

where, C_1 is a constant factor. $S(x)$ and $C(x)$ are Fresnel integral functions defined as

$$S(x) = \int_0^x \sin\left(\frac{\pi}{2} t^2\right) dt \quad (4.13)$$

and

$$C(x) = \int_0^x \cos\left(\frac{\pi}{2} t^2\right) dt. \quad (4.14)$$

If we choose the pixel spacing in reconstruction the same as the pixel pitch of CCD, then

$$U_I(n) = \sum_{m=0}^{N-1} U_{AV}[m] DR[n-m], \quad (4.15)$$

with

$$\begin{aligned} DR[n] &= \exp\{i\pi\beta^2 n^2\} \{C(\sqrt{2}/\beta - \sqrt{2}\beta n) - C(-\sqrt{2}/\beta - \sqrt{2}\beta n)\} \\ &\quad - i \exp\{i\pi\beta^2 n^2\} \{S(\sqrt{2}/\beta - \sqrt{2}\beta n) - S(-\sqrt{2}/\beta - \sqrt{2}\beta n)\}, \end{aligned} \quad (4.16)$$

where, some nonsignificant constant has been dropped. Equation (4.15) is also linear discrete convolution. Figure 2 gives the plots of $DR[n]$. The values of β are in accordance with the experimental conditions: the pixel pitch of CCD 6.45 μm , the wavelength 0.6328 μm and the reconstruction distances 15 mm.

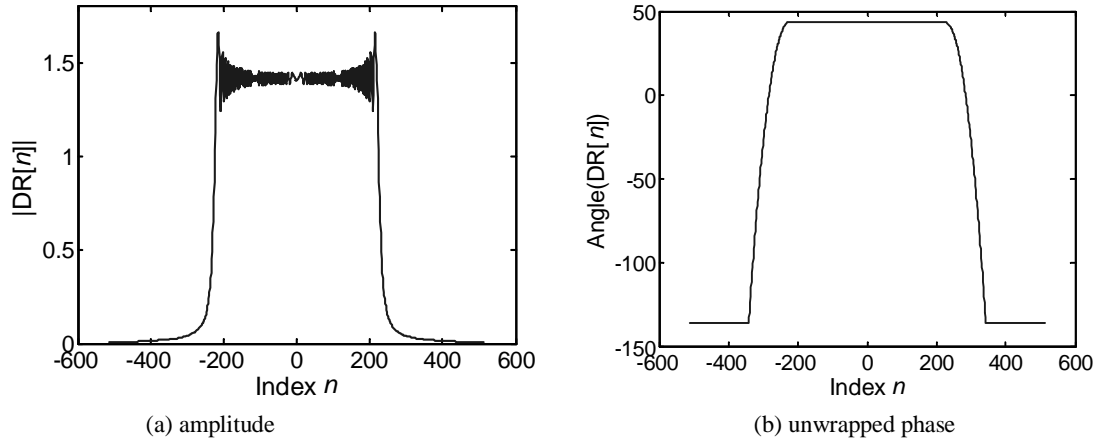


Fig. 2 (a) amplitude and (b) phase of $DR[n]$ with $\beta^2=6.45^2/(0.6328*15e3)$.

From Fig. 2, one can see that $DR[n]$ has a dominant value for a closed range of n . The range can be estimated as $[-1/\beta^2, 1/\beta^2]=[-\lambda z/\Delta x^2, \lambda z/\Delta x^2]$. The differences between (4.15) and (4.4) are: (a) impact of boundary effects. In (4.4) the length of kernel sequence is always the same as the number of CCD pixels, however, it depends on the recordings parameters in (4.15); (b) the phase of kernel $DR[n]$ may lead to a more accurate reconstructed phase map. Figure 3 shows the reconstruction results of holograms recorded at a distance 59 mm. Other experiment conditions are the same as before. For comparison, we also show the reconstruction by the conventional convolution algorithm in Fig 4.

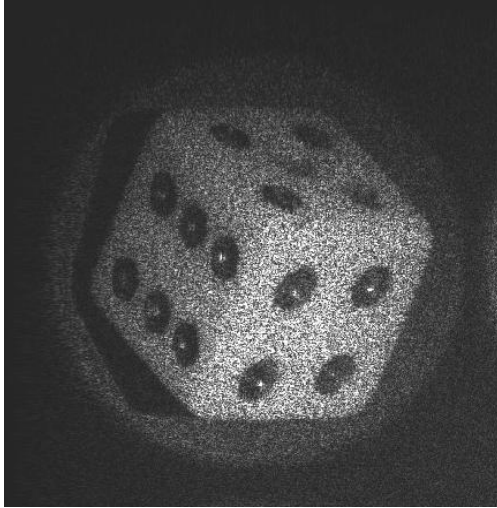


Fig.3. Reconstruction based on the general model with a sinc basis function and Fresnel diffraction formula.

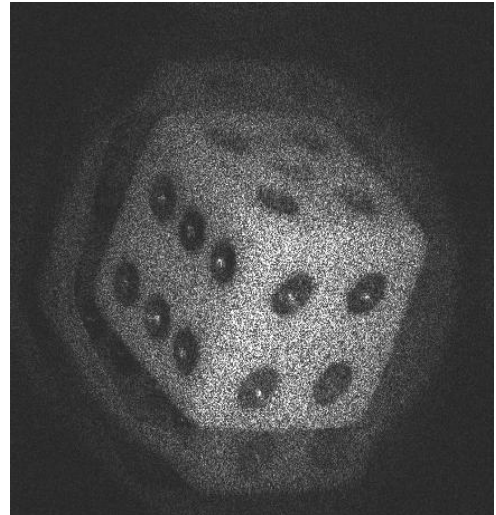


Fig.4. Reconstruction with convolution algorithm.

It is obvious that Fig. 3 have a better image quality than Fig. 4 in which aliasing effect is observable.

5. CONCLUSION

In this paper, we have proposed a general model for the design of reconstruction algorithm. This model finds its foundation in the broad sense sampling theory. By adopting a basis function that provides the best match with the diffraction performance of the object and a proper formula for describing the diffraction process, optimal reconstruction algorithm can be developed for a certain system. Furthermore, compensation methods for the averaging effect due to non-zero fill factor of an image acquisition device can be integrated into this model as well. Several cases of the basis

function are given. When a Dirac delta function is taken as the basis function, the model leads to an algorithm similar to the conventional algorithms, but now it is applicable to objects larger than the image acquisition device and can obtain reconstruction at arbitrary lattice. The reconstruction from real hologram data shows that results obtained based on this model are better than previous methods. Finally, we believe that the general unifying view of reconstruction is beneficial because it offers a common framework for understanding different imaging techniques: direct imaging and transformed imaging. [6]

REFERENCES

1. M. A. Kronrod, N. S. Merzlyakov, and L. P. Yaroslavskii, "Reconstruction of a hologram with a computer," *Sov. Phys. Tech. Phys.* **17**, 333–334 (1972).
2. T. H. Demetrakopoulos and R. Mittra, "Digital and Optical Reconstruction of images from suboptical Diffraction patterns," *Appl. Opt.* **13**, 665-670 (1974).
3. I. Yamaguchi and Tong Zhang, "Phase-shifting digital holography," *Opt. Lett.* **22**, 1268-1270 (1997).
4. C. Wagner, S. Seebacher, and W. Jüptner, "Digital recording and numerical reconstruction of lensless Fourier holograms in optical metrology," *Appl. Opt.* **38**, 4812-4820 (1999).
5. Alan V. Oppenheim, Ronald W. Schaffer, *Discrete-Time Signal Processing* (Prentice-Hall, New Jersey, 1989).
6. L. Yaroslavsky, *Digital Holography and Digital Image Processing: Principles, Methods, Algorithm* (Kluwer, Boston, 2003).