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TARGET HIT INTERCEPTOR MID-COURSE GUIDANCE SCHEME FOR BALLISTIC MISSILE INTERCEPTION

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Abstract- A Mid-course guidance algorithm has been developed for an exo-atmospheric interceptor to neutralize an incoming high speed ballistic Target. The guidance scheme positions the interceptor ahead of the target so that the velocity vectors of the target and interceptors are in the same direction. As the interceptors used in ballistic missile defence have a lower velocity than the incoming target, the target approaches the interceptor. The guidance scheme reduces the closing velocity compared to a head on approach thereby increasing the homing time for a given terminal sensor detection range. The guidance law is validated through numerical simulation.

Keywords- *Ballistic Missile Defence, THI Guidance, Mid-Course Trajectory Shaping, Exo-atmospheric Intercept.*

Nomenclature
S_r Seeker detection range
V_C Closing Velocity
V_m Missile Velocity
γ_p Flight path angle in elevation plane
γ_y Flight path angle in azimuth plane
μ Geocentric latitude
λ Geocentric longitude
m Vehicle mass
T Missile Thrust
C_{Do}, $C_{N\alpha}$ aerodynamic coefficients
ρ density
S Effective surface area
α, β angle of attack in elevation and azimuth planes
ω Earth angular speed
ΔDR Intercept point down range error
ΔCR Intercept point cross range error

1. INTRODUCTION

The interception of high-speed ballistic missiles has been engaging the attention of the guidance designers for the last two decades. The trajectory dynamics of ballistic missiles have been covered in many texts[1, 2]. The basic of ballistic missile intercept has been explained by Zarchan[3]. The paper by Lukacs and. Yakimenko[4] covers the existing guidance laws and develops a mid course trajectory shaping law. A sub-optimal mid course guidance was developed by Song and Takht[5].

The engagement process of the interceptor normally consists of the following phases namely (a) Ground Guidance Computation (b) Launch process (c) Mid-Course Guidance (d) Coast phase and (e) Terminal Homing Phase. The ground guidance computation determines the appropriate launch time of the interceptor, its initial flight azimuth along with the initial guidance parameters of the interceptor. The launch process consists of the actual launch of the missile, a vertical rise period culminating in a pitch down in the desired azimuth. The mid course phase is the trajectory shaping phase based on ground uplinked target state vector, lasting till burnout. During the coast phase the kill vehicle covers ground range and gains altitude. The terminal phase is when the interceptor acquires the target using its own onboard sensor and performs maneuvers to nullify the handing over errors.

A new approach for the terminal phase guidance called ‘Head Pursuit Guidance’ was introduced by Shima[6]. In this scheme, the interceptor velocity matched with that of the target by a preliminary maneuver. The interceptor trajectory is so shaped such that it is ahead of the target and flying in the same direction. As the speed of the interceptor is significantly lower than that of the target, the target catches up with the interceptor. After acquisition of the target using onboard sensors, the interceptor carries out the required terminal maneuvers to hit the target. As the target is overtaking the interceptor, the phase ‘Target Hit Interceptor’ (THI) seems more appropriate than ‘Head Pursuit’, which is therefore used in this study. The preliminary interceptor maneuver as the mid course phase guidance is the area of study of this paper.

A major advantage of THI guidance scheme is that the homing time for a given scenario for an onboard terminal sensor is significantly increased. The classical intercept geometry is shown in Fig1.

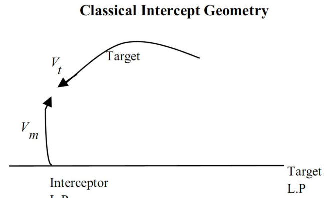


Fig1:

Let V_t and V_m be the velocities of the target and interceptor respectively, then the closing velocity V_c is approximately

$$V_c \approx V_t - V_m \qquad [1]$$

If S_r is the sensor detection range, the homing time t_h is given by

$$t_h = \frac{S_r}{V_c} \qquad [2]$$

The geometry for THI guidance is shown below in Fig2.

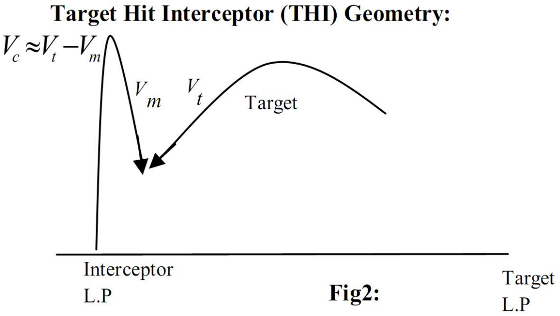


Fig2:

The closing velocity is now $V_c \approx V_t - V_m$ [3]

$$t_h = \frac{S_r}{V_c} \qquad [4]$$

As closing velocity for the second case is markedly lower than the first, one obtains a larger homing time. Larger homing time naturally reduces the divert capability requirement thus leading to a much lighter kill vehicle. The above stated advantage not withstanding a major penalty is the increased flight time of the interceptor. This in turn implies a larger weapon system reaction time along with increased ground sensor range.

The reminder of the paper is organized as follows. The next section covers the coordinate frames and the interceptor mathematical model is explained in section 3. The guidance law and the solution methodology are covered in section 4 and 5. The numerical simulation results are presented and followed by the conclusion section.

2. REFERENCE CO-ORDINATE FRAMES

The co-ordinate frames used in the study are Earth Centered Earth Fixed (ECEF), Local East-North-Vertical (ENV) and the Down range-Cross range-Height (DCH) frame. The ECEF frame has its origin at the center of the earth and rotates with the earth. The X-axis lies in the equatorial plane going through the intersection of Greenwich meridian and the equator, the Z-axis goes through the North pole and the Y-axis completes the right-handed triad. The Local ENV frame has its center at the interceptor's center of gravity with its X-axis pointing towards local east, Y-axis pointing towards local north (Fig 3) and the Z-axis pointing in the local vertical direction. The DCH frame has its center at the interceptor's Launch point with its X-axis pointing towards intercept point, the Z-axis pointing in the local vertical direction and the Y-axis completes the right-handed triad.

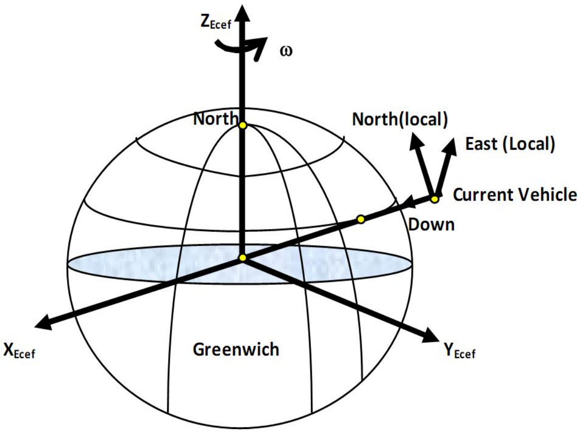


Fig: 3 Reference Frames- ECEF and Local ENV

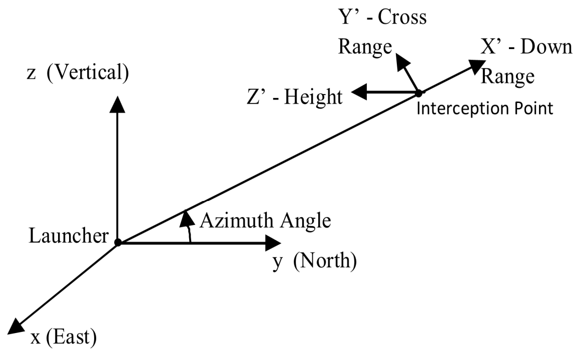


Fig4: Reference Frames- DCH

3. VEHICLE MATHEMATICAL MODEL

The target and the interceptor are modeled as point mass, traveling over a spherical rotating earth. As the target is considered in the exo-atmospheric phase, the propulsive and aerodynamic terms are zero. The equations of motion are given below equations from 5 to 12

$$\dot{V} = \frac{\left(T\cos\alpha - \frac{1}{2}\rho V^2 S C_{D0}\right)}{m} - g\sin\gamma_p \qquad [5]$$

$$\dot{\gamma}_p = \dot{\gamma}_{pdemand} \qquad if \quad t \le t_{guid} \qquad [6]$$

$$\dot{\gamma}_p = \frac{-g\cos\gamma_p}{V} + \frac{V}{r}\cos\gamma_p + 2\omega\cos\gamma_y\cos\lambda + \omega^2 r\cos\lambda(\cos\gamma_p\cos\lambda + \sin\gamma_p\sin\gamma_y\sin\lambda)$$

$$if \quad t > t_{guid} \qquad [7]$$

$$\dot{\gamma}_y = \dot{\gamma}_{ydemand} \quad if \quad t \le t_{guid} \qquad [8]$$

$$\dot{\gamma}_y = -\frac{V\tan\lambda\cos\gamma_p\cos\gamma_y}{r} + 2\omega(\tan\gamma\sin\gamma_y\cos\lambda - \sin\lambda) - \frac{\omega^2 r}{\cos\gamma_p}\cos\gamma_y\sin\lambda\cos\lambda$$

$$if \quad t > t_{guid} \qquad [9]$$

$$\dot{r} = V\sin\gamma_p \qquad [10]$$

$$\dot{\mu} = \frac{V\cos\gamma_p\cos\gamma_y}{r\cos\lambda} \qquad [11]$$

$$\dot{\lambda} = \frac{V\cos\gamma_p\sin\gamma_y}{r} \qquad [12]$$

4. GUIDANCE LAW

The mid course trajectory shaping guidance law is developed using an optimal control formulation and then converted to non-linear programming problem by discretizing the control vector. The problem can be stated as

$$\min \quad j = m_d(t_f) + k*(\gamma_{tf} - \gamma_{mf})$$

Where m_d is the miss distance γ_{tf}, γ_{mf} are the flight path angles of the target and miss distance t_f is the time of flight which is free k is the penalty weighting factor

The system differential equations are given by

$$\dot{x}^T = f(x^T, t) \qquad [13]$$

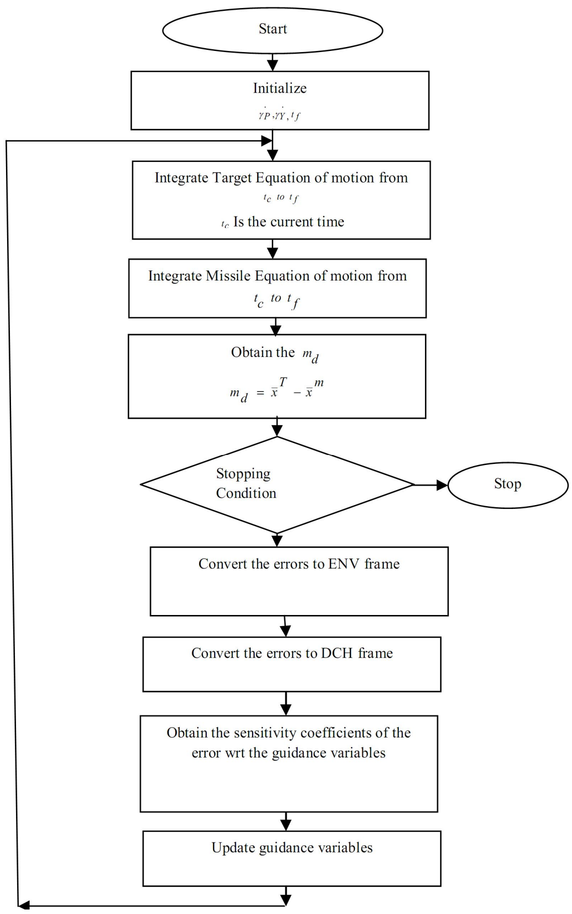
$$\dot{x}^m = f(x^m, u, t) \qquad [14]$$

Where x^T and x^m are the state vectors of the target and missile and the governing equations are given by equations 5 to 12.

The t_f is the flight time during ballistic phase.

In trajectory optimization problems, usually the angle of attack α, β are the control variables while in guidance problems, the lateral accelerations η_p, η_y are the control variables. In this study, the control variable is chosen as the flight path rates $\dot{\gamma}_p$ and $\dot{\gamma}_y$. Under trim conditions all the three are equivalent and the choice of $\dot{\gamma}_p$ and $\dot{\gamma}_y$ give direct control over the trajectory profile. The continuous optimal time problem is converted to a NLP format by assuming a set of control values at ‘n’ node points (Linearly interpolating between them). The present study uses a single $\dot{\gamma}$ value for simplicity. The simulation results confirm the adequacy of this assumption.

Essentially the guidance algorithm proceeds as follows as explained in the flow chart.



The guidance vector that is chosen here is $u = [\dot{\gamma}_p, \dot{\gamma}_y]^T$, where $\dot{\gamma}_p, \dot{\gamma}_y$ are the flight path angle rates of the missile in elevation and azimuth plane respectively. The flight path rates are the basic trajectory parameters which can be altered for lateral correction by the guidance system to achieve the desired interception point.

The guidance obtains initialization of the variables $\dot{\gamma}_p, \dot{\gamma}_y$ from the Ground guidance computation algorithm computed before the launch of interceptor. The following procedure is carried out at every guidance cycle to minimize the predicted miss distance. The initial values for $\dot{\gamma}_p, \dot{\gamma}_y, t_f$ are obtains from Ground guidance computation algorithm computed before the launch of interceptor. The guidance algorithm updates the above said parameters at every guidance cycle to minimizing the predicted miss distance.

The equations of motion of target and interceptor are integrated till t_f to calculate the final state vectors in ECEF frame. These are then transforms to DCH frame via an intermediate local ENV frame. The errors in DCH frame are related to the control variables $\gamma_p \,\&\, \gamma_y$

The down range error (ΔDR) and the cross range error (ΔCR) are the functions of the flight path angles γ_p & γ_y respectively and the time of correction is a function of range error in the velocity direction. Therefore,

$$\Delta\gamma_p = \left(\frac{\partial\gamma_p}{\partial DR}\right) * \Delta DR \qquad [15]$$

$$\Delta\gamma_y = \left(\frac{\partial\gamma_y}{\partial CR}\right) * \Delta CR \qquad [16]$$

$$\Delta t_c = -\left(\frac{\Delta x \Delta V_x + \Delta y \Delta V_y + \Delta z \Delta V_z}{\Delta V_x^2 + \Delta V_y^2 + \Delta V_z^2}\right) \qquad [17]$$

Where $\frac{\partial\gamma_p}{\partial DR}$ & $\frac{\partial\gamma_y}{\partial CR}$ are the sensitivity of down range & cross range w.r.to flight path angles. $\Delta\gamma_p \,\&\, \Delta\gamma_y$ are the angular changes required in the flight path angles to minimize the down range and cross range errors respectively. The required angular change is distributed from current time to closed loop guidance time (t_{guid}). Therefore the demanded flight path rate is

$$\dot{\gamma}_p = \dot{\gamma}_p + \frac{\Delta\gamma_p}{t_{guid} - t} \qquad [18]$$

$$\dot{\gamma}_y = \dot{\gamma}_y + \frac{\Delta\gamma_y}{t_{guid} - t} \qquad [19]$$

6. NUMERICAL SIMULATION RESULTS

The guidance algorithm was evaluated numerically using missile data published in ref4 and reproduced in the table 1.0. The following table 2.0 summarizes the miss-distance achieved for different initial conditions of the target. The error convergence is shown in Fig6&7 and variation of the guidance parameters are shown in Fig 8 & 9.

Table 1 Missile Data

$$m_0 = 907.2kg, g_0 = 9.81\frac{m}{s^2}, I_{sp} = 270\sec$$

$$T = \dot{m} g_0 I_{sp}, \dot{m} = \begin{cases} 27.06\frac{kg}{\sec}, 0 \le t \le 10\sec \\ 9.02\frac{kg}{\sec}, 10 \le t \le 57\sec \end{cases}$$

Aerodynamic Derivatives					
M	**0.00**	**0.60**	**1.00**	**1.07**	**1.14**
$C_{L\alpha}$	**10.04**	**10.80**	**13.21**	**14.16**	**13.04**
M	**1.20**	**1.50**	**2.00**	**2.50**	**≥ 3.00**
$C_{L\alpha}$	**12.60**	**11.50**	**10.49**	**9.58**	**8.62**
M	**0.00**	**0.80**	**0.90**	**1.00**	**1.05**
C_{D0}	**0.26**	**0.27**	**0.28**	**0.31**	**0.38**
M	**1.25**	**1.50**	**2.00**	**2.50**	**≥ 3.00**
C_{D0}	**0.36**	**0.34**	**0.29**	**0.26**	**0.21**

Table 2. Miss Distance data.

Ballistic Target Interception	Miss-Distance(m)
Case1: High Altitude	**9.15**
Case2: Medium Altitude	**2.53**
Case3: Low Altitude	**7.13**

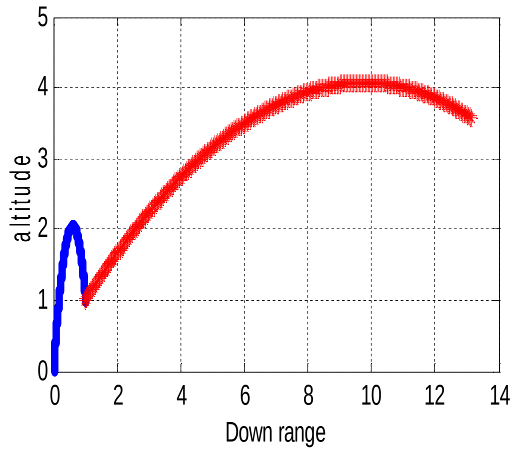


Fig5: Interception Geometry Pitch Elevation plane

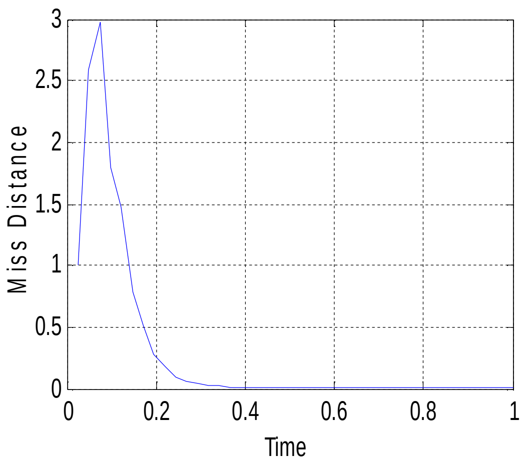


Fig6: Position Error Convergence

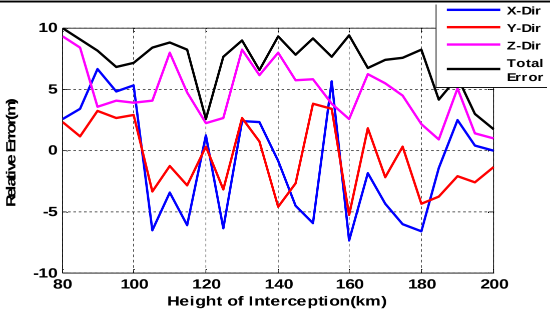


Fig7: Impact Error Vs Height of Interception

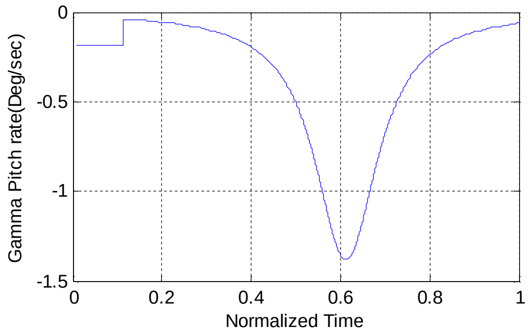


Fig8: Flight Path Rate in convergence

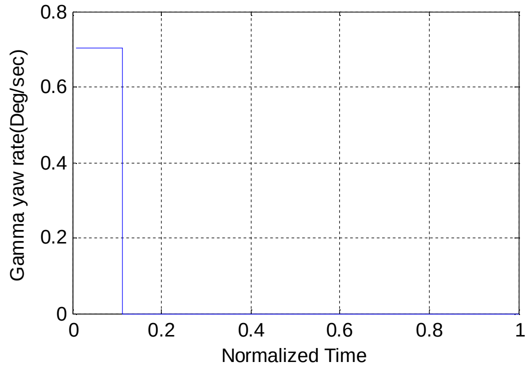


Fig9: Flight Path Rate in yaw

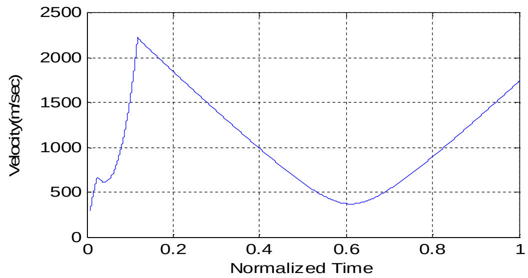


Fig10: Velocity Profile

7. CONCLUSION

A 'Target Hit Interceptor', THI guidance midcourse law has been developed for intercepting exo-atmospheric ballistic missiles. The law performs well and brings the miss distance to small values. It is proposed to evaluate the guidance algorithm using a higher fidelity 6-dof simulation test bed.

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