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## Taylor Rules and the Deutschmark–Dollar Real Exchange Rate

We explore the link between an interest rate rule for monetary policy and the behavior of the real exchange rate. The interest rate rule, in conjunction with some standard assumptions, implies that the deviation of the real exchange rate from its steady state depends on the present value of a weighted sum of inflation and output gap differentials. The weights are functions of the parameters of the interest rate rule. An initial look at German data yields some support for the model.

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THIS PAPER EXPLORES the link between an interest rate rule for monetary policy and the behavior of the real exchange rate.

A large body of research has studied the connection between monetary policy or interest rates on the one hand and exchange rates on the other. The vintage monetary model of the exchange rate takes the money supply as the indicator of policy. (See Frankel and Rose, 1995, for a survey.) Some recent literature (Bergin 2003) also assumes exogenous monetary policy, while estimating a general equilibrium sticky-price open-economy macroeconomic model. Yet an ongoing literature has argued, persuasively in our view, that recent monetary policy can be better modeled as taking the interest rate as the instrument of policy, with policy described by a feedback rule. The empirical literature for the United States includes Taylor (1993) as a relatively early contribution, English, Nelson, and Sack (2003) as a recent study. Open-economy interest rate rules that include exchange rates have been studied

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quantitatively (e.g., Clarida, Galí, and Gertler, 1998, Meredith and Ma, 2002) and in terms of welfare properties (e.g., Ball, 1999, Clarida, Galí, and Gertler, 2001, Svensson, 2000, and Kollman, 2002). These papers do not, however, consider the positive effects of interest rate rules on the exchange rate.

At least four strands of literature do specifically analyze real or nominal exchange rates in a framework that treats the interest rate–exchange rate interaction in detail. One is the literature on identified VARs. Kim (2002), who includes an exchange rate in his interest rate equation, is an example. A second strand of the literature tests or examines interest parity, in a relatively unstructured way, decomposing real exchange rate movements into components that can be linked to interest rates and those that cannot. Examples include Campbell and Clarida (1987), Edison and Pauls (1993), and Baxter (1994). A third strand of the literature develops general equilibrium sticky-price models, and uses calibration. Examples include Benigno (1999) and Benigno and Benigno (2001). The papers in these three strands typically, though not always, find a statistically or quantitatively strong connection between interest rates and exchange rates. This encourages us to study the connection. We use an approach that is shared by a fourth strand of the literature.

This strand is a long-standing one that models the exchange rate as a present value. Traditional econometric techniques are used, and the present value is estimated using atheoretical forecasting equations. Examples include Woo (1985), Frankel and Meese (1987), and West (1987). In terms of mechanics, our empirical approach is similar to that of the present value literature, though this literature has yet to consider Taylor rules.

Several of the papers cited above have added terms in exchange rates to otherwise standard Taylor rules (e.g., Clarida, Galí, and Gertler, 1998, Benigno, 1999). We, too, take this approach, adding the deviation of the real exchange rate from its steady state value to a Taylor rule that also includes standard terms in inflation and output. We do so with the aim of evaluating the effect such a term has on time series properties of aggregate variables, including in particular exchange rates. We show that in conjunction with interest parity, this modified rule delivers a relation between current and expected real exchange rates on the one hand and inflation and output on the other.

Our empirical work solves this relationship forward, expressing the real exchange rate as the present value of the difference between home and foreign output gaps and inflation rates. The discount factor depends on the weight that the real exchange rate receives in the Taylor rule. The weights on output and inflation are those of the Taylor rule. We attempt to gauge the congruence between the variable implied by this present value and the actual Deutschmark–dollar real exchange rate, 1979–98. We focus on Germany because Clarida, Galí, and Gertler (1998) found that the real exchange rate entered an interest rate rule for Germany with a coefficient that is statistically significant, albeit small.

To construct this present value—what we call a “model-based” real exchange rate—we compute forecasts of German–U.S. inflation and output gaps differentials from an unconstrained vector autoregression and impose rather than estimate Taylor

rule coefficients. We generate a model-based nominal exchange rate by adding actual inflation to changes in the model-based real exchange rate.

We find that the model-based exchange rates display two well-known properties of actual exchange rates. First, for both nominal and real model-based rates, autocorrelations of the growth rates are quite close to zero. In this sense, they follow processes not too far from random walks. Second, the correlation between changes in nominal and real model-based rates is nearly one. For the model-based real exchange rate, this follows because discounting a sum of highly persistent series (in our case, output and annual inflation) produces a series far more variable than the series that are being discounted. So constructing the model-based nominal exchange rate by adding inflation does little to change the behavior of the series.

We also compare the implied time series of the model-based real exchange rate with that of the actual series. Here the results are more modest. The correlation between levels of the two variables is about 0.3, between growth rates is about 0.1. The correlation between model-based and actual changes in nominal exchange rates is also about 0.1. Some investigation suggests that the 0.3 figure results because this is essentially the correlation between the linear combination of inflation and output that enters the Taylor rule on the one hand, and the real exchange rate on the other.

These correlations admittedly are not particularly large. And whatever success our approach does achieve empirically comes after making many simplifying, and admittedly debatable, assumptions. These include among others: model-consistent (rational) expectations; expectations that are homogeneous across participants in exchange rate markets and monetary policy makers; stable regimes (no bumps from reunification, no bumps—even in 1998—from the prospective introduction of the euro). Hence we recognize that these results should be interpreted with caution.

Section 1 describes our model. Section 2 outlines data, econometric model, and estimation technique. Section 3 presents basic results. Section 4 presents additional results and discussion. Section 5 concludes. An Appendix outlines a stylized sticky-price general equilibrium model similar to that in Galí and Monacelli (2003). The paper occasionally references the Appendix when interpreting results.

## 1. MODEL

We use a two-country model, with most variables defined as the difference between a home country (Germany, in our empirical work) and a foreign country (the U.S., in our empirical work). Define:

- $i_t$ : difference between home and foreign interest rates; for  $i_t$  and other variables, an increase indicates a rise in home relative to foreign rates; all interest rates are expressed at annual rates;
- $y_t$ : difference between home and foreign deviation of log output from trend;
- $p_t$ : difference between home and foreign log price levels;

- $\pi_t$ : difference between home and foreign inflation (with inflation first difference of log consumer price level [CPI]);
- $u_{mt}$ : difference between home and foreign shocks to monetary policy rule;
- $s_t$ : log nominal exchange rate (e.g., DM/\$, when Germany is the home country);
- $q_t = s_t - p_t$ : log real exchange rate;
- $E_t$ : mathematical expectations conditional on a period  $t$  information set.

For convenience, we will generally omit the qualifier “difference” when referring to differences between home and foreign variables:  $y_t$  and  $p_t$  will be called plain old “output” and “inflation.” For clarity, this section omits factors of 12 necessary in certain equations to convert monthly to annual rates. These are inserted in the next section and in the empirical work.

Let an “ $h$ ” superscript denote the home country, a “ $*$ ” superscript the foreign country. The monetary rules in the foreign and home countries are:

$$i_t^* = \gamma_\pi E_t \pi_{t+1}^* + \gamma_y y_t^* + u_{mt}^* , \quad (2)$$

$$i_t^h = \gamma_q q_t + \gamma_\pi E_t \pi_{t+1}^h + \gamma_y y_t^h + u_{mt}^h . \quad (3)$$

In Equations (2) and (3),  $i_t^*$  is the interest rate in the foreign country,  $i_t^h$  the interest rate in the home country, and the variable  $i_t$  referenced in Definition (1) is defined as  $i_t = i_t^h - i_t^*$ . The other variables in Equations (2) and (3) relate similarly to those defined in Definition (1). Here and throughout we omit constant and trend terms. Use of expected inflation and current output slightly simplifies some of our calculations below (relative to use of expected output along with expected inflation, or current inflation and current output), but seems unlikely to have important qualitative effects.<sup>1</sup>

Equation (2) is a standard Taylor rule, assumed in our empirical work to apply to the U.S. Equation (3) is a Taylor rule with the real exchange rate included. This equation is assumed in our empirical work to apply to Germany. The assumption that the two countries have the same monetary policy parameters  $\gamma_\pi$  and  $\gamma_y$  is for convenience; empirical work reported below briefly experiments with distinct parameters, finding that results with homogenous parameters are representative. We assume  $\gamma_\pi > 1$ ,  $\gamma_y > 0$ , and  $\gamma_q > 0$ .<sup>2</sup>

With  $\gamma_q > 0$ , the monetary authority is assumed to raise interest rates when the real exchange rate is above (the currency is depreciated relative to) its long-run level. (Recall that since we are omitting constants and trends, we write Equation (3) in a form that gives the long-run level as zero.) We take this as a reasonable

1. Of course, the details of the solution will be different when actual rather than expected inflation appears in the monetary rule, and, since conditions for stability and uniqueness are different in the two cases, it is possible that qualitative characteristics of the solution will be radically different. The statement in the text merely means that for a range of parameters, nothing central turns on the use of expected rather than actual inflation.

2. The Appendix relaxes these conditions, in particular allowing  $\gamma_q = 0$ .

description of monetary policy in some open economies.<sup>3</sup> Perhaps the most pertinent reference is Clarida, Galí, and Gertler (1998), who find that a term in the real exchange rate is statistically significant in Taylor rules estimated for Germany and Japan, with  $\hat{\gamma}_q \approx 0.1$ .

More generally, we take Equation (3) to be a specific form of a Taylor rule that includes a term of the form

$$\gamma_q(s_t - \bar{s}_t) \quad (4)$$

where  $s_t$  is the nominal exchange rate and  $\bar{s}_t$  is a target for  $s_t$ . In Equation (3), the target is  $(p_t^h - p_t^*) + \text{constant}$ , where the constant has been omitted from Equation (3) for simplicity. In Cho and West (2003), the target  $\bar{s}_t$  followed an unobserved random walk. The Taylor rules for Italy, France, and the UK that were estimated by Clarida, Galí, and Gertler (1998) fall into this framework with  $\bar{s}_t$  set to central parity within the ERM.

Subtract Equation (2) from Equation (3), obtaining

$$i_t = \gamma_q q_t + \gamma_\pi E_t \pi_{t+1} + \gamma_y y_t + u_{mt} . \quad (5)$$

Next, write uncovered interest parity as

$$i_t = E_t s_{t+1} - s_t . \quad (6)$$

Upon subtracting the expected value of next period's inflation from both sides of Equation (6), and using the definition of  $q_t$ , we obtain

$$i_t - E_t \pi_{t+1} = E_t q_{t+1} - q_t . \quad (7)$$

Use Equation (7) to substitute out for  $i_t$  on the left-hand side of Equation (5). The result may be written

$$q_t = b E_t q_{t+1} + b E_t (1 - \gamma_\pi) \pi_{t+1} - b \gamma_y y_t - b u_{mt} . \quad (8)$$

In Equation (8),  $b = 1/(1 + \gamma_q)$ ,  $0 < b < 1$ .

We note that if we add an exogenous risk premium shock to Equation (6), then Equation (8) still results but with  $u_{mt}$  redefined to be the difference between the monetary policy shock and the risk premium shock. (That is, if we call the risk premium shock  $u_{dt}$ , rewriting Equation (6) as  $i_t = E_t s_{t+1} - s_t + u_{dt}$ , and rename the monetary policy shock to  $\tilde{u}_{mt}$ , then Equation (8) holds with  $u_{mt} = \tilde{u}_{mt} - u_{dt}$ ). One can therefore interpret  $u_{mt}$  as incorporating such shocks. The effects of a risk premium shock are the opposite of those of a monetary policy shock.

After once again suppressing a risk premium shock, we observe that by combining the Taylor rule with uncovered interest parity we have a relationship that is rather richer than uncovered interest parity. To test uncovered interest parity, one needs a model for the exchange rate. Often such a model is supplied in what might be called

3. Here and throughout, we abstract from operational difficulties in identifying the long-run level of the real exchange rate, just as we abstract from the raft of other practical problems involved with implementing a monetary policy rule (e.g., data availability).

nonparametric form, with minimal assumptions made about the exchange rate. An example is when one tests or estimates interest parity under rational expectations by replacing expectations with realizations, but one does not spell out the process followed by the exchange rate. See Lewis (1995). Another example is when one models  $q_t$  as the expected present discounted value of real interest differentials (e.g., Campbell and Clarida 1987). Our empirical work not only assumes model-consistent (rational) expectations, but also maintains an additional set of assumptions in the form of the Taylor rule. We expect additional assumptions to on balance be helpful if they are consistent with the data. And in our view, the empirical results are consistent with this expectation.

In our empirical work, we do not estimate a Taylor rule. Instead, relying on Equation (8), we focus on the relationship between  $q_t$  on the one hand and  $y_t$  and  $\pi_t$  on the other. Equation (8) contains three variables that are endogenous in general equilibrium ( $q_t$ ,  $\pi_t$ , and  $y_t$ ). Our empirical work does not rely on any particular set of structural equations or restrictions to close the model. Rather, we use atheoretical forecasting equations, as described in the next section. But for intuition, it is helpful to work through a specific structural model. In the Appendix, we combine Equation (8) with: (1) a market clearing condition relating the output gap to the real exchange rate, called the IS curve for convenience, and (2) a price adjustment equation (Phillips curve) that relates inflation and output. These three equations determine equilibrium output, the real exchange rate, and prices/inflation. The nominal exchange rate is determined via  $\Delta s_t = \Delta q_t + \pi_t$ .

The Appendix system has the following properties in terms of responses to shocks:

- A positive monetary policy shock ( $u_{mt} \uparrow$ —i.e., an exogenous monetary tightening) causes the real and nominal exchange rates  $q_t$  and  $s_t$  to fall (i.e., appreciation). Inflation  $\pi_t$  and output  $y_t$  also fall. The response of exchange rates is consistent with interest parity. The response of inflation and output is consistent with conventional closed economy models.
- Consider a positive Phillips curve shock that, given output and expected inflation, raises inflation transitorily. A real appreciation will result. This follows from the combination of interest parity and the parameter restriction  $\gamma_\pi > 1$  in the Taylor rule (Equation 5). With  $\gamma_\pi > 1$ , incipient increases in inflation cause the real interest rate to increase. From interest parity, the real exchange rate falls. Output falls as well.
- Consider a positive real shock to the IS curve that, given the real exchange rate  $q_t$ , raises output  $y_t$ . Then in equilibrium, output rises, the real exchange rate falls, and the inflation rate of home produced goods rises relative to the foreign produced goods. The impact on  $\pi_t$  (home inflation  $\pi_t^h$  relative to foreign inflation  $\pi_t^*$ ), however, is ambiguous: inflation of home relative to foreign goods rises (pushing  $\pi_t$  up), while the real exchange rate falls (pushing  $\pi_t$  down). But we can say that  $\pi_t$  unambiguously falls for a sufficiently large interest rate response to output  $\gamma_y$ . For in this case, the rise in output will cause an increase in interest rates sufficiently large to dampen inflation.

This discussion of the Appendix model is intended to give intuition to how a Taylor rule affects propagation. In our empirical work, we do not attempt to identify and trace through the effects of structural shocks. Instead, we aim to compare certain properties of data generated according to Equation (8) and the actual German–U.S. data.

## 2. DATA, EMPIRICAL MODEL, AND ECONOMETRIC TECHNIQUE

### 2.1 Data

In our empirical work, Germany is the home country, U.S. the foreign country. We focus on Germany because Clarida, Galí, and Gertler (1998) found that a Taylor rule, with a real exchange rate term included, well characterizes German monetary policy. We use monthly data; apart from lags, our sample runs from 1979:10 to 1998:12. The start of the sample is chosen to coincide with the beginning of the Volcker regime shift, the end with the introduction of the euro.

Data were obtained from the International Financial Statistics CD-ROM and from the web site of the Bundesbank. Output is measured as the log of seasonally adjusted industrial production (IFS series 66.c), prices as the log of the CPI (series 64), inflation as the first differences of log prices, interest rates by a money market rate (series 60b), exchange rate as the log of the end of month rate (series ae). (Use of monthly average exchange rates in preliminary work led to little difference in results.) Following Clarida, Galí, and Gertler (1998), the output gap  $y_t$  was constructed as the residual from quadratically detrended output. U.S. and German output were detrended separately, before the output gap differential was constructed. Output, prices, and exchange rates were multiplied by 100 so differences are interpretable as percentage changes.

The IFS data combine data for West Germany (1979–90) and unified Germany (1991–98). We obtained a continuous series for West German industrial production and CPI from the Bundesbank. To smooth a break in the price and output levels between 1990:12 and 1991:1, we proceeded as follows. We assumed that West German inflation and growth rates of output in 1991:1 (the first year of reunification) also applied to Germany as a whole, and used that ratio to scale up the level of post-1990 data on German output and price level. Thus growth rates of inflation and output match those for West Germany through 1991:1, Germany as a whole afterward. The scaling still affects our empirical results, because we use the level rather than change of output (quadratically detrended) and the price level figures into the real exchange rate. One final adjustment was to smooth out a one month fall and then rise of over 10% in industrial production in 1984:6 that was present in both IFS and Bundesbank data. We simply set the 1984:6 figure to the average 1984:5 and 1984:7. (We observed this downward spike in an initial plot of the data. If the spike is genuine rather than a data error, it presumably reflects a strike known to be transitory and our smoothing likely makes our VAR forecasts more reasonable.)

Figure 1 plots the data. The real exchange rate has been adjusted to have a mean of 50. The strong appreciation of the dollar in the early 1980s is apparent. Initially, inflation in the U.S. was higher than that in Germany. The output gap in U.S. relative to Germany peaked in the early 1990s, and fluctuated a good deal both before and after.

2.2 Empirical Model and Econometric Technique

In accordance with Clarida, Galí, and Gertler (1998), we assume that expected annual inflation appears in the monetary policy rules Equations (2) and (3). Equation (5) becomes

$$i_t = \gamma_q q_t + \gamma_\pi E_t(p_{t+12} - p_t) + \gamma_y y_t + u_{mt} . \tag{5}'$$

As well, we calibrate Taylor rule parameters on the conventional assumption that annualized interest rates appear on the left-hand side. With  $i_t$  measured at annual rates, and monthly data used throughout, uncovered interest parity (Equation 6) is  $(i_t/12) = E_t s_{t+1} - s_t$ , implying that in real terms

$$(i_t/12) - E_t \pi_{t+1} = E_t q_{t+1} - q_t . \tag{7}'$$

Upon combining Equations (5)' and (7)' and rearranging, we get

$$q_t = b E_t q_{t+1} + (12 + \gamma_q)^{-1} [12 E_t \pi_{t+1} - \gamma_\pi E_t (p_{t+12} - p_t) - \gamma_y y_t - u_{mt}] . \tag{8}'$$

where  $b$  has now been redefined as  $b = 12/(12 + \gamma_q)$ . Upon imposing the terminal condition that  $q_t$  is non-explosive, the solution to Equation (8)' may be written

$$q_t = (12 + \gamma_q)^{-1} \sum_{j=0}^{\infty} b^j E_t [12 \pi_{t+j+1} - \gamma_\pi (p_{t+j+12} - p_{t+j}) - \gamma_y y_{t+j} - u_{mt+j}] . \tag{9}$$

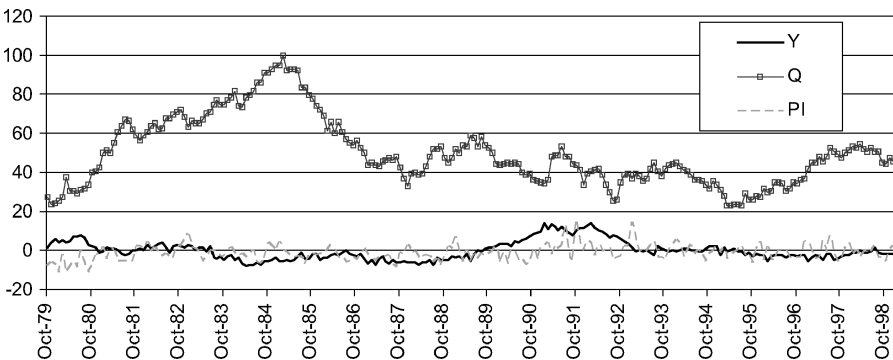


FIG. 1. Real Exchange Rate  $q$ , Annualized Output Gap  $y$ , and Annualized Inflation  $\pi$  (Notes:  $y$  and  $\pi$  are the difference between German and U.S. output gap (log industrial production, deviation from quadratic trend) and German and U.S. inflation (log CPI, annualized);  $q$  is the log real exchange rate, with a larger value indicating depreciation of the Deutschmark relative to the dollar, and with the mean arbitrarily chosen to make the figure readable.)



We aim to construct a “model-based” or “fitted” real exchange rate, call it  $\hat{q}_t$ , and compare its properties with those of the actual exchange rate  $q_t$ . We proceed by imposing values for  $\gamma_q$ ,  $\gamma_p$ , and  $\gamma_y$ , setting  $b = 12/(12 + \gamma_q)$ . It would be of interest to search for monetary policy rule parameters that lead to the best possible fit to the exchange rate, but we defer that task to future work. Using the imposed values, we compute forecasts of monthly inflation and output gaps with a vector autoregression (VAR). This autoregression relies on a vector, call it  $z_t$  that in our baseline specification includes the interest rate along with monthly inflation and the output gap:  $z_t = (\pi_t, y_t, i_t)'$ . Thus, in making the forecasts, we allow for the possibility that past interest rates help predict future inflation and output gaps. We do not, however, allow for a direct effect from the shock  $u_{mt}$ . We omit this shock because of lack of an independent time series for it. Our intuition is that this omission biases the results against our model. Also, while we experiment with alternative definitions of the forecasting vector  $z_t$  (for example, adding commodity prices), we do not at any point include the exchange rate itself, because our aim is to explain exchange rates from fundamental variables. Our intuition again is that this makes it more difficult to make the model fit well.

Let  $n$  denote the order of the vector autoregression, let  $Z_t$  denote the  $(3n \times 1)$  vector that results when the VAR is written in companion form as a vector AR(1),  $Z'_t \equiv (z_t, z_{t-1}, \dots, z_{t-n+1})$ . It is straightforward to show that  $\hat{q}_t$  is linear in  $Z_t$ ,

$$\begin{aligned} \hat{q}_t &= (12 + \gamma_q)^{-1} \sum_{j=0}^{\infty} b^j E[12\pi_{t+j+1} - \gamma_\pi(p_{t+j+12} - p_{t+j}) - \gamma_y y_{t+j} | Z_t] \\ &= (\text{say}) c'_q Z_t, \end{aligned} \tag{10}$$

where  $c_q$  is a  $(3n \times 1)$  vector that is mapped from  $b$ ,  $\gamma_p$ ,  $\gamma_y$  and the estimates of the VAR parameters. As long as inflation and output gap differentials are mean reverting, so, too, is  $\hat{q}_t$ .

Define a “model-based” nominal exchange rate change as  $\Delta \hat{s}_t \equiv \Delta \hat{q}_t + \pi_t$ . We use  $c_q$  and the VAR parameters to compute autocorrelations of  $\hat{q}_t$ ,  $\Delta \hat{q}_t$ , and  $\Delta \hat{s}_t$  as well as their cross-correlations with  $\pi_t$ , and  $y_t$ . We compare these to the corresponding values for  $q_t$ ,  $\Delta q_t$ , and  $\Delta s_t$ . Finally, we construct time series for  $\hat{q}_t$  from Equation (10) and then for  $\Delta \hat{q}_t$  and  $\Delta \hat{s}_t$  ( $\equiv \Delta \hat{q}_t + \pi_t$ ), and compute their correlation with  $q_t$ ,  $\Delta q_t$ , and  $\Delta s_t$ .

For inference about the correlation between model-based and actual  $q_t$ ,  $\Delta q_t$ , and  $\Delta s_t$ , we construct 95% confidence intervals from the percentile method of a nonparametric bootstrap. For inference about  $\text{corr}(\hat{q}_t, q_t)$  and  $\text{corr}(\Delta \hat{q}_t, \Delta q_t)$ , we proceed as follows.

1. We estimate a bivariate VAR(4) in  $(\hat{q}_t, q_t)'$ . We use Kilian’s (1998) procedure to estimate the bias in the least squares estimator of the coefficients of the VAR. (There is a downward bias because the data are highly positively serially correlated.)
2. We generate 1000 artificial samples, each of size 227 (227 because that is the number of monthly observations running from 1980:2 to 1998:12). In each of the 1000 repetitions, we:

- a. Generate a sample using bias-adjusted VAR coefficients and sampling with replacement from the residuals of the bias-adjusted VAR. We use the actual and model-based data, 1979:10–1980:1, for initial conditions.
  - b. Estimate the VAR on the generated sample.
  - c. Compute the two correlations.
3. Finally, we sort the 1000 values of each of the statistics. We construct 95 percent confidence intervals by reporting the 25th smallest and 25th largest statistics (25 = 2.5 percent of 1000).

To construct a confidence interval for the correlation between  $\Delta\hat{s}_t$  and  $\Delta s_t$ , we used a bivariate VAR(1) in  $(\Delta\hat{s}_t, \Delta s_t)'$ , omitting the Kilian (1998) bias adjustment and setting the lag length to 1 because  $(\Delta\hat{s}_t, \Delta s_t)'$  shows almost no serial correlation.

### 3. BASIC EMPIRICAL RESULTS

To construct  $\hat{q}_t$ , we set the lag length of the vector autoregression to 4 ( $n = 4$ ), and used the following parameters:  $\gamma_q = 0.1 (\Rightarrow b \approx 0.99)$ ,  $\gamma_\pi = 1.75$ ,  $\gamma_y = 0.25$ . The value for the exchange rate parameter  $\gamma_q$  is roughly that estimated for Germany by Clarida, Galí, and Gertler (1998). The values for the inflation and output gap differentials are roughly those estimated for Germany by Clarida, Galí, and Gertler (1998) and those estimated for the U.S. by a number of authors, including Rudebusch (2002) and English, Nelson, and Sack (2003).

Panel A of Table 1 has autocorrelations of model-based data in columns (2)–(4), with figures for actual data in columns (5)–(9). Begin with the actual data. The exchange rate pattern is familiar. The real exchange rate is highly autocorrelated ( $\rho_1 = 0.98$  [column (5), row(1)]). Growth rates of the real and nominal exchange rates are approximately serially uncorrelated (columns (6) and (7)). (Indeed, there is a considerable body of evidence dating back to Meese and Rogoff (1983) that the “approximately” can be dropped for nominal exchange rates.) The output gap is highly serially correlated (column (9)); monthly inflation less so (column (8)).

Good news for the model is that the model-based exchange rates display properties similar to those of the actual exchange rate data. See columns (2)–(4) in panel A. The first order serial correlation coefficient of  $\hat{q}_t$  is 0.97 (column (2)). The growth rates of the model’s real and nominal exchange rates are essentially serially uncorrelated (see columns (3) and (4)). Now, as stated in Equation (10),  $\hat{q}_t$  is a linear combination of the VAR variables, which include  $\pi_t$  and  $y_t$ . As argued in the next section, high serial correlation of  $\hat{q}_t$  (column (2)) reflects high serial correlation of the output gap and annual inflation; low serial correlation of  $\Delta\hat{q}_t$  and  $\Delta\hat{s}_t$  reflects the fact that  $\hat{q}_t$  is constructed as a present value with a discount factor near 1.

Panel B presents data on cross-correlations. It is well known that real and nominal exchange rates are highly correlated with one another. Indeed, in our data, when rounded to two digits, the correlation is 1.00 (row (3), column (7)). The high correlation between real and nominal exchange rates is also captured by the model,

TABLE 1  
MOMENTS OF MODEL-BASED AND ACTUAL DATA

A. Autocorrelations									
Model-based series				Data					
(1) Lag	(2) $\hat{q}_t$	(3) $\Delta\hat{q}_t$	(4) $\Delta\hat{s}_t$	(5) $q_t$	(6) $\Delta q_t$	(7) $\Delta s_t$	(8) $\pi_t$	(9) $y_t$	
1	0.97	-0.03	0.02	0.98	0.05	0.06	0.36	0.94	
2	0.93	0.00	0.03	0.96	0.06	0.04	0.12	0.92	
3	0.90	-0.02	0.02	0.94	0.04	0.04	0.21	0.90	

B. Cross-correlations										
Model-based series (actual $\pi$ and $y$ )					Data					
(1)	(2)	(3)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
	$\hat{q}_t$	$\Delta\hat{q}_t$	$\Delta\hat{s}_t$	$\pi_t$	$y_t$	$q_t$	$\Delta q_t$	$\Delta s_t$	$\pi_t$	$y_t$
(1) $\hat{q}_t$	1.00					$q_t$	1.00			
(2) $\Delta\hat{q}_t$	0.13	1.00				$\Delta q_t$	0.07	1.00		
(3) $\Delta\hat{s}_t$	0.09	0.94	1.00			$\Delta s_t$	0.07	1.00	1.00	
(4) $\pi_t$	-0.07	0.10	0.44	1.00		$\pi_t$	0.04	-0.08	0.02	1.00
(5) $y_t$	-0.98	-0.18	-0.11	0.14	1.00	$y_t$	-0.37	-0.03	-0.01	0.14
										1.0

C. Correlations Between Actual and Model-Based Exchange Rate Series		
corr( $\hat{q}_t, q_t$ )	corr( $\Delta\hat{q}_t, \Delta q_t$ )	corr( $\Delta\hat{s}_t, \Delta s_t$ )
0.32 (-0.49, 0.82)	0.09 (-0.05, 0.22)	0.10 (-0.03, 0.23)

NOTES: 1. Variable definitions:  $q$  = real exchange rate (DM/\$),  $s$  = nominal exchange rate,  $p$  = inflation differential (CPI, U.S.-Germany),  $y$  = output gap differential (industrial production, U.S.-Germany, constructed by quadratic detrending). All data are monthly. The sample period is 1979:10-1998:12. West German data are used 1979-90, with price and output levels adjusted post-1990 to smooth the break in the series caused by reunification. See text for details. 2. A "hat" over a variable indicates that it was constructed according to the model described in the paper. See Equation (10). Parameters of the monetary policy rule:  $\gamma_q = 0.1$ ,  $\gamma_\pi = 1.75$ ,  $\gamma_y = 0.25$ . In constructing  $\hat{q}_t$ , a fourth order VAR in  $(\pi, y, i)$  was used to construct the present value defined in the text, where  $i$  = U.S.-German interest differential. 3. In Panel C, 95% bootstrap confidence intervals are in parentheses.

with a figure that also rounds to 1.00 (row (3), column (2)). The high correlation results because  $\Delta\hat{q}_t$  is much more variable than  $\pi_t$ , so movements in  $\Delta\hat{s}_t$  ( $\equiv \Delta\hat{q}_t + \pi_t$ ) are dominated by movements in  $\Delta\hat{q}_t$ . More generally, cross-correlations of  $\hat{q}_t$ ,  $\Delta\hat{q}_t$ , and  $\Delta\hat{s}_t$  are very similar to those of  $q_t$ ,  $\Delta q_t$ , and  $\Delta s_t$ .

On the other hand, the model is less successful in reproducing the correlations between the real exchange rate on the one hand and the output gap and inflation on the other. We see in panel B that  $\hat{q}_t$  and  $y_t$  are sharply negatively correlated—specifically, -0.98 (row (5), column (1)), which is rather different from the correlation between  $q_t$  and  $y_t$  of -0.37 (row (5), column (6)). The correlation between  $\hat{q}_t$  and  $\pi_t$  (-0.07) is also below that of  $q_t$  and  $\pi_t$  (0.04). According to the model sketched in the Appendix, the negative correlation between output and the real exchange rate indicates that real shocks have played an important role. The positive correlation between inflation and the real exchange rate indicates a role for monetary policy or risk premium shocks. (See the bullet points and the closing paragraphs

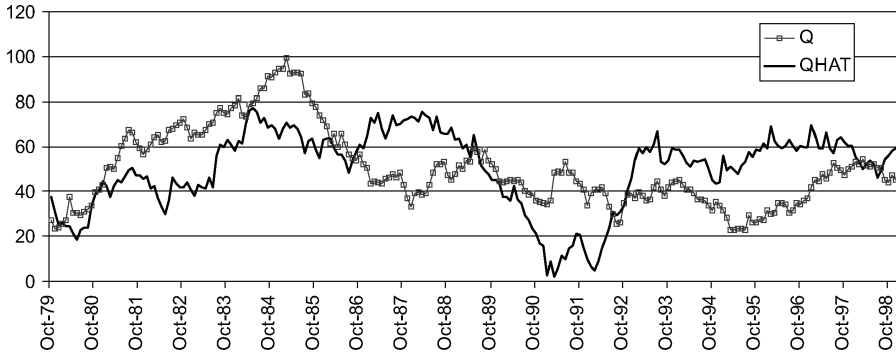


FIG. 2. Actual ( $q$ ) and Model-Based ( $\hat{q}_t$ ) Real Exchange Rates (Notes:  $\hat{q}_t$  is the fitted value of the real exchange rate from the baseline specification presented in Table 1, scaled to have the same mean and standard deviation as the actual real exchange rate  $q$ .)

of Section 1 above.) That we did not have data to directly account for the effects of such shocks might therefore partly explain the negative correlation between  $\hat{q}_t$  and  $\pi_t$  and the far too negative correlation between  $\hat{q}_t$  and  $y_t$ .

Panel C presents the correlations between actual and model-based exchange rates. These are 0.32 for the real exchange rate, 0.09 and 0.10 for growth rates of real and nominal exchange rates, respectively. The standard deviation of  $\hat{q}_t$  is about one-fifth that of  $q_t$  (not reported in any table): as is common in empirical work on asset prices, the fitted value is much less variable than the actual value.<sup>4</sup> Figure 2 plots the model-based and actual real exchange rate series. The two series have been scaled to have the same mean and standard deviation. We scale up the model-based series  $\hat{q}_t$  to make it easier to track the co-movements of the two series. We focus on co-movements because our model does not aim to pin down the level of the exchange rate, nor, as just noted, does it successfully generate the volatility seen in actual exchange rates. In any event, that the two series track each other is apparent, with a better match at the beginning and end than at the middle of the sample.<sup>5</sup>

The predicted real exchange rate from the model matches relatively well during the period of the great appreciation of the dollar in the late 1970s and early 1980s. In that period, the U.S. raised interest rates (relative to those in Germany and other industrialized countries) to combat high inflation. This tight U.S. monetary policy has frequently been cited as a cause of the dollar's strength. Although the U.S. may not have been closely following a Taylor rule in that period, the high U.S. relative

4. See Engel and West (2004a) on accounting for variability in exchange rates.

5. Here and in the rest of this section, our discussion relies on our having scaled the model-based exchange rate to have the same standard deviation as the actual exchange rate. With such scaling, the ability of our model-based series to co-move with the actual series is reflected not only in whether the two series move in the same direction, but also in the relative magnitudes of the movements in the two series.

interest rates that arose in response to high U.S. inflation captures the essence of one element of our model.<sup>6</sup>

The model does not capture the continuing appreciation of the dollar in late 1984 and early 1985. The appreciation of the dollar in 1984 has frequently been labeled a “bubble”. (See Frankel (1994).) In 1985, U.S. interest rates began to decline gradually, contributing to the fall in the dollar.

The model also does not match the data well around reunification (1989-92). And this despite the fact that this is a period in which the Taylor rule for the U.S. fits unusually well (e.g., Fig. 2 on p1051 of Clarida, Galí, and Gertler (1998)). During this period, output gaps and inflation were increasing rapidly in Germany relative to the U.S., but, nonetheless, and in contradiction to our model, the Deutschmark appreciated (i.e., fell) modestly rather than dramatically. One possible explanation is that special events—specifically, reunification, and ongoing stresses in the EMS—offset the forces that our model incorporates. (See Clarida and Gertler 1997.) But in later periods, our model-based series seems broadly similar to the actual series. During this period, German growth lagged U.S. growth. U.S. interest rates rose relative to German rates. Our fitted model captures the strong appreciation of the dollar over this period.

In light of the long history of difficulty in modeling exchange rates, our gut sense is that match between model and data is respectable though not overwhelming. We acknowledge, however, that the relevant standard is not clear-cut and there is much movement in actual exchange rates not captured in our model-based series.

#### 4. ADDITIONAL EMPIRICAL RESULTS AND DISCUSSION

Table 2 presents results under some alternative specifications. All such alternatives are identical to the baseline specifications whose results are presented in Table 1, apart from the variation described in panel A. Alternative specification *a* raises the coefficients on expected inflation and output ( $\gamma_\pi = 2.0$ ,  $\gamma_y = 0.5$  rather than  $\gamma_\pi = 1.5$ ,  $\gamma_y = 0.25$ ). Specification *b* uses West German data on prices and output throughout. (Recall that the baseline uses West German data up to 1990:12, data for unified Germany after 1990:12.) It may be seen in panel B of Table 2 that these variations yield little change in the behavior of the model’s exchange rates.

Specifications *c*, *d*, and *e* vary the VAR used to forecast future inflation and output. Specification *c* uses six rather than four lags, specification *d* drops the interest rate *i* from the VAR, and specification *e* adds commodity price inflation to the VAR. There is little change in results.

Specifications *f* and *g* use VARs in which U.S. data on inflation and output are added to the VAR. (Since  $\pi$  and  $y$  are simply the difference between German and

6. By one measure, our results are sensitive to inclusion of the period from the late 1970s and early 1980s. We re-estimated from scratch (detrending regressions to construct the output gap, as well as the VARs), starting the sample in 1982:10. Correlations between actual and fitted changes in real and nominal exchange rates remain essentially unchanged. But the correlation between the levels of  $q_t$  and  $\hat{q}_t$  falls dramatically, to 0.02.

TABLE 2  
RESULTS OF ALTERNATIVE SPECIFICATIONS

A. Description of How Alternative Specifications Vary from Baseline							
Mnemonic and Description							
a	$\gamma_\pi = 2.0, \gamma_y = 0.5$						
b	West German data						
c	sixth order VAR in $(\pi, y, i)$ used to forecast $(\pi, y)$						
d	fourth order VAR in $(\pi, y)$ used to forecast $(\pi, y)$						
e	fourth order VAR in $(\pi, y, i, \text{commodity price inflation})$ used to forecast $(\pi, y)$						
f	fourth order VAR in $(\pi, y, \pi^*, y^*, i)$ used to forecast $(\pi, y, \pi^*, y^*)$						
g	$\gamma_\pi^h = 1.3, \gamma_y^h = 0.3, \gamma_\pi^* = 2.0, \gamma_y^* = 0.2$ ; fourth order VAR in $(\pi, y, \pi^*, y^*, i)$ used to forecast $(\pi, y, \pi^*, y^*)$						
h	Hodrick-Prescott detrending						
B. Summary of Results Under Alternative Specifications							
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Mnemonic	$\hat{q}_t$	$\rho_1$	$\Delta \hat{s}_t$	$(\Delta \hat{q}_t, \Delta \hat{s}_t)$	Cross-correlation		$(\Delta \hat{s}_t, \Delta \hat{s}_t)$
		$\Delta \hat{q}_t$			$(\hat{q}_t, \hat{q}_t)$	$(\Delta \hat{q}_t, \Delta \hat{q}_t)$	
Baseline	0.97	-0.03	0.02	0.94	0.32	0.09	0.10
a	0.97	-0.02	0.00	0.98	0.32	0.09	0.11
b	0.97	-0.02	0.02	0.94	0.32	0.06	0.08
c	0.96	-0.02	0.03	0.94	0.36	0.10	0.11
d	0.97	-0.04	0.01	0.94	0.38	0.10	0.11
e	0.97	-0.03	0.02	0.94	0.32	0.08	0.09
f	0.98	0.03	0.08	0.88	0.47	0.08	0.08
g	0.98	0.01	0.07	0.87	0.49	0.11	0.11
h	0.96	-0.07	-0.06	0.78	0.42	0.20	0.20

NOTES: 1. The parameters, data, sample period, and VAR variables for the baseline specification are reported in the notes to Table 1. The alternative specifications differ from the baseline only in the indicated fashion. 2. The panel B figures for the baseline specification are repeated from Table 1. 3. In panel B,  $\rho_1$  is the first order autocorrelation coefficient.

U.S. variables, results for these specifications would be identical were the VAR to replace  $\pi$  and  $y$  with German inflation and output.) Specification *f* keeps Taylor rule parameters the same in both countries while specification *h* allows them to differ. The parameters in specification *g* are chosen to be roughly consistent with those estimated by Clarida, Galí, and Gertler (1998): in the German Taylor rule, the weights on expected inflation and output are  $\gamma_\pi^h = 1.3, \gamma_y^h = 0.3$ ; in the U.S. Taylor rule, the weights are  $\gamma_\pi^* = 2.0, \gamma_y^* = 0.2$ . One would expect this to result in a better fit. And indeed it does. While most moments remain unchanged, the correlation between  $q_t$  and  $\hat{q}_t$  climbs to nearly 0.5.

In our view, the improvement from the baseline is doubly reassuring. More information in the VAR and allowance for separate Taylor rule coefficients improves model fit. As well, the simplified baseline approach of a smaller dimension VAR and identical Taylor rule parameters yields qualitatively similar results.

Specification *h* uses a Hodrick-Prescott filter rather than a quadratic time trend to construct the output gap. Relative to baseline, this results in a modest fall in  $\text{corr}(\Delta \hat{q}_t, \Delta \hat{s}_t)$  and a rise in all three correlations between model-based and actual data.

That the alternative specifications in Table 2 give similar results perhaps reflects the following mechanics. First, the high correlation between  $q_t$  and  $\hat{q}_t$  reflects the

high correlation between the exchange rate and the linear combination of variables in the Taylor rule. Specifically, the correlation between  $q_t$  and  $E[12\pi_{t+1} - 1.75(p_{t+12} - p_t) - 0.25y_t | Z_t]$  is 0.43 (not reported in any table). Note that this correlation is higher than the absolute value of the correlation of  $y_t$  and  $q_t$  ( $= -0.37$ ) or between  $\pi_t$  and  $q_t$  ( $= 0.04$ ) (Table 1B, column (6), rows (5) and (4)). The correlation is also larger than the 0.09 correlation between annual inflation  $p_t - p_{t-12}$  and  $q_t$  (not reported in any table).

Second, the low autocorrelations of  $\Delta\hat{q}_t$  reflect the fact that discounting a persistent series (output and inflation, in our model) with a discount factor near one tends to produce a series ( $\hat{q}_t$ ) with random walk like characteristics. This result is developed at length in Engel and West (2004b). We will limit ourselves here to stressing that the result follows even when the persistent series do not themselves display random walk like behavior.

Third, the high correlation between  $\Delta\hat{q}_t$  and  $\Delta\hat{s}_t$  reflects the fact that discounting a persistent series tends to produce a series that is more variable than the series being discounted. Indeed, not only  $\hat{q}_t$  but also  $\Delta\hat{q}_t$  is notably more variable than  $\pi_t$ , by a factor of about seven in the baseline specification. Since  $\Delta\hat{s}_t$  is constructed by summing  $\Delta\hat{q}_t$  and  $\pi_t$ ,  $\Delta\hat{s}_t$  is dominated by movements in  $\Delta\hat{q}_t$  and the high correlation follows.<sup>7</sup>

More importantly, at an economic rather than mechanical level, the consistency between model and actual data reported in Tables 1 and 2 reflects two things: a Taylor rule well describes monetary policy, and the mark tends to strengthen relative to the dollar when German interest rates rise relative to U.S. rates. Our model can therefore account for the finding that when German inflation and output are relatively high, the mark tends to appreciate. It is a notable feature of our model that it captures the correlation in the data that high German inflation is associated with a strong mark. Traditional monetary models (Frankel, 1979, or Engel and Frankel, 1984, for example) have tended to predict the opposite—that high inflation is associated with a weak currency.

## 5. CONCLUSION

We view our study as a promising initial approach to investigating the empirical implications of Taylor rules for exchange rate behavior. Our model reproduces many significant features of the real and nominal dollar/DM exchange rates: both are very persistent, with differences that are nearly serially uncorrelated; both are very

7. These mechanics continue to apply when we set the weight on the real exchange rate in the Taylor rule ( $\gamma_q$ ) to a very small value such as 0.01, implying a discount factor nearer to 1. Consistent with the first and second point,  $\text{corr}(\hat{q}_t, q_t)$  stays high (it falls slightly to 0.30) and autocorrelations of  $\Delta\hat{q}_t$ 's stay near zero in absolute value. Consistent with the third point, the variances of  $\hat{q}_t$  and  $\Delta\hat{q}_t$  rise, as does  $\text{corr}(\Delta\hat{q}_t, \Delta\hat{s}_t)$  (to 0.95). On the other hand, and consistent with the discussion in the text, when we push  $\gamma_q$  upward to the implausibly high value of 1000, implying a discount factor near zero, only the first result continues to apply. The value of  $\text{corr}(\hat{q}_t, q_t)$  stays high (in fact it rises modestly to 0.43). But  $\hat{q}_t$  is no longer random walk like (first order autocorrelation of  $\Delta\hat{q}_t$  is  $-0.49$ ). And the variances of  $\hat{q}_t$  and  $\Delta\hat{q}_t$  fall, as does  $\text{corr}(\Delta\hat{q}_t, \Delta\hat{s}_t)$  (to 0.34). Incidentally, as in Mark (2005),  $\gamma_q = 0$  could be allowed if the unconditional mean of the real interest differential is zero.

volatile; differences of the two are highly correlated. We had less success in reproducing the correlations of exchange rates with output and inflation, perhaps because our empirical work omitted shocks to the Taylor rule itself and to interest parity. Finally, the model-based real and nominal exchange rates we construct from functions of VAR forecasts of output and inflation are correlated with the actual real and nominal exchange rates. The correlation is modest, but perhaps is acceptable by industry standards.

Our empirical model takes the time series process for inflation and output as given. A priority for future work is to explicitly interpret these variables in terms of behavioral equations. One advantage of doing so is that we would be able to measure the Taylor rule shocks. In order to improve the fit for exchange rates, the structural model would probably need to do a good job explaining inflation and output as well.

The focus of much of the existing quantitative literature on Taylor rules in open economics is on optimality properties of different rules. We believe there is promise in further exploration of the implications of such models for the empirical behavior of exchange rates.

## APPENDIX

This Appendix outlines the model used to discuss impulse responses at the end of Section 1 and to help interpret the empirical results in Section 3.

Our New Keynesian open-economy model is similar to the ones in Benigno (1999), McCallum and Nelson (1999), and, especially, Galí and Monacelli (2002). We imagine a two-country world. Each of the countries produces one good and consumes two. The foreign country is large, in the sense that its aggregate price level and consumption is indistinguishable from the price and consumption of the good it produces. The home country, by contrast, is small. The price of the home country's domestically produced good is  $p_{dt}$ , of the imported good is  $p_{ft}$ . Corresponding inflation rates are  $\pi_{dt}$  and  $\pi_{ft}$ . Preferences are logarithmic in a Cobb-Douglas aggregate over the two goods. In the Cobb-Douglas aggregate, the weight on the home produced good is  $1 - \alpha$ , on the foreign produced good  $\alpha$ . From familiar logic, then, the aggregate inflation rate  $\pi_t^h$  obeys

$$\pi_t^h = (1 - \alpha)\pi_{dt} + \alpha\pi_{ft}. \quad (\text{A1})$$

For the foreign country,  $\alpha \approx 1$ . Otherwise, all parameters are identical in the two countries. The law of one price holds, so

$$\pi_{ft} = \Delta s_t + \pi_t^*. \quad (\text{A2})$$

Adjustment of prices of the domestically produced good takes place according to

$$\pi_{dt} = \beta E_t \pi_{dt+1} + \kappa y_t^h + u_{ct}^h, \quad \pi_t^* = \beta E_t \pi_{t+1}^* + \kappa y_t^* + u_{ct}^* \quad (\text{A3})$$



In Equation (A3),  $0 < \beta < 1$ , and  $\kappa > 0$ , and  $u_{ct}^h$  and  $u_{ct}^*$  are cost shocks. We note that such cost shocks are absent in the model of Galí and Monacelli (2002); we allow them for empirical relevance and consistency with the broader literature on open-economy macroeconomics. Thus,

$$\pi_{dt} - \pi_t^* = \beta E_t(\pi_{dt+1} - \pi_{t+1}^*) + \kappa(y_t^h - y_t^*) + u_{ct}^h - u_{ct}^*$$

which, for notational simplicity, we write as

$$\overset{\circ}{\pi}_t = \beta E_t \overset{\circ}{\pi}_{t+1} + \kappa y_t + u_{ct}, \quad \overset{\circ}{\pi}_t \equiv \pi_{dt} - \pi_t^*, y_t = y_t^h - y_t^*, u_{ct} \equiv u_{ct}^h - u_{ct}^*. \quad (A4)$$

The output gap differential  $y_t$  is linearly related to the real exchange rate and an exogenous disturbance

$$y_t = \theta q_t + u_{yt}, \quad (A5)$$

with  $\theta > 0$ . Equation (A5) can be motivated from first principles, as in Galí and Monacelli (2002), in which  $\theta = 1/(1 - \alpha)$ , for  $\alpha$  defined in Equation (A1), and  $u_{yt}$  is the difference between home and foreign productivity shocks. Alternatively, it can be taken as a textbook IS curve in an open economy. (To prevent confusion, we note that Equation (A5) and the rest of our model is consistent with an open-economy dynamic IS curve relating expected output growth to a real interest rate; see Galí and Monacelli 2002.)

For analytical convenience, we assume that the Taylor rule involves one period ahead rather than 12 period ahead expected inflation, and that all variables are measured at monthly rates. Then, if we follow the logic in the text, we see that interest parity and Taylor rules lead to

$$(1 + \gamma_q)q_t = E_t q_{t+1} + E_t(1 - \gamma_\pi)\pi_{t+1} - \gamma_y y_t - u_{mt} \quad (A6)$$

where, as in the text,  $q_t \equiv s_t - p_t^h + p_t^*$  is the real exchange rate,  $\pi_t \equiv \pi_t^h - \pi_t^*$  is the inflation differential, and  $u_{mt}$  is the exogenous monetary policy shock.

Identities (A1), (A2) and  $\Delta q_t \equiv \Delta s_t - \pi_t^h + \pi_t^*$  imply that  $\overset{\circ}{\pi}_t$  and  $\pi_t$  are related via

$$\overset{\circ}{\pi}_t + \frac{\alpha}{1 - \alpha} \Delta q_t \equiv \pi_{dt} - \pi_t^* + \frac{\alpha}{1 - \alpha} \Delta q_t = \pi_t^h - \pi_t^* = \pi_t. \quad (A7)$$

Use Equation (A5) to substitute out for  $y_t$  in Equations (A4) and (A6). Use Equation (A7) led one period to substitute out for  $\pi_{t+1}$  in Equation (A6). Upon defining

$$\gamma = \gamma_q + \gamma_y \theta, \eta \equiv 1 + \frac{(1 - \gamma_\pi)\alpha}{1 - \alpha} = \frac{(1 - \alpha\gamma_\pi)}{1 - \alpha}$$

one may write the two resulting stochastic equations in the two variables  $q_t$  and  $\overset{\circ}{\pi}_t$  as

$$\beta E_t \overset{\circ}{\pi}_{t+1} - \overset{\circ}{\pi}_t + \kappa \theta q_t = -\kappa u_{yt} - u_{ct} \quad (\text{A8a})$$

$$(1 - \gamma_\pi) E_t \overset{\circ}{\pi}_t + \eta E_t q_{t+1} - (\gamma + \eta) q_t = u_{mt} + \gamma_y u_{yt}. \quad (\text{A8b})$$

Use Equations (A8a) and (A8a) led one period to substitute out for  $q_t$  and  $E_t q_{t+1}$  in Equation (A8b). The result is a second order stochastic difference equation in  $\overset{\circ}{\pi}_t$ . Given restrictions on parameters, the difference equation has a unique stationary solution, with  $\overset{\circ}{\pi}_t$  the present value of future shocks. (We do not write the restrictions because they are not particularly enlightening. But we do note that there can be a unique stationary solution even if  $\gamma_q = 0$ .) This solution can be put into (A8a) to solve for  $q_t$ . The other variables in the system, including  $\pi_t$  and  $y_t$ , can then be constructed.

Some closing notes:

1. The discussion in the text of signs of the impulse response functions in Section 1 presumes: (a) The shocks  $u_{mt}$ ,  $u_{yt}$ , and  $u_{ct}$  follow stationary AR(1) processes with positive parameters, and (b)  $\alpha \gamma_\pi < 1$ , a condition consistent with our small country assumption.
2. While this model with AR(1) shocks is certainly too simple to reproduce all of the serial correlation and second moments, it is consistent with some notable features of the data. This includes higher first order autocorrelations in  $q_t$  and  $y_t$  than in  $\pi_t$ , along with very high correlation between  $\Delta s_t$  and  $\Delta q_t$ .

The model's persistence properties are to a certain extent exogenous—when the shocks follow AR(1) processes, the endogenous variables are linear in those shocks. (Thus this model shares the property of many New Keynesian models of generating little endogenous persistence. For example, to generate plausible persistence in real exchange rates, Benigno (1999) requires that a lag of the interest appear in the monetary rule with a large coefficient. While we have not checked the details, it appears that such a lag would also generate persistence in our model.) But if the IS or monetary policy shocks are persistent and volatile while the Phillips curve shock is not, the properties described in the previous paragraph will result if, as well, the Phillips curve is flat and the share of imports is not too large. Low serial correlation in  $\overset{\circ}{\pi}_t$  follows when  $\overset{\circ}{\pi}_t$  is not very responsive to the output gap, and Phillips curve shocks have low serial correlation. CPI inflation  $\pi_t$  will behave much like PPI inflation  $\overset{\circ}{\pi}_t$  when import shares are low. Further, high serial correlation in relative output and the real exchange rate will be reproduced in the model when Taylor rule shocks and IS shocks are highly serially correlated and volatile, for such shocks will dominate the behavior of  $y_t$  and  $q_t$  when inflation is not persistent. As well, there will be high correlation between innovations to real and nominal exchange rates since neither series will be much affected by inflation.

3. Increasing the weight  $\gamma_q$  put on the real exchange rate in the monetary rule decreases the variance of the real exchange rate. In particular, in the limit, as  $\gamma_q \rightarrow \infty$ , the variance of  $q_t$  goes to zero, while those of  $y_t$  and  $\pi_t$  stay finite.

4. The model can be generalized easily to the case in which the foreign country consumes a nontrivial amount of the home country good. Assume a Cobb-Douglas utility function for consumption, in which the foreign residents put a weight of  $\epsilon$  on foreign goods (i.e., the goods produced in the foreign country). Equations (A8a) and (A8b), which describe the dynamics of  $\overset{\circ}{\pi}_t$  and  $q_t$ , still hold, except  $\eta$  is now defined as:  $\eta = 1 + [(1 - \gamma_\pi)(1 + \alpha - \epsilon)/(\epsilon - \alpha)]$ . Equation (A7), relating relative PPI inflation  $\overset{\circ}{\pi}_t$  to relative CPI inflation  $\pi_t$  becomes:  $\overset{\circ}{\pi}_t + [(1 + \alpha - \epsilon)/(\epsilon - \alpha)] = \pi_t$ .

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