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TABLES FOR TWO NORMAL-SCORES RANK TESTS FOR THE TWO-SAMPLE LOCATION PROBLEM

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Introduction. In this paper we present the tables of scores and of critical values for the Fisher-Yates-Terry-Hoeffding test and for the van der Waerden test for cases when the pooled sample size $m + n$ lies within the bounds $6 \leqq m + n \leqq 20$ and the onesided significance level lies near 0·5%, 1%, 2·5%, 5%. These tests are optimal (in a sense to be stated later) for the two-sample location problem when the underlying distributions are normal.

In the sequel we shall use the terminology and the results from the book by Hájek-Šidák [1].

Description of the tests. Let us have two random samples $X_1, \ldots, X_m$ and $Y_1, \ldots, Y_n$ with densities f_1 and f_2, respectively. Suppose that the notation is chosen so that $m \leqq n$, and put $N = m + n$. Our aim is to test the hypothesis H_0 that f_1 and f_2 are identical but otherwise arbitrary against the alternatives of shift in location expressed by $f_1(x) = f(x - \Delta)$, $f_2(x) = f(x)$, where $\Delta > 0$, or $\Delta < 0$ (one-sided alternatives), or $\Delta \neq 0$ (two-sided alternative).

Let $R_1, \ldots, R_m$ denote the ranks of $X_1, \ldots, X_m$ in the pooled sample of all observations $X_1, \ldots, X_m, Y_1, \ldots, Y_n$ arranged in order of their magnitude. The theoretical Fisher-Yates-Terry-Hoeffding test employs the statistic $S' = \sum_{i=1}^{m} a'_N(R_i)$, where $a'_N(k)$ are the normal scores, i.e. $a'_N(k) = \mathsf{E} V_N^{(k)}$ where $V_N^{(1)} < V_N^{(2)} < \ldots < V_N^{(N)}$ is an ordered random sample of size N from the standardized normal distribution. The test with the critical region $\{S' \geqq c\}$, c being some critical value, is the locally most powerful rank test of H_0 against $\Delta > 0$ if f is the normal density; it is also asymptotically optimum for this case. Of course, in practice we must round off the scores $a'_N(k)$. Since, moreover, the test is not affected by the multiplication of S' by any positive constant, we employ in this paper, for the sake of simplicity, the test statistic

$$S = \sum_{i=1}^{m} a_N(R_i)\,,$$

where $a_N(k) = 100a'_N(k)$ rounded off to integer values. If we wish to test against the one-sided alternative $\Delta > 0$ (or $\Delta < 0$) indicating that the first density f_1 is shifted to the right (to the left) with respect to the second density f_2, we use a one-sided critical region $\{S \geqq c\}$ (or $\{S \leqq -c\}$, respectively). In testing against $\Delta \neq 0$ we use the critical region $\{|S| \geqq c\}$.

The theoretical van der Waerden test employs similarly the statistic $T' = \sum_{i=1}^{m} b'_N(R_i)$ where $b'_N(k)$ are approximate normal scores, $b'_N(k) = \Phi^{-1}(k/(N+1))$ with Φ^{-1} being the inverse of the standardized normal distribution function. This test is asymptotically optimum for the mentioned shift alternatives with f being the normal density. In this paper we consider the test statistic

$$T = \sum_{i=1}^{m} b_N(R_i)\,,$$

where $b_N(k) = 100b'_N(k)$ rounded off to integer values. Otherwise, everything is analogous as for the preceding Fisher-Yates-Terry-Hoeffding test.

Description of the tables. The scores $a_N(k)$ for the Fisher-Yates-Terry-Hoeffding test are given in Table 1 for $6 \leqq N \leqq 20$. They have been obtained from Teichroew's tables [4] (and checked by means of Harter's tables [2]) after multiplying the relevant numbers by 100 and rounding them off to integer values.

Table 2, having four double-columns for $\alpha_1 = 0{\cdot}5\%$, $\alpha_2 = 1\%$, $\alpha_3 = 2{\cdot}5\%$, $\alpha_4 = 5\%$, contains in its i-th double-column the upper-tail critical values c_i of the statistic S and the corresponding exact probabilities $P\{S \geqq c_i\}$ in per cents such that this probability is the closest possible to the value α_i $(i = 1, 2, 3, 4)$. Precisely in formulas: First, let there be two consecutive possible values $c_i^- < c_i^+$ of the statistic S such that $P\{S \geqq c_i^+\} < \alpha_i \leqq P\{S \geqq c_i^-\}$; then, if $P\{S \geqq c_i^-\} - \alpha_i \geqq \alpha_i - P\{S \geqq c_i^+\}$ we tabulate c_i^+ and $P\{S \geqq c_i^+\}$; if, conversely, $P\{S \geqq c_i^-\} - \alpha_i < \alpha_i - P\{S \geqq c_i^+\}$ we tabulate c_i^- and $P\{S \geqq c_i^-\}$. Second, let there be no c_i^+ with $P\{S \geqq c_i^+\} < \alpha_i$; then we take for c_i^- the maximal possible value of S and tabulate this c_i^- and $P\{S \geqq c_i^-\}$ in the i-th double-column corresponding to the largest α_i (among $\alpha_1, \alpha_2, \alpha_3, \alpha_4$) for which $\alpha_i \leqq P\{S \geqq c_i^-\}$ is satisfied; the double-columns for smaller α_j's (if there are any) are then filled by dashes, the double-columns for larger α_j's (if there are any) are filled according to the first rule. Third, if this second case occurs for α_1, the value c_1^- is preceded by a star indicating that the tabulated c_1^- is the largest possible value (so that it can be distinguished from the first case).

This table (except the maximal values of c, i.e. except those c preceded by dashes or by a star) can be used in two somewhat different ways: First, if we have no objections against a slight exceeding of the significance level α_i, we can use the critical region $\{S \geqq c_i\}$ with the exact significance level $P\{S \geqq c_i\}$ given in Table 2. Second, if we insist on having a significance level not exceeding α_i but we find in the table

$P\{S \geqq c_i\} > \alpha_i$, we can use the critical region $\{S > c_i\}$ which has the significance level $<\alpha_i - (P\{S \geqq c_i\} - \alpha_i)$.

Since the statistic S has under H_0 a symmetric distribution about 0, its lower-tail critical regions are $\{S \leqq -c\}$ with the significance levels $P\{S \leqq -c\} = P\{S \geqq c\}$, and its two-sided critical regions are $\{|S| \geqq c\}$ with the significance levels $P\{|S| \geqq c\} = 2P\{S \geqq c\}$.

Further, concerning the van der Waerden test, the scores $b_N(k)$ for it are presented in Table 3; they have been computed anew, and checked by means of van der Waerden-Nievergelt's tables [6].

Table 4 contains the upper-tail critical values c_i of the statistic T and the exact probabilities $P\{T \geqq c_i\}$ in the same arrangement as in the preceding case of the statistic S, so that all remarks made above for S continue to hold also for T.

Tables 2 and 4 have been computed by direct combinatorial procedures, so that the probabilities in these tables are exact (except, of course, their natural rounding off). The author is deeply indebted to M. Nosál for programming these computations, and to J. Hájek for suggesting the way of tabulation used here.

Previous tabulations for the Fisher-Yates-Terry-Hoeffding test are as follows: Terry [5] tabulated the whole distribution for $2 \leqq N \leqq 10$ and some critical values for $6 \leqq N \leqq 10$ using in the test statistic the scores $a'_N(k)$ rounded off to two decimal places. Further, Klotz [3] tabulated the critical values for $6 \leqq N \leqq 20$ and the corresponding exact significance levels near 0·1%, 0·5%, 1%, 2·5%, 5%, 7·5%, 10%, but using the scores $a'_N(k)$ from Teichroew [4] rounded off to five decimal places. As for the number of necessary decimal places, our point of view in this paper is more practical than that of Klotz (though, of course, theoretically we would like to use the precise scores $a'_N(k)$). In practice one never meets a precise normal distribution, so that hunting for optimality for such a distribution by means of a high precision of the scores in a non-parametric test is somewhat illusory. In our opinion, the precision to five decimal places may bring rather more clumsiness into practical calculations than a real gain in efficiency. Therefore, we considered it sufficient to round off the scores only to two decimal places (followed by a multiplication by 100), which, of course, requires a new tabulation of critical values and probabilities.

As for the van der Waerden test, its conservative critical values for $6 \leqq N \leqq 50$, $|m - n| \leqq 5$ and for the significance levels not exceeding 0·5%, 1%, 2·5% were tabulated by van der Waerden-Nievergelt [6] using the scores $b'_N(k)$ rounded off to two decimal places.

Example 1. Let the X sample be 11; 39; 67; 82; 127, and the Y sample 58; 95; 112; 130; 139; 149; 167; 191; 219, so that $m = 5$, $n = 9$, $N = 14$. We wish to test H_0 against $\Delta < 0$, i.e. against the alternative that the density of X is shifted to the left, by means of the Fisher-Yates-Terry-Hoeffding test at the significance level not exceeding 1%. In Table 2 we find the critical value $c_2 = 381$ with the level

$P\{S \geqq 381\} = 1{\cdot}10\%$. However, since we do not want to exceed 1%, we can use the critical region $\{S > 381\}$ with the level $<1\% - (1{\cdot}10\% - 1\%) = 0{\cdot}90\%$. Since we are testing against $\Delta < 0$, we use actually the critical region $\{S < -381\}$. An easy calculation in our example gives now the value $S = -394$ so that we reject H_0 at a significance level $<0{\cdot}90\%$.

Example 2. Let the X sample be 44; 69; 124 and the Y sample -39; -8; 10; 21; 42; 70, so that $m = 3$, $n = 6$, $N = 9$. We want to test H_0 against the two-sided alternative $\Delta \neq 0$ by means of the van der Waerden test. We calculate the actual value of the statistic $T = 205$. Since Table 4 shows the upper-tail critical value $c_4 = 205$ with the level $P\{T \geqq 205\} = 4{\cdot}76\%$, the two-sided critical region $\{|T| \geqq 205\}$ has the level $2P\{T \geqq 205\} = 9{\cdot}52\%$, and we could reject H_0 at this level but at no smaller level.

Remark on asymptotic normality. The statistic S has, under H_0, the mean value $\mathsf{E}S = 0$, the variance

$$\text{var } S = \frac{mn}{N(N-1)} \sum_{k=1}^{N} [a_N(k)]^2 ,$$

and the standardized variable $S/(\text{var } S)^{1/2}$ has asymptotically the standardized normal distribution whenever $m \to \infty$, $n \to \infty$ in an arbitrary manner. As for the statistic T, completely analogous assertions and formula for var T (where only $a_N(k)$ is replaced by $b_N(k)$) are valid.

Table 5 illustrates the closeness of the normal approximation for the Fisher-Yates-Terry-Hoeffding statistic S for all pairs m, n with $m + n = N = 20$. Its arrangement is similar to that of Table 2, but the first number in each double-column is the approximate level in per cents (i.e. $1 - \Phi(c_i/(\text{var } S)^{1/2})$ in per cents where Φ is the standardized normal distribution function and c_i is the critical values from Table 2), the second number in each double-column is the exact level $P\{S \geqq c_i\}$ in per cents taken from Table 2.

Remark on ties. If there are some equal observations, so that their ranks are not well defined, we can either randomize their ordering by means of an artificial experiment, or use the following method of average scores: Suppose that we consider a group of τ equal observations, and that these observations would have the ranks (say) $r + 1, \ldots, r + \tau$ if they were distinct but if they had otherwise the same position in the ordered pooled sample; then we assign to each of these observations the artificial average score $\tau^{-1} \sum_{i=1}^{\tau} a_N(r + i)$ in the case of S, or $\tau^{-1} \sum_{i=1}^{\tau} b_N(r + i)$ in the case of T. If there are not many tied observations, we can still use our Tables 2 and 4 as approximations.

Table 1. Scores $a_N(k)$ for the Fisher-Yates-Terry-Hoeffding statistic S

k \ N	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
1	−127	−135	−142	−149	−154	−159	−163	−167	−170	−174	−177	−179	−182	−184	−187
2	−64	−76	−85	−93	−100	−106	−112	−116	−121	−125	−128	−132	−135	−138	−141
3	−20	−35	−47	−57	−66	−73	−79	−85	−90	−95	−99	−103	−107	−110	−113
4	20	0	−15	−27	−38	−46	−54	−60	−66	−71	−76	−81	−85	−89	−92
5	64	35	15	0	−12	−22	−31	−39	−46	−52	−57	−62	−66	−71	−75
6	127	76	47	27	12	0	−10	−19	−27	−34	−40	−45	−50	−55	−59
7		135	85	57	38	22	10	0	−9	−17	−23	−30	−35	−40	−45
8			142	93	66	46	31	19	9	0	−8	−15	−21	−26	−31
9				149	100	73	54	39	27	17	8	0	−7	−13	−19
10					154	106	79	60	46	34	23	15	7	0	−6
11						159	112	85	66	52	40	30	21	13	6
12							163	116	90	71	57	45	35	26	19
13								167	121	95	76	62	50	40	31
14									170	125	99	81	66	55	45
15										174	128	103	85	71	59
16											177	132	107	89	75
17												179	135	110	92
18													182	138	113
19														184	141
20															187

Table 2. Critical values c_i of the Fisher-Yates-Terry-Hoeffding statistic S and significance levels $P\{S \geqq c_i\}$ in per cents

n	m	0·5%		1·0%		2·5%		5·0%	
5	1	—	—	—	—	—	—	127	16·67
4	2	—	—	—	—	—	—	191	6·67
3	3	—	—	—	—	211	5·00	211	5·00
6	1	—	—	—	—	—	—	135	14·29
5	2	—	—	—	—	211	4·76	211	4·76
4	3	—	—	—	—	246	2·86	211	5·71
7	1	—	—	—	—	—	—	142	12·50
6	2	—	—	—	—	227	3·57	227	3·57
5	3	—	—	274	1·79	274	1·79	212	5·36
4	4	—	—	289	1·43	259	2·86	227	5·71
8	1	—	—	—	—	—	—	149	11·11
7	2	—	—	—	—	242	2·78	206	5·56
6	3	—	—	299	1·19	269	2·38	233	4·76
5	4	* 326	0·79	326	0·79	272	2·38	242	4·76
9	1	—	—	—	—	—	—	154	10·00
8	2	—	—	254	2·22	254	2·22	220	4·44
7	3	* 320	0·83	320	0·83	266	2·50	232	5·00
6	4	358	0·48	332	0·95	282	2·38	246	5·24
5	5	370	0·40	346	0·79	292	2·78	258	4·76
10	1	—	—	—	—	—	—	159	9·09
9	2	—	—	265	1·82	265	1·82	205	5·45
8	3	* 338	0·61	311	1·21	278	2·42	232	4·85
7	4	360	0·61	338	0·91	292	2·42	247	5·15
6	5	384	0·43	360	0·87	300	2·60	260	4·98
11	1	—	—	—	—	—	—	163	8·33
10	2	—	—	275	1·52	242	3·03	217	4·55
9	3	354	0·45	329	0·91	273	2·73	244	5·00
8	4	385	0·40	344	1·01	300	2·42	255	5·05
7	5	395	0·51	370	0·88	310	2·53	266	5·05
6	6	387	0·54	364	1·08	319	2·60	275	5·19
12	1	—	—	—	—	—	—	167	7·69
11	2	—	—	283	1·28	252	2·56	206	5·13
10	3	368	0·35	322	1·05	283	2·45	244	4·90
9	4	382	0·56	351	0·98	304	2·52	262	5·03
8	5	407	0·47	370	0·93	323	2·41	277	4·97
7	6	408	0·52	382	1·05	329	2·51	283	5·13

Table 2 (continued)

n	m	0·5%		1·0%		2·5%		5·0%	
13	1	—	—	—	—	—	—	170	7·14
12	2	—	—	291	1·10	260	2·20	211	5·49
11	3	357	0·55	326	1·10	282	2·75	245	4·95
10	4	390	0·50	364	1·00	311	2·50	267	5·00
9	5	417	0·50	381	1·10	332	2·45	282	5·19
8	6	428	0·47	391	1·00	340	2·50	293	4·83
7	7	436	0·50	399	1·05	344	2·53	295	5·01
14	1	—	—	—	—	—	—	174	6·67
13	2	* 299	0·95	299	0·95	245	2·86	220	4·76
12	3	370	0·44	333	1·10	291	2·42	247	5·05
11	4	394	0·51	368	1·03	318	2·49	271	4·98
10	5	426	0·50	392	1·00	340	2·50	289	5·03
9	6	445	0·50	409	1·00	352	2·50	300	4·98
8	7	448	0·50	412	1·01	358	2·49	306	4·99
15	1	—	—	—	—	—	—	177	6·25
14	2	* 305	0·83	305	0·83	253	2·50	217	5·00
13	3	362	0·54	333	1·07	293	2·50	248	5·00
12	4	404	0·49	373	0·99	324	2·47	276	5·00
11	5	432	0·50	400	0·98	345	2·56	294	4·99
10	6	452	0·50	415	1·00	359	2·50	307	5·01
9	7	464	0·50	429	0·99	368	2·51	315	4·99
8	8	466	0·50	429	1·01	373	2·49	317	4·96
16	1	—	—	—	—	—	—	179	5·88
15	2	* 311	0·74	311	0·74	260	2·21	213	5·15
14	3	373	0·44	341	1·03	296	2·50	250	5·00
13	4	408	0·50	377	1·01	327	2·48	279	5·00
12	5	439	0·50	407	1·00	348	2·54	298	4·91
11	6	461	0·51	425	0·99	366	2·50	312	5·05
10	7	476	0·51	439	0·99	378	2·49	322	5·01
9	8	484	0·50	444	1·02	382	2·51	327	5·01
17	1	—	—	—	—	—	—	182	5·56
16	2	* 317	0·65	289	1·31	248	2·61	217	5·23
15	3	374	0·49	339	0·98	296	2·57	253	5·02
14	4	417	0·49	381	1·01	331	2·52	281	5·00
13	5	447	0·49	411	1·02	355	2·50	302	5·01
12	6	469	0·50	433	1·02	374	2·52	318	4·98
11	7	488	0·51	447	1·01	386	2·50	328	5·01
10	8	497	0·50	455	1·01	394	2·48	335	4·98
9	9	502	0·50	460	0·99	396	2·51	338	5·00

Table 2 (continued)

n	*m*	0·5%		1·0%		2·5%		5·0%	
18	1	—	—	—	—	—	—	184	5·26
17	2	* 322	0·58	294	1·17	255	2·34	210	5·26
16	3	377	0·52	344	1·03	298	2·48	254	4·95
15	4	419	0·52	385	1·01	333	2·50	283	4·95
14	5	451	0·51	417	1·00	359	2·50	306	5·02
13	6	479	0·50	439	1·00	378	2·52	323	4·97
12	7	497	0·50	458	1·00	393	2·52	335	5·01
11	8	509	0·50	468	0·99	402	2·50	342	5·01
10	9	517	0·49	474	1·00	407	2·49	346	5·02
19	1	—	—	—	—	187	5·00	187	5·00
18	2	* 328	0·53	300	1·05	254	2·63	216	5·26
17	3	375	0·53	347	0·96	298	2·54	255	5·00
16	4	422	0·52	390	0·99	335	2·52	286	4·99
15	5	458	0·50	422	0·99	362	2·51	309	5·00
14	6	487	0·50	448	0·99	384	2·50	327	5·04
13	7	508	0·50	466	1·01	400	2·51	340	5·01
12	8	521	0·50	478	1·00	411	2·50	350	4·98
11	9	529	0·50	486	0·99	417	2·51	355	5·00
10	10	533	0·50	489	0·99	420	2·49	357	4·99

Table 3. Scores $b_N(k)$ for the van der Waerden statistic T

k \ N	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
1	−107	−115	−122	−128	−134	−138	−143	−147	−150	−153	−156	−159	−162	−164	−167
2	−57	−67	−76	−84	−91	−97	−102	−107	−111	−115	−119	−122	−125	−128	−131
3	−18	−32	−43	−52	−60	−67	−74	−79	−84	−89	−93	−97	−100	−104	−107
4	18	0	−14	−25	−35	−43	−50	−57	−62	−67	−72	−76	−80	−84	−88
5	57	32	14	0	−11	−21	−29	−37	−43	−49	−54	−59	−63	−67	−71
6	107	67	43	25	11	0	−10	−18	−25	−32	−38	−43	−48	−52	−57
7		115	76	52	35	21	10	0	−8	−16	−22	−28	−34	−39	−43
8			122	84	60	43	29	18	8	0	−7	−14	−20	−25	−30
9				128	91	67	50	37	25	16	7	0	−7	−13	−18
10					134	97	74	57	43	32	22	14	7	0	−6
11						138	102	79	62	49	38	28	20	13	6
12							143	107	84	67	54	43	34	25	18
13								147	111	89	72	59	48	39	30
14									150	115	93	76	63	52	43
15										153	119	97	80	67	57
16											156	122	100	84	71
17												159	125	104	88
18													162	128	107
19														164	131
20															167

Table 4. Critical values c_i of the van der Waerden statistic T and significance levels $P\{T \geq c_i\}$ in per cents

n	m	0·5%		1·0%		2·5%		5·0%	
5	1	—	—	—	—	—	—	107	16·67
4	2	—	—	—	—	—	—	164	6·67
3	3	—	—	—	—	182	5·00	182	5·00
6	1	—	—	—	—	—	—	115	14·29
5	2	—	—	—	—	182	4·76	182	4·76
4	3	—	—	—	—	214	2·86	182	5·71
7	1	—	—	—	—	—	—	122	12·50
6	2	—	—	—	—	198	3·57	198	3·57
5	3	—	—	241	1·79	241	1·79	184	5·36
4	4	—	—	255	1·43	227	2·86	198	5·71
8	1	—	—	—	—	—	—	128	11·11
7	2	—	—	—	—	212	2·78	180	5·56
6	3	—	—	264	1·19	237	2·38	205	4·76
5	4	* 289	0·79	289	0·79	239	2·38	212	4·76
9	1	—	—	—	—	—	—	134	10·00
8	2	—	—	225	2·22	225	2·22	194	4·44
7	3	* 285	0·83	285	0·83	236	2·50	205	5·00
6	4	320	0·48	296	0·95	250	2·38	218	5·24
5	5	331	0·40	309	0·79	260	2·78	229	4·76
10	1	—	—	—	—	—	—	138	9·09
9	2	—	—	235	1·82	235	1·82	181	5·45
8	3	* 302	0·61	278	1·21	248	2·42	207	4·85
7	4	323	0·61	302	0·91	259	2·42	226	5·15
6	5	345	0·43	323	0·87	269	2·60	232	4·98
11	1	—	—	—	—	—	—	143	8·33
10	2	—	—	245	1·52	217	3·03	193	4·55
9	3	319	0·45	295	0·91	246	2·73	216	5·00
8	4	348	0·40	309	1·01	269	2·42	232	5·05
7	5	358	0·51	334	0·88	279	2·53	240	5·05
6	6	350	0·54	329	1·08	285	2·60	245	5·19
12	1	—	—	—	—	—	—	147	7·69
11	2	—	—	254	1·28	226	2·56	186	5·13
10	3	333	0·35	291	1·05	254	2·45	222	4·90
9	4	348	0·56	320	0·98	276	2·52	236	5·03
8	5	370	0·47	338	0·93	292	2 49	250	4·97
7	6	371	0·52	348	1·05	298	2·45	256	4·95

Table 4 (continued)

n	m	0·5%		1·0%		2·5%		5·0%	
13	1	—	—	—	—	—	—	150	7·14
12	2	—	—	261	1·10	234	2·20	193	5·49
11	3	323	0·55	296	1·10	257	2·47	220	4·95
10	4	353	0·50	329	1·00	282	2·50	242	5·00
9	5	378	0·50	345	1·10	304	2·45	257	4·95
8	6	389	0·47	356	1·00	307	2·50	265	4·93
7	7	396	0·50	363	0·99	312	2·48	269	4·98
14	1	—	—	—	—	—	—	153	6·67
13	2	* 268	0·95	268	0·95	220	2·86	202	4·76
12	3	335	0·44	300	1·10	268	2·42	226	4·84
11	4	358	0·51	333	1·03	287	2·49	247	4·98
10	5	389	0·50	357	1·03	307	2·53	261	5·00
9	6	406	0·50	371	1·00	320	2·46	271	5·03
8	7	408	0·50	375	0·98	325	2·49	276	5·02
15	1	—	—	—	—	—	—	156	6·25
14	2	* 275	0·83	275	0·83	228	2·50	194	5·00
13	3	329	0·54	303	1·07	266	2·68	227	5·00
12	4	369	0·49	340	0·99	293	2·47	251	4·89
11	5	397	0·50	366	0·98	314	2·47	268	5·01
10	6	414	0·50	380	1·00	328	2·50	280	4·98
9	7	428	0·50	391	1·00	336	2·52	287	4·99
8	8	427	0·50	393	0·99	340	2·49	290	4·97
16	1	—	—	—	—	—	—	159	5·88
15	2	* 281	0·74	281	0·74	235	2·21	198	5·15
14	3	340	0·44	309	1·03	267	2·50	230	5·00
13	4	375	0·50	346	0·97	297	2·52	255	5·00
12	5	403	0·50	372	0·99	320	2·50	273	4·99
11	6	423	0·51	390	1·00	336	2·50	286	4·99
10	7	438	0·49	402	1·01	346	2·50	295	5·01
9	8	444	0·50	407	1·01	351	2·49	299	4·98
17	1	—	—	—	—	—	—	162	5·56
16	2	* 287	0·65	262	1·31	242	1·96	196	5·23
15	3	342	0·49	310	0·98	273	2·45	232	5·02
14	4	380	0·49	349	1·01	302	2·52	258	5·00
13	5	412	0·49	378	0·99	325	2·52	277	5·01
12	6	434	0·50	398	1·01	343	2·49	291	5·02
11	7	449	0·51	412	1·01	354	2·51	302	4·97
10	8	458	0·50	421	1·01	361	2·51	308	4·97
9	9	461	0·49	423	1·01	363	2·51	309	5·02

Table 4 (continued)

n	m	0·5%		1·0%		2·5%		5·0%	
18	1	—	—	—	—	—	—	164	5·26
17	2	* 292	0·58	268	1·17	232	2·34	195	5·26
16	3	344	0·52	316	1·03	273	2·48	234	4·95
15	4	384	0·49	355	1·01	306	2·50	261	4·95
14	5	417	0·50	383	1·01	331	2·51	281	5·01
13	6	442	0·50	405	1·00	349	2·48	297	4·97
12	7	460	0·50	422	1·00	362	2·50	308	4·97
11	8	470	0·51	431	1·01	370	2·51	315	5·02
10	9	477	0·50	436	1·01	375	2·50	319	4·98
19	1	—	—	—	—	167	5·00	167	5·00
18	2	* 298	0·53	274	1·05	238	2·63	197	5·26
17	3	345	0·53	326	0·96	276	2·54	237	5·00
16	4	392	0·50	361	0·99	311	2·48	263	4·99
15	5	426	0·50	390	1·00	335	2·51	286	4·97
14	6	451	0·50	413	1·00	356	2·50	302	5·00
13	7	470	0·50	431	1·00	370	2·48	315	4·96
12	8	483	0·50	444	0·99	380	2·52	323	5·00
11	9	491	0·50	450	1·01	386	2·52	328	5·00
10	10	494	0·50	453	1·00	388	2·49	330	5·00

Table 5. Approximate and exact significance levels of the Fisher-Yates-Terry-Hoeffding test for $N = 20$

n	m	0·5%		1·0%		2·5%		5·0%	
19	1	—	—	—	—	—	—	2·35	5·00
18	2	0·57	0·53	1·03	1·05	2·49	2·63	4·78	5·26
17	3	0·75	0·53	1·22	0·96	2·66	2·54	4·91	5·00
16	4	0·73	0·52	1·20	0·99	2·63	2·52	4·89	4·99
15	5	0·72	0·50	1·20	0·99	2·64	2·51	4·92	5·00
14	6	0·69	0·50	1·18	0·99	2·61	2·50	4·92	5·04
13	7	0·68	0·50	1·18	1·01	2·61	2·51	4·94	5·01
12	8	0·69	0·50	1·19	1·00	2·60	2·50	4·91	4·98
11	9	0·69	0·50	1·18	0·99	2·61	2·51	4·93	5·00
10	10	0·68	0·50	1·18	0·99	2·59	2·49	4·92	4·99

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Souhrn

TABULKY PRO DVA POŘADOVÉ TESTY S NORMÁLNÍMI SKÓRY PRO DVOUVÝBĚROVÝ PROBLÉM POLOHY

ZBYNĚK ŠIDÁK

Publikují se tabulky skórů a kritických hodnot pro Fisher-Yates-Terry-Hoeffdingův test a pro van der Waerdenův test pro případy, kdy rozsah $m + n$ dvou spojených výběrů leží v mezích $6 \leqq m + n \leqq 20$ a jednostranná hladina významnosti leží blízko 0,5%, 1%, 2,5%, 5%. Tyto testy jsou optimální pro dvouvýběrový problém polohy, jsou-li základní rozložení normální.

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