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Teacher as Designer: A Framework for Teacher Analysis of Mathematical Model-Eliciting Activities

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Abstract

The study investigated tool development by three middle school mathematics teachers. The tools they designed were intended to support the use of model-eliciting activities (a form of instruction related to problem-based learning) and particularly the students' presentations of their solutions for the whole class. The study examined the design and purposes for the presentation tools and resulted in a framework for categorizing teachers' purposes for tools. The framework addressed the unit of analysis for the tools (individual students or groups of students) and the nature of teachers' purposes for the tools. Design research was used as a theoretical perspective for conducting the investigation.

Keywords: mathematics, design, tools, models, presentation

Background

John Dewey (1990) stated, "The fundamental point in the psychology of an occupation is that it maintains a balance between the intellectual and the practical phases of experience" (p. 133). Although many studies have examined the nature of mathematics teachers' knowledge (e.g., Ball, 2000; Shulman, 1986) and in particular their understanding of problem solving or other methods related to inquiry-based teaching, less investigation has been conducted about the teacher as a designer of tools for practice or the "practical phases." Some studies of teaching in mathematics education have focused on teacher development where researchers have an ideal notion of teaching in mind and investigate how teachers carry out that ideal (e.g., Carpenter, Fennema, Peterson, Chang, & Loef, 1989; Romberg, 1997). Other studies have focused on the interactions between teacher professional development facilitators and teachers and the impact on teaching (e.g., Borko & Putnam, 1995; Simon & Tzur, 1999; Swafford, Jones, Thornton, Stump, & Miller, 1999). However, the studies do not focus particularly on the products and objects teachers design for their classrooms. Rather, emphasis is placed on teachers' behaviors or teachers' cognition related to their practice and understanding of content.

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MacNab (2000) and Clements (2002) described the need for research at multiple stages of instruction from the ideal curriculum to the implemented curriculum to the experienced curriculum. One interpretation of "curriculum" is what students should know and be able to do as represented in a particular set of instructional materials and the related teaching methods. For problem-based learning, this interpretation means considering learning and instruction from the intentions of the original designers to the teachers' interpretations and implementation of practices to the students' experiences and learning. Teachers often need to design support structures in order to implement curriculum or activities in their classrooms. Gutstein (2003), for example, discussed how he modified and supplemented a reform-based curriculum in order to better serve his urban, Latino student population. Maher (1988) explicitly refers to the teacher as designer and investigated both the tools teachers developed and the teachers' conceptual learning. Two points are important to note. First, that materials and products, particularly for inquiry-based teaching or problem-based teaching, may be revised, modified, or supplemented by the teachers in order to fit their needs and contexts. Second, that modification is a cyclical process of design, testing, and revision as teachers learn more about what works and what does not work according to their perceptions of classroom needs.

Limited research on problem-based learning has been carried out at the K-12 level (Savery, 2006). In addition, a challenge in problem-based mathematics teaching is developing a classroom environment that uses engaging, problem-based activities for students that elicit complex, creative mathematical thinking (Ertmer & Simons, 2006). The study described here focused on how teachers designed tools to support teaching and learning using small group, mathematical modeling activities. For instance, one task asked students to generate a procedure for determining the number of boxes required to pack 1 million books collected in a school book drive. After the activity, groups presented their work in a whole-class presentation. Sometimes, their classmates may be encouraged to ask questions after the presentation.

The presentations raised assessment issues for the teachers in terms of what and how to assess what students presented and discussed. As a result, teachers developed products for orchestrating and assessing the presentations. These products are referred to here as "presentation tools" and include any tools the teachers designed that were associated with the students' final presentations of their solutions (e.g., a scoring rubric, presentation outlines). We refer to the products as "tools" to emphasize the functional nature of the objects the teachers designed. Any tool is designed with a particular purpose and function in mind. The most fundamental test of a tool is whether it carried out the function as desired. This leads to a design process as tools are designed and tested multiple times to bring them closer to filling the intended purpose.

The study addressed the following question: What is a framework for analyzing the characteristics of the tools teachers design for implementation with mathematical modeling activities? To answer this question, a design research lens was taken in assembling case studies of teacher practice around middle school mathematics activities. The three teachers designed tools (referred to as presentation tools) related to the students' oral presentations and discussions of their solutions to the problem-solving activities. Multiple model-eliciting activities were used by each teacher so the tools were revised and modified over time to meet their intended purposes for implementing the activities. Based on the common characteristics that emerged, a framework was developed to sort, classify, and track the tools over time.

Design Research

Due to the complexity and the constraints of teaching mathematics, we draw a parallel to design research in other contexts in order to examine tool development by middle school mathematics teachers and to develop a framework for characterizing such tools. Design research and experiments as initially conceived by Brown (1992) and others (Collins, 1992; 1999; Edelson, 2002) focused on investigating teaching methods in real classroom situations. We focused on presentation tools as one artifact of mathematics teachers' design process associated with complex, problem-based activities. The design of the tools for teaching is similar to the design of tools in engineering contexts in that multiple variables must be considered, the problems that exist are human problems in a changing environment, and the design situation faces various constraints. Revision of the tools and processes over time occurred as the teachers used different activities and learned more about how students would solve the given problems. Tools were developed by the teachers that solved local problems but with characteristics transferable to other, similar situations. The notion of the "best" tool was locally defined by the needs of the designer (in this case, the teacher), and tools were refined over time to become better suited to the teachers' purposes and to accommodate changing needs in the local context.

Design research encompasses both the study of a product and the design process. The design metaphor is related to engineering design where products are designed to solve human problems within particular contexts. In engineering, problems are identified and engineers need to creatively integrate knowledge from multiple disciplines. Any problem-solving situation (in teaching or in engineering) will face constraints and opportunities. Engineering, like teaching, includes the design of products within constraints where revision is a natural part of the design process.

In the case of mathematics teaching, knowledge about mathematics, students' learning, pedagogy, and knowledge about the local context are integrated. Teachers engineer, or design, their classrooms in order to maximize learning. The classroom environment and tools are open to modification to continuously improve their functioning. The design process becomes more complex as the tasks assigned to students become more complex and teachers work in unfamiliar territory where students' responses are not always predictable. The lack of predictability is where design becomes useful as a metaphor for teaching. Because the students' solutions to a problem-based task are unknown to a degree, the teacher needs to design tools that are responsive to multiple solutions. The open-ended nature of the tasks and work in small groups increase the complexity of the analysis and the number of pedagogical decisions to be made by the teacher. Hence, designing tools for a changing context (where teachers learned more about students' thinking and changed their expectations of students) and complex work by students becomes a process of integrating knowledge of teaching and mathematics, determining which variables are important to assess, and developing tools to aid teaching.

Design research was used as a lens to investigate the design of an artifact, to examine the underlying assumptions of that artifact, and to generate frameworks to facilitate further analysis of teacher-designed tools. Other examples of design research include work in software (Bannan-Ritland, 2003; Oshima et al., 2004), curriculum (Barab & Luehmann, 2003), and other collections of educational products (Cobb, Confrey, diSessa, Lehrer, & Schauble, 2003; Gersten, 2005; Gorard, Roberts, & Taylor, 2004). This study is distinct from previous work in design in that the teachers were the principal tool designers. Design research projects often incorporate teachers as partners in the design process. Their role is often to implement and to provide feedback about products designed by researchers (e.g., Bannan-Ritland, 2003; Edelson, 2002; Gersten, 2005; Jitendra, 2005). The teacher as designer metaphor has been explored previously in mathematics education (Maher, 1988) but not in relationship to design experiments or design research.

Problem-Based Learning and Model-Eliciting Activities

The goal of a model-eliciting activity is for students to develop a model that solves a problem or makes a decision for a client. The model-eliciting activities used in this study were developed using six principles defined by Lesh, Hoover, Hole, Kelly, and Post (2000). For instance, in the Million-Book Challenge activity, students had to determine how many boxes would be needed to pack and ship 1 million books collected in a school-based book drive. The complexity in the task was that the size of the books was unknown and varied. The books collected ranged from elementary picture books to novels to reference books. This variation meant students needed to collect data about typical books, perform estimates about book size and volume, and generate a model for making the estimate for the number of boxes needed. This task is similar to tasks in engineering design and other applied fields where statistical inference is needed to estimate and generate a model for predicting a value. The task required students to use area, volume, data analyses, and statistics to generate their models.

As a problem-solving process, the students had to draw on their own knowledge and experience with packing and estimation in order to develop a solution. Traditional mathematics teaching proceeds with the teacher demonstrating a concept or topic and the students completing a series of exercises to practice the algorithm or skill that was demonstrated. Model-eliciting activities work in the opposite direction.

Previous studies have examined students' solutions to model-eliciting activities where a range of models are developed using multiple mathematical topics (Carmona-Dominguez, 2004; Doerr & English, 2003). In an interview with Katy, an eighth-grade teacher in this study, she described the strength of the activities as follows:

> … having to actually just kind of look at a situation and pull from their knowledge base an actual mathematical concept that they could apply to that. Kind of the opposite of what we normally do where we say, "Ok here's the concept, now apply it to this situation." Giving them the situation and letting them pull out a concept—opposite of the way we usually present the material—which is just so much more valuable. To have them be able to look at the situation and actually come up some mathematical concept that would work.

The students typically spent three to five days on a single activity. The first day was an introduction and orientation to the problem and its context. The following days, the students worked in small groups to develop their final solution. The final day comprised presentations of students' solutions to the whole class. During the final day, each group presented its solution and explained what they had done to solve the problem. Then, the teacher and the rest of the class had an opportunity to ask questions of the group. Teachers structured and conducted the presentations differently. The differences in structure and emphasis are the focus of this study.

Model-eliciting activities and problem-based learning are related instructional approaches. Both approaches include problems situated in realistic, meaningful contexts that require students to design approaches to solving a task. Both approaches also include the integration of knowledge and skills toward the solution of the task. Model-eliciting activities for middle school are usually designed for students to complete in a few days so students are not required to do as much background research as they might do in a problem-based learning activity. The model-eliciting activities in this study were used on their own; however, model-eliciting activities could be used as the introduction to a longer unit (e.g., Lesh, Cramer, Doerr, Post, & Zawojewski, 2003). A distinctive characteristic of model-eliciting activities is that students are designing a model as their final product. The model is a procedure or process rather than a single numeric answer. The model includes representations of the conceptual systems used to develop a procedure or explanation for a purpose (Lesh, Doerr, Carmona, & Hjalmarson, 2003). Problem-based learning does not necessarily have such an explicit focus on the development of procedures or processes.

Researcher Principles

Three principles guided the design of this study and its focus on the teacher as designer of the presentation tools. First, the teachers have the most expertise about their classrooms and teaching contexts. Second, the teachers should be the owners and developers of their products. Finally, because the teachers were the owners of the products, Hjalmarson (the first author) functioned as a consultant providing suggestions and advice only as requested. The goal was not to be a professional development facilitator intending to teach them about presentations and classroom discourse, but rather to provide advice if requested and document their design processes in order to find out how the presentations naturally fit into their goals for their students' learning. The teachers were the primary designers of the tools for their own classrooms.

The first principle, teacher as expert, represents the core of the study. When requesting their participation, teachers were told the study would investigate tools surrounding the students' presentations. In this instance, the teacher as expert was important because he or she had the most experience in the classroom context. All the teachers had previously used similar activities. We wanted to work with teachers who had reached a point where they wanted to use the activities with some regularity and to integrate them into regular classroom practice.

The second principle, teacher as owner, was critical to understanding the needs teachers naturally had within the context and the types of solutions they deemed appropriate to identified problems. For instance, student work can be assessed as the students present in class. Tools that are designed for this purpose need to be sensitive to the classroom expectations and the nature of assessing oral presentations. The teacher's ownership of the products also meant that she had an authentic interest in revising the tools with successive activities. She could determine what was useful and not useful even as the operational definition of "useful" might change over time. The teacher was responsible for revisions based on her understanding and analyses.

The final principle, researcher as consultant, was important for Hjalmarson's relationship with the teachers as principal designers of the tools. Hjalmarson did not determine the purpose of the presentation tools and provided input only when requested from the teachers. Part of the rationale was to be able to document the design of tools in as natural an environment as possible when a researcher is present. The study investigated how the teachers designed tools based on their own needs and from their own resources rather than a tool imposed from the outside. This principle depends on the first two in that seeing them as experts and principal designers enabled the role of researcher as consultant.

Methodology for Data Collection and Analysis

Methods

Three middle school teachers in the urban fringe of a large city participated in the study. Two teachers taught sixth grade (Amy and Abby), and the third teacher taught eighth grade (Katy). They were selected based on the following criteria. Each teacher agreed to implement at least three model-eliciting activities over the course of one school year. All three teachers had been using activities for at least two years and had participated in summer institutes or professional development workshops about model-eliciting activities. They agreed to let the first author observe in their classrooms throughout each model-eliciting activity and provided us with copies of the presentation tools. Two teachers' tools are described in more detail in subsequent sections (Amy and Katy).

Data Sources

Four primary sources of data were collected for this study: presentation tools, observation notes from contact with the teachers throughout the school day, transcripts of classroom interactions, and semistructured interviews. Contact summary sheets and document summary forms were used for observations and tool collection for preliminary analysis as the data were collected (Miles & Huberman, 1994). The forms were designed to provide responses to the research questions about the type of tool used and the purposes for the tool. Hjalmarson completed the data collection and analyses in between fall 2002 and spring 2004.

Each teacher completed at least three model-eliciting activities during the course of the study. Because the presentation tools are the focus of this study, they were the most important source of data. The other data supported the interpretation of the presentation tools. Hjalmarson collected the tangible tools each teacher used. Tools were often being designed as the activity was underway (on-the-spot design) so having the tools in advance of the activity or the presentations was sometimes neither possible nor desirable. The questions asked by the teacher of each presentation were noted and audiotaped. Tapes were transcribed and the transcripts were filed with the field notes for the day. Notes for each tool included when the tool had been implemented, the type of tool it was, and any observations made about student or teacher reaction to the tool using a document summary form (Miles & Huberman, 1994).

In many cases, because Hjalmarson was in the classroom, the teachers discussed the tools with her informally throughout the school day. Daily notes included as much about these spontaneous, informal discussions as possible (Merriam, 2001; Patton, 1990). These spontaneous conversations provided insight into the teachers' on-the-spot decision making and impressions of the activity. They served as short, unstructured interviews within the school day to discuss events as they unfolded (Fontana & Frey, 1998; Patton, 1990).

The study also included one semistructured interview (Merriam, 2001) with each teacher at the end of the data collection in order to document their impressions of the model-eliciting activities in general and of the presentations in particular. Each teacher was asked slightly different questions based on the tools they had implemented and on our observations of the classroom. All of the interviews included questions about what they valued about the model-eliciting activities, their goals for the presentations, unexpected results from the students, and their implementation of the activities and the presentations. Specifically, the interviews asked what the teachers wanted to hear from the students during the presentation and what they expected the students to be doing during the presentations (either as presenters or audience members) to document the purpose behind the presentation tools. Their responses supported later analysis of the transcripts of whole-class discussions, the tools, and other field notes about the presentations in their classrooms. An interview before the start of the activities was not possible because we did not have enough notice from the teachers before the start of the activity to schedule a time to meet. However, the data gathered in the classroom as the teachers worked (in particular the tools they designed) helped document their purposes for using the activities and their initial purposes for the presentation tools.

Data Analysis

As with the data collection, data analysis was guided by chronologically documenting the presentation tools as they were designed. Consistent with the need to document the design process for developing the case studies of each teacher, data analysis for the study was divided into two major phases: during and after data collection. The analysis during the data collection includes all analyses from the first day in the first teacher's classroom until the last day of data collection. Data analysis about the entire set of data collected from all teachers was categorized as occurring after data collection.

Data analysis was also divided into two approaches: within-case and cross-case analysis (Merriam, 2001; Miles & Huberman, 1994). Each teacher in the study comprised one case including at least three model-eliciting activities for within-case analysis. A preliminary analysis of the tools occurred as they were collected from each teacher. Transcripts and the field notes supported the analyses of the purpose for each presentation tool. Constant comparison analysis (Merriam, 2001; Strauss & Corbin, 1998) occurred between activities within each teacher case study.

The cross-case analysis built on the results from the within-case analysis. Constant comparison of results for the cross-case analysis aided the analysis for individual cases as well as the models we designed for generating results across teachers (Merriam, 2001; Strauss & Corbin, 1998). Constant comparison for the cross-case analysis during the data

Table 1

Tool characteristic categories.

Characteristics	Description	Example
Structure	What tool looks like	Scoring guide, presentation outline
Form	How the tool is implemented	Written, verbal
Content	What is included in the tool	Questions about final product
Time of use	When the tool is used or discussed in the classroom	Introduction to activity, presentations

collection occurred as we collected data in one classroom and then spent time in a different classroom. In addition, constant comparison occurred as we collected a larger set of tools and gained more experience within the classroom settings. The constant comparison analysis situated the individual teachers' set of tools within the set of all teachers' tools and provided a description of the range and types of presentation tools.

The initial data analysis occurred as the data collection progressed (Merriam, 2001). The summary forms captured the preliminary data analysis during the data to record impressions of the tool design and implementation (Miles & Huberman, 1994) and were supported by the field notes for each day. The field notes recorded as much information as possible about what the teacher said about the tools and about how they were used in the classroom. As discussed previously, this data collection included recording comments they made in informal discussions with the first author throughout the school day as well as the use of the tool with their students. The documentation forms noted how the tool had been used in the classroom and any potential changes that the teacher mentioned. Analysis of the current presentation tool, its purposes, and its effectiveness at meeting the purpose supported the analysis of tool revision.

The second phase of data analysis occurred after data collection. First, the tools were organized by type and categorized using the scheme in Table 1.

Next, the teachers' purposes for the presentation tool and the presentation itself were identified. The purposes for the presentation tools were triangulated by examining the interview transcripts, the field notes, and transcripts from classroom observations. For example, Katy described what each category meant on her scoring guide to the students at the start of the activity. When Katy used the scoring guide to assess responses, it provided further data about her purposes for the tools and about how well the tool met those purposes. The scoring process revealed how well the guide captured her expectations for student responses and how the scoring guide served to differentiate high-quality and low-quality responses.

Design Case: Presentation Outline

In order to illustrate the design process, one of the teacher cases is described here in more detail. This example was selected because Amy's purposes were clear, and she designed a unique tool. Unlike the other two teachers who primarily designed scoring rubrics, Amy designed a presentation outline she revised over time. From the presentation, she wanted to learn about her students' thinking processes for solving the problem. The outline was classified as a presentation tool because Amy had represented a purpose in a tangible way that she shared with students. The outlines facilitated the students' presentations and communicated her expectations for the activity to them. Amy wanted to learn more about her students' solution processes for the activity so she designed an outline for each activity that consisted of a series of questions or statements that students should respond to in their presentations (Hjalmarson, 2003, 2004). As she stated in the following interview excerpt, she was interested in the path between their first idea for a solution and their final product.

> The reason that I started doing the questions is I just didn't feel I was gettin' enough out of 'em. They would get up and either present their letter or if it was a poster board, they'd get up … introduce themselves, present their poster board, and I just felt like "Ok, but how did you get there?" I wasn't finding how they were getting there to be able to make that presentation. That wasn't gettin' shared. And, that's what I was more interested in than anything. Especially since so often the results are not always correct. So, I was more interested in their thinking.

As she stated, the tool was developed to meet a need she identified in order to enrich the experience for her students and to help her learn about their thinking. She had been designing presentation outlines before the start of the study during the previous school year. During this study, she designed four outlines.

Amy was interested in her students' thinking both as they presented their own solutions and as they responded to presentations by other teams. She reminded them during the question phase that she would ask questions, but that she wanted them to ask questions of the group as well. The first outline was for the mini-golf activity. The students had to design a miniature golf course using specified shapes for the holes. The holes were not drawn to scale and the task required the students to scale up the holes and make them all the same scale in order to determine how to position them in the given area. All of the groups produced a letter and a miniature golf course layout on a poster board. The outline included the obstacles the group may have encountered

(“Getting started struggles”), the moment they knew how to solve the problem (“Aha! Moment”), and the final product (reading the letter and describing the layout). “Who did what” asked them to describe how tasks were divided amongst the members of the group (e.g., writing the letter, cutting out shapes). Questions came either from Amy or other students in the class.

Amy was interested in how each group solved the activity from their initial ideas to their final product. Based on her experience, she felt the students had not explained clearly or specifically the solution paths they had followed. She was trying to determine a way to word a question to elicit each group’s solution path more clearly. Even though the outlines were improving, she was still not entirely satisfied with their effectiveness in eliciting students’ solution processes even toward the end of the study when we discussed them. She explained:

> [Be]cause some case studies lend [themselves] more to talking about struggles they had getting started. What I would like to hear them do more—and this is another question I haven’t figured out exactly how to word—is paths that they started to follow. Like they thought, “Oh, this’ll work,” and then what caused them to realize that it didn’t. And, so then what did they start thinking next or move to next?

For the next activity (Million Book Challenge described previously), she changed the introduction of the task. The students read the problem individually and wrote about their initial ideas and thoughts about the problem in a notebook. The change to the introduction created a change in the presentation. When each group presented, students spoke about their individual initial ideas before the group started working together. Describing the individual ideas led to a clearer description of the path from initial thoughts to final product when they responded to the first statement in the outline in Figure 1 (“First thoughts”). The presentation tool was affected by an instructional decision that occurred early in the solution process. The interaction between the tool and instructional decisions illustrates how changes were made both to the tool and the classroom context.

Million-Book Presentation Outline

- First thoughts
- How you chose your group
- Troubles
- Plan to solve
- How confident are you about your answer?
- Read your letter
- Questions

Figure 1. *Amy’s revised presentation outline.*

Another change between the two outlines was the addition of "How confident are you about your answer?" as a statement in the outline. This change occurred when Amy used the Million-Book Challenge activity first with her advanced class and then with her other classes. The outline for the presentations in the advanced class did not contain the question, but she asked it to groups after their presentations and found it elicited information about their processes. In answering the question, the groups had to describe the mathematics of their solutions and could compare their mathematical methods with methods from other groups that had presented. They could also describe strengths and weaknesses of their procedures. Such descriptions explained their mathematical thinking related to the task. As a result, Amy added the question to the outline when she used it with the rest of her classes. This change was the shift of an informal question to a formal part of the outline, and it provides another illustration of changes to the tool that were a direct result of classroom practices in situ.

The use of the presentation outlines structured the students' presentations and communicated Amy's expectations to the students. After a few activities, the students knew what to expect from the outline even before she wrote it on the board for them to follow. They knew the types of questions she was going to ask them about their product and process. From a design perspective, to improve the value of the presentation, Amy's outline facilitated her learning about students' thinking and mathematical solutions. The outline developed as she learned more about her students' interaction with the activities. Amy's design process for her presentation outlines was grounded in her purpose for the presentations (hearing about solution process) and in her classroom practice.

Design Case: Scoring Guides

In contrast to Amy's focus on process, Katy's scoring guide focused exclusively on the final product (i.e., the mathematical model the students developed) rather than group interaction, metacognition, and so on. Figure 2 shows the scoring guide she designed for the Lawnmower activity. In referring to the "method" and how they used the data (the second and third items in the rubric), she intended for them to describe their mathematical model or procedure. Her other scoring guides were similar. The task in the Lawnmower activity was to design a procedure for hiring employees based on information about individuals' prior performance mowing lawns. The students needed to aggregate the data statistically and determine which variables were relevant to the hiring question. She used the scoring guide during class as each group presented. She asked the first author to score using the rubrics as well and we discussed the scoring after school.

Lawnmower Case Study Scoring Guide

Which 4 employees should be hired? (3 points)

Explain the method you used in detail to make this decision. (5 points)

How did you use the data provided to make your decision? (5 points)

Is your response in the form of a letter? (2 points)

Figure 2. *Katy's scoring guide.*

Because the guide needed to be completed in class as Katy was listening to the presentations and monitoring classroom activity, the guide needed to be simple and short. Katy used the task description as a guide for developing the rubric so the task and the assessment would be closely aligned. Time was an important variable because the class periods were only 45 minutes long. If all the groups were going to present, then Katy did not have a lot of time between groups to take notes about their work or to complete a complex rubric or scoring guide. Katy also collected the letters the students wrote (unlike Amy who did not collect the letters). Katy did no assessment of individual students in the group. She also did not incorporate their problem solving or group interaction as part of the scoring requirement. The central focus of all her rubrics was the group product that had emerged.

Katy's rubrics served a fundamentally different purpose, and the type of product designed to meet that purpose was different from Amy's outlines. Katy was more focused on the quality of the final product and whether the students created a usable procedure. The changes to Katy's rubrics were not as significant as the changes to Amy's outlines throughout the study, but both teachers did refine their presentation tools over time. The two sets of tools represent different yet equally valid goals for presentation tools. The tools accomplished some goals well, but were never intended to meet other goals (e.g., the presentation outline was not intended to quantify student performance on the activity, the scoring rubric was not intended for understanding student process). As a result of the qualitatively different goals represented in the tools designed by each teacher, the framework for tool analysis was developed.

Development of Framework for Presentation Tool Analysis

To support the development of a framework for analyzing the collection of all the presentation tools, the tools were categorized based on the characteristics in Table 2.

Table 2

Description of tool characteristics.

Tool	Structure	Form	Content	Time of Use
Amy				
Mini-Golf Outline	Set of 6 points for an outline	Orally by students	Initial struggles, aha, group interaction, description of final product	Presentation
Million-Book Outline	Set of 6 points for an outline	Orally by students	Impressions, struggles, group interaction, aha, final product	Presentation
Million-Book Outline (Revised)	Set of 7 points for an outline	Orally by students	Impressions, struggles, group interaction, confidence in solution, final product	Presentation
Snowflake Outline	Set of 5 points for an outline	Orally by students	Impressions, struggles, group interaction, final product	Presentation
Katy				
Summer Reading Scoring Guide	Scoring guide with 3 categories	Written by teacher	System content, usability, letter	Introduction, working, presentation
Departing on Time Scoring Guide	Scoring guide with 3 categories	Written by teacher	Detailed ranking process, ranking, letter	Introduction, working, presentation
Lawnmower Scoring Guide	Scoring guide with 4 categories	Written by teacher	Detailed ranking process, ranking, use of data letter	Introduction, working, presentation
Abby				
Mini-Golf Scoring Guide	Scoring guide with 4 categories	Written by teacher	Group work, work shown, model, letter	Introduction, presentation
Mini-Golf Long Follow-up	Worksheet with 8 categories	Written by students	Construction of scale models, generalizing golf course, problem solving, reflection about methods, group interaction	Post-presentation
Mini-Golf Short Follow-up	Worksheet with 4 categories	Written by students	Reflection about methods, group interaction-	Post-presentation
Quilt Rubric	Scoring guide with 3 categories	Written by teacher	Final product, explanation of process, specific mathematical content	Post-presentation
Snowflake Rubric	Scoring guide with 5 categories	Written by teacher	Explanation, visuals, knowledge, final product, reasonableness of solution	Post-presentation

Table 3

Examples of tools in each quadrant of the framework.

	Process	Product
Group	Questions in a presentation about how the group came to its conclusions	Rubric analyzing the final solution produced by the group
Individual	Questions in a presentation about individual contributions to the process	Questions in a presentation individual contributions to the final solution (e.g., What were your initial ideas?)

This analysis resulted in the development of a framework for categorizing the tools designed by the teachers. The function of the framework was both to classify and examine the holistic nature of a set of tools. The framework had two dimensions. The first dimension was between individual and group products. The students worked in groups, but teachers may have asked questions about individual contributions or assigned individual assessment scores. The second dimension was between the process and the product. Each item in the tool was classified according to whether it was focused on the final product (e.g., the letter or solution) or the students' solution process (e.g., group functioning, reflection on idea development). Table 3 shows examples of tools that fit within each quadrant of the framework.

The dimensions work in concert with each type of focus used by at least one teacher. A single tool may have components that fit within different quadrants of the framework. All quadrants were represented by at least one tool designed by the teachers. The framework is one means of organizing a diverse set of tools designed by teachers and classifying them based on their purpose. Table 4 shows how the components of Amy's tools were categorized within the framework. As shown, her focus was largely on the group process with less emphasis on the final product or individual products. In contrast, Katy's tools (shown in Table 5) all fall within the group product quadrant. She revised her tools for that purpose.

Where Amy's tools have components distributed across multiple quadrants of the framework, Katy's tools are centralized in one quadrant. The point here is not to say that one teacher's tools are better than the other's, but rather to illustrate the diversity of tool purposes and types and how teacher practice can reflect different types of tools to support the same type of activity. The point is also to recognize that teachers may move between multiple purposes for the same type of activity or they may be more focused on a single purpose given the time constraints of the classroom environment.

Table 4

Analysis of Amy's presentation outlines on the analysis framework.

Amy	Process	Product
Group	MG-Outline-1 MG-Outline-2 MG-Outline-3 MB1-Outline-2 MB1-Outline-3 MB1-Outline-4 MB1-Outline-5 MB2-Outline-2 MB2-Outline-3 MB2-Outline-4 S-Outline-1 S-Outline-2 S-Outline-3	MG-Outline-4 MG-Outline-5 MB1-Outline-6 MB2-Outline-5 MB2-Outline-6 S-Outline-4
Individual	MB1-Outline-1 MB2-Outline-1	

MG = Mini-Golf activity
MB1 = Million-Book Challenge activity (version 1)
MB2 = Million-Book Challenge activity (version 2)
S = Snowflake activity

Table 5

Analysis of Katy's scoring guides on the analysis framework.

Katy	Process	Product
Group		SR-Rubric-1 SR-Rubric-2 SR-Rubric-3 DOT-Rubric-1 DOT-Rubric-2 DOT-Rubric-3 L-Rubric-1 L-Rubric-2 L-Rubric-3 L-Rubric-4
Individual		

SR = Summer Reading Activity Rubric
DOT = Departing on Time Activity Rubric
L = Lawnmower Activity Rubric

A strong point of the framework is the organization of tools by purpose and function. It does not organize them by quality because quality is a characteristic locally determined by the teacher using and designing the tool. A tool for analyzing problem-solving processes might not be good for analyzing final products, but that determination is up to the teacher who designs the tools. The framework organizes the tools based on what the tools are intended to do rather than an external characterization of what aspects of the model-eliciting activity or the presentation are valuable. For instance, the framework presents two different types of teachers' purposes when Katy's tools and Amy's tools are classified. Realistically, as represented by the placement of components of the tools on the framework, teachers are interested in multiple aspects of the model-eliciting activity (or any other problem-solving activity). The tools represent both an interest in mathematical content and mathematical process. The framework could be used for other types of tools related to complex problem-solving activities.

A second strength of the framework is the documentation of varying units of analysis used by the teacher. The unit of analysis is critical as teachers shift between individual learning, group learning, and whole-class learning. As an example of whole-class analysis, Chamberlin studied student-thinking sheets that teachers developed to organize class sets of student work by solution type (Zawojewski, Chamberlin, Lewis, & Hjalmarson, in preparation). Moving between the three levels is critical for teachers as they plan and analyze instruction. With a different set of tools and further study, teachers' aggregated analyses of whole-class work could be captured. Informally, the teachers in the present study did analyze information about the whole class but no tools were developed specifically for this purpose. The tools focused on either the individual's work or a group's work. Further work could investigate the whole-class level of analysis.

In terms of analyzing teacher practice and the development of tools, the framework highlights a few characteristics. First, a shifting focus within practice is a natural part of teaching. Sometimes teachers wanted information about a group as the unit of analysis. Sometimes they were interested in the individual unit of analysis. More detailed study could examine teachers shifting between individual, group, and the whole class. For secondary teachers, there is the added dimension of multiple whole classes divided into block periods that have their own characteristics. The question is not whether a teacher should focus on individuals or groups, but when and why the unit of analysis needs to be a group or the individual. The purpose dimension of the framework highlights how the function of a tool varies within an activity and across activities. Some teachers had a constant purpose from activity to activity and some teachers had purposes that varied across and within activities. Recommendations by the National Council of Teachers of Mathematics stress that mathematical processes are part of mathematics standards alongside content (2000). This framework for analysis of teacher tools encompasses both types of standards: process and content. The framework also highlights how purposes change and how teachers' views of their classrooms change. At various points, a teacher is interested in students as individuals, as groups, or as a class, depending on the purpose for the activity at hand. In parallel to the question about the unit of analysis, the question is not whether teachers should focus on process or product, but when and how they focus on either dimension.

Conclusions and Implications

The study emphasized what the teachers designed in terms of external products rather than their behaviors in the classroom. The "action" of the classroom was not the primary focus of the investigation. Even though the teachers' uses of the tools with students were part of the data collection, the design research lens emphasized the products teachers designed more than their actions in the classroom or what they said. We wanted to know what aspects of their teaching were elicited by the tool design so data collection was situated around the presentation tools. Similar to thought-revealing activities for students, this project was conceived as a thought-revealing activity for teachers by examining the design and development of tools for their classrooms. The products enabled us to see how they perceived the purpose of the model-eliciting activities in the classroom and how they perceived what aspects of the activity were valuable. However, every tool has limitations; no tool can capture all aspects of a classroom activity. The presentation tools demonstrated what kinds of design choices the teachers made for the particular phase of the activity (presentations) and what aspects of the model-eliciting activity were valuable from their perspective.

The primary contribution of this study is the development of a framework for analyzing and examining change in teachers' tools designed to support practice for model-eliciting activities. Problem-based learning activities can also include the assessment of students' final products as well as their problem-solving processes (e.g., group interaction, reflection, problem-solving strategies). Given a greater number of teachers or tools, the framework could be used to investigate the change in teacher focus (individuals or groups) depending on the type of activity. The teachers' unit of analysis could be useful for understanding how and when they analyze their students' learning. Both levels are important for teachers to understand. What is not clear is how, when, and why teachers shift their views from one level to another. A third level not addressed in the tools in this study is the whole class. Secondary teachers may compare and contrast different classes of students as they teach multiple classes of students in a single day. For the purpose dimension of the framework (product and process), further investigation could focus on how teachers practice changes related to the purposes. In addition, how do teachers relate students' problem-solving processes to their final products?

In conclusion, this study examined teacher purposes for instruction from the perspective of the tools they designed to support students' presentations of solutions to complex problems. Across the three teachers, four separate types of purposes in two dimensions occurred (individual vs. group analysis, focus on product or process). The framework for analyzing the tools could extend beyond the given set of tools to other tools designed by teachers using model-eliciting activities or forms of problem-based learning where students are designing products in a small group within a realistic context.

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