SYSTEMS OF ORDINARY DIFFERENTIAL EQUATIONS: An Introduction

JACK L. GOLDBERG ARTHUR J. SCHWARTZ The University of Michigan

HARPER & ROW, PUBLISHERS NEW YORK, EVANSTON, SAN FRANCISCO, LONDON

CONTENTS

Preface xi

CHAPTER 1 VECTORS AND MATRICES

1.1. The Solution of Simultaneous Equations

1.2. The Arithmetic of Matrices 4

1.3. Matrix Multiplication 8

1.4. The Inverse of a Matrix 13

1.5. Determinants of $n \times n$ Matrices 19

1.6. Vector Spaces of *n*-Tuples 26

1.7. Linear Independence 30

1.8. Bases and Dimension 34

1.9.* Vector Norms 40

1.10.* Matrix and Vector Calculus 45

CHAPTER 2 LINEAR HOMOGENEOUS SYSTEMS AND EIGENVECTORS

- 2.1. The First-Order Equation in One Dimension 50
- 2.2. The Origin of Linear Systems of Linear Differential Equations 52

2.3. Eigenvalues, Eigenvectors, and Systems of Equations 56

1

- 2.4. Initial-Value Problems 62
- 2.5. Symmetric Matrices 71
- 2.6. Dilute Solutions: A Symmetric Example 73
- 2.7. Complex Eigenvalues and Eigenvectors 76
- 2.8. A Summary 79

CHAPTER 3 LINEAR SYSTEMS AND ROOT VECTORS

- 3.1. New Solutions of $\mathbf{x}' = A\mathbf{x}$ 82
- 3.2. Root Vectors 87
- 3.3. The Root Vector and Solutions of $\mathbf{x}' = \mathbf{A}\mathbf{x}$ 94
- 3.4. A Recapitulation 102
- 3.5. Another View of the Initial-Value Problem 104
- 3.6.* The General Theory 110
- 3.7. Electrical Networks 117

CHAPTER 4 APPLICATIONS OF THE THEORY OF THE HOMOGENEOUS EQUATION

- 4.1. Fundamental Matrices 123
- 4.2. The Uniqueness Theorem 130
- 4.3. Some Consequences of the Uniqueness Theorem 135
- 4.4. The Inhomogeneous Problem: Variation of Parameters 140
- 4.5. The Inhomogeneous Problem: Some Special Methods 145
- 4.6.* Gronwall's Inequality 150
- 4.7.* The Growth of Solutions 153

CHAPTER 5 HIGHER ORDER EQUATIONS

- 5.1. The *n*th-Order Linear Equation in One Variable 156
- 5.2. The Companion System 157
- 5.3. The Auxiliary Equation: Distinct Roots 164
- 5.4. The Auxiliary Equation: Repeated Roots 170
- 5.5.* The General Theory of the *n*th-Order Linear Equation with Constant Coefficients 173
- 5.6.* An Example from Mechanics 180
- 5.7.* The General System of Second-Order Equations 181
- 5.8.* An Important Special Case 187

CHAPTER 6 AN INTRODUCTION TO THE THEORY OF ANALYTIC DIFFERENTIAL SYSTEMS

- 6.1. Introduction 192
- 6.2. Power Series Expansions of Analytic Functions 194
- 6.3. Series Solutions of $\mathbf{x}' = \mathbf{A}(t)\mathbf{x}$ 201
- 6.4. The Legendre Equation: An Example of the Ordinary Point for One-Dimensional Equations 209
- 6.5. The Exponential of a Matrix 211
- 6.6.* The Regular Singular Point 215
- 6.7.* The Bessel Equation 223

CHAPTER 7 AN INTRODUCTION TO NONLINEAR EQUATIONS

- 7.1. What is a Nonlinear Equation? 227
- 7.2. Some Predators and Their Prey 229
- 7.3. Nonlinear Equations in One Dimension 231
- 7.4. The Euler Method in One Dimension: An Example 234
- 7.5. The Euler Method 236
- 7.6.* Existence and Uniqueness 240
- 7.7.* The Accuracy of the Euler Method 244

CHAPTER 8 TWO-DIMENSIONAL AUTONOMOUS SYSTEMS: AN INTRODUCTION TO THE QUALITATIVE THEORY OF DIFFERENTIAL EQUATIONS

- 8.1. Introduction: The Phase Plane 249
- 8.2. Orbits and Their Properties: I 252
- 8.3. Orbits and Their Properties: II 256
- 8.4. The Geometry of Critical Points of $\mathbf{x}' = A\mathbf{x}$ 261
- 8.5. Stability of Critical Points 270

8.6. Perturbations of Linear Systems 274

8.7.* The Equation of First Variation 284

8.8.* Stability of Periodic Orbits 289

Answers to Odd-Numbered Problems 292

Index 313