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Task Space Position Control of Slider-Crank Mechanisms Using Simple Tuning Techniques Without Linearization Methods

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ABSTRACT In this work, a position control in task space for slider-crank mechanisms is presented. In order to apply linear controllers it is required to linearize the mechanism dynamics at an equilibrium point. However, complete dynamic knowledge is needed and the linearization technique gives an oversimplified model that affects the control performance. In this work, it is proposed a novel method to design task space controllers without using the complete knowledge of the mechanism dynamics and linearization methods. From the extended dynamic model of parallel robots, it can be seen that the end-effector (slider) dynamics is expressed as a linear system that can be used directly for the control design instead of the complete mechanism linear dynamics. The approach requires a minimal knowledge of the mechanism dynamics and avoids linearization methods. To verify our approach, it is used pole placement and sliding mode controllers whose gains are tuned according to the slider dynamics. A linear sensor is mounted at the slider to measure its position and avoids considering noise and disturbances at links before the slider. Simulations and experiments are presented to validate our approach using two kinds of slider-crank mechanisms.

INDEX TERMS Slider-crank mechanism, extended dynamic model, linear system, controllable, disturbance, pole placement, sliding mode control.

I. INTRODUCTION

Position control is a well known problem for mechanical systems, specially in robots manipulators and mechanisms. The dynamic model of these systems is very non-linear [1], and the controllers design need to compensate the nonlinearities [2]. To satisfy the control objectives there are developed different controllers, such as PD+ [3], PID [4], [5], adaptive [6], [7], sliding mode [7]–[9], neural networks [10], among others.

Linear controllers such as PD or PD+ [11], [12] do not guarantee stability in tracking problems, meanwhile non-linear controllers such as adaptive or sliding mode can guarantee it [7], [13]. For simplicity, controllers without non-linear terms are preferred because only depends on state measures and gain tunings [11], such as linear controllers. There is a wide theory for linear systems [14] to achieve tracking tasks, but require a linear robot/mechanism dynamics.

PID control is the most popular controller for industrial applications. The gain tuning requires knowledge of the complete mechanism dynamics and its linear model obtained from a linearization method at a point of interest. However the integral term reduces the bandwidth and can destroy the closed-loop performance [7], [15]. Other controller widely used at the literature is sliding mode control (SMC) [8], [9] where the position control is guarantee by choosing a big enough gain such that the disturbances are compensated, however the sliding manifolds and controller gains are chosen arbitrarily because the mechanism dynamics is non-linear.

There exist several methods to linearize robot/mechanism models. The simplest method is when velocity and gravity are neglected, then the Coriolis matrix and the gravity forces vector are zero [16], however this is an oversimplified model [11]. If gravity is taken into account, it is obtained

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another way to linearize the dynamics [17], but it has been shown that the Coriolis effect even at low speeds should be accounted for [18]. The most used method is Taylor series expansion [19], but requires the knowledge of the robot dynamics model. All the above methods make the linearization at one operating point or equilibrium point, then the controller is restricted to areas near that point. In the special case of mechanisms, the linearization gives an oversimplified model and many information is lost since all the generalized coordinates depend on the independent coordinate. For this reason it is preferred to avoid linearization methods for closed chain mechanisms.

There exists a kind of mechanisms that has a linear movement at its end-effector, which are called slider-crank mechanisms [20]. This mechanisms are widely used for machine tools or molding machines [21] for cutting tasks. One of the most used slider-crank mechanism is the Whitworth mechanism [6], [21], [22] which is of our interest. Generally the Whitworth mechanism is controlled in joint space [23] such that the slider is controlled indirectly. Nevertheless, the precision of the controller is affected by noise measures, joint clearance and disturbances of the links before the slider [24], therefore it is preferred to control the mechanism dynamics in task space. The transformation from joint space to task space requires the Jacobian matrix. The task space model is more complex model than the joint space model and does not take advantage of the linear behavior of the slider.

In this work a novel method to design task space controllers of slider-crank mechanisms is presented. From the extended dynamic model is obtained the slider dynamics which is a simple double integrator system with a constant disturbance that requires minimal knowledge of the mechanism dynamics and avoids linearization methods. Also the precision problem is avoided by using real measures of a linear sensor mounted at the slider [25]. To prove our approach it is used two well known controllers, pole placement and sliding mode control, where its gains are tuned using the slider dynamics and linear control theory. The controllers are tested in simulations and experiments in two kind of slider-crank mechanisms. The results show the effectiveness of our approach.

The paper outline is as follows: Section II shows the non-linear dynamic model of a 1-degree of freedom (DOF) slider-crank mechanism and its linear model using Taylor series approximation; Section III gives the extended dynamic model of slider-crank mechanisms and the slider dynamics; in Section IV the controllers design (pole placement and SMC) is presented; Section V gives two examples of slider-crank mechanisms and their respective control gains tuning; Section VI shows the experimental results and the conclusions are given in Section VII.

II. DYNAMIC MODEL OF A 1-DOF CLOSED-CHAIN SLIDER-CRANK MECHANISM

The dynamic model in joint space of a 1-DOF closed-chain slider-crank mechanism is of the form:

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + G(q) = \tau, \tag{1}$$

where $M(q) \in \mathbb{R}$ is the mechanism inertia, $C(q,\dot{q}) \in \mathbb{R}$ is the Coriolis term, $G(q) \in \mathbb{R}$ is the gravitational term, $\tau \in \mathbb{R}$ is the driven torque and $q, \dot{q}, \ddot{q} \in \mathbb{R}$ are the position, velocity and acceleration of the generalized coordinate. In order to transform the model (1) to task space it is required the slider Jacobian term $\rho_x(q) \in \mathbb{R}$ as:

$$M_x\ddot{x} + C_x\dot{x} + G_x = \rho_x^{-1}(q)\tau = u, \tag{2}$$

where $x, \dot{x}, \ddot{x} \in \mathbb{R}$ are the position, velocity and acceleration of the slider. The terms M_x, C_x and G_x are obtained from the velocity kinematics relation $\dot{x} = \rho_x(q)\dot{q}$. The model (2) does not consider the main advantage of slider-crank mechanisms, i.e., the linear movement of the slider. The transformation from joint space to task space considers all the dynamics from the input link to the output link and yields a more complex model in comparison to the joint space model (1). The model (2) in state space is expressed as

$$\dot{z} = \frac{d}{dt}\begin{bmatrix} z_1 \\ z_2 \end{bmatrix} = \underbrace{\begin{bmatrix} z_2 \\ -M_x^{-1}(C_x z_2 + G_x) \end{bmatrix}}_{f(z)} + \underbrace{\begin{bmatrix} 0 \\ M_x^{-1} \end{bmatrix}}_{g(z)} u \tag{3}$$

The linear version of the mechanism dynamics is obtained using Taylor series as follows

$$\dot{z} = Az + Bu, \tag{4}$$

where $A = \left.\frac{\partial f(z)}{\partial z}\right|_{z=z(0)}$ and $B = \left.\frac{\partial g(z)}{\partial u}\right|_{u=u(0)}$, $z(0)$ and $u(0)$ are the linearization points. Also the models (1),(2) and (4) require knowledge of the complete dynamics and mainly its parameters for the controller design. However in most cases we do not have knowledge of the dynamics neither the parameters for the linear model and controller design, therefore we need to use identification methods or model-free controllers to overcome this issue. For system identification, the parameterization of the mechanism dynamics must be in joint space and requires knowledge of the kinematics equations because the secondary variables depends on the generalized coordinates; on the other hand, the gains of the model-free controllers need a manual tuning procedure which is not the aim of this work.

In the following section is shown one special property of slider-crank mechanisms which helps us in the controller design and avoids the knowledge of the complete mechanism dynamics and the use of linearization methods.

III. EXTENDED DYNAMIC MODEL OF A 1-DOF CLOSED-CHAIN SLIDER-CRANK MECHANISMS

In order to avoid the model (2) and the linear model (4) it is used the Euler-Lagrange formulation [26] for the extended dynamic model. The extended dynamic model of a 1-DOF closed-chain slider-crank mechanism is of the form:

$$\boldsymbol{M}'(\boldsymbol{q}')\ddot{\boldsymbol{q}}' + \boldsymbol{C}'(\boldsymbol{q}',\dot{\boldsymbol{q}}')\dot{\boldsymbol{q}}' + \boldsymbol{G}'(\boldsymbol{q}') = \boldsymbol{\rho}^{-\top}(\boldsymbol{q}')\tau \tag{5}$$

where $\boldsymbol{q}' \in \mathbb{R}^{n'}$ are the extended coordinates whose components are the generalized coordinate q and all the n secondary variables. Here $n' = n + 1$. $\boldsymbol{M}'(\boldsymbol{q}') \in \mathbb{R}^{n' \times n'}$ is the inertia matrix, $C'(q', \dot{q}') \in \mathbb{R}^{n' \times n'}$ represents the Coriolis and centrifugal terms, $G'(q') \in \mathbb{R}^{n'}$ is the gravity vector, $\tau \in \mathbb{R}$ is the control input and $\rho(q') \in \mathbb{R}^{n'}$ is the extended Jacobian vector. Notice that (5) is presented with the generalized coordinates q' instead of using the independent coordinate q. The extended dynamics is obtained by using the relation

$$\dot{q}' = \rho(q')\dot{q}.$$

For slider-crank mechanisms, the position of the slider x is the main control objective and a component of the generalized coordinate vector, i.e. $q' = [q \cdots x]^\top$. In matrix form we have:

$$M'(q') = \begin{bmatrix} M_{(n'-1)\times(n'-1)} & \mathbf{0}_{n'-1} \\ \mathbf{0}_{1\times(n'-1)} & m \end{bmatrix}$$
$$C'(q', \dot{q}') = \begin{bmatrix} C_{(n'-1)\times(n'-1)} & \mathbf{0}_{n'-1} \\ \mathbf{0}_{1\times(n'-1)} & 0 \end{bmatrix} \quad (6)$$
$$G'(q') = \begin{bmatrix} G_{n'-1} \\ v \end{bmatrix}$$
$$\rho(q') = \begin{bmatrix} \rho_{n'-1} \\ \rho_x(q') \end{bmatrix} \quad (7)$$

where m is the slider mass, v is the slider gravity force component that depends on the mechanism configuration, and $\rho_x(q')$ is the Jacobian component that gives the mapping between the joint velocity $\dot{q}$ and the slider velocity $\dot{x}$. For a task position control problem, we want to control the slider position to a desired position, then the control problem simplifies to:

$$m\ddot{x} + v = \left(\sum_{i=1}^{n'} \rho_i^2(q')\right)^{-1} \rho_x(q')\tau = u, \quad (8)$$

the above model only requires knowledge of the slider mass; also avoids the knowledge of the complete mechanism dynamics. By using this fact the following proposition is derived:

Proposition 1: For any slider-crank mechanism controlled in task space, its dynamics can be simplified as a linear system of the form (8) without any kind of linearization and minimal knowledge of its parameters.

The above proposition holds if it is considered that we have real measures of the slider position x, otherwise the controller is affected by noise and disturbances of the links before the slider. It is clear that (8) is a linear system with a disturbance [27]. In state space we have:

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= bu + d \end{aligned} \quad (9)$$

where $b = 1/m$ and $d = -v/m$. In matrix form:

$$\dot{X} = \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \underbrace{\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}}_{A} X + \underbrace{\begin{bmatrix} 0 \\ b \end{bmatrix}}_{B} u + \underbrace{\begin{bmatrix} 0 \\ 1 \end{bmatrix}}_{D} d \quad (10)$$

where $X = [x_1, x_2]^\top$. Since it is obtained a linear system it is possible to use any controller from linear system theory as it is demonstrated in the following section.

IV. CONTROLLERS DESIGN

It is designed two well known controllers, pole placement and sliding mode control. Before starting the controllers design, it is necessary to verify that the system is controllable. The controllability matrix of the system (10) is

$$\mathcal{C} = \begin{bmatrix} B & AB \end{bmatrix} = \begin{bmatrix} 0 & b \\ b & 0 \end{bmatrix} \quad (11)$$

which is full rank since $b \neq 0$, then the system is controllable.

A. POLE PLACEMENT

We want to design a controller such that the system follows a desired time-varying reference $r \in C^2$. Position error is defined as:

$$e = r - x = r - x_1 \quad (12)$$

Since the perturbation v only lies in the gravity force component of the slider, then d is bounded by the acceleration of gravity $g = 9.81\text{m/s}^2$. Therefore, the disturbance is known and can be compensated. The control law is of the form:

$$u = \frac{1}{b}(-d + k_1 e + k_2 \dot{e} - \ddot{r}) \quad (13)$$

where $k_1, k_2 > 0$. The closed-loop system of (10) under the control law (13) is:

$$\dot{E} = \frac{d}{dt}\begin{bmatrix} e \\ \dot{e} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -k_1 & -k_2 \end{bmatrix} E \quad (14)$$

where $E = [e, \dot{e}]^\top$. Whose characteristic polynomial is:

$$\lambda^2 + k_2\lambda + k_1$$

the closed loop system is stable and achieve the control objective by assigning the controller gains strictly positive. The desired system response can be obtained by:

$$\lambda^2 + 2\xi\omega_n\lambda + \omega_n^2 = \lambda^2 + k_2\lambda + k_1$$

where ω_n is the undamped natural frequency and ξ is the damping factor. One way to obtain the control gain is by means of the Ackermann formula [14]. Notice that we can apply different linear controllers for a non-linear system without any kind of linearization.

B. SLIDING MODE CONTROL

If we assume that the disturbance is unknown and bounded, then a simple way to compensate it is by using sliding mode control. For the sliding surface design gain C we can use the Ackermann-Utkin formula [28]. We have that the system in terms of the error is:

$$\begin{aligned} \dot{E} &= AE - B(u + F) + R \\ s &= CE \end{aligned} \quad (15)$$

where $R = [0, \ddot{r}]^\top$, $F = B^+ D$ and B^+ stands for the Moore-Penrose Pseudo-inverse of B, which verifies that the disturbance is coupled to the control. Taking the time derivative of the sliding surface (15) and equating to zero yields the equivalent control:

$$\dot{s} = \boldsymbol{C}\left(\boldsymbol{AE} - \boldsymbol{B}(u+F) + \boldsymbol{R}\right) = 0$$
$$u_{eq} = (\boldsymbol{CB})^{-1}\boldsymbol{C}\left(\boldsymbol{AE} + \boldsymbol{R}\right) - F \tag{16}$$

For our control design we have that $\boldsymbol{CB} = 1$. Then the closed-loop system under the equivalent control is:

$$\dot{\boldsymbol{E}} = (\boldsymbol{I} - \boldsymbol{BC})(\boldsymbol{AE} + \boldsymbol{R}) \tag{17}$$

Consider the next Lyapunov function:

$$V(s) = \frac{1}{2}\|s\|^2 \tag{18}$$

Taking the time derivative of V:

$$\dot{V} = s^\top\left(\boldsymbol{C}(\boldsymbol{AE} + \boldsymbol{R}) - u - F\right) \tag{19}$$

If we choose the control u as:

$$u = \boldsymbol{C}(\boldsymbol{AE} + \boldsymbol{R}) + Ksign(s) \tag{20}$$

then we have:

$$\dot{V} = s^\top\left(-Ksign(s) - F\right)$$
$$\leq -\|s\|(K - \bar{F}) \tag{21}$$

where $\|F\| \leq \bar{F}$, if we choose $K = \bar{F} + K_0$, with $K_0 > 0$, then the time derivative of V is:

$$\dot{V} \leq -K_0\|s\| \tag{22}$$

We can use more sophisticated algorithms to accomplish the control objective, but they are not relevant since the purpose of this work is to show that we can control this type of mechanisms using simple linear controllers based on the slider dynamics.

When the control u in (13) or (20) is applied to the mechanism dynamics, it needs to be transformed into control torque as:

$$\tau = \rho_x^{-1}(\boldsymbol{q}')u, \tag{23}$$

also the Jacobian component can be compensated by using the Jacobian compensator of our previous work [25]. Here we do not use the transformation $\tau = \rho_x(q)u$ since we do not want to control the force of the slider dynamics.

V. SLIDER-CRANK MECHANISM EXAMPLES

In this section two slider-crank mechanisms are analyzed, whose configuration differs in the location of the slider.

A. WHITWORTH MECHANISM

A widely used mechanism is the Whitworth mechanism shown in Figure 1.

The generalized coordinate vector is given by $\boldsymbol{q}' = [q, r_3, \theta_4, \theta_5, x]^\top$, the extended dynamic model is

$$\boldsymbol{M}'(\boldsymbol{q}') = \begin{bmatrix} M_{11} & 0 & 0 & 0 & 0 \\ 0 & M_{22} & 0 & 0 & 0 \\ 0 & 0 & M_{33} & M_{34} & 0 \\ 0 & 0 & M_{34} & M_{44} & 0 \\ 0 & 0 & 0 & 0 & m_6 \end{bmatrix}$$

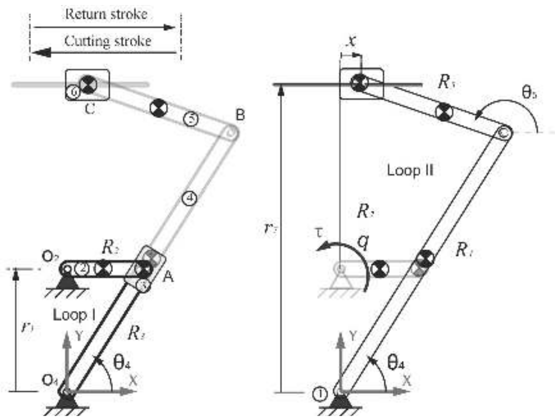

FIGURE 1. **Whitworth mechanism.**

$$\boldsymbol{C}'(\boldsymbol{q}', \dot{\boldsymbol{q}}') = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -C_{23} & 0 & 0 \\ 0 & C_{23} & C_{33} & C_{34} & 0 \\ 0 & 0 & C_{43} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$
$$\boldsymbol{G}(\boldsymbol{q}') = \begin{bmatrix} G_1 & G_2 & G_3 & G_4 & 0 \end{bmatrix}^\top$$
$$\boldsymbol{\rho}(\boldsymbol{q}') = \begin{bmatrix} 1 \\ \rho_{r_3} \\ \rho_{\theta_4} \\ \rho_{\theta_5} \\ -\dfrac{r_2 r_4 \cos(q-\theta_4)\sin(\theta_4-\theta_5)}{r_3\cos(\theta_5)} \end{bmatrix} \tag{24}$$

where m_6 is the slider mass, $v = 0$, and $\rho_x(\boldsymbol{q}') = -\frac{r_2 r_4}{r_3}\cos(q-\theta_4)\sin(\theta_4-\theta_5)\sec(\theta_5)$; the other terms are not relevant for this approach. Finally we have that the slider dynamic equation is:

$$m_6\ddot{x} = u, \tag{25}$$

that is a simple double integrator system without perturbation. The solution of $\boldsymbol{q}'$ is given in our previous work [25].

B. SIMULATIONS

The simulations are made using Matlab/Simulink® in a time of 10 seconds. It is used the slider dynamics (25) to design the pole placement and sliding mode control. The slider mass is $m_6 = 1$ kg and there it is not considered any disturbance.

For pole placement controller, it is proposed $\omega_n^2 = 15$ and $\xi = 1$, then:

$$k_1 = 15 \quad k_2 = 8.$$

For the SMC surface design, it is used the Ackermann-Utkin formula with a desired pole in $\lambda = -10$. Then the surface is given by:

$$C = \begin{bmatrix} 10 & 1 \end{bmatrix}.$$

The desired trajectory is:

$$r = -0.2 - 0.2\sin(\pi t) \tag{26}$$

The SMC gain is proposed as $K = 1$ since the desired trajectory and derivatives are bounded as $\|r\| \leq 1$.

Figure 2 and Figure 3 show the slider position tracking and the position error, respectively. Both controllers have good tracking results, where the pole placement controller presents more tracking error in comparison to the SMC. The main reason of this difference is that pole placement control law uses a feedforward term that depends on the value of b which affects the controller precision. On the other hand, the SMC avoids this problem by using a large enough control gain.

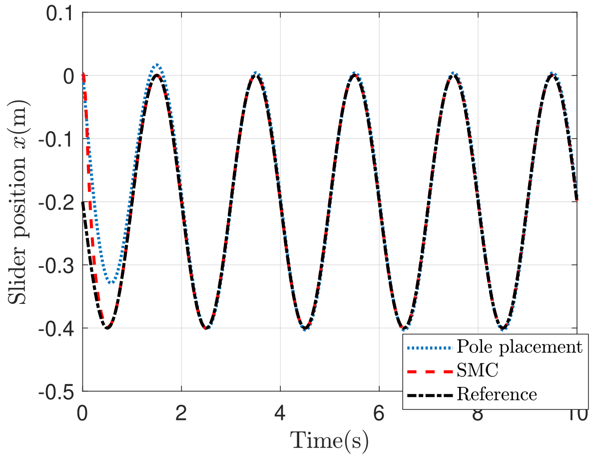

FIGURE 2. **Position tracking.**

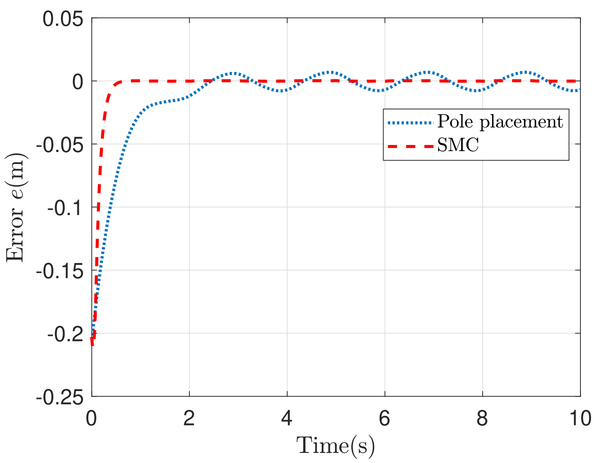

FIGURE 3. **Position error.**

It is used the Integral Squared Error (ISE) to make the comparisons between both controllers behavior. The ISE is given by (27)

$$ISE = \int_{t_0}^{t} (ke(\sigma))^2 d\sigma, \tag{27}$$

where k is a scaling factor. It is used a scaling factor of $k = 100$, and the integrator is reset in each period of the sine function to show how the position error decreases. The ISE result is given in Figure 4 where it is shown that both controllers achieve the control task by using the slider dynamics for the gain tuning. The pole placement can be improved by choosing other values for ω_n and ξ. SMC presents good performance since there is no disturbance and the slider dynamics is compensated. When there exists other disturbances, the gain of the SMC needs to be large enough such that $\|K\| \geq \bar{F}$.

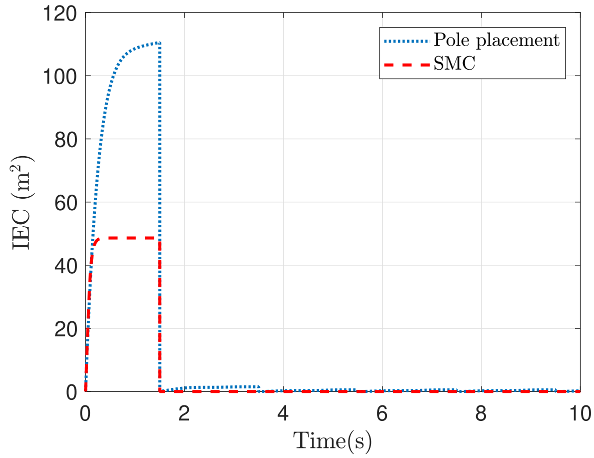

FIGURE 4. **ISE comparisons.**

1) SENSITIVITY ANALYSIS

Our slider dynamics model is very simple, then we can think that it cannot represent accurately the real dynamics [29]. Therefore the controller cannot guarantee good tracking performance because it was designed according to parameters that contain parametric error. The robustness of the controllers are tested considering the sensitivity of the control system to parametric errors [30]. Since the slider dynamics is a double integrator system we have only one parameter to evaluate. The parameter b has a random error of 50% of its real value: $\bar{b} = b + 0.5b\zeta(t)$, where $\zeta(t)$ is a normally distributed random function [31].

Figure 5 shows the tracking error of the Whitworth without/with parametric error. The results show similar

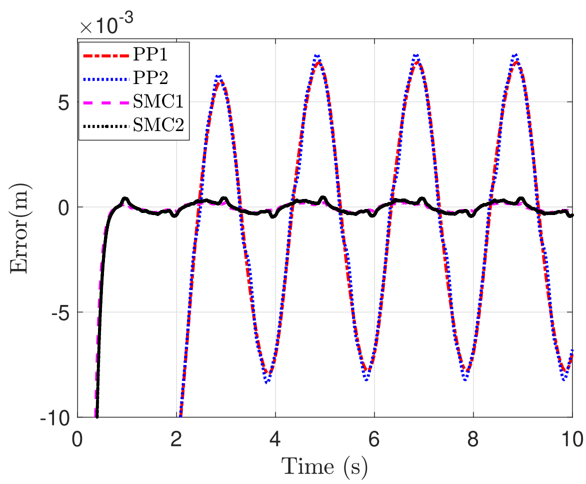

FIGURE 5. **Sensitivity analysis. *PP* = pole placement. 1 without parametric errors, 2 with parametric errors.**

responses since the parameter b only increases or decreases the amplitude of the control input. The control gains of the pole placement controller only depends of the value of b and since the random error is bounded then it does not affect the robustness of the controller. However as we already mentioned, the pole placement uses a feedforward control term which affects the tracking performance. On the other hand, SMC has robust performance even the parameter b has parametric error. The hyperplane gain C is designed such that it satisfies $CB = 1$, therefore small variations of b does not affect the control performance.

C. INVERTED WHITWORTH MECHANISM

The inverted Whitworth mechanism is shown in Figure 6.

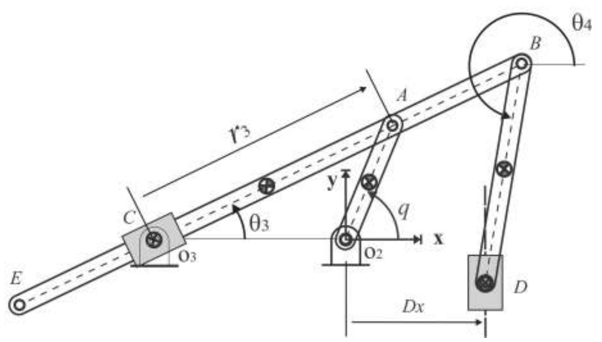

FIGURE 6. Inverted whitworth mechanism.

The generalized coordinate vector is given by $\boldsymbol{q}' = [q, r_3, \theta_3, \theta_4, y]^\top$, the extended dynamic model is

$$\boldsymbol{M}'(\boldsymbol{q}') = \begin{bmatrix} M_{11} & 0 & 0 & 0 & 0 \\ 0 & M_{22} & 0 & M_{24} & 0 \\ 0 & 0 & M_{33} & M_{34} & 0 \\ 0 & M_{24} & M_{34} & M_{44} & 0 \\ 0 & 0 & 0 & 0 & m_5 \end{bmatrix}$$

$$\boldsymbol{C}'(\boldsymbol{q}', \dot{\boldsymbol{q}}') = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -C_{23} & C_{24} & 0 \\ 0 & C_{23} & C_{33} & C_{34} & 0 \\ 0 & C_{42} & C_{43} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\boldsymbol{G}(\boldsymbol{q}') = \begin{bmatrix} G_1 & G_2 & G_3 & G_4 & m_5 g \end{bmatrix}^\top$$

$$\boldsymbol{\rho}(\boldsymbol{q}') = \begin{bmatrix} 1 \\ \rho_{r_3} \\ \rho_{\theta_3} \\ \rho_{\theta_4} \\ \dfrac{AB\sin(2-2\theta_3+\theta_4)-(AB+2r_3)\sin(q-\theta_4)}{2r_2^{-1}r_3\sin(\theta_4)} \end{bmatrix} \tag{28}$$

where m_5 is the slider mass, $v = m_5 g$, and $\rho_x(\boldsymbol{q}') = \frac{AB\sin(2-2\theta_3+\theta_4)-(AB+2r_3)\sin(q-\theta_4)}{2r_2^{-1}r_3\sin(\theta_4)}$. Finally we have that the slider dynamic equation is:

$$m_5\ddot{y} + m_5 g = u \tag{29}$$

that is a simple double integrator system with a gravity perturbation.

VI. EXPERIMENTAL RESULTS

We use an inverted Whitworth mechanism prototype (see Figure 7) to test our approach with the well known PID control. The real time environment is Matlab/Simulink® using a Sensoray model 626 for data acquisition. For slider position measures it is used a US-digital strip sensor with a resolution of 300 cpi which avoids noise measures and disturbances of links before the slider [25].

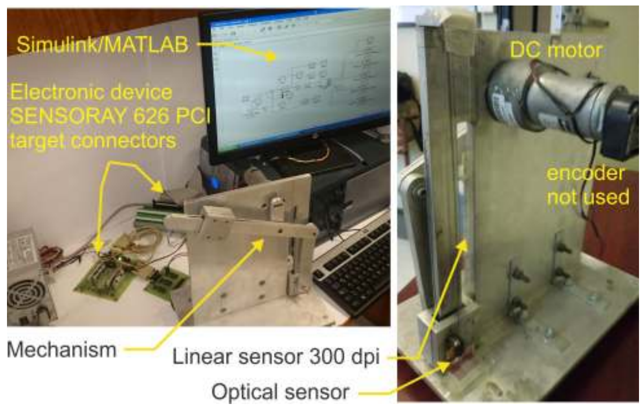

FIGURE 7. Inverted whitworth mechanism prototype.

The desired trajectory is given by the following expression:

$$y_d(t) = -0.22 + 0.03\sin\left(\frac{\pi}{2}t\right) \tag{30}$$

First it is designed the PID controller. The linearized dynamics of the inverted Whitworth mechanism in task space is:

$$\ddot{y} = -0.0281396y + 0.253427u. \tag{31}$$

Note that it is required the complete knowledge of the mechanism dynamics. The PID controller in frequency domain is of the form:

$$u(s) = K_p + K_i\frac{1}{s} + K_d\frac{Ns}{s+N}, \tag{32}$$

where K_p, K_i, K_d are the proportional, integral and derivative gains, respectively, and N is the filter coefficient. The gains are tuned using the Matlab control toolbox. The tuned gain values are: $K_p = 15.344$, $K_i = 1.097$, $K_d = 48.006$ and $N = 1391.169$.

Now it is designed the pole placement and sliding mode controller using our approach. The slider mass is $m_6 = 4.5 \times 10^{-4}$ kg. The slider dynamics is given by

$$\dot{\boldsymbol{X}} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}\boldsymbol{X} + \begin{bmatrix} 0 \\ 2222.22 \end{bmatrix}u - \begin{bmatrix} 0 \\ 9.81 \end{bmatrix} \tag{33}$$

For pole placement controller, we choose $\omega_n^2 = 40$ and $\xi = 1$, then

$$k_1 = 40, \quad k_2 \approx 13$$

However the prototype presents friction, then k_2 requires to be tuned manually via trials for a desired response. The final derivative gain is $k_2 = 5$. For the SMC surface design, it is used Ackerman-Utkin formula with a desired pole in $\lambda = -5$. Then the surface gain is given by

$$\boldsymbol{C} = \begin{bmatrix} 0.0023 & 0.0005 \end{bmatrix}$$

Since $F = \boldsymbol{B}^{+}\boldsymbol{D} = 0.0044$, then the sliding gain is proposed as $K = 0.0045$. We use a Jacobian compensator [25] instead of the real Jacobian, whose value is $\hat{\rho}_x = 0.1$.

The tracking results are given in Figure 8 and Figure 9. Both controllers achieve good tracking performance with small error values. Since the Jacobian compensation is constant, then the singularities at the beginning and end of the slider stroke are avoid. Here the performance of the controllers are similar due the friction at the slider groove. The main disadvantage of the PID controller is that it requires complete knowledge of the mechanism dynamics to make the linearization and obtain the values of its gains. It has been proved that our approach can guarantee position tracking of the slider using simple controllers with minimal knowledge of the mechanism dynamics and linearization methods.

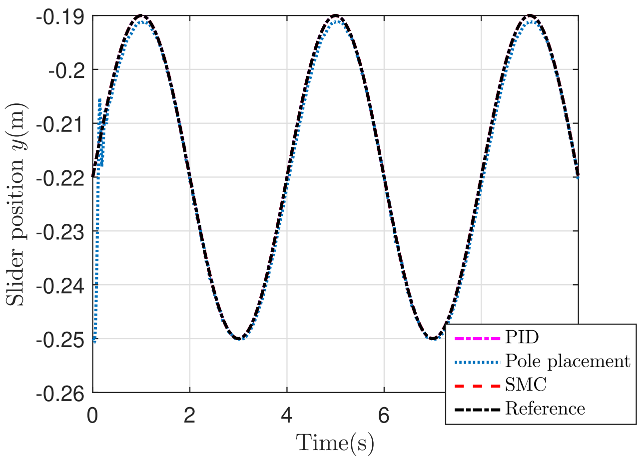

FIGURE 8. Position tracking.

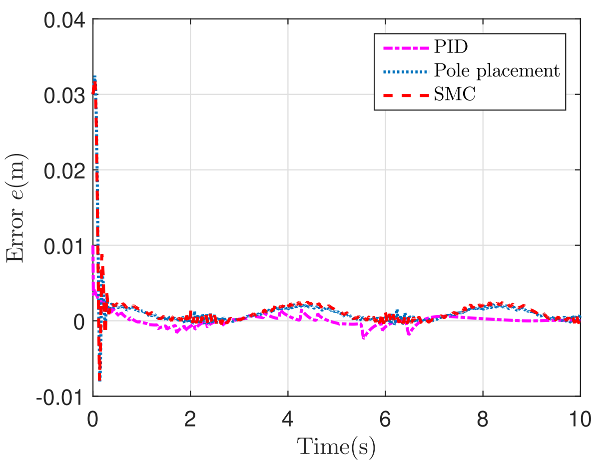

FIGURE 9. Position error.

VII. CONCLUSION

In this work a position control in task space for slider-crank mechanisms that avoids linearization and complete knowledge of the mechanism dynamics is presented. The extended dynamic model gives the information about the slider dynamics which helps to avoid any kind of linearization and transformation between the joint and task space. The resulting system is a double integrator system with a disturbance which is simple to control by using any controller from linear systems control theory. Two slider-crank mechanism are presented as examples to show the linearity of the slider dynamics. A linear sensor is used to avoid precision problems due to noise and disturbances at the links before the slider. Simulations and real time experiments were carried out using an inverted Whitworth mechanism prototype to verify our approach using pole placement and sliding mode controllers.

Our future research focuses on generating a constant velocity profile of the slider for cutting tasks which guarantees the complete turn of the crank and avoids the singularities points at the beginning and end of the slider stroke.

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