TABLES OF EXPECTED VALUES OF ORDER STATISTICS AND PRODUCTS OF ORDER STATISTICS FOR SAMPLES OF SIZE TWENTY AND LESS FROM THE NORMAL DISTRIBUTION[1]

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1. Summary. Tables of the means, variances, and covariances, to five decimal places, of order statistics from samples of size ten or less have been given by Godwin [3]. In this paper basic expected values are given of order statistics and products of order statistics, for samples of size twenty and less to 10 decimal places (DP).[3] In addition, certain other functions are tabulated to 25 DP to facilitate extension to larger sample sizes.

2. Introduction. Let $x_1, x_2, \cdots, x_N$ be a sample from $N(0, 1)$ arranged in order of size so that

$$x_1 \geqq x_2 \geqq \cdots \geqq x_N .$$

The means, variances, and covariances of these "order statistics" may be obtained from the following expressions for expected values and product moments:

$$E(x_j; N) = \frac{N!}{(j-1)!(N-j)!} B(j-1, N-j),$$

$$E(x_j^2; N) = \frac{N!}{(j-1)!(N-j)!} D(j-1, N-j),$$

$$E(x_i x_j; N) = \frac{N!}{(i-1)!(j-i-1)!(N-j)!} G(i-1, N-j, j-i-1),$$

where

$$B(m, n) = \int_{-\infty}^{\infty} x f(x)[F(x)]^m [1 - F(x)]^n \, dx,$$

$$D(m, n) = \int_{-\infty}^{\infty} x^2 f(x)[F(x)]^m [1 - F(x)]^n \, dx,$$

$$G(m, n, p) = \int_{-\infty}^{\infty} \int_{-\infty}^{y} xy f(x) f(y) [F(x)]^m [1 - F(y)]^n [F(y) - F(x)]^p \, dx \, dy,$$

$$f(x) = \frac{e^{-x^2/2}}{\sqrt{2\pi}}, \qquad F(x) = \int_{-\infty}^{x} f(t) \, dt.$$

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[3] The letters "DP" will be used for "decimal places." Values to 20 DP are available; see the concluding paragraph of this article.

Interest in numerical values of order statistics from normal samples has arisen from three main sources. The first is the need for "normalizing" data so that techniques for the normal distribution can be used for the analysis. Fisher and Yates [2] give the means of the order statistics for $N \leqq 50$ for this purpose. The second is the search for a less laborious way to compute an estimate of the standard deviation. The range, which is a function of order statistics, was suggested for this purpose but, as is well known, it loses efficiency rapidly as the sample size increases. Other functions of order statistics have been studied and it has been shown by Godwin [4] that a linear function of the ordered observation can be obtained which will for $N \leqq 10$ estimate the standard deviation with efficiency at least as great as 0.9883. Cadwell [1] has examined the efficiency of quasi ranges, W_r, where

$$W_r = x_{r+1} - x_{N-r},$$

and combinations of the form $W_r + \lambda W_s$, where λ is an arbitrary constant.

The last and most recent interest in the order statistics has arisen from the study of censored samples. In this field it is natural to consider ordered observations because it is usually the larger or smaller values which are missing. Papers in this field are by Gupta [5] and by Sarhan and Greenberg [10].

The practical usefulness of the proposed estimates is usually judged by the efficiency which is obtained by comparing the variance of the estimate with the variance of the optimum statistic. Most of the estimates are linear functions of the order statistics and their variance can, therefore, be computed if $E(x_j ; N)$ $E(x_j^2 ; N)$, and $E(x_i x_j ; N)$ are known.

The first attempt to compute these quantities was undertaken by Hastings, Mosteller, Tukey, and Winsor [6]. They give for $N \leqq 10$ the variances to 5 DP and the covariances to 2 DP. They mention the difficulty in computing these values and state that more accurate values are required.

Jones [7] evaluated the variances and covariances for $N \leqq 4$ in closed form. Godwin [3] extended these results to $N \leqq 6$ and computed by numerical integration the variance and covariance for $N \leqq 10$ to 5 DP. The variances (and higher moments) of the extreme were computed by Tippett [12] and have been given by Ruben [9] to 9 DP for $N \leqq 50$.

In 1948, while computing efficiencies of various linear functions of order statistics, Professor W. J. Dixon found that the number of decimal places given in Ref. [6] was not sufficient for practical purposes. He also came to the conclusion that the tables should be extended beyond $N = 10$. He proposed the task of computing the means, variances, and covariances for $N \leqq 20$ to the National Bureau of Standards. Computation was started at its Institute for Numerical Analysis in Los Angeles in 1949 under the sponsorship of the Office of Naval Research, USN. This paper presents the report on that project.

In view of the large number of people who have contributed to this project, it seems desirable to present briefly the history of the computation and to give proper credit for the work.

The original method of computation consisted in integration of B, D, and G, as defined above, using 10-digit floating-point arithmetic and an integration interval of 0.25. The results of this computation were not satisfactory, and in 1950, J. Barkley Rosser, who was then director of the Institute for Numerical Analysis, studied the problem extensively and proposed the method of calculation which was followed. This method is outlined in Section 3. Rosser, in a National Bureau of Standards report, gave a method for computing moments of order statistics and gave numerical values for the $E(x_j\,; N)$ and $E(x_j^2\,; N)$ for N up to 21 to 18 digits. He also gave constants required to compute the first 8 moments for N up to 20. These computations were carried out to the full accuracy (about 24 DP) obtainable from Sheppard's table of the normal probability integral [11].

The problem of loss of accuracy in the combination of integrals was studied in detail by A. D. Hestenes and by differencing the Sheppard table it was found that the ψ's (defined below) could not be computed accurately enough from that table. It was, therefore, necessary to produce a more accurate table of the normal probability integral. Coding was started on a triple precision (108 binary digits) interpretative routine for the SWAC (Bureau of Standards Western Automatic Computer). This routine could handle addition, subtraction, multiplication, and integration; it had been coded but was not completely checked out by July 1, 1954, when the Institute for Numerical Analysis ceased to exist as a section of the National Bureau of Standards and became the Numerical Analysis Research Project at the University of California, Los Angeles. Sponsorship by the Office of Naval Research continued.

In addition to J. Barkley Rosser, a larger number of people contributed to the project during the period when it was administered by the National Bureau of Standards. Among those who should be mentioned are the following: A. D. Hestenes, who directed the project for a number of years; E. C. Yowell, who supervised the computation and coding; G. Blanch, who directed some of the checking; and M. Howard, S. Marks, O. Mock, and A. Rosenthal who did the computation and coding.

The author completed the problem and, of course, assumes full responsibility for all statements made about the accuracy of the results. The $E(x_j\,; N)$ and $E(x_j^2\,; N)$ values were given by Rosser in his report and are published here with his gracious permission. Rosser did not publish his paper because he felt it would be better to wait until the covariances were computed. He now expects to improve the theoretical parts of his paper before publishing it.

This paper is to be regarded only as a report of the result of some lengthy computations. No attempt was made to use these results to answer statistical problems or to determine whether the computations should have been carried beyond $N = 20$. However the $\psi(a, b)$ values are listed to permit the calculation of some variances and covariances beyond $N = 20$. Some additional variances can also be computed by the differencing method mentioned by Ruben [9]. In view of the increasing difficulties of the computation as N increases, it would seem advisable to investigate techniques such as the one used by Cadwell [1] before attempting further computations.

3. Computation and Checks. The means and variances are relatively simple to compute since they require merely the evaluation of one-dimensional integrals in which the integrands vanish at both ends of the range of integration. The values given in Tables I and II were computed by the method developed by Rosser [8]. Rosser's method is applicable to higher moments; for the means and variances, it is equivalent to the formulas given by Godwin [3]. The numerical integration was performed on values of the normal integral obtained from Sheppard's table [11]. The computations were done in 1952 and 1953, using mainly punched-card equipment, under the direction of A. D. Hestenes. The values were checked by sum checks. In addition, the D's appear in the computation of the G's, and the B's appear in the checks for the G's.

The computation of the covariances is more complicated because it involves either the evaluation of a large number of double integrals $G(m, n, p)$ or the more accurate evaluation of a smaller number of double integrals $\psi(a, b)$, which requires more elaborate integration formulas. Godwin [3] has given a method for the computation of the B's, D's, and G's which is based on two basic sets of integrals, $\psi(a)$ and $\psi(a, b)$, where

$$\psi(a) = \int_{-\infty}^{\infty} [F(x)]^a[1 - F(x)]^a \, dx,$$

$$\psi(a, b) = \int_{-\infty}^{\infty} [F(x)]^a \, dx \int_{x}^{\infty} [1 - F(y)]^b \, dy.$$

The method of computation used in this project is analogous to Godwin's; however, it was developed independently by Rosser. All of the computations were performed using fixed-point, triple-precision arithmetic on the SWAC.

The steps in the computation were as follows:

(1) The normal density function $f(x)$ was computed for $x = -12.00(.02)0$. At least 27 DP are accurate.

(2) The distribution function $F(x)$ was obtained by numerical integration, using Everett's central difference formula (rewritten in terms of ordinates) with terms up to and including δ^{22}; i.e., 24 ordinates were used in each sum.

(3) $H(x; b) = \int_{-\infty}^{x}[F(t)]^b \, dt$, for $b = 1(1)19$, was obtained by the same numerical integration routine as that used in (2).

(4) $\psi(a, b) = \int_{-\infty}^{\infty}[F(t)]^a H(-t, b) \, dt$, for $a = 1(1)19$, was obtained by multiplying ordinates and summing.

(5) $$G(m, n, 0) = \frac{D(n + 1, m)}{2(n + 1)} + \frac{D(n, m + 1)}{2(m + 1)} - \frac{\psi(n + 1, m + 1)}{(n + 1)(m + 1)}.$$

(6) $$G(m, n, p + 1) = G(m, n, p) - G(m + 1), n, p) - G(m, n + 1, p).$$

(7) $$E(x_i x_j; N) = \frac{N!}{(i - 1)!(j - i - 1)!(N - j)!} G(i - 1, N - j, j - i - 1).$$

The formulas in (5), (6), and (7) are equivalent to Godwin's [3] formula 4, page 281. His formula is useful for computing an isolated $E(x_i x_j)$; ours are useful if all the $E(x_i x_j)$ are needed. It may be noted that for $N \leqq 20$, $\psi(a, b)$ is required only for $a + b \leqq 19$; the extra values were computed for checking.

The following checks were used:

(1) The normal density function checked at intervals of 0.1, with the values given by Sheppard [11]. This, together with the fact that $\psi(1, 1)$ turned out to be $\frac{1}{2}$, to 27 DP, leads us to believe that $f(x)$ is correct to at least 27 DP.

(2) The above remarks apply also to $F(x)$, in addition, $F(0) = \frac{1}{2}$, to 27 DP.

(3) Our faith in $H(x; b)$ depends on the accuracy of $\psi(a, b)$.

(4) The ψ's must satisfy the formula

$$\psi(a)\psi(b) = 2\sum_{u=0}^{a}\sum_{v=0}^{b}(-)^{u+v}\binom{a}{u}\binom{b}{v}\psi(a+u, b+v).$$

As mentioned above, the $\psi(a)$'s used for the left-hand side were computed on IBM equipment using values from Sheppard's tables and are therefore completely independent of the $\psi(a, b)$. In the computation of the right-hand side, individual terms could be computed to only 26 DP. The identity was satisfied to within 1 unit in the 25th DP. We can therefore conclude that the $\psi(a)$'s and $\psi(a, b)$ are correct to 25 DP for $i \leqq 9$, $a, b \leqq 18$. (Unfortunately, $\psi(a, b)$ for either a or $b = 19$ cannot be checked until the values for a or $b = 20$ are also available.) An additional check is that obtained from the symmetry relation $\psi(a, b) = \psi(b, a)$. This check was satisfied to more than 25 DP.

(5) Once the $\psi(a, b)$'s have been checked, the problem becomes one of applying the recurrence relations correctly. This part of the computation can be checked by the formula

$$G(0, 0, 2p) = \sum_{i=0}^{p-1}(-)^i\binom{2p}{i}B(2p - i, 0)B(i, 0) + \tfrac{1}{2}(-)^p\binom{2p}{p}[B(p, 0)]^2.$$

This identity was satisfied to 25 DP. It may be noted that $G(0, 0, 18)$ is a function of all $G(m, n, 0)$ and also of all $G(m, n, p)$ for $p \leqq 17$, and the correctness of $G(0, 0, 18)$ verifies the correctness of all $G(m, n, p)$ except possibly for mistakes in punching. This source of error was eliminated in the next check.

(6) The final values are checked by the identity

$$\sum_{i=1}^{j} E(x_i x_j; N) = 1.$$

The sum includes the variances, which satisfy the identity

$$\sum_{j=1}^{N} E(x_j x_j; N) = N.$$

These two relations are satisfied to within one or two units in the 20th DP.

(7) In addition, for $N \leqq 6$, the closed-form expressions as given by Jones [7] and Godwin [3] were computed to 32 DP and used as checks.

4. Tables. Table I gives $E(x_i\,;N)$, and Table II gives $E(x_ix_j\,;N)$, rounded to 10 DP. Missing values may be obtained by symmetry relations, e.g.,

(i) $E(x_i\,;N) = -E(x_{N-i+1}\,;N)$

(ii) If N is odd, the center order statistic has its mean equal to zero.

(iii) $E(x_ix_j\,;N) = E(x_j\,,x_i\,;N)$

(iv) $E(x_ix_j\,;N) = E(x_{N-i+1}\,,x_{N-j+1}\,;N)$.

Table III gives $\psi(a)$ and Table IV gives $\psi(a, b)$ rounded to 25 DP. Again, missing values can be obtained by symmetry

(v) $\psi(a, b) = \psi(b, a)$.

The values given in Tables I and II were computed to 20 DP; they are given in this paper to 10 DP mainly because this should be sufficient for most purposes. The complete table can be obtained on 925 punched cards by writing the Director, Numerical Analysis Research, University of California, Los Angeles 24, Calif., requesting table number 5024. A charge of three new cards for each one used will be made. A typed version of the tables can be borrowed from UMT of MTAC by writing the Editor, Mathematical Tables and Other Aids to Computation, University of California, Los Angeles 24, Calif. A similar arrangement holds for table number 5023 which gives on 1180 punched cards the values for $f(x)$ and $F(x)$ for $x = -12.00$ (.02) 12.00 to 27 DP. Tables of $[F(x)]^k$ and $H(x; k)$ for $k = 1(1)19$ and $x = -12.00(.02)$ 12.00 are also available; correspondence regarding these should be sent directly to the author.

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TABLE I

Expected values of order statistics

N	i	$E(x_i, N)$	N	i	$E(x_i, N)$	N	i	$E(x_i, N)$
2	1	.56418 95835	12	5	.31224 88787	17	5	.61945 76511
3	1	.84628 43753	12	6	.10258 96798	17	6	.45133 34467
4	1	1.02937 53730	13	1	1.66799 01770	17	7	.29518 64872
4	2	.29701 13823	13	2	1.16407 71937	17	8	.14598 74231
5	1	1.16296 44736	13	3	.84983 46324	18	1	1.82003 18790
5	2	.49501 89705	13	4	.60285 00882	18	2	1.35041 37134
6	1	1.26720 63606	13	5	.38832 71210	18	3	1.06572 81829
6	2	.64175 50388	13	6	.19052 36911	18	4	.84812 50190
6	3	.20154 68338	14	1	1.70338 15541	18	5	.66479 46127
7	1	1.35217 83756	14	2	1.20790 22754	18	6	.50158 15510
7	2	.75737 42706	14	3	.90112 67039	18	7	.35083 72382
7	3	.35270 69592	14	4	.66176 37035	18	8	.20773 53071
8	1	1.42360 03060	14	5	.45556 60500	18	9	.06880 25682
8	2	.85222 48625	14	6	.26729 70489	19	1	1.84448 15116
8	3	.47282 24949	14	7	.08815 92141	19	2	1.37993 84915
8	4	.15251 43995	15	1	1.73591 34449	19	3	1.09945 30994
9	1	1.48501 31622	15	2	1.24793 50823	19	4	.88586 19615
9	2	.93229 74567	15	3	.94768 90303	19	5	.70661 14847
9	3	.57197 07829	15	4	.71487 73983	19	6	.54770 73710
9	4	.27452 59191	15	5	.51570 10430	19	7	.40164 22742
10	1	1.53875 27308	15	6	.33529 60639	19	8	.26374 28909
10	2	1.00135 70446	15	7	.16529 85263	19	9	.13072 48795
10	3	.65605 91057	16	1	1.76599 13931	20	1	1.86747 50598
10	4	.37576 46970	16	2	1.28474 42232	20	2	1.40760 40959
10	5	.12266 77523	16	3	.99027 10960	20	3	1.13094 80522
11	1	1.58643 63519	16	4	.76316 67458	20	4	.92098 17004
11	2	1.06191 65201	16	5	.57000 93557	20	5	.74538 30058
11	3	.72883 94047	16	6	.39622 27551	20	6	.59029 69215
11	4	.46197 83072	16	7	.23375 15785	20	7	.44833 17532
11	5	.22489 08792	16	8	.07728 74593	20	8	.31493 32416
12	1	1.62922 76399	17	1	1.79394 19809	20	9	.18695 73647
12	2	1.11573 21843	17	2	1.31878 19878	20	10	.06199 62865
12	3	.79283 81991	17	3	1.02946 09889			
12	4	.53684 30214	17	4	.80738 49287			

TABLE II

Expected values of products of normal order statistics

N	i	j	$E(x_i x_j ; N)$	N	i	j	$E(x_i x_j ; N)$	N	i	j	$E(x_i x_j ; N)$
1	1	1	1.00000 00000	8	1	1	2.39953 49747	10	1	6	−.13035 66254
2	1	1	1.00000 00000	8	1	2	1.39953 49747	10	1	7	−.52928 83257
2	2	2	1.00000 00000	8	1	3	.79907 62783	10	1	8	−.96842 82811
3	1	1	1.27566 44477	8	1	4	.31184 25735	10	1	9	−1.50680 02399
3	1	2	.27566 44477	8	1	5	−.14235 45216	10	1	10	−2.34106 10315
3	1	3	−.55132 88954	8	1	6	−.61290 27315	10	2	2	1.21724 00737
3	2	2	.44867 11046	8	1	7	−1.16492 90242	10	2	3	.80357 20248
4	1	1	1.55132 88954	8	1	8	−1.98980 25239	10	2	4	.48797 62226
4	1	2	.55132 88954	8	2	2	.96568 82621	10	2	5	.21257 70424
4	1	3	−.14772 81323	8	2	3	.56614 69584	10	2	6	−.04863 46765
4	1	4	−.95492 96586	8	2	4	.25323 98948	10	2	7	−.31404 67778
4	2	2	.44867 11046	8	2	5	−.03241 18438	10	2	8	−.60464 26847
4	2	3	.14772 81323	8	2	6	−.32422 86175	10	2	9	−.95934 47746
5	1	1	1.80002 04360	8	2	7	−.66304 06044	10	3	3	.60541 68331
5	1	2	.80002 04360	8	3	3	.42432 99017	10	3	4	.38032 60958
5	1	3	.14814 77252	8	3	4	.22447 06701	10	3	5	.18822 18294
5	1	4	−.46991 74988	8	3	5	.04885 15166	10	3	6	.00874 81054
5	1	5	−1.27827 10984	8	3	6	−.12574 39762	10	3	7	−.17160 54566
5	2	2	.55656 27332	8	4	4	.21044 68615	10	3	8	−.36738 03049
5	2	3	.20843 54440	8	4	5	.12591 48488	10	4	4	.29913 80219
5	2	4	−.09510 11144	9	1	1	2.56261 74183	10	4	5	.17360 31403
5	3	3	.28683 36616	9	1	2	1.56261 74183	10	4	6	.05969 16062
6	1	1	2.02173 90694	9	1	3	.97012 95851	10	4	7	−.05225 29049
6	1	2	1.02173 90694	9	1	4	.49898 17432	10	5	5	.16610 12814
6	1	3	.39483 66863	9	1	5	.07274 22354	10	5	6	.11055 15903
6	1	4	−.15297 20358	9	1	6	−.34819 14908	11	1	1	2.85002 77414
6	1	5	−.73587 22832	9	1	7	−.80030 77349	11	1	2	1.85002 77414
6	1	6	−1.54947 05060	9	1	8	−1.34438 03016	11	1	3	1.26861 57614
6	2	2	.69142 72690	9	1	9	−2.17420 88731	11	1	4	.81841 62399
6	2	3	.31832 96521	9	2	2	1.09487 54256	11	1	5	.42562 33724
6	2	4	.01032 03642	9	2	3	.68736 32588	11	1	6	.05720 07586
6	2	5	−.30594 40716	9	2	4	.37294 55079	11	1	7	−.30839 96597
6	3	3	.28683 36616	9	2	5	.09344 77394	11	1	8	−.69165 68331
6	3	4	.14265 16716	9	2	6	−.17939 36730	11	1	9	−1.12114 69906
7	1	1	2.22030 41356	9	2	7	−.47001 14367	11	1	10	−1.65524 31199
7	1	2	1.22030 41356	9	2	8	−.81746 39387	11	1	11	−2.49346 50118
7	1	3	.60903 83042	9	3	3	.51353 31898	11	2	2	1.33286 42755
7	1	4	.09848 68607	9	3	4	.29909 87825	11	2	3	.91427 62554
7	1	5	−.40036 28885	9	3	5	.11376 80176	11	2	4	.59773 16558
7	1	6	−.96418 63986	9	3	6	−.06365 82663	11	2	5	.32525 83655
7	1	7	−1.78358 41490	9	3	7	−.24991 53959	11	2	6	.07192 05024
7	2	2	.83034 86720	9	4	4	.24592 33257	11	2	7	−.17792 83736
7	2	3	.44161 45034	9	4	5	.13699 13669	11	2	8	−.43863 19457
7	2	4	.13072 98656	9	4	6	.03730 27039	11	2	9	−.72971 16588
7	2	5	−.16517 61670	9	5	5	.16610 12814	11	2	10	−1.09056 36980
7	2	6	−.49363 46110	10	1	1	2.71210 37899	11	3	3	.69693 11658
7	3	3	.34412 37617	10	1	2	1.71210 37899	11	3	4	.46367 52869
7	3	4	.16555 98429	10	1	3	1.12577 18388	11	3	5	.26655 00636
7	3	5	.00520 26434	10	1	4	.66645 83784	11	3	6	.08551 78832
7	4	4	.21044 68615	10	1	5	.25949 67065	11	3	7	−.09143 52295

TABLE II—*Continued*

N	i	j	$E(x_i x_j ; N)$	N	i	j	$E(x_i x_j ; N)$	N	i	j	$E(x_i x_j ; N)$
11	3	8	−.27482 06666	12	6	6	.13716 24335	14	1	1	3.20923 88213
11	3	9	−.47845 18709	12	6	7	.09786 99407	14	1	2	2.20923 88213
11	4	4	.36137 86128	13	1	1	3.09739 66149	14	1	3	1.63813 45584
11	4	5	.22376 99938	13	1	2	2.09739 66149	14	1	4	1.20610 76773
11	4	6	.10003 46585	13	1	3	1.52340 67031	14	1	5	.83996 85490
11	4	7	−.01901 81895	13	1	4	1.08641 29990	14	1	6	.50901 53339
11	4	8	−.14087 88129	13	1	5	.71318 92732	14	1	7	.19625 86980
11	5	5	.19021 69879	13	1	6	.37261 38249	14	1	8	−.11005 46103
11	5	6	.11674 49805	13	1	7	.04688 33088	14	1	9	−.42009 46865
11	5	7	.04861 76885	13	1	8	−.27717 83904	14	1	10	−.74496 56899
11	6	6	.13716 24335	13	1	9	−.61230 31771	14	1	11	−1.09989 97992
12	1	1	2.97801 90896	13	1	10	−.97461 57510	14	1	12	−1.51105 65052
12	1	2	1.97801 90896	13	1	11	−1.39066 20939	14	1	13	−2.03701 04294
12	1	3	1.40064 48721	13	1	12	−1.91878 35177	14	1	14	−2.88488 07386
12	1	4	.95770 81654	13	1	13	−2.76375 64087	14	2	2	1.64344 79321
12	1	5	.57581 29501	13	2	2	1.54548 87851	14	2	3	1.21455 21950
12	1	6	.22313 53113	13	2	3	1.11947 86969	14	2	4	.89599 83466
12	1	7	−.11947 98445	13	2	4	.80149 03085	14	2	5	.62879 95664
12	1	8	−.46770 36483	13	2	5	.53292 13392	14	2	6	.38887 15475
12	1	9	−.83919 55827	13	2	6	.28959 88669	14	2	7	.16317 73868
12	1	10	−1.26121 26489	13	2	7	.05804 57285	14	2	8	−.05711 69005
12	1	11	−1.79198 71742	13	2	8	−.17146 74891	14	2	9	−.27950 69979
12	1	12	−2.63376 05793	13	2	9	−.40813 32366	14	2	10	−.51204 55279
12	2	2	1.44212 29110	13	2	10	−.66340 38591	14	2	11	−.76565 95612
12	2	3	1.01949 71285	13	2	11	−.95595 83375	14	2	12	−1.05900 48647
12	2	4	.70216 89575	13	2	12	−1.32667 39000	14	2	13	−1.43374 15141
12	2	5	.43189 07057	13	3	3	.87361 06037	14	3	3	.95773 39030
12	2	6	.18424 85733	13	3	4	.62859 26962	14	3	4	.70831 46326
12	2	7	−.05500 35423	13	3	5	.42447 04964	14	3	5	.50164 08601
12	2	8	−.29717 48037	13	3	6	.24120 59303	14	3	6	.31754 40043
12	2	9	−.55469 83245	13	3	7	.06792 82354	14	3	7	.14535 11045
12	2	10	−.84647 59479	13	3	8	−.10299 14878	14	3	8	−.02200 85032
12	2	11	−1.21260 75731	13	3	9	−.27856 77916	14	3	9	−.19040 07284
12	3	3	.78657 10977	13	3	10	−.46735 91289	14	3	10	−.36600 58138
12	3	4	.54683 59754	13	3	11	−.68315 45225	14	3	11	−.55709 77133
12	3	5	.34582 33989	13	4	4	.49643 94108	14	3	12	−.77769 71294
12	3	6	.16355 98631	13	4	5	.34235 43058	14	4	4	.56515 85063
12	3	7	−.01121 56206	13	4	6	.20584 27845	14	4	5	.40517 01872
12	3	8	−.18712 43766	13	4	7	.07801 73339	14	4	6	.26424 37333
12	3	9	−.37334 70932	13	4	8	−.04713 55097	14	4	7	.13349 25589
12	3	10	−.58355 66485	13	4	9	−.17494 01662	14	4	8	.00719 05256
12	4	4	.42801 13701	13	4	10	−.31169 54238	14	4	9	−.11927 53893
12	4	5	.28119 74136	13	5	5	.27404 82785	14	4	10	−.25063 68524
12	4	6	.15023 90816	13	5	6	.17772 22865	14	4	11	−.39310 68524
12	4	7	.02616 74273	13	5	7	.08904 34754	14	5	5	.32464 16720
12	4	8	−.09754 90483	13	5	8	.00336 97700	14	5	6	.22054 62157
12	4	9	−.22753 83422	13	5	9	−.08317 48535	14	5	7	.12521 59995
12	5	5	.22811 30981	13	6	6	.15461 68094	14	5	8	.03405 57967
12	5	6	.14165 47372	13	6	7	.10168 24204	14	5	9	−.05648 46831
12	5	7	.06166 16395	13	6	8	.05212 01841	14	5	10	−.14990 02795
12	5	8	−.01660 20662	13	7	7	.11679 89950	14	6	6	.18298 01703

TABLE II—*Continued*

N	i	j	E(x_i x_j ; N)	N	i	j	E(x_i x_j ; N)	N	i	j	E(x_i x_j ; N)
14	6	7	.11970 52573	15	4	9	−.06206 91293	16	2	15	−1.62998 91758
14	6	8	.06039 70132	15	4	10	−.18987 67942	16	3	3	1.11697 54048
14	6	9	.00245 92097	15	4	11	−.32433 82747	16	3	4	.86061 26454
14	7	7	.11679 89950	15	4	12	−.47169 95126	16	3	5	.64998 26930
14	7	8	.08753 66786	15	5	5	.37781 74644	16	3	6	.46457 54507
15	1	1	3.31443 70586	15	5	6	.26743 31303	16	3	7	.29383 42833
15	1	2	2.31443 70586	15	5	7	.16683 37346	16	3	8	.13121 04477
15	1	3	1.74582 84742	15	5	8	.07143 31681	16	3	9	−.02809 89274
15	1	4	1.31802 46931	15	5	9	−.02212 21836	16	3	10	−.18827 94941
15	1	5	.95779 69592	15	5	10	−.11683 30234	16	3	11	−.35370 26425
15	1	6	.63469 79582	15	5	11	−.21603 47915	16	3	12	−.52983 60638
15	1	7	.33225 18229	15	6	6	.21829 00871	16	3	13	−.72482 70563
15	1	8	.03957 36673	15	6	7	.14689 22656	16	3	14	−.95327 73058
15	1	9	−.25204 03438	15	6	8	.08014 07559	16	4	4	.70028 92373
15	1	10	−.55108 35331	15	6	9	.01543 42647	16	4	5	.53126 35264
15	1	11	−.86769 12702	15	6	10	−.04944 10103	16	4	6	.38373 20754
15	1	12	−1.21655 26558	15	7	7	.13001 52951	16	4	7	.24869 15226
15	1	13	−1.62362 41465	15	7	8	.09004 99964	16	4	8	.12065 61178
15	1	14	−2.14777 39615	15	7	9	.05235 02295	16	4	9	−.00432 37162
15	1	15	−2.99828 17812	15	8	8	.10169 46521	16	4	10	−.12962 66569
15	2	2	1.73646 34988	16	1	1	3.41373 54094	16	4	11	−.25872 32978
15	2	3	1.30507 20833	16	1	2	2.41373 54094	16	4	12	−.39590 09182
15	2	4	.98602 72992	16	1	3	1.84731 12087	16	4	13	−.54749 81070
15	2	5	.71995 26573	16	1	4	1.42314 99069	16	5	5	.43226 23745
15	2	6	.48276 68106	16	1	5	1.06794 02886	16	5	6	.31667 39395
15	2	7	.26168 92701	16	1	6	.75138 84716	16	5	7	.21178 86399
15	2	8	.04842 38833	16	1	7	.45735 36462	16	5	8	.11300 34551
15	2	9	−.16355 24188	16	1	8	.17550 84595	16	5	9	.01708 18433
15	2	10	−.38050 99695	16	1	9	−.10195 11721	16	5	10	−.07867 66926
15	2	11	−.60984 62515	16	1	10	−.38202 22664	16	5	11	−.17698 22446
15	2	12	−.86220 53501	16	1	11	−.67219 06141	16	5	12	−.28112 23697
15	2	13	−1.15632 39995	16	1	12	−.98198 37155	16	6	6	.25803 86623
15	2	14	−1.53462 06103	16	1	13	−1.32575 02291	16	6	7	.18008 04100
15	3	3	1.03884 67484	16	1	14	−1.72935 17286	16	6	8	.10744 70169
15	3	4	.78569 53136	16	1	15	−2.25197 62116	16	6	9	.03753 15039
15	3	5	.57688 47969	16	1	16	−3.10489 68626	16	6	10	−.03176 42139
15	3	6	.39208 32453	16	2	2	1.82496 17979	16	6	11	−.10247 13993
15	3	7	.22070 74184	16	2	3	1.39138 59986	16	7	7	.15204 24618
15	3	8	.05601 36122	16	2	4	1.07191 00598	16	7	8	.10368 42572
15	3	9	−.10720 30892	16	2	5	.80677 53414	16	7	9	.05793 54921
15	3	10	−.27386 03347	16	2	6	.57185 42866	16	7	10	.01325 33917
15	3	11	−.44968 15299	16	2	7	.35451 43842	16	8	8	.10169 46521
15	3	12	−.64283 01302	16	2	8	.14679 55938	16	8	9	.07905 57704
15	3	13	−.86760 84623	16	2	9	−.05723 07939	17	1	1	3.50776 08345
15	4	4	.63328 25216	16	2	10	−.26280 91651	17	1	2	2.50776 08345
15	4	5	.46839 54140	16	2	11	−.47548 74045	17	1	3	1.94326 81594
15	4	6	.32386 61474	16	2	12	−.70227 00971	17	1	4	1.52228 66799
15	4	7	.19076 28673	16	2	13	−.95365 51101	17	1	5	1.17139 83159
15	4	8	.06351 75907	16	2	14	−1.24851 49137	17	1	6	.86039 87122

TABLE II—*Continued*

N	i	j	$E(x_i x_j ; N)$	N	i	j	$E(x_i x_j ; N)$	N	i	j	$E(x_i x_j ; N)$
17	1	7	.57337 10807	17	4	14	−.62058 23190	18	2	7	.52579 71599
17	1	8	.30036 02504	17	5	5	.48712 82192	18	2	8	.32629 69700
17	1	9	.03414 41055	17	5	6	.36715 49498	18	2	9	.13364 37283
17	1	10	−.23135 40122	17	5	7	.25870 89814	18	2	10	−.05636 68282
17	1	11	−.50210 08789	17	5	8	.15715 34507	18	2	11	−.24755 81181
17	1	12	−.78494 23030	17	5	9	.05931 87706	18	2	12	−.44393 11712
17	1	13	−1.08900 90086	17	5	10	−.03730 72491	18	2	13	−.65029 63787
17	1	14	−1.42843 26533	17	5	11	−.13505 67988	18	2	14	−.87326 71257
17	1	15	−1.82904 55966	17	5	12	−.23648 48775	18	2	15	−1.12325 89453
17	1	16	−2.35036 31645	17	5	13	−.34489 02158	18	2	16	−1.41948 44863
17	1	17	−3.20550 13557	17	6	6	.30058 43470	18	2	17	−1.80640 17049
17	2	2	1.90932 86076	17	6	7	.21720 87085	18	3	3	1.26467 64541
17	2	3	1.47382 12827	17	6	8	.13980 20328	18	3	4	1.00305 35893
17	2	4	1.15396 29569	17	6	9	.06574 42736	18	3	5	.78948 03338
17	2	5	.88962 66310	17	6	10	−.00698 59665	18	3	6	.60308 20649
17	2	6	.65661 02659	17	6	11	−.08021 38072	18	3	7	.43325 69926
17	2	7	.44236 27797	17	6	12	−.15588 96202	18	3	8	.27363 82127
17	2	8	.23913 96759	17	7	7	.18003 82402	18	3	9	.11984 10480
17	2	9	.04139 28192	17	7	8	.12491 29753	18	3	10	−.03157 75063
17	2	10	−.15549 06731	17	7	9	.07281 54074	18	3	11	−.18371 62727
17	2	11	−.35599 25332	17	7	10	.02217 32128	18	3	12	−.33978 91153
17	2	12	−.56521 21375	17	7	11	−.02837 24403	18	3	13	−.50363 38299
17	2	13	−.78991 25546	17	8	8	.11204 84927	18	3	14	−.68050 28532
17	2	14	−1.04052 60200	17	8	9	.08080 00267	18	3	15	−.87864 62568
17	2	15	−1.33608 86697	17	8	10	.05114 76686	18	3	16	−1.11326 04489
17	2	16	−1.72042 01008	17	9	9	.09004 65814	18	4	4	.82988 20810
17	3	3	1.19221 07252	18	1	1	3.59704 61702	18	4	5	.65422 63222
17	3	4	.93305 05306	18	1	2	2.59704 61702	18	4	6	.50196 17902
17	3	5	.72084 96577	18	1	3	2.03427 65308	18	4	7	.36390 60479
17	3	6	.53491 52167	18	1	4	1.61609 97450	18	4	8	.23461 65634
17	3	7	.36467 94144	18	1	5	1.26897 78155	18	4	9	.11039 26076
17	3	8	.20370 91772	18	1	6	.96275 44762	18	4	10	−.01163 48391
17	3	9	.04745 55487	18	1	7	.68166 51889	18	4	11	−.13401 60252
17	3	10	−.10781 56686	18	1	8	.41601 09008	18	4	12	−.25936 68761
17	3	11	−.26569 04144	18	1	9	.15896 16815	18	4	13	−.39078 45980
17	3	12	−.43021 07197	18	1	10	−.09496 18007	18	4	14	−.53248 36936
17	3	13	−.60670 27090	18	1	11	−.35079 10772	18	4	15	−.69106 12193
17	3	14	−.80335 02164	18	1	12	−.61383 47108	18	5	5	.54186 03092
17	3	15	−1.03505 57182	18	1	13	−.89051 42554	18	5	6	.41813 66297
17	4	4	.76587 72428	18	1	14	−1.18969 36460	18	5	7	.30668 07870
17	4	5	.59330 28054	18	1	15	−1.52536 57101	18	5	8	.20281 14988
17	4	6	.44322 64848	18	1	16	−1.92337 42230	18	5	9	.10339 39287
17	4	7	.30655 90118	18	1	17	−2.44355 91207	18	5	10	.00603 60908
17	4	8	.17785 06544	18	1	18	−3.30074 41351	18	5	11	−.09135 44997
17	4	9	.05330 57575	18	2	2	1.98991 01268	18	5	12	−.19089 31496
17	4	10	−.07014 40479	18	2	3	1.55267 97662	18	5	13	−.29505 55083
17	4	11	−.19540 29393	18	2	4	1.23247 94168	18	5	14	−.40718 36001
17	4	12	−.32570 55597	18	2	5	.96883 03606	18	6	6	.34482 47854
17	4	13	−.46527 83685	18	2	6	.73744 01802	18	6	7	.25689 37505

TABLE II—*Continued*

N	i	j	$E(x_i x_j ; N)$	N	i	j	$E(x_i x_j ; N)$	N	i	j	$E(x_i x_j ; N)$
18	6	8	.17553 00021	19	2	14	−.73123 82925	19	6	11	−.02134 96632
18	6	9	.09809 30677	19	2	15	−.95274 96978	19	6	12	−.09877 04413
18	6	10	.02260 96299	19	2	16	−1.20223 32941	19	6	13	−.17836 20745
18	6	11	−.05260 93423	19	2	17	−1.49908 24322	19	6	14	−.26205 43418
18	6	12	−.12923 63965	19	2	18	−1.88835 35110	19	7	7	.24693 38145
18	6	13	−.20919 60677	19	3	3	1.33451 10082	19	7	8	.18154 56358
18	7	7	.21210 34702	19	3	4	1.07070 03889	19	7	9	.12004 79540
18	7	8	.15139 92491	19	3	5	.85591 60662	19	7	10	.06082 97030
18	7	9	.09415 84056	19	3	6	.66910 59363	19	7	11	.00259 85845
18	7	10	.03878 84044	19	3	7	.49962 04556	19	7	12	−.05582 13320
18	7	11	−.01603 11780	19	3	8	.34112 70806	19	7	13	−.11565 43329
18	7	12	−.07156 68585	19	3	9	.18934 87555	19	8	8	.15239 43086
18	8	8	.12965 00217	19	3	10	.04103 65629	19	8	9	.10850 51122
18	8	9	.09146 89512	19	3	11	−.10659 12312	19	8	10	.06669 58229
18	8	10	.05509 64106	19	3	12	−.25623 38217	19	8	11	.02595 95147
18	8	11	.01955 77328	19	3	13	−.41086 52514	19	8	12	−.01458 51042
18	9	9	.09004 65814	19	3	14	−.57419 50647	19	9	9	.09837 66271
18	9	10	.07201 04387	19	3	15	−.75144 37762	19	9	10	.07327 03911
19	1	1	3.68204 78516	19	3	16	−.95094 41729	19	9	11	.04933 12957
19	1	2	2.68204 78516	19	3	17	−1.18817 56533	19	10	10	.08079 09751
19	1	3	2.12082 70910	19	4	4	.89222 54987	20	1	1	3.76315 97146
19	1	4	1.70514 62523	19	4	5	.71386 54325	20	1	2	2.76315 97146
19	1	5	1.36134 13030	19	4	6	.55969 65138	20	1	3	2.20334 06878
19	1	6	1.05927 66874	19	4	7	.42044 02316	20	1	4	1.78989 83552
19	1	7	.78329 22736	19	4	8	.29064 30231	20	1	5	1.44904 08118
19	1	8	.52386 95189	19	4	9	.16666 14590	20	1	6	1.15063 48881
19	1	9	.27445 15628	19	4	10	.04575 76598	20	1	7	.87909 21502
19	1	10	.02996 34144	19	4	11	−.07438 76891	20	1	8	.62502 36800
19	1	11	−.21401 84895	19	4	12	−.19600 29194	20	1	9	.38206 78460
19	1	12	−.46185 59408	19	4	13	−.32152 30588	20	1	10	.14543 27711
19	1	13	−.71841 79950	19	4	14	−.45396 68711	20	1	11	−.08889 26380
19	1	14	−.98983 53826	19	4	15	−.59756 57833	20	1	12	−.32465 42591
19	1	15	−1.28478 86665	19	4	16	−.75905 79000	20	1	13	−.56576 49939
19	1	16	−1.61718 28951	19	5	5	.59609 42648	20	1	14	−.81678 99399
19	1	17	−2.01289 86080	19	5	6	.46912 18881	20	1	15	−1.08365 51006
19	1	18	−2.53209 20354	19	5	7	.35508 47179	20	1	16	−1.37491 30326
19	1	19	−3.39117 37939	19	5	8	.24925 07652	20	1	17	−1.70441 51707
19	2	2	2.06701 59055	19	5	9	.14849 89037	20	1	18	−2.09809 45742
19	2	3	1.62823 66660	19	5	10	.05051 41639	20	1	19	−2.61641 25315
19	2	4	1.30772 81241	19	5	11	−.04663 86868	20	1	20	−3.47725 83786
19	2	5	1.04467 74622	19	5	12	−.14479 56323	20	2	2	2.14092 24544
19	2	6	.81469 35030	19	5	13	−.24594 14349	20	2	3	1.70074 14812
19	2	7	.60527 67432	19	5	14	−.35251 67925	20	2	4	1.37995 34182
19	2	8	.40891 41917	19	5	15	−.46792 44975	20	2	5	1.11742 89274
19	2	9	.22048 14684	19	6	6	.39000 52335	20	2	6	.88867 43301
19	2	10	.03604 90040	19	6	7	.29818 53404	20	2	7	.68118 45644
19	2	11	−.14777 85386	19	6	8	.21348 33614	20	2	8	.48750 54400
19	2	12	−.33432 31434	19	6	9	.13323 26697	20	2	9	.30263 12966
19	2	13	−.52726 99747	19	6	10	.05548 77905	20	2	10	.12282 27822

TABLE II—*Concluded*

N	i	j	$E(x_i x_j; N)$	N	i	j	$E(x_i x_j; N)$	N	i	j	$E(x_i x_j; N)$
20	2	11	−.05502 56801	20	4	6	.61628 48185	20	6	9	.17022 63337
20	2	12	−.23379 34563	20	4	7	.47597 85179	20	6	10	.09058 72810
20	2	13	−.41646 98104	20	4	8	.34573 32605	20	6	11	.01240 45909
20	2	14	−.60652 56628	20	4	9	.22193 82389	20	6	12	−.06569 00797
20	2	15	−.80845 17036	20	4	10	.10194 29857	20	6	13	−.14506 55681
20	2	16	−1.02871 53688	20	4	11	−.01641 62785	20	6	14	−.22726 43343
20	2	17	−1.27777 82114	20	4	12	−.13511 33623	20	6	15	−.31423 93531
20	2	18	−1.57521 34603	20	4	13	−.25616 84132	20	7	7	.28361 37564
20	2	19	−1.96663 85237	20	4	14	−.38190 08258	20	7	8	.21423 29399
20	3	3	1.40185 69655	20	4	15	−.51528 76108	20	7	9	.14914 96896
20	3	4	1.13608 73612	20	4	16	−.66059 43627	20	7	10	.08673 36466
20	3	5	.92022 49684	20	4	17	−.82470 02582	20	7	11	.02571 07728
20	3	6	.73304 61715	20	5	5	.64959 18260	20	7	12	−.03503 06974
20	3	7	.56384 55907	20	5	6	.51977 46692	20	7	13	−.09658 24633
20	3	8	.40630 41631	20	5	7	.40349 64454	20	7	14	−.16015 53620
20	3	9	.25621 53876	20	5	8	.29597 10292	20	8	8	.17881 39224
20	3	10	.11046 28149	20	5	9	.19407 70951	20	8	9	.13013 82496
20	3	11	−.03352 11508	20	5	10	.09554 84059	20	8	10	.08383 50290
20	3	12	−.17809 92725	20	5	11	−.00144 47474	20	8	11	.03887 50395
20	3	13	−.32570 85570	20	5	12	−.09855 34351	20	8	12	−.00561 46394
20	3	14	−.47916 42651	20	5	13	−.19745 10463	20	8	13	−.05046 69633
20	3	15	−.64209 57155	20	5	14	−.30004 37127	20	9	9	.11276 48879
20	3	16	−.81971 78215	20	5	15	−.40876 40897	20	9	10	.08184 33087
20	3	17	−1.02045 47229	20	5	16	−.52708 49053	20	9	11	.05222 70111
20	3	18	−1.26005 60520	20	6	6	.43560 15809	20	9	12	.02326 98571
20	4	4	.95288 39168	20	6	7	.34041 91897	20	10	10	.08079 09751
20	4	5	.77212 83437	20	6	8	.25285 97019	20	10	11	.06608 30803

TABLE III

Table of ψ (a)

a	ψ (a)
1	.56418 95835 47756 28694 80786
2	.09900 37940 91548 44183 91722
3	.02015 46833 80170 42508 13857
4	.00435 75542 71626 37697 33560
5	.00097 35535 89558 23860 49576
6	.00022 20555 84024 11010 03862
7	.00005 13748 33363 61937 65236
8	.00001 20104 83187 26023 22116
9	.00000 28302 16707 39166 75167
10	.00000 06711 15270 89017 24362

TABLE IV

Table of ψ (a, b)

a	b	ψ (a, b)
1	1	.50000 00000 00000 00000 00000
1	2	.19550 11094 77885 32095 55017
1	3	.11216 77761 44551 98762 21684
1	4	.07565 42297 25579 61175 38235
1	5	.05580 73499 73723 90255 21453
1	6	.04357 46097 43013 49083 54940
1	7	.03538 45804 63369 80517 52981
1	8	.02956 98892 94334 34717 07091
1	9	.02525 68141 89549 41207 16719
1	10	.02194 71151 86726 20600 60009
1	11	.01933 76592 27269 66560 41992
1	12	.01723 43142 35855 87962 30710
1	13	.01550 74682 41634 55219 43757
1	14	.01406 75480 05471 95895 85675
1	15	.01285 08246 43521 88637 11122
1	16	.01181 08168 58215 58441 63317
1	17	.01091 28876 84164 70642 54955
1	18	.01013 07382 60934 15624 57146
1	19	.00944 40684 94693 48610 45567
2	2	.05015 71620 47665 97767 98872
2	3	.02132 62753 45802 91857 50860
2	4	.01138 67208 79221 55199 98228
2	5	.00693 43673 07328 01837 19221
2	6	.00460 22186 96898 56707 40120
2	7	.00324 55846 91894 21587 66344
2	8	.00239 45873 02798 23952 58542
2	9	.00182 94811 85965 13734 14246
2	10	.00143 71193 31165 89338 99162
2	11	.00115 47563 77785 94265 26607
2	12	.00094 54804 67238 23095 16447
2	13	.00078 65051 45993 09801 74959
2	14	.00066 31883 56560 56285 52932
2	15	.00056 57969 39184 89389 61034
2	16	.00048 76700 93061 07253 01059
2	17	.00042 41332 79783 95640 18455
2	18	.00037 18327 83094 89632 43507
2	19	.00032 83152 14978 92123 21206
3	3	.00720 90995 02436 29975 65961
3	4	.00319 21562 48137 39221 17817
3	5	.00165 92448 49199 72636 06090
3	6	.00095 98747 05188 13651 02138
3	7	.00059 95944 31303 32285 18810
3	8	.00039 68460 74933 83813 31343
3	9	.00027 47966 29506 92694 00407
3	10	.00019 73157 72790 13119 65464
3	11	.00014 59658 57472 21653 05727
3	12	.00011 07018 05271 17245 59531

TABLE IV—*Continued*

a	b	$\psi(a, b)$
3	13	.00008 57494 20745 29354 41430
3	14	.00006 76374 31474 50937 57570
3	15	.00005 41981 35043 80420 32791
3	16	.00004 40328 83934 46386 89237
3	17	.00003 62132 28079 75117 99695
3	18	.00003 01072 80833 83210 31975
3	19	.00002 52753 96848 34490 95730
4	4	.00120 76001 82167 82352 26105
4	5	.00054 77852 11336 04939 03183
4	6	.00028 10310 06320 11589 21149
4	7	.00015 76600 19741 38803 68994
4	8	.00009 46764 05000 10155 78414
4	9	.00005 99853 69007 17101 90664
4	10	.00003 96902 11683 28478 11889
4	11	.00002 72194 71775 97915 22185
4	12	.00001 92375 16174 56717 72636
4	13	.00001 39495 48566 95416 17691
4	14	.00001 03414 61725 18665 97688
4	15	.00000 78159 05356 05535 22392
4	16	.00000 60081 58453 34511 61894
4	17	.00000 46884 59596 75746 80849
4	18	.00000 37080 29284 86122 61652
4	19	.00000 29681 47498 09241 60418
5	5	.00022 04352 81924 64868 68374
5	6	.00010 16119 70163 35119 22870
5	7	.00005 17462 48306 80576 76418
5	8	.00002 84461 98484 02076 92270
5	9	.00001 66157 36979 80989 52093
5	10	.00001 01966 85130 01531 28746
5	11	.00000 65193 63290 35670 04499
5	12	.00000 43150 18311 90904 55499
5	13	.00000 29418 67127 84332 51278
5	14	.00000 20577 79587 34297 33162
5	15	.00000 14720 09436 42762 11034
5	16	.00000 10740 09945 28233 99675
5	17	.00000 07975 08312 69926 99696
5	18	.00000 06015 70599 45933 50524
5	19	.00000 04602 34345 95088 33995
6	6	.00004 25243 42919 47828 83531
6	7	.00001 98283 69359 42453 27161
6	8	.00001 00513 90400 16675 62086
6	9	.00000 54466 38456 73171 00400
6	10	.00000 31169 02107 77339 59195
6	11	.00000 18667 24719 96959 55595
6	12	.00000 11619 46631 41218 49052
6	13	.00000 07476 11073 65175 47454
6	14	.00000 04950 58796 83923 47338
6	15	.00000 03361 94671 97672 93207
6	16	.00000 02334 58274 02965 95783

TABLE IV—*Continued*

a	b	ψ (a, b)
6	17	.00000 01653 68708 77040 26861
6	18	.00000 01192 42176 55398 09875
6	19	.00000 00873 73236 45446 11208
7	7	.00000 85263 15970 45567 62570
7	8	.00000 40100 97465 04941 91140
7	9	.00000 20265 99629 82427 26539
7	10	.00000 10865 07155 13142 50967
7	11	.00000 06120 37985 00067 05510
7	12	.00000 03595 78188 47635 74603
7	13	.00000 02190 51850 55752 58827
7	14	.00000 01377 23312 93196 02647
7	15	.00000 00890 26140 99352 28361
7	16	.00000 00589 80231 05959 92095
7	17	.00000 00399 41744 31029 13351
7	18	.00000 00275 87288 77426 51883
7	19	.00000 00193 96542 56110 57069
8	8	.00000 17590 42868 28919 40610
8	9	.00000 08328 88124 97958 28581
8	10	.00000 04200 22631 65991 36053
8	11	.00000 02233 38303 77997 47451
8	12	.00000 01242 45102 77378 15424
8	13	.00000 00718 70359 09459 99756
8	14	.00000 00430 15369 82998 01350
8	15	.00000 00265 30223 60700 01871
8	16	.00000 00168 05142 13726 33829
8	17	.00000 00109 01922 50194 42794
8	18	.00000 00072 25796 62989 05235
8	19	.00000 00048 83175 48637 35200
9	9	.00000 03709 61389 15222 86735
9	10	.00000 01765 94102 88575 28247
9	11	.00000 00889 17735 02404 15631
9	12	.00000 00469 72562 05520 73927
9	13	.00000 00258 67365 43949 17592
9	14	.00000 00147 72632 94183 79478
9	15	.00000 00087 11889 78661 48239
9	16	.00000 00052 86627 59071 61725
9	17	.00000 00032 91278 73157 81782
9	18	.00000 00020 96874 99904 65848
9	19	.00000 00013 64148 37299 24226
10	10	.00000 00796 01729 04764 44595
10	11	.00000 00380 64032 71975 08928
10	12	.00000 00191 43513 61245 61666
10	13	.00000 00100 59508 83756 36067
10	14	.00000 00054 93334 86740 22140
10	15	.00000 00031 03648 84426 10042
10	16	.00000 00018 07512 14271 91775
10	17	.00000 00010 81707 19966 49114
10	18	.00000 00006 63448 61871 47304
10	19	.00000 00004 16088 73981 22512

TABLE IV—*Concluded*

a	b	$\psi(a, b)$
11	11	.00000 00173 27384 86156 31589
11	12	.00000 00083 15615 35388 09490
11	13	.00000 00041 78423 30143 32371
11	14	.00000 00021 86071 81467 08134
11	15	.00000 00011 85372 23133 22632
11	16	.00000 00006 63617 77190 91648
11	17	.00000 00003 82338 97530 22544
11	18	.00000 00002 26072 77303 26465
11	19	.00000 00001 36863 22260 79079
12	12	.00000 00038 16117 27130 13665
12	13	.00000 00018 37120 27936 28823
12	14	.00000 00009 22465 26120 70723
12	15	.00000 00004 80842 22868 05723
12	16	.00000 00002 59164 95414 97468
12	17	.00000 00001 43952 02115 97277
12	18	.00000 00000 82164 91287 93300
12	19	.00000 00000 48074 38746 06020
13	13	.00000 00008 48820 90858 46154
13	14	.00000 00004 09726 88769 84710
13	15	.00000 00002 05618 54654 90472
13	16	.00000 00001 06844 58427 48192
13	17	.00000 00000 57288 72363 36801
13	18	.00000 00000 31603 35816 41955
13	19	.00000 00000 17891 30211 80750
14	14	.00000 00001 90409 96774 61853
14	15	.00000 00000 92124 85117 15084
14	16	.00000 00000 46211 04276 26404
14	17	.00000 00000 23947 60524 37080
14	18	.00000 00000 12782 39493 37525
14	19	.00000 00000 07009 09859 73929
15	15	.00000 00000 43027 27493 21081
15	16	.00000 00000 20860 01282 43064
15	17	.00000 00000 10459 71110 66198
15	18	.00000 00000 05407 72128 71865
15	19	.00000 00000 02874 98777 38563
16	16	.00000 00000 09785 31923 99285
16	17	.00000 00000 04752 54436 40160
16	18	.00000 00000 02382 29028 10025
16	19	.00000 00000 01229 11411 40161
17	17	.00000 00000 02237 95808 85931
17	18	.00000 00000 01088 67316 53745
17	19	.00000 00000 00545 57053 31324
18	18	.00000 00000 00514 39826 97744
18	19	.00000 00000 00250 59182 84180
19	19	.00000 00000 00118 76445 88778