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# Systematic network design for liner shipping services

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## Publication Details

Wang, S., Liu, Z. & Meng, Q. (2013). Systematic network design for liner shipping services. *Transportation Research Record*, (2330), 16-23.

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# Systematic network design for liner shipping services

## **Abstract**

A systematic design for a liner shipping network was addressed. Many practical features in real-world operations, including multiple types of containers, container transshipment operations, empty container repositioning, origin-to-destination transit time constraints, consistent services with the current network, and joint services with other liner shipping companies, were considered. Given a set of candidate ship routes, some of these routes had to be used, and the use of others was optional. A mixed-integer linear programming model was proposed for the selection of optional ship routes. The solution of this model provided the laden and empty container flow on the selected ship routes. On the basis of the results of this model, techniques were proposed for refining ship routes by changing existing routes, designing new routes, and removing some routes. A large-scale numerical test based on the global shipping network of a liner shipping company, consisting of 166 ports, was performed.

## **Keywords**

era2015

## **Disciplines**

Engineering | Science and Technology Studies

## **Publication Details**

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# SYSTEMATIC NETWORK DESIGN FOR LINER SHIPPING SERVICES

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To the committees of Ports and Channels (AW010) for review

Paper submitted to the 92<sup>st</sup> Annual Meeting of Transportation Research Board to be considered for presentation and publication in Transportation Research Record

Submission Date: July 30, 2012

Number of words in text:			= 6078 words
Number of tables:	1	(1×250)	= 250 words equivalent
Number of figures:	3	(3×250)	= 750 words equivalent
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Total number of words:			= 7078 words equivalent

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**ABSTRACT**

This paper addresses a systematic design of liner shipping network. Many practical features in real-world operations are considered, which include multi-type containers, container transshipment operations, empty container repositioning, origin-to-destination transit time constraint, consistent services with the current network, and joint services with other liner shipping companies. Given a set of candidate ship routes, some of these routes must be used while the others are optional for use. Hence, a mixed-integer linear programming model is first proposed for the selection of the optional ship routes. Solving this model also gives the laden and empty container flow on the selected ship routes. Based on the results of this model, some techniques are proposed to refine the ship routes, by changing existing ship routes, designing new ship routes, and removing some ship routes. Finally, a large scale numerical test is performed, based on the global shipping network of a liner shipping company, consisting of 166 ports.

**KEY WORDS**

Liner Shipping Services; Network Design; Container Transshipment Operation; Empty Container Repositioning

## INTRODUCTION

Alike to the bus service on road network, liner shipping companies provide regular shipping services with fixed port rotations and ship deployment, to attract and transport containerized cargo. The aim of liner shipping network design is to determine which ports to visit and in what order they should be visited by each ship route, as well as the type and number of ships deployed on this ship route. A well designed liner shipping network would largely reduce the operating cost for a shipping company and meanwhile fulfill most of its container demand.

The liner shipping network design problem is NP-hard (1). Research on liner shipping network design can be classified into four categories. The first category examines the feeder shipping network design problem, which only consists of a hub port and its feeder ports. Transshipment is excluded within the feeder network. Fagerholt (2) proposed a set-partitioning model by enumerating all possible shipping service routes and combining these single shipping service routes into multiple shipping service routes. This study is extended by Fagerholt (3) via assuming heterogeneous ship fleets. Sambracos et al. (4) proposed a list-based threshold acceptance meta-heuristic method to solve the feeder ship route design problem. This work was later generalized by Karlaftis et al. (5) to account for container pickup and delivery operations as well as time deadlines.

The second category aims to design one or a few liner service routes without container transshipment operations. Rana and Vickson (6) built a mixed-integer linear programming model for a single ship route design problem. Rana and Vickson (7) later extended this model to design multiple ship routes. Shintani et al. (8) relaxed the assumption of port calling precedence relations and incorporated empty container repositioning to design a single ship route, and a genetic algorithm is used to solve the problem.

The third group of studies designs a hub-and-spoke (H&S) liner shipping network. Gelareh et al. (9) examined a H&S network design problem in a competitive environment with a newcomer liner shipping company and an existing operating dominator. Based on this work, Gelareh and Pisinger (10) presented a mixed-integer linear programming formulation for the simultaneous design of a H&S network and deployment of containerships. A Benders decomposition based algorithm is developed.

The fourth line of research investigates the general liner shipping network design problem, which usually involves more ports in the network and allows for container transshipment operations. Agarwal and Ergun (1) proposed a multi-commodity-based space-time network model for the liner shipping service network design problem with cargo routing. This model covers a heterogeneous fleet, a weekly service frequency, multiple ship routes, and cargo transshipment operations, but transshipment cost is not considered in the network design stage. Results on 20 ports and 100 ships are reported. Alvarez (11) formulated the transshipment cost in network design and also applied a column generation-based heuristic to design the service network with 120 ports and 5 types of ships. Meng and Wang (12) designed the liner network by choosing the optimal ship routes to operate in a given candidate sets. Meng et al. (13) further extended the study by incorporating the inland origin

and destinations. Reinhardt and Pisinger (14) presented a model that allows a port to be visited twice in a route, stating that such butterfly routes are common in real-world situations. Their model also incorporates transshipment costs and route dependent capacities. An exact branch-and-cut algorithm is developed to solve instances with up to 15 ports. Wang and Meng (15) examined a special type of network design by optimally reversing the port rotation directions of ship routes in a network.

The above studies have the following deficiencies. (i) Some studies do not allow container transshipment operations; (ii) Most of these studies do not consider empty container repositioning; (iii) Many of the problems addressed are of a much smaller scale than those encountered in practice. In our study, we not only consider container transshipment and empty container repositioning in addressing large-scale problems, but also incorporate the following characteristics in liner shipping that are not incorporated in most of the previous studies. (a) We consider multiple type containers; (b) Both the volume capacity and weight capacity of containerships are modeled; (c) The origin-to-destination transit time of containers, which is an important differentiating factor of liner services, is captured; (d) Liner shipping services that are jointly operated with other companies, which are common in practice, are incorporated. (e) The error in demand prediction is considered. (f) Human expertise could be combined with the algorithm.

## PROBLEM DESCRIPTION

The liner shipping network design (LSND) problem aims to construct the ship routes (port rotations, type and number of ships to deploy) that constitute the network, which fulfills the container shipment demand at minimum cost. Let  $\mathcal{P}$  represent the set of ports in the shipping network. A *ship route*  $r$  is a set of ports that are visited in a predetermined sequence expressed as  $p_{r1} \rightarrow p_{r2} \rightarrow \dots \rightarrow p_{rN_r} \rightarrow p_{r1}$  where  $N_r$  is the number of ports of call on the ship route and  $p_{ri} \in \mathcal{P}$  is the port corresponding to the  $i$ th call. We further let  $I_r = \{1, 2, \dots, N_r\}$  be the set of all the ports of call sequences for the ship route  $r$ . A *container route* is a path from the origin port to the destination port in the network that records the itinerary of a container from the origin to destination. For the ease of presentation, the following notation is introduced in the first place:

$\mathcal{H}$	Set of container routes
$\mathcal{H}^{od}$	Set of container routes for the origin-to-destination (O-D) port pair $(o, d) \in \mathcal{W}$
$\mathcal{K}$	Set of container types
$\mathcal{V}$	Set of ship types
$\mathcal{R}_p$	Set of ship routes that call at port $p \in \mathcal{P}$
$\mathcal{R}_v$	Set of ship routes that use ships of type $v \in \mathcal{V}$
$\mathcal{V}_r$	Set of ship types available for the ship route $r$

$\mathcal{V}_p$	Set of ship types that port $p \in \mathcal{P}$ has enough draft to accommodate
$\mathcal{W}$	Set of O-D port pairs
$\text{Cap}_v$	Container capacity (twenty-foot equivalent units, or TEUs) of a ship with type $v \in \mathcal{V}$
$\text{Wei}_v$	Weight capacity (tons) of a ship with type $v \in \mathcal{V}$
$c_h$	Overall container handling and berth occupancy cost (USD/TEU) associated with delivering one TEU in the container route $h \in \mathcal{H}$
$c_r$	Fixed operating cost (USD/week) for a particular ship route $r \in \mathcal{R}$
$c_v^{\text{in}}$	Price (USD/week) for chartering in one ship in type $v \in \mathcal{V}$
$c_{od}$	Penalty (USD) for each unshipped TEU for O-D pair $(o, d) \in \mathcal{W}$
$\hat{c}_{pk}^{\text{EMP}}$	Load cost (USD) for each container in type $k \in \mathcal{K}$ at port $p \in \mathcal{P}$
$\tilde{c}_{pk}^{\text{EMP}}$	Discharge cost (USD) for each container in type $k \in \mathcal{K}$ at port $p \in \mathcal{P}$
$\bar{c}_{pk}^{\text{EMP}}$	Relay cost (USD) for each container in type $k \in \mathcal{K}$ at port $p \in \mathcal{P}$
$c_{pk}^{\text{EMP}}$	Penalty (USD) for each unshipped empty container in type $k \in \mathcal{K}$ at port $p \in \mathcal{P}$
$E_k$	The volume (TEUs) of a container in type $k \in \mathcal{K}$
$n_{od}$	Weekly container shipment demand (TEUs/week) for port pair $(o, d) \in \mathcal{W}$
$n_{od}^k$	Weekly number of laden containers in type $k$ to be transported for the O-D pair $(o, d) \in \mathcal{W}$
$N_v^{\text{OWN}}$	Number of ships in type $v \in \mathcal{V}$ owned by the liner shipping company
$W_{od}$	Average weight (tons) per TEU for the O-D pair $(o, d) \in \mathcal{W}$
$\rho_{hri}$	A binary indicator which equals 1 if and only if container route $h \in \mathcal{H}$ contains leg $i$ of ship route $r \in \mathcal{R}$
$\gamma_{pk}$	Number of surplus empty containers in type $k \in \mathcal{K}$ at port $p \in \mathcal{P}$
$\Lambda_r$	Percentage of ship slots on ship route $r \in \mathcal{R}$ controlled by the liner shipping company

### Container Demand

There are many types of containers to transport for each O-D pair, such as dry 20-ft, dry 40-ft, reefer 20-ft, and reefer 40-ft. Different types of containers are different in volume and port handling cost. Let  $\mathcal{K}$  represent the set of container types. Denote by  $E_k$  the twenty-foot equivalent volume (TEUs) of a container in type  $k \in \mathcal{K}$ . For example, a dry 40-ft is 2 TEUs.

Let  $W_{od}^k$  be the average weight (tons) of a laden container in type  $k$  to be transported for the O-D pair  $(o, d) \in \mathcal{W}$ .

In reality, different types of containers are shipped together. Therefore, we assume that whenever containers of the same O-D are shipped, different types of containers must be shipped proportional to the container shipment demand  $n_{od}^k$ . This assumption significantly simplifies the modeling difficulties because we can treat the containers of the same O-D as a single type of containers, which is elaborated below.

We can use TEU to compute the container shipment demand of port pair  $(o, d) \in \mathcal{W}$ . Thus, in terms of TEU, the container shipment demand  $n_{od}$  can be calculated by

$$n_{od} = \sum_{k \in \mathcal{K}} E_k n_{od}^k, \forall (o, d) \in \mathcal{W} \quad (1)$$

The average weight (tons) per TEU,  $W_{od}$ , is computed by

$$W_{od} = \sum_{k \in \mathcal{K}} W_{od}^k n_{od}^k / n_{od}, \forall (o, d) \in \mathcal{W} \quad (2)$$

Other parameters can be computed in a similar manner. For example, let  $\mathcal{V}$  be the set of ship types, suppose that the productivity of port  $p \in \mathcal{P}$  is  $M_{pv}$  moves/hour for ships of type  $v \in \mathcal{V}$ . Then the average container handling time (hour) per TEU of containers in the port pair  $(o, d) \in \mathcal{W}$  at port  $p \in \mathcal{P}$  for ships of type  $v \in \mathcal{V}$  denoted by  $t_{pv}^{od}$ , can be calculated by

$$t_{pv}^{od} = \sum_{k \in \mathcal{K}} n_{od}^k / (M_{pv} n_{od}), \forall (o, d) \in \mathcal{W}, \forall p \in \mathcal{P}, \forall v \in \mathcal{V}_p, \quad (3)$$

Regarding the demand for empty containers, due to trade imbalance, some locations have surplus and other locations are deficit in empty containers. Define  $\gamma_{pk} := \sum_{o \in \mathcal{P}} n_{op}^k - \sum_{d \in \mathcal{P}} n_{pd}^k$ . Let  $\mathcal{P}_k^{\text{SUR}}$  be the set of ports with surplus empty containers in type  $k$ , that is,  $\mathcal{P}_k^{\text{SUR}} := \{p \in \mathcal{P} \mid \gamma_{pk} > 0\}$ . Similarly, let  $\mathcal{P}_k^{\text{BAL}}$  and  $\mathcal{P}_k^{\text{DEF}}$  be the set of ports with balanced and deficit empty containers in type  $k$ , respectively. We further represent by  $W^k$  the weight (tons) of an empty container in type  $k$ .

### Three Types of Ship Routes

The liner shipping company does not design a shipping network from scratch. In fact, it usually designs the network based on its current shipping network. Therefore, we use the current network as an input of the network design. We assume that the company has an initial network which consists of three types of ship routes: ship routes that must be used (type 1) denoted by  $\hat{\mathcal{R}}$ ; ship routes that at least  $N^{\text{SR}}$  of them must be used (type 2) denoted by  $\bar{\mathcal{R}}$ ; and ship routes that are optional (type 3) denoted by  $\tilde{\mathcal{R}}$ .  $\mathcal{R} = \hat{\mathcal{R}} \cup \bar{\mathcal{R}} \cup \tilde{\mathcal{R}}$ .

There are two constituents of ship routes in type 1: ship routes that are jointly operated with other shipping companies and ship routes that are proven to be very profitable. Each ship route of type 1 has a specified type of ship deployed on it. It should be mentioned



that if a ship route is operated with other companies, it is possible that only partial of the capacity is owned by the focal company; e.g. six 9k-TEU ships are deployed, while the focal company controls only two of them. Therefore, the capacity of focal company on this ship route is only 3k-TEU. To capture this feature, we use  $\Lambda_r$  to represent the percentage of ship slots of ship route  $r \in \hat{\mathcal{R}}$  controlled by the focal company. Note that the number of ships deployed as well as the arrival and departure time at each port of call are fixed for ship routes of type 1.

Ship routes of type 2 are important and the liner shipping company does not intend to change all of them. Hence, it requires that at least  $N^{\text{SR}}$  of them must be used,  $N^{\text{SR}} \leq |\bar{\mathcal{R}}|$ . Ship routes of type 3 are less important. Each ship route of type 2 and type 3 also has a specified type of ships to deploy. However, the ship type can be changed. The number of ships deployed on a ship route of type 2 and type 3 is a decision variable. When we say that a ship route of type 2 is used, we mean that the pre-specified ship type is not changed.

### Transit Time Constraints

The liner container shipping company must provide a certain level of service in terms of the maximum allowable transit time for each O-D pair  $(o, d) \in \mathcal{W}$ , denoted by  $\hat{T}_{od}$  (hours), for shippers. This is one of the most important constraints for container routes.

To capture the transit time of containers, the arrival and departure time at each port of call on each ship route in  $\mathcal{R}$  must be known. As mentioned above, the arrival and departure times of ship routes in type 1 are already given. However, the time components and the number of ships deployed on ship routes in type 2 and type 3 are not known. Before routing containers, we have to set the schedules for these ship routes. This is implemented as follows. We first estimate the time spent at each port of call based on historical data, and the sea time based on the voyage distance and the design speed of ships. After that, we obtain the round-trip time, thereby calculating the number of ships required. If this number is not an integer, we round it up or down by adjusting the speed of ships; for example, if the round-trip time is 50 days, which is 7.14 weeks, then only 7 ships are deployed on this route. After that, we can obtain the sea time on each leg and the port time at each port of call.

It should be mentioned that the transit time of containers includes the connection time (or dwell time) at transshipment ports. This connection time is dependent on the schedules of the two ship routes. However, we do not capture so many details in the model and simply assume that the connection time at a port is a constant value  $\hat{t}_p$  hours. We make this simplified assumption because otherwise we have to consider the available berth time window at each port of call, yet this type of issue is usually not taken into consideration at the planning level.

### LINER SHIPPING NETWORK DESIGN (LSND) MODEL

Let  $\mathfrak{R}$  denote the set of all the possible ship routes, then any ship route plan  $\mathcal{R}$  is a subset of  $\mathfrak{R}$ . The objective of LSND problem is to search for the optimal ship route plan  $\mathcal{R}^* \subset \mathfrak{R}$  that can bring maximal benefit to the shipping company. Due to the large number of possible ship routes, it is impossible to directly take  $\mathfrak{R}$  for optimization. Thus, a heuristic process is proposed as follows to cope with the LSND problem.

First, an initial network  $\mathcal{R} \subset \mathfrak{R}$  is taken, and an optimization model is proposed to evaluate the network  $\mathcal{R}$ . Note that  $\mathcal{R}$  can be classified into three groups  $\hat{\mathcal{R}}$ ,  $\bar{\mathcal{R}}$ , and  $\tilde{\mathcal{R}}$ , and some ship routes in  $\bar{\mathcal{R}} \cup \tilde{\mathcal{R}}$  are optional. Thus, the optimization model can also decide which optional ship route should be taken and which should not, in order to minimize the total cost of transporting both laden and empty containers. For the ease of presentation, this optimization model is termed as network design model (NDM). It should be noted that since the predicted container shipment demand cannot match the true demand exactly, we allow some laden and empty containers not shipped while incurring a penalty cost. In practice, this modeling approach is reasonable. For example, suppose that the predicted demand is 5001 TEUs. If we had imposed that all containers must be shipped, we would have to deploy a 6000-TEU ship. Evidently, deploying a 5000-TEU ship is more profitable in this case.

Second, let  $\mathcal{R}'$  denote the set of selected ship routes from set  $\mathcal{R}$ , some approaches are proposed to design new ship routes. For example, refining the ship routes in  $\mathcal{R}$  by deleting some port-calls or legs with low utilization, which gives a new ship route; or designing new ship routes by connecting the ports with large volume of un-shipped demand. Setting the newly designed ship routes as optional and combing them together to  $\mathcal{R}'$  gives a new set  $\mathcal{R}''$ . Then, evaluating the set  $\mathcal{R}''$  by the NDM model gives us the final ship route plan.

Note that the above two steps can be iteratively conducted to further improve the ship route plan. The network design model (NDM) and its solution method are first presented in this section as follows, and the approach for designing new ship routes are discussed in the following section.

Based on the ship route plan  $\mathcal{R}$ , to fully capture the transshipment property of the container shipping operations, we aim to build a route-based network design model. Thus, the generation of container route set is first addressed. Any container route that can fulfill the transit time constraint is regarded as “feasible”. In view of the size of the global shipping network, there may be an infinite number of feasible paths for each O-D pair. However, practically speaking, the number of paths in use is quite limited because of operational constraints and business considerations. Thus, in this study, only the top  $\bar{n}$  (say,  $\bar{n} = 20$ ) feasible container routes with the minimal shipping cost is included in the container route set. The generation of the top  $\bar{n}$  feasible container routes can be formulated as integer linear programming models and addressed by CPLEX (Wang et al., 16). All the generated container routes for O-D pair  $(o, d) \in \mathcal{W}$  are grouped into set  $\mathcal{H}^{od}$ .

### Decision Variables

The NDM has the following decision variables:

- $x_r$ : Binary decision variable which equals 1 if and only if ship route  $r \in \mathcal{R}$  is used;
- $n_v^{\text{in}}$ : Number of ships in type  $v \in \mathcal{V}$  that are chartered in;
- $y_h$ : Number of laden containers (TEUs) shipped on container route  $h \in \mathcal{H}$ ;
- $y_{od}$ : Number of laden containers (TEUs) unfulfilled for the O-D pair  $(o, d) \in \mathcal{W}$ ;
- $f_{ri}^k$ : Number of empty containers in type  $k$  flowing on leg  $i$  of ship route  $r \in \mathcal{R}$ ;
- $\hat{z}_{ri}^k$ : Number of empty containers in type  $k$  loaded at the port of call  $i$  of ship route  $r \in \mathcal{R}$ ;
- $\tilde{z}_{ri}^k$ : Number of empty containers in type  $k$  discharged at the port of call  $i$  of ship route  $r \in \mathcal{R}$ ;
- $\hat{z}_p^k$ : Number of loading operations for empty containers in type  $k$  at port  $p \in \mathcal{P}$ ;
- $\tilde{z}_p^k$ : Number of discharge operations for empty containers in type  $k$  at port  $p \in \mathcal{P}$ ;
- $\bar{z}_p^k$ : Number of transshipments for empty containers in type  $k$  at port  $p \in \mathcal{P}$ ;
- $z_p^k$ : Number of unshipped empty containers in type  $k$  at port  $p \in \mathcal{P}$ .

### Mixed-integer Linear Programming Model

Let  $\hat{c}_{pk}^{\text{EMP}}$ ,  $\tilde{c}_{pk}^{\text{EMP}}$ , and  $\bar{c}_{pk}^{\text{EMP}}$  be the load, discharge, and relay cost of empty containers in type  $k$  at port  $p \in \mathcal{P}$ , respectively. Let  $c_{od}$  (USD/TEU) be the penalty for each unshipped container (TEU) between O-D pair  $(o, d) \in \mathcal{W}$ . We define  $\Lambda_r = 1$  for ship routes  $r \in \bar{\mathcal{R}} \cup \tilde{\mathcal{R}}$  because ship routes in  $\bar{\mathcal{R}} \cup \tilde{\mathcal{R}}$  are operated solely by the liner shipping company. The type of ship deployed on ship route  $r \in \mathcal{R}$  is  $v(r)$ , and the number of ships is denoted by  $m(r)$ . Let  $\mathcal{R}_v := \{r \in \mathcal{R} \mid v(r) = v\}$  be the set of ship routes deployed with ships of type  $v \in \mathcal{V}$  and  $\mathcal{R}_p \subseteq \mathcal{R}$  be the set of ship routes that call at port  $p \in \mathcal{P}$ . We define binary indicator  $\rho_{hri}$  which equals 1 if and only if container route  $h \in \mathcal{H}$  contains leg  $i$  of ship route  $r \in \mathcal{R}$ . The network design model (NDM) is:

$$\begin{aligned}
 \text{[NDM]} \quad & \min \sum_{r \in \mathcal{R}} c_r x_r + \sum_{h \in \mathcal{H}} c_h y_h + \sum_{k \in \mathcal{K}} \sum_{p \in \mathcal{P}} \left( \hat{c}_{pk}^{\text{EMP}} (\hat{z}_p^k - \bar{z}_p^k) + \tilde{c}_{pk}^{\text{EMP}} (\tilde{z}_p^k - \bar{z}_p^k) + \bar{c}_{pk}^{\text{EMP}} \bar{z}_p^k \right) \\
 & + \sum_{v \in \mathcal{V}} c_v^{\text{in}} n_v^{\text{in}} + \sum_{(o,d) \in \mathcal{W}} c_{od} y_{od} + \sum_{k \in \mathcal{K}} \sum_{p \in \mathcal{P}} c_{pk}^{\text{EMP}} z_p^k
 \end{aligned} \tag{4}$$

$$y_{od} + \sum_{h \in \mathcal{H}^{od}} y_h = n_{od}, \forall (o, d) \in \mathcal{W} \tag{5}$$

$$\hat{z}_p^k = \sum_{r \in \mathcal{R}_p} \sum_{i \in I_r, p_i = p} \hat{z}_{ri}^k, \forall k \in \mathcal{K}, \forall p \in \mathcal{P} \quad (6)$$

$$\tilde{z}_p^k = \sum_{r \in \mathcal{R}_p} \sum_{i \in I_r, p_i = p} \tilde{z}_{ri}^k, \forall k \in \mathcal{K}, \forall p \in \mathcal{P} \quad (7)$$

$$\hat{z}_p^k - \tilde{z}_p^k = 0, \forall k \in \mathcal{K}, \forall p \in \mathcal{P}_k^{\text{BAL}} \quad (8)$$

$$\hat{z}_p^k - \tilde{z}_p^k + z_p^k = \gamma_{pk}, \forall k \in \mathcal{K}, \forall p \in \mathcal{P}_k^{\text{SUR}} \quad (9)$$

$$\hat{z}_p^k - \tilde{z}_p^k - z_p^k = \gamma_{pk}, \forall k \in \mathcal{K}, \forall p \in \mathcal{P}_k^{\text{DEF}} \quad (10)$$

$$\hat{z}_{ri}^k + f_{ri}^k = \tilde{z}_{ri}^k + f_{r,i-1}^k, \forall k \in \mathcal{K}, \forall r \in \mathcal{R}, \forall i \in I_r \quad (11)$$

$$\bar{z}_p^k = \min\{\hat{z}_p^k, \tilde{z}_p^k\}, \forall k \in \mathcal{K}, \forall p \in \mathcal{P} \quad (12)$$

$$\sum_{h \in \mathcal{H}} \rho_{hri} y_h + \sum_{k \in \mathcal{K}} E_k f_{ri}^k \leq \Lambda_r \text{Cap}_{v(r)} x_r, \forall r \in \mathcal{R}, \forall i \in I_r \quad (13)$$

$$\sum_{(o,d) \in \mathcal{W}} \sum_{h \in \mathcal{H}_{od}} \rho_{hri} W_{od} y_h + \sum_{k \in \mathcal{K}} W_k f_{ri}^k \leq \Lambda_r \text{Wei}_{v(r)} x_r, \forall r \in \mathcal{R}, \forall i \in I_r \quad (14)$$

$$\sum_{r \in \mathcal{R}_v} m(r) x_r \leq N_v^{\text{own}} + n_v^{\text{in}}, \forall v \in \mathcal{V} \quad (15)$$

$$\sum_{r \in \bar{\mathcal{R}}} x_r \geq N^{\text{SR}} \quad (16)$$

$$x_r = 1, \forall r \in \hat{\mathcal{R}} \quad (17)$$

$$x_r \in \{0,1\}, \forall r \in \bar{\mathcal{R}} \cup \tilde{\mathcal{R}} \quad (18)$$

$$n_v^{\text{in}} \geq 0, \forall v \in \mathcal{V} \quad (19)$$

$$y_h \geq 0, \forall h \in \mathcal{H} \quad (20)$$

$$y_{od} \geq 0, \forall (o,d) \in \mathcal{W} \quad (21)$$

$$\hat{z}_{ri}^k \geq 0, \tilde{z}_{ri}^k \geq 0, f_{ri}^k \geq 0, \forall k \in \mathcal{K}, \forall r \in \mathcal{R} \quad (22)$$

$$\hat{z}_p^k \geq 0, \tilde{z}_p^k \geq 0, \bar{z}_p^k \geq 0, z_p^k \geq 0, \forall k \in \mathcal{K}, \forall p \in \mathcal{P} \quad (23)$$

The objective function (4) minimizes the total cost. The first component is the cost associated with ship routes, the second term represents the laden container routing cost, the third term reflects empty container handling cost, the fourth term is the cost for chartering in more ships, and the last two terms are penalty cost for not fulfilling the laden and empty container shipment demand. Eq. (5) stands for the laden container flow conservation equation. Eqs. (6)-(7) define the load and discharge volumes of empty containers, respectively. Eqs. (8)-(11) are empty container flow conservation equations. Eq. (12) defines the transshipped empty containers. Eqs. (13)-(14) impose the ship volume and capacity constraints, respectively. Eq. (15) is the constraint for ship numbers. Eq. (16) requires that at least  $N^{\text{SR}}$  ship routes of type 2 must be chosen. Eq. (17) requires that all ship routes of type 1 must be used. Eqs. (18)-(23) define the decision variables.

The NDM can be transformed to a mixed-integer linear programming model, when replacing Eq. (12) by

$$\bar{z}_p^k \leq \hat{z}_p^k, \forall k \in \mathcal{K}, \forall p \in \mathcal{P} \quad (24)$$

$$\bar{z}_p^k \leq \tilde{z}_p^k, \forall k \in \mathcal{K}, \forall p \in \mathcal{P} \quad (25)$$

This is because, assuming that  $\bar{z}_p^k > \min\{\hat{z}_p^k, \tilde{z}_p^k\}$  at the optimal solution, we can further decrease the objective function (4) by reducing the value of  $\bar{z}_p^k$  while not violating any constraint.

### A Heuristic Solution Approach for Solving the Network Design Model (NDM)

NDM cannot be solved directly by off-the-shelf solver if the cardinality of  $\bar{\mathcal{R}} \cup \tilde{\mathcal{R}}$  is large because it has hundreds of thousands of continuous decision variables. To obtain a high quality solution efficiently, we propose a heuristic approach as follows.

First, denoting by  $y_h^*$  and  $f_{ri}^{*k}$  the resulting flow of laden and empty containers, respectively, in a given network, we define the capacity utilization of leg  $i$  of ship route  $r \in \mathcal{R}$ , denoted by  $U_{ri}$ , as follows:

$$U_{ri} := \max \left\{ \begin{array}{l} \frac{\sum_{h \in \mathcal{H}} \rho_{hri} y_h^* + \sum_{k \in \mathcal{K}} E_k f_{ri}^{*k}}{\Lambda_r \text{Cap}_{v(r)}} \\ \frac{\sum_{(o,d) \in \mathcal{W}} \sum_{h \in \mathcal{H}_{od}} \rho_{hri} W_{od} y_h^* + \sum_{k \in \mathcal{K}} W_k f_{ri}^{*k}}{\Lambda_r \text{Wei}_{v(r)}} \end{array} \right\}, \forall r \in \mathcal{R}, \forall i \in \mathcal{I}_r \quad (26)$$

We further define the capacity utilization of ship route  $r \in \mathcal{R}$ , denoted by  $U_r$ , as follows:

$$U_r := \frac{\sum_{i \in \mathcal{I}_r} U_{ri} L_i}{\sum_{i \in \mathcal{I}_r} L_i}, \forall r \in \mathcal{R} \quad (27)$$

where  $L_i$  is the nautical distance of leg  $i$ . The hit-haul leg is the voyage leg with the highest load and the hit-haul utilization is the ratio of the load on the hit-haul leg over the ship capacity. The hit-haul capacity utilization of ship route  $r \in \mathcal{R}$ , denoted by  $U_r^{\max}$ , can be calculated as follows:

$$U_r^{\max} := \max_{i \in \mathcal{I}_r} \{U_{ri}\}, \forall r \in \mathcal{R} \quad (28)$$

A heuristic algorithm that obtains a high quality solution is as follows:

#### **Algorithm 1: Optimizing the initial network**

*Step 0:* Specify the value of  $N^{\text{OPT}}$  (e.g.  $N^{\text{OPT}}=20$ ), which represents the number of ship routes that can be set as optional in the optimization models. Solve NDM by requiring  $x_r = 1$  for any  $r \in \mathcal{R}$ . Obtain the container flow  $y_h^*$  and  $f_{ri}^{*k}$ .

*Step 1:* Compute the capacity utilization of ship route  $r \in \bar{\mathcal{R}} \cup \tilde{\mathcal{R}}$ . Set  $x_r \in \{0,1\}$  for the  $N^{\text{OPT}}$  ship routes with the lowest capacity utilization.

*Step 2:* Set  $x_r = 1$  for any  $r \in \mathcal{R}$  except the  $N^{\text{OPT}}$  ship routes identified in NDM. Therefore NDM has exactly  $N^{\text{OPT}}$  binary decision variables. Solve NDM and obtain the container flow  $y_h^*$  and  $f_{ri}^{*k}$ . If all the  $N^{\text{OPT}}$  ship routes are chosen, that is, the optimal solution  $x_r^* = 1$  for all ship routes, stop. Otherwise remove the ship routes that are not chosen, and go to Step 1.

Algorithm 1 terminates in a finite number of iterations, because at least one ship route in  $\mathcal{R}$  is removed each time the algorithm repeats Step 2 and the number of ship routes in  $\mathcal{R}$  is limited. In the above algorithm, the number  $N^{\text{OPT}}$  is used to balance the trade-off between the solution quality and computational time. For instance, if  $N^{\text{OPT}} = |\mathcal{R}|$ , we obtain the optimal solution. If  $N^{\text{OPT}}$  is small, NDM can be solved efficiently in each iteration. Nevertheless, some potentially good ship routes may be removed at early stages of the algorithm.

## APPROACHES FOR NEW SHIP ROUTE DESIGN

Based on the initial network, the algorithm 1 chooses a set of high quality ship routes. However, no new ship routes are generated. As a result, it may not be sufficient for designing the liner shipping network. Therefore, we propose some approaches for designing new ship routes, which are shown in Stages 1, 2 and 3 of Figure 1.

*<Insert Figure 1 here>*

### Generate New Ship Routes

If the projected container shipment demand is much larger than the current demand, or if the demand covers shipping markets that are not serviced by the current network, or if there is no initial network, the resulting network after optimizing the initial network may have a large number of unshipped containers. In such a setting, we need to design ship routes from scratch, which are entirely different from the ship routes in the initial network.

The line-haul ship routes are first designed, which are classified according to different trade lanes such as intra-Asia, Asia to American West Coast, trans-Atlantic, Asia to American East Coast, etc. For each trade lane, if the volume of unshipped containers exceeds a threshold value, one or more line-haul ship routes will be designed.

To design a line-haul ship route, first, we sort the ports in the regions covered by the trade lane according to their unshipped laden containers to and from other ports in the regions. It should be mentioned that if a port is a regional hub, in the algorithm 30% of its feeders' unshipped containers are added to this hub. The top  $n'$  (say  $n' = 10$ ) ports with

largest volume of unshipped containers are selected to design a new ship route. The port calling sequence is determined such that the round-trip journey distance is minimized. We design three line-haul ship routes with the same port rotations yet different types of ships. The type of ship is determined based on 1/3, 1/2, and 100% of the total unshipped demand of all the ports on the ship route.

Feeder ship routes are designed accordingly, by connecting the feeder ports to the regional hubs on the line-haul ship route if necessary. All these newly designed line-haul and feeder ship routes are set as optional and added to the set of ship routes. We then solve the NDM based on this new set of ship routes. This process is repeated until there are no new line-haul ship routes designed. After this stage, the volume of unshipped containers is usually less than 5% of the overall demand, which can be acceptable by the liner shipping company.

### Network Refinement

The network refinement stage aims to improve the designed network without adding ports of call. The five steps in this stage are elaborated as follows. Note that each refined ship route is taken as optional and added to the existing route set, and then the NDM is solved based on the new set of ship routes.

In step 1, if the number of laden and empty containers handled at some ports of call is very small, for example, less than 10 containers, then these ports of call can be removed. The number of empty containers handled can be obtained directly from the values of  $\hat{z}_{ri}^k$  and  $\tilde{z}_{ri}^k$ . The number of laden containers handled can be derived by examining the value of  $y_h$  and the property of the container route  $h \in \mathcal{H}$ .

In step 2 of the network refinement stage, if the capacity utilization  $U_r$  of ship route  $r \in \bar{\mathcal{R}} \cup \tilde{\mathcal{R}}$  is very large or small, we consider replacing the ships with a string of larger or smaller ships, respectively. Still, we let the algorithm determine whether the ship size should be changed.

An example of step 3 of the network refinement stage is shown in Figure 2. If the leg capacity utilizations  $U_{ri}$  of the legs Yantian to Pusan, Pusan to Shanghai and Shanghai to Yantian are all very low, we consider removing all these legs altogether.

*<Insert Figure 2 here>*

In step 4 of the network refinement stage, we choose those ship routes from  $\bar{\mathcal{R}} \cup \tilde{\mathcal{R}}$  with the lowest capacity utilization, set them as optional, and re-optimize.

In step 5 of the network refinement stage, we choose the  $\lfloor N^{\text{OPT}} / 2 \rfloor$  ship routes from  $\bar{\mathcal{R}} \cup \tilde{\mathcal{R}}$  with the lowest capacity utilization, set them as optional, set the new ship routes by reversing their port calling sequences as optional, too, and re-optimize.

### Ship Route Alteration

The ship route alteration refinement stage aims to improve the designed network by adding or removing ports of call in an intelligent manner. Still, all the newly designed ship routes here are added to the existing ship route set, and then evaluated together by solving the NDM.

Step 1 deals with the feeder ship route, which is defined as a ship route consisting of a regional hub and its feeders. If the capacity utilization of a feeder ship route is low, the feeder ports included in the feeder ship route are then removed from existing line-haul ship routes. The rationale is that as there is space on the feeder service, there is no need to call at a feeder port on line-haul ship routes.

In step 2, if the capacity utilization of a feeder ship route is low, we then consider adding a new feeder port, which is assigned to the regional hub in the feeder ship route, to the feeder ship route, based on the unshipped demand.

In step 3, if the capacity utilization of a line-haul ship route is low, we then consider adding a new port to the ship route based on the unshipped demand.

It should be highlighted that in any stage of the above algorithm, liner service planners can design or change ship routes based on their expertise, and the designed/changed ship routes could be set as optional and optimized by solving the NDM. Therefore the successive optimization heuristic could incorporate the human expertise.

### CASE STUDY

We apply the network design models and solution algorithms to a global liner shipping network. The network has a total of more than 150 ports, with total demand of 250,000+ laden TEUs and 100,000+ empty TEUs to ship per week. There are four types of containers: dry 20-ft, dry 40-ft, reefer 20-ft, and reefer 40-ft. Three types of ships are used: 1500-TEU, 3000-TEU, and 5000-TEU ships. All other parameters and inputs are provided or estimated by the global liner shipping company. The initial network is based on the current network that the global liner shipping company is operating. The mixed-integer linear programming model NDM is solved by CPLEX-12.1 running on a 3.2 GHz Dual Core PC with 4 GB of RAM.

The algorithm finishes after 7 minutes. A total of 81 ship routes are designed in the final network, as shown in Table 1. 304 ships are deployed in the network, with a total of ship board capacity of 1.157 million TEUs.

*<Insert Table 1 here>*

Two indicators are used by the liner shipping company to evaluate the quality of the designed network. The first one is the ratio of ship board capacity over the demand, and the second one is the hit-haul utilization of ship routes. The designed liner shipping network outperforms the current network of the company in terms of these two indicators.



Software based on Lua and C++ have been developed for the global liner shipping company that integrates the proposed model and solution algorithms. The interface of the software is shown in Figure 3. Different ship routes are indicated by lines of different colors. The software is currently in use by the global liner shipping company.

<Insert Figure 3 here>

## CONCLUSIONS

This paper addressed a realistic liner shipping network design problem while considering practical operations and features. Based on the generated set of feasible container routes, a network design model (NDM) was proposed to evaluate any give set of ship routes. A heuristic procedure was then introduced to deal with the network design problem: an initial network was first evaluated by the NDM, and then some approaches were used to design new ship routes based on the evaluation results. Each newly designed ship routes were combined to the existing network and then evaluated again by the NDM.

A real case study based on the global shipping network of a liner shipping company, consisting of more than 150 ports, was reported. The algorithm efficiently designed a liner shipping network. Two indicators, the ratio of ship board capacity over the demand and the hit-haul capacity utilization of ship routes, demonstrate that the design network is of high quality.

Future work would be devoted to the distributed implementation of the model (17) and incorporation of more general problem settings, such as the availability of berths (e.g., 18-21), the competition between shipping companies (e.g., 22) and ship scheduling (e.g., 23).

## ACKNOWLEDGMENTS

This study is supported by the research grants - WBS No. R-302-000-014-720 and WBS No. R-702-000-007-720 - from the NOL Fellowship Programme of Singapore.

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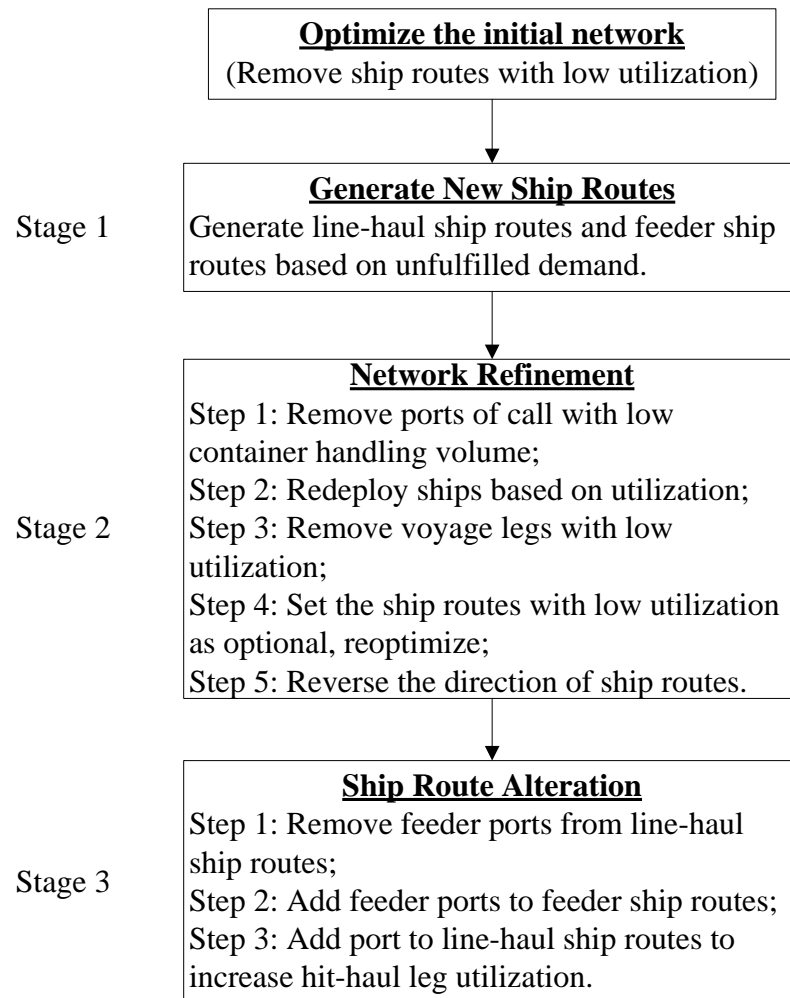
**FIGURE 1** Flowchart of the procedure for designing new ship routes.

**FIGURE 2** Removal of voyage legs with low utilization.

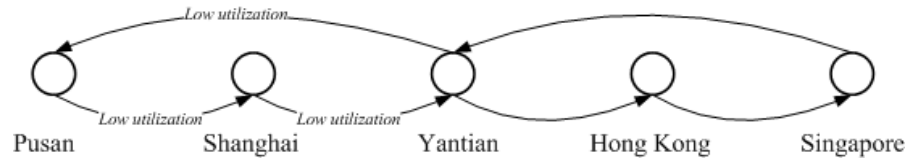
**FIGURE 3** Interface of network design software.

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**TABLE 1** Ship Routes and Ships Deployed



**FIGURE 1** Flowchart of the procedure for designing new ship routes.



**FIGURE 2 Removal of voyage legs with low utilization.**

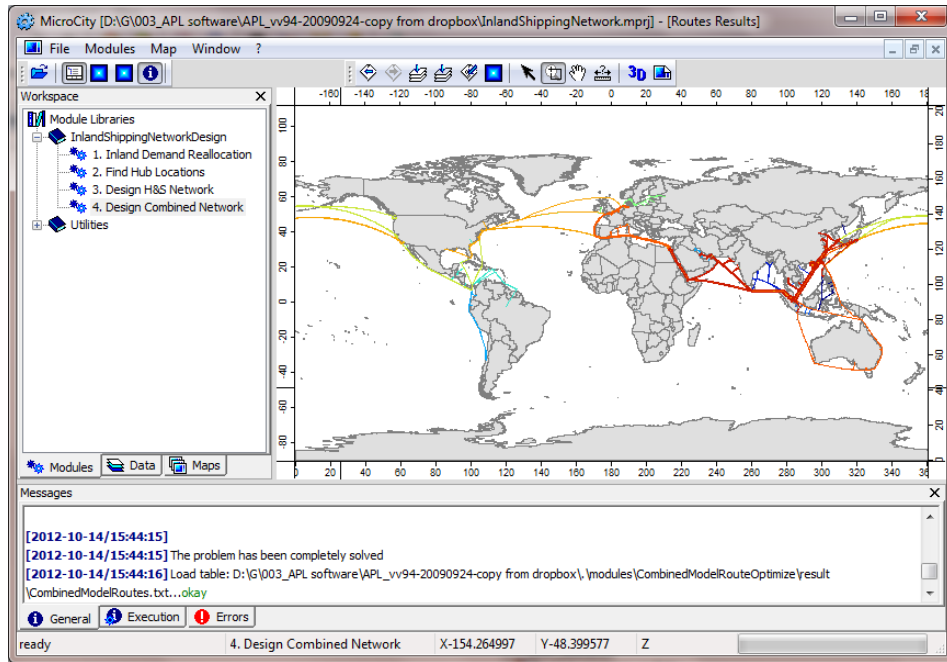


FIGURE 3 Interface of network design software.

**TABLE 1 Ship Routes and Ships Deployed**

Region-to-region	Number of ship routes	Number of ships		
		1500-TEU	3000-TEU	5000-TEU
Intra-Asia	50	49	22	46
Asia-Europe	6	8	0	43
Trans-Pacific	10	3	7	82
Trans-Atlantic	2	0	10	0
Intra-Europe	5	4	3	2
Intra-America	7	14	3	0
Asia-US-Europe	1	0	0	8
<b>Total</b>	<b>81</b>	<b>78</b>	<b>45</b>	<b>181</b>