The Mathematics Enthusiast

Volume 10
Number 3 *Number 3*

Article 5

7-2013

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Jennifer M. Lewis

Davida Fischman

Iris Riggs

Kelli Wasserman

Recommended Citation

Lewis, Jennifer M.; Fischman, Davida; Riggs, Iris; and Wasserman, Kelli (2013) "Teacher Learning in Lesson Study," *The Mathematics Enthusiast*: Vol. 10 : No. 3 , Article 5.
DOI: https://doi.org/10.54870/1551-3440.1281
Available at: https://scholarworks.umt.edu/tme/vol10/iss3/5

Teacher Learning in Lesson Study[1]

Jennifer M. Lewis[2]
Wayne State University

Davida Fischman, Iris Riggs & Kelli Wasserman[3]
California State University San Bernardino

Abstract: This article documents teacher learning through participation in lesson study, a form of professional development that originated in Japan and is currently practiced widely in the US. Specifically, the paper shows how teachers in three different lesson study teams 1) expanded their mathematical content knowledge, 2) grew more skillful at eliciting and analyzing student thinking, 3) became more curious about mathematics and about student thinking, 4) emphasized students' autonomous problem-solving, and 5) increasingly used multiple representations for solving mathematics problems. These outcomes were common across three lesson study teams, despite significant differences among the teams' composition, leadership, and content foci.

Keywords: Professional development; Teacher learning; Mathematics education; Lesson study; Mathematics instruction

In this article we report on some outcomes of lesson study as part of a professional development effort to improve mathematics teaching and learning in a large, exurban, diverse elementary and middle school district. In lesson study, a group of teachers identifies a problem from practice on which they would like to make progress in their teaching. Over an extended period of time—several months to a year—the teachers study

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[1] This research is funded by the National Science Foundation, DUE-0962778. Findings presented in this paper represent the work of the authors and not necessarily the views of the funding agency.
The authors wish to express gratitude and appreciation to the teachers and school administrators of the Noether Project for their dedication and ongoing commitment to enhanced student learning.
[2]jmlewis@wayne.edu
[3] fischman@csusb.edu, iriggs@csusb.edu, kwasserman@sbcglobal.com

the topic as well as students' perceptions of it, and plan a lesson to address this topic. They bring in other professionals as needed during this process. One member of the group then teaches the lesson while the others observe and record student actions and reactions during the lesson. The group reflects afterwards on the design and teaching of the lesson, its outcomes for student learning, and implications for student learning more generally. The cycle repeats, building teachers' mathematical content knowledge and their shared views of pedagogy simultaneously and over time (Lewis, 2002a; Lewis, Perry & Murata, 2006; Stigler & Hiebert, 1999).

This article documents[4] the experience of three school-based lesson study teams of teachers in their efforts to address the development of teachers' mathematical content knowledge, pedagogical skill, and leadership capacity through a combination of professional development activities, with an emphasis on the learning that occurs through the lesson study process. The "Noether" Project,[5] an NSF-funded Math and Science Partnership program, involves 60 teachers from 16 schools (with teams varying in size

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[4] We should note that the descriptions below are mainly extracted from field notes with some video transcriptions. When quotations are extracted from field notes, they may be incomplete in some cases; however, we have endeavored to convey the intent of the message accurately in all instances; the notes were taken by the facilitators as they were participating in the discussions, and were not taken for research purposes at the time.

[5]All names in this paper are pseudonyms, and we take our Project and school names from some of our favorite mathematicians. Amalie Emmy Noether (1882-1935), in the words of Einstein: "In the judgment of the most competent living mathematicians, Fraulein Noether was the most significant creative mathematical genius thus far produced since the higher education of women began." http://www.awmmath.org/noetherbrochure/AboutNoether.html

Euclid of Alexandria (about 325BCE-about 265 BCE) was a mathematician and author of the second most printed book in the world, *The Elements.* The contents include plane and spatial geometry, ratios, proportions, and elementary number theory. http://aleph0.clarku.edu/~djoyce/java/elements/elements.html

Hyman Bass is a research mathematician whose work in algebra connects to geometry, topology and number theory. In mathematics education his research focuses on knowledge and resources needed for effective teaching of elementary mathematics. http://www.soe.umich.edu/people/profile/hyman_bass/

Miriam Cohen is a research mathematician working in ring theory, Hopf algebras and quantum groups and their applications to physics. Director of the Center for Advanced Studies in Mathematics, and former president of the Israel Mathematical Union. http://www.math.bgu.ac.il/~mia/#Additional_Information from four to fourteen, and some teams drawing teachers from multiple sites) who study mathematics and pedagogy in multiple formats. Each year, teachers participate in an intensive two-week summer institute, academic year monthly seminars, self-facilitated monthly collaboration time, and lesson study. All teams meet for ten full days of lesson study during the academic year. The program began with 45 participating teachers and subsequently expanded to include 60. The district is a high needs district with 89% of students eligible for free and reduced lunch. 88% of students are Hispanic, with large numbers of English learners (51%), primarily Spanish-speaking, and many parents have limited academic backgrounds. In each middle school at most one or two teachers have a math credential that qualifies them to teach algebra or single subject mathematics.

The theory of action in the Noether Project is that teachers who participate in lesson study will become increasingly knowledgeable in mathematics and more skillful in teaching mathematics, and this expanded teacher learning will lead to improved student learning. This logic of expected improvement follows recent research (Dudley, 2012) indicating that schools where lesson study is conducted show higher levels of student learning in mathematics relative to comparable schools. By "expanded teacher learning," we mean teachers' increased content knowledge, confidence in mathematical skills and abilities to help children learn mathematics, and a growing expertise in teaching mathematics. The Project measures increases in student mathematics learning and proficiency, using standardized test scores as well as outcomes on alternative assessments such as the STAR assessments (California Department of Education, 2010). The alternative assessments gauge student performance on the Standards for Mathematical Practice (Common Core State Standards Initiative, 2010), which represent for the facilitators and instructors a close approximation of the kinds of student learning in mathematics towards which the Noether Project is working. These Standards for Mathematical Practice figure heavily in teachers' work during the Summer Institutes and academic year seminars, and often constitute learning objectives chosen by Project teams in lesson study.

The facilitators and instructors of this project include mathematicians, mathematics and science educators, and an elementary educator with little formal mathematics background. Administrators play a significant role in support of teachers' participation. At one school, the principal regularly participates in the research lessons and drops in for other phases of lesson study. At another school site the principal and vice principal have observed one research lesson, and at the other, the principal and vice principal made a brief appearance at one lesson study meeting, in addition to maintaining correspondence throughout the year with team members and the lesson study facilitator. Two to three times per year, the project convenes meetings of administrators to discuss project goals and activities, and for teachers to demonstrate the types of work they are doing in the project. In all sites, the principals support teachers' release from the classroom for ten days per year for lesson study sessions.

We have chosen to show lesson study through the prism of three different school-based teams who all participate in the Noether Project within one school district. Throughout the project, teachers have commented repeatedly on the extent to which they are deepening their understanding of mathematics concepts and how students conceptualize mathematics, and report that as a consequence they are enhancing their ability to teach effectively. As we reflected on the work being done by the teachers of these lesson study teams, it became clear that each of the teams' stories exemplifies and highlights a particular aspect of teacher learning.

The characters of the teams are very different, both by their size and grade levels and by the backgrounds of the teachers involved; and the facilitators have different mathematical and educational backgrounds and work experience. However, the facilitators have in common core goals and beliefs that are central to the lesson study process. The project facilitators share a set of strong convictions regarding mathematics teaching. They believe that key aspects of high quality mathematics teaching include deep content knowledge (Ball & Bass, 2000), and that strong links between conceptual understanding and procedural fluency are essential to learning mathematics (Kilpatrick, Swafford & Findell, Eds., 2001).They also believe that frequent and varied use of formative assessment is central to good mathematics instruction (Black & William, 1998) and that thoughtful listening to students' mathematical ideas is fundamental (Carpenter, Fennema, Franke & Empson, 1999). Furthermore, they are committed to fostering teachers' autonomy regarding their own learning and teaching, and see this as a requirement of good professional development (Little, 2000). The facilitators also share a belief in nurturing students' desire to learn in order to yield long-term improvements (Kilpatrick, Swafford & Findell, Eds., 2001). All three teams follow a fairly standard form of lesson study. By this we mean that participation by teachers is voluntary; that teachers set the goals and topic of work for the lesson study cycle, and that the team studies the topic using curriculum and other supplementary materials extensively to plan a research lesson. All team members observe the research lesson in person and participate in a face-to-face debriefing session afterwards. Outside experts are included at several stages of the cycle. A wide variety of professional development practices are referred to as "lesson study" in the US, but in our project we have hewed fairly close to the canonical model as implemented in Japan (Lewis, 2002a), with one exception worth noting: Since the US culture of teaching does not normally accommodate a long period of study and planning as is done in Japan, the teams began with shorter cycles of lesson study. However, the teams in the project have been expanding the amount of time for curriculum investigation and planning for each lesson, and consequently reducing the number of research lessons per year. As the teachers and administrators become more comfortable with this extended study and planning time, the time frame more closely approximates the standard Japanese model. In our conclusion we say more about the fundamental ways in which we have followed the Japanese model from the inception of the project and why this matters.

The teams studied mathematical content as well as mathematical practices. One team focused on multiplicative structure for students in the middle grades. Two teams focused on developing student problem solving skills using contextual problems, also in the middle grade. The third team focused on students' argumentation skills in mathematics. Each team participated in extended study of the content area across almost two years, drawing on summer learning institutes and monthly seminars, monthly lesson study sessions, the independent reading of articles and books, and the presence of content and pedagogy experts. The research lessons developed in this process reflect teachers' progress in the areas of mathematics content, mathematical practices, pedagogical skill, and dispositions to teach and do mathematics.

Team "Euclid" is composed of six teachers from two school sites: Two fourth grade teachers and one fourth-fifth grade combination teacher from one site, and a fourth, fifth and second grade teacher from another site. One of the teachers is new to the team and the project this year, and the second grade teacher was reassigned from his original grade (the project as a whole is geared to fourth to eighth grade teachers). This team's facilitator was also new to the team in Year Two, and is a former secondary mathematics teacher with a master's degree in mathematics education.

Team "Bass" is composed of fourteen teachers of grades four through six, including one special education teacher and the school's instructional coach (who had started the program as a teacher.) Of the fourteen teachers, two joined the project in its second year. The team's facilitator is a professor of mathematics education.

Team "Cohen" includes five teachers, three fifth grade teachers and two teachers of sixth grade gifted and talented students. Four of the teachers work at one school while another joins from a nearby site. Four of the five teachers have been members of this team from its inception, and were joined in the second year by a teacher who had participated on another team during Year One. This team has two facilitators, a mathematics professor and an education professor.

Looking across the three teams, different affordances of lesson study coalesce. Across the three teams, we see themes emerging regarding *why* teachers learn from lesson study, *what* they learn, and *how* teachers learn in the context of lesson study. In Team Euclid, teachers' understanding of the crucial role their own learning, and the value of listening to student thinking, became especially salient. The Team Euclid story told below describes the process through which the team learned to expose student thinking and respond to it, and what motivates the teachers to continue to learn. Theirs is the story of *why* teachers learn.

Team Bass is a large team, and has wide variation in teacher knowledge and approach to learning. By examining teachers' comments following research lessons, we discern *what* teachers learn in lesson study.

Team Cohen's story illustrates *how* teachers learn. The team has worked to predict what students might think, including any misconceptions, and to design assessments and lessons so that they will highlight anticipated student responses. Teachers' struggles with the task of guiding students to discover mathematical ideas were a key factor leading to their own personal mathematical growth.

Team Euclid

Team Euclid is a case where we can see the development of teachers' internal motivations to learn mathematics and, at the same time, how teachers deepened their ability to understand students' interactions with mathematics. Team Euclid was driven to understand what instruction might look like as guided by the Standards for Mathematical Practice of the Common Core State Standards (Common Core State Standards Initiative, 2010). As teachers learned to reflect more deeply on students' understandings, they gained a deeper appreciation for their own need for content knowledge and meaningful understanding of mathematics. Below we describe a series of turning points in which teachers increased their own understanding and the desire to learn even more.

The Summer Institute introduced teachers to the Common Core State Standards (Common Core State Standards Initiative, 2010), in particular the Standards for Mathematical Practice. In light of these Standards, the team started by asking the following questions:

- What content do we want the students to understand?
- How will we know that they understand the content?

- How do we teach students perseverance, reasoning, modeling, structure and conjecturing when there is so much content to teach? Is there enough time?

First Attempt

The content focus for the first research lesson was writing and evaluating expressions. Teachers were concerned about students' ability to find entry points into contextual math problems. They were interested in incorporating some exploratory aspects into the lesson, and providing a variety of manipulatives for the students to use while solving the problems. Fifth grade students were presented with the following problem:

Sonya spent $7 and $9 at Target. She gave the cashier a $20 bill. Write and simplify an expression to show the change that Sonya should receive.[6]

Students were asked to work with a partner to write and simplify an expression, and be prepared to explain their thinking to the class. When called upon, students would come to the front of the class, show what they did with their selected manipulatives, and briefly explain how they solved the problem. This was the team's first attempt at having students verbalize their solutions, in a classroom where student explanations were not commonly elicited. Students' comments proceeded as follows:

Pair 1: "We knew we had $20, and 20 minus 7 is 13 and 13 minus 9 is 4."

Pair 2: "We thought it was easier to add 7 and 9 and get 16. And 20 minus 16 is 4."

Both of the above examples indicate a correct solution, as well as two different, but correct, approaches for solving the problem. So, at first glance, it seemed as though the

[6] This problem was based on one in the textbook *California Math*, Houghton Mifflin, 2009, p.113.

students understood the lesson -- they got *the right answer.* However, further examination of student work indicated that students were still not writing expressions correctly, which was the goal of the lesson. Students were able to compute the arithmetic either mentally or using manipulatives, but while the arithmetic process was correct, the students did not demonstrate an understanding of the underlying structure. For instance, in discussing the lesson afterwards, one teacher reported that one pair of students had the problem written 16 - 20, and when a student was asked if there was a difference between her expression and that on the board (20 - 16), she didn't notice a difference.

Teachers learned that a student *computing a right answer* does not necessarily indicate that the student has an accurate *understanding* of the target content. This prompted the team to return to the content themselves and discuss more deeply the questions that would help them better understand the students' understanding. Teachers considered the responses that were given and what else they could have done to help students deepen their understanding of this concept. With some guidance from the facilitator, the team discussed the missed opportunity for comparing the two strategies written as expressions. For instance, the first pair of students solved the problem by illustrating the expression 20 – 7 - 9. They then subtracted from left to right, resulting in the answer of $4. The second pair of students illustrated the expression 20 - (7 + 9), though their expression was written as 20 - 16. Teachers assumed that the students understood the equivalence of these expressions. They concluded that to develop understanding of equivalence of expressions and order of operations, they might have asked questions such as: How did you get to 16 and how is that represented symbolically? How did you show that you added 7+9 first? How is 20-(7+9) different from 20-7+9?

The Turning Point

Teachers had come to the realization that facilitating a mathematical discussion is a complex process and this spurred them to further learning. Based on their conclusions from the first cycle of lesson study, they began the next cycle with a new question: What are the kinds of questions that I need to ask students to facilitate a productive mathematical discussion?

The team began to utilize more curricular resources in their study and planning. These resources included *Young Mathematicians at Work Constructing Algebra* by Fosnot and Jacob (2010) and *Classroom Discussions: Using Math Talk to Help Students Learn* by Chapin, O'Connor and Anderson (2009). Additionally, this team embraced the use of Mathematics Assessment Resource Service (MARS) assessment tasks and often referred to websites, such as http://insidemathematics.org, for such tasks. With an increased focus on questioning and discourse, some teachers began to work on creating a collaborative environment in their classrooms and asking probing questions more regularly. Developing questions was a main focus in the planning of the second research lesson. Teachers wanted to structure opportunities for students to demonstrate their thinking and verbalize their processing while continuing to work on developing problem solving skills. In the next research lesson, the team posed the following question to the class:

> *The Rodriguez family decided to make tamales and give them to their friends and neighbors as Christmas presents. They made beef, chicken and cheese tamales. They made four dozen tamales and they are going to wrap them in bags of 5. How many bags do they need?*

Expected student sticking points included understanding of the term "dozen," inclusion of extraneous information, and consideration of what to do with the three leftover tamales. As the students worked on the problem independently, the teacher noticed two different solutions arising. Some students had illustrated nine bags of five tamales with three left over, while others had illustrated five bags of nine tamales with three leftover. The teacher determined that this would be an excellent starting point for a class discussion and had students present both of the solutions in order to use comparison as the basis for the discussion, expecting that the students would come to a consensus about the solution. The teacher then used newly developed facilitation skills to re-voice each of the arguments and she gave the students the power to navigate their own learning by asking, "How do *you* decide?" After much debate, a new misunderstanding was revealed: one student read the question to mean that the family had five bags, and did they need them *all*?

The Euclid Team teachers found this lesson eye-opening. Through the teacher's perseverance in patiently questioning students without providing answers, she demonstrated to the students her interest in their thinking. Teachers realized that without the extended discussion during class, this student misunderstanding—whether linguistic or mathematical—would not have surfaced and thus would never have been identified or addressed.

Continued reflection at the next meeting revealed even more. Teachers' initial thoughts were that, linguistically, the English learners had difficulty with the translation of "bags of 5." However, there were some cultural aspects to the question the team had not considered that may have led to the misread of the problem (e.g., tamales often come packaged in dozens). Additionally, "bags of five" in Spanish sounds much like "five bags," perhaps another source of confusion in this primarily English learner class. Also, the problem that was created was a measurement type of division problem, as opposed to a partitive type of division problem that is typically more familiar to students.

Teachers came to a deeper understanding both of the students' thinking and of the importance of ongoing reflection on their practice. As a consequence, the teachers' focus in lesson study shifted. From a global concern about students' lack of understanding, the team progressed to a desire to learn about specific aspects of students' understanding and misunderstanding through purposeful questioning.

Transforming thinking

In the third cycle of lesson study, teachers' perspectives and planning questions changed to:

- What can we learn about student understanding of content before we plan the lesson?
- How can we get our students to explain their thinking more clearly? What questions do we ask for this purpose?

To begin answering these questions, the team for the first time developed a pre-assessment to administer to small groups of students at different grade levels. They chose a broad focus--number lines-- and wanted to investigate current student conceptions of number lines; in particular whether students could:

- Identify intervals on a number line
- Construct a number line on their own
- Correctly plot and label rational numbers on a number line given endpoints and intervals

If students could do these things, could they explain how they did them? If not, at what grade level did specific misunderstandings occur?

The team members were in agreement that they wanted to know not only whether students could solve the problems correctly, but also how the students thought about the problems. They determined a very specific process of administering the pre-assessment in order to obtain as much useful data as possible. Students would be brought in in small groups according to grade level; one teacher would lead the students through the pre-assessment, answering questions about the items on the instrument to ensure they were learning about student understanding of the math, as opposed to difficulties with language. A different teacher worked with each age group of students. Teachers planned to pose the following questions to students after they completed the pre-assessment, in order to learn more about how the students thought about the problem:

- Why did you write _______? How did you get that answer?
- Have you seen a number line like this before? Where? What do you know about number lines?
- Which problem was the hardest for you? The easiest? Why?

The assessing teacher would then ask the observers if there were any additional questions that they wanted to ask. Finally, they videotaped each assessment in order to reference it later for clarification if needed.

In the first assessment (of third grade students), it became clear that students did not have a deep understanding of the meaning of the points on the number line. For example, students were asked, “How many units are there between each point?” on the number line below:

One student responded "300." The teacher asked, "So, how did you decide it was 300?" And the student responded, "I put the 100 and the 200 together and got 300." Teachers conjectured that the confusion might have stemmed partly from the wording of the problem.

In the second assessment (of fourth grade students), the assessing teacher asked if there were any words that needed to be clarified. This time, one student asked about "units." The teacher (mistakenly) indicated that "unit" is the space between the markings on a number line. On the assessment, two students wrote, "There are two units between each point." When asked "Why did you decide to write that?" one student responded, "There's only two spaces on the line," thus indicating that the description of the unit had led to a misunderstanding of unit for these students. The team realized there was a need to revise their own idea of what "unit" means, and then decided that the teacher should introduce units by talking about measurement.

Before the last assessment (of fifth grade students), this exchange took place:

Teacher: Let's look at the question together. You read it. Are there any words that don't make sense?

Student: Units.

Teacher: Think about how we measure. I could say how many steps to get to the desk, steps can be units; or arm lengths, arm lengths to the pad, how many arm lengths.

Think about a ruler, can you think of how we measure with a ruler?

Student: Inches, centimeters, millimeters

Based on the results of this last assessment, this description worked much better. Four out of the five students wrote that there were 100 units between each pair of points. One student drew ten tick marks between each pair of points and wrote, “I think this way because I am going by tens,” indicating that he understood that ten 10’s make 100, yet not understanding how to label it on a number line (his tenth tick mark should have fallen on the second point).

In reflecting on students’ answers in the pre-assessments, teachers concluded that student understanding of intervals on the number line grew across the three grade levels (though the improvement of the question posed was probably also a factor), perhaps in part because number lines are not included in the third grade curriculum. Other concerns emerged regarding students’ ability to construct a number line and to plot rational numbers accurately on a number line. The team then conjectured that students’ difficulty with placing rational numbers on the number line could either be a reflection of difficulty with rational numbers, or with understanding the ordering of numbers on a number line. The team then decided to experiment with teaching these concepts using Cuisenaire rods as instructional tools.

Although the team’s earlier work was strictly focused on producing and teaching lessons, the team’s work evolved towards investing significant time in deepening their understanding of students’ thinking about content and consequently teachers’ own understanding of that content, before delving into lesson planning.

Team Euclid Conclusion

Over the course of this year, the team has shifted what it means by "need for understanding." The "need for understanding" no longer means only, "What do our students need to understand?" Over the course of this second year, the question has become, *"What do **we** need to understand about the mathematics and about our students to be able to progress to a desired level of understanding?"*

Team Cohen

Study of the Cohen team is based on analysis of facilitator field notes taken during planning sessions in addition to scripting notes of teacher and student talk during research lessons. This case study shows *how* the Cohen team members learned in lesson study. Through their experience with research lessons, teachers came to realize that students did not fully understand the concepts their lessons were designed to teach. Based on analyses of student (mis)understandings, teachers designed a series of mini-lessons on the same content. Through careful listening to students' explanations, they ultimately reconceived the content for themselves in a more meaningful way and revised their approach to instruction of this content.

Teacher Content Learning Through Anticipating Student Behavior

In Year One, Cohen Team teachers studied student learning by first looking at benchmark test results and then observing student performance during research lessons. By Year Two, teachers expanded their study of student learning to include both pre-assessments and piloting of draft research lessons with small groups of students. The teachers agreed to focus on teaching through mathematical investigations as part of their annual research goal, and this appears to have altered their approach to pre-assessment.

Where formerly they focused only on skills, they came to also assess students' ability to approach problems and solve them. For example, the Cohen Team decided their first investigative pre-assessment would ask a group of students to respond to the following prompt:

> *Polly works in a zoo and needs to build pens where animals can live and be safe. The walls of the pens are made out of cubes that are connected together. Polly has 40 cubes and wants to make the largest pen possible, so the animals can move around freely but not get loose. Build the largest area using all 40 cubes. Use the grid paper to show the shape of the pen. Explain to Polly why you believe your pen is the largest one that can be made.*

Prior to implementing the assessment, teachers tried to predict how students might respond to the prompt. No longer were they simply thinking about teaching the area formula with already-created shapes, but they were considering how students might design shapes and explore ways to maximize area. Additionally, teachers realized that students often confuse area and perimeter and hypothesized that students would count the centimeter cubes as a part of a shape's area rather than see the cubes as the "fence." They thought students would most likely not plan for dimensions of a shape but would randomly place cubes to see what they could create. Thus, they expected to see some students struggle to use all the required cubes or run out of cubes as they created their shapes. Teachers also predicted that some students would create irregular shapes. Teachers discussed whether they should address these possibilities with students at the start of the lesson but decided to allow students the opportunity to investigate the prompt without any direct instruction in the hopes that students might be able to develop their own insights into the difference between area and perimeter and how shape might relate to area.

The conversation about what students might do prompted teachers to frame teacher observations during the research lesson. Since the teachers had engaged in discussions about their students' potential interactions with the content, they were able to consider content more specifically as they planned for their observation of student talk and action. Observing teachers were not just going to watch for correct answers and errors. They were going to watch for behaviors that portrayed specific conceptual understandings. They decided that observers would watch to see whether students traced the outside or inside of the cubes that outlined their pens. One observer would attempt to track different approaches used such as including cubes in the total area, narrow vs. wide shapes, and irregular shapes. This information would guide the selection of students for sharing in the class discussion so that different approaches might be viewed and analyzed by the class as a whole.

These considerations are a change from Year One in which observers were assigned to watch students with varied characteristics such as language needs or behavior challenges. Additionally, choosing students to share during the whole-class discussion based upon their approach to the problem is also a change for these teachers as many of them reported typically drawing name cards at random to select students to share, irrespective of the content of student's mathematical work.

A pre-assessment using the "Polly" problem was implemented with five sixth graders. Individual students first worked with 20 cubes to explore the problem. Prior to using 40 cubes with a partner, students were to predict and draw the shape they thought would provide the largest space for elephants to live. Partners then drew as many shapes as they could, using the cubes and grid paper. Students were able to see one another's drawings and discuss findings.

Each and every student began working on the problem by including the cubes within their area totals. Only one student eventually recognized that the cubes' inclusion in the count made for inaccurate areas. Additionally, the teachers were surprised to find that students had different concepts of "largest." For example, two students said their pen was "largest" because it had included a bend so that each elephant had a private area. Thus teachers learned that a context can get in the way of mathematical understanding, and every aspect of the context needs to be considered carefully in advance.

Student Outcomes as a Basis for Teacher Content Learning

Team Cohen teachers recognized that students, as predicted, did struggle with the concepts of area and perimeter. While it might have seemed easier to address misconceptions directly with students, the teachers wished to maintain an investigative stance in instruction rather than returning to a direct instruction approach. Still, they expressed frustration about how to help struggling students without simply telling them what to do. They wanted to have students reach conclusions about the essence of area rather than hear students repeat back a formula or a definition. As part of this planning process, a visiting math professor taught the group a mini-lesson on the area model for multiplication of fractions, which helped the teachers consider how students might record findings and look for patterns as a way to reflect on learning throughout and after an investigation.

Teachers liked the idea of having students record their findings, look for patterns, and make connections. They thought they might try this approach, and after a few iterations and related pre-assessments, the team developed a lesson in which students would use Geoboards to create rectangles of assigned sizes. The teacher would not indicate whether students were right or wrong but would record areas and the rectangle dimensions on chart paper. Finally, students would be asked to consider a problem in which a shape with an area of 3 is viewed by a fictional student (Paul), who says its area is actually 8 square units (counting points rather than spaces). Students were to discuss that response, how they thought Paul had reached the conclusion, and what they would say to help Paul see the area in a different way. To assess student understanding at the end of the lesson a short assessment question was developed that asked students to draw as many six-square-unit rectangles as they could, record the area, length and width of each in a table, and describe any pattern they saw in their table.

The Cohen Team teachers were willing to give time for students to develop connections between dimensions and area without direct instruction from the teacher. In fact, the teachers noted that students had already had direct instruction on the formula for area of a rectangle during fourth grade. Realizing that teacher "telling" did not seem to guarantee student understanding, they wanted students to construct their own view of how the length and width of a rectangle connects to its area, and through this gain a better understanding of the concept of area.

Learning about the "Big Ideas" of Content

In order to learn more about how students think about area and perimeter, teachers decided to ask four sixth-grade students to teach four fifth-grade students about the two concepts. The sixth graders were told that the teachers had been struggling with ways to help students understand area and perimeter of shapes. As a way for the teachers to consider how students learn about area, they wanted the students to think about how they might work with a younger student who didn't know multiplication but wanted to understand area.

The team was intrigued to find that three of the four students independently came up with approaches that started from the whole shape and progressed to the unit rather than moving from the unit to the whole as the teachers had taught their classes. For example, Roberto started by creating three congruent rectangles. He held up the first, which had no grid, and said, "Here is a shape. If we want to know the size of a shape, how might we go about finding out the size? It's not like measuring just a line. We need something else. We might want to divide it into equal spaces (units) and count them." He held up a congruent rectangle on which he'd traced square units from the graph paper. He then showed a third congruent rectangle that he'd cut up into the square units, sliding them apart and then pushing them back together. Roberto's emphasis was on measurement and the need for a way to determine size of spaces, and he implicitly utilized the concept of conservation of area. He emphasized *why* we need a means for measuring space since it's different from measuring a line. He then moved to a Geoboard in which he'd created a rectangle and used different colors of tiles for each row. He planned to have his student find the area of the rectangle by counting tiles.

When actually teaching, Roberto's student had a lot of difficulty. Roberto responded by taking out a row of tiles at a time, trying to deal with area of a smaller region, spreading out the row for the count and then putting the tiles back together and asking his student if the area was still the same (now explicitly checking on conservation of area!). The student truly grappled with the ideas throughout the lesson, and following the lesson he was able to determine the area of a rectangle.

Esteban created an irregular shape because he wanted to emphasize that any shape can be measured, even one that looks like "a scary sixth grade shape." He worked with his student to count whole square units and then combine partial square units, documenting the adding of units as they worked and also shading in units as they were counted to be sure not to double count. He commented, "See, when you cut up space into square units, you can count the units easily, and even a strange shape isn't scary." When asked how he might help students who are confused by area and perimeter, Esteban quickly defined the irregular shape as the footprint of their school. He cut out a "Fred" character and placed Fred in the school and outside of the school to help his student determine where the area of the school was. He had Fred walk the perimeter of the school. He made the observation that the area--the inside space of the school--is measured in squares while the perimeter is measured in length.

George tried to demonstrate the difference between area and perimeter. He showed a picture of a rectangle outlined in black with a green interior. He made a rectangle of tiles, then placed the tiles on graph paper and outlined them. He commented that the tracing is the perimeter and what's inside is the area. He then made another rectangle with tiles and stood up tiles around the edge as though they were walls (perimeter). He used the example of carpet or grass and walls and fence as contexts for area and perimeter.

After this "teaching" event, the fifth grade students debriefed the experience with the Cohen Team teachers, followed by a separate debriefing with the sixth grade students. Based upon student recommendations, teachers felt that they would make major changes in their future approach to the teaching of area and perimeter. They stated that they would teach the concepts together rather than separately, because they realized the connections between area and perimeter. They saw the linear dimensions of a rectangle stemming from the perimeter's linear measure, and they came to believe that students needed to compare and contrast them in order to differentiate between linear and square unit measures.

Team Cohen also discussed the way the student-teachers started with the "big picture" of measurement of space instead of simply defining a unit and moving into counting square units. As one teacher said, "I've probably been teaching the concept of area backwards—we always start with the unit of measurement and build on that. The kids today worked from the blank shape and had a focus on the fact that we're trying to measure space before considering square units as a means for measuring space."

Teachers also agreed that they'd have a context for area--one that relates more to students' lives, such as their school building, rather than animal pens. They thought it was important to get kids to talk about where and when they'd interacted with a concept such as area in order to hear students' present understandings. They also discussed the effects of putting restrictions on students' use of formulaic language such as "length times width" in order to help students try to define a *concept* rather than rely on surface-level application of a formula.

Team Cohen Conclusion

These changes in the team's approach to teaching most likely did not result solely from readings or discussions or observations of students. The process of lesson study appears to support risk-taking in implementing new approaches to teaching and learning by providing a collegial and safe environment. Team Cohen teachers used this process to focus both on the specifics of student learning and on the long-term effects of their instruction on students' content understanding. In turn, this created an intrinsic need to know more about student thinking, and influenced the development of teachers' own content understanding.

Team Bass

As with Team Cohen and Team Euclid, teachers on Team Bass are working on understanding student thinking and, as a result, considering their own understandings of mathematical concepts. For our study of Team Bass, we analyzed the field notes taken during the discussion sessions following each research lesson. The post-lesson discussion session is a post-hoc analysis and discussion about the jointly conceived research lesson that all team members create and observe. Typically the teacher of the lesson speaks first, and team members endeavor to provide evidence with specific data they collected regarding any conclusions or observations they offer about the research lesson. We chose to study this phase of lesson study for Team Bass because teachers' comments in this activity offered a window onto what teachers were thinking about and processing, and their comments are sometimes summative in the sense that they make observations that span the team's efforts together from the beginning of the lesson study cycle through to this point.

Our analysis of the field notes in the post-research lesson discussion sessions was conducted by labeling categories of teachers' comments using each separate teacher turn as the unit of analysis. Each turn was given a label. Turns were mostly considered instances of some kind of thinking or offering; these kinds of thinking or offering were the labels used. So, for example, many teacher turns during this debriefing discussion concerned details about mathematical work that a specific child had done.

Using a version of Yin's (2009) cross-case study method, treating each post-discussion session as a separate case, we created word tables that described teachers' comments, and then worked across them. We sought to identify patterns of practice that emerged across the teachers' turns in this set where trends and patterns were noted and new labels assigned to clusters of related teacher actions. For example, one teacher commented early on in a debriefing, "Didn't Malena skip a step? She took 12 and divided it." This turn was labeled as "student problem-solving specifics" and was later subsumed into the category of "student work and student thinking."

We noticed five categories of teachers' comments during the post-lesson discussion sessions, and we offer these to indicate *what teachers are learning* during lesson study. These five overarching categories are:

- Teachers' instructional moves
- Student work and student thinking
- Understanding the math
- Big ideas about mathematics and learning
- About the lesson study process

We discuss three of these, and provide illustrative examples.

Teachers' Instructional Moves

Teachers in Team Bass offered frequent comments, or posed questions, about actual or possible instructional moves. Some of these were offered as repairs to the planning of the observed lesson, for example, "We might have moved to the whiteboard or a table in the center of the room to show the ways students modeled the problem." This is phrased as a suggestion for how this aspect of the lesson might have been conducted during the research lesson, but it also represents a tinkering with instructional materials that will serve future lessons, and in this particular case underlines the importance of including all students in the presentation of ideas, and the team's emphasis on modeling. Team Bass had been working this year on modeling in a number of ways: the notion of *mathematical models*, that is, various representations of mathematical ideas; as well as the pedagogical form called *modeling*, where the teacher or a student provides an exemplar for other students to follow.

In other comments regarding this category of teacher learning, teachers' instructional moves, teachers are conducting thought experiments about instruction, playing with possibilities that are prompted by the lesson they observed and considering a range of alternatives and what those instructional alternatives might have generated. The team had been encouraging students to model problems with drawings and other materials, and they also wanted to see how they might best prepare students for the kind of word problems they encounter on standardized tests. The focus had been multiplicative structure, so they formulated the following problem:

> *We have 4 boxes of pencils. Each box has a dozen pencils in it. If 6 people share all of these pencils equally, how many pencils will each person receive?*

After observing the research lesson where students worked on the problem, a succession of teachers' turns included a string of these:

> *Teacher 1: Would it have been different if we had had "12" instead of a dozen?*
>
> *Teacher 2: Might we have just presented the 4 x 12?*
>
> *Principal: Would there have been an advantage to use real pencil boxes?*

These comments reveal a care with wording, weighing the use of numeric symbols, and a consideration of various representations that could be used in this problem.

Student Work and Student Thinking

About half of all teacher turns have to do with student work and student thinking. The Japanese, who originated the formal practice we call lesson study, say that lesson study "gives teachers eyes to see students" (Lewis, 2002b). The teacher turns in this category show how this transpires. During the research lesson, teachers are encouraged to collect data on individual students, and these data are shared readily at the debriefing sessions. Teachers share specifics of the mathematical work that individual students did during the lesson, and then often interpret the meaning of their work. Here is a typical comment of this category, on the same problem we discussed above.

> *Teacher: A girl immediately made one stack of 12 and was about to make another, but then made four stacks one at a time. I realized that you had to destroy the original representation to finish the problem.*

The comments often contain highly specific details about what a particular student did, as in the case here, where the actual numbers and methods of problem solving are mentioned, and the sequence of the child's work in solving the problem.

Notice, too, that in the next sentence, the teacher adds a comment about how watching this student solve the problem led her to realize something about the deployment of models in this problem. Thus, teachers move from specific understandings in the context of this particular problem, to realizations that might be relevant to other problems as well. Here the teacher offers an idea—that the construction of the mathematical model here had to be destroyed in order to finish solving the problem—that may be useful in work on another problem. We anticipate that such thinking is accessible to teachers when they are alone in their classrooms and outside the framework of the research lesson.

Understanding the Math

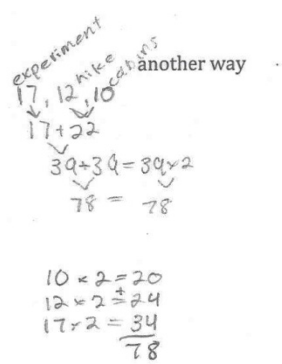
While there are only a few teacher turns in this category, over the three post-lesson discussion sessions, teachers' expressions about understanding the mathematics in the lessons are significant. Specifically, teachers say that they did not fully understand the mathematics until they watched students work on the problems during the lesson, or participated in the teachers' analysis of student work during these debriefing sessions. Another research lesson was designed for students to work on the distributive property, and teachers devised the following problem for students:

> *At science camp, 17 students are doing an experiment, 12 students are taking a hike, and 10 students are in their cabins. There are twice as many students in the dining room as are doing an experiment, on a hike, and in their cabins put together. How many students are in the dining room?*

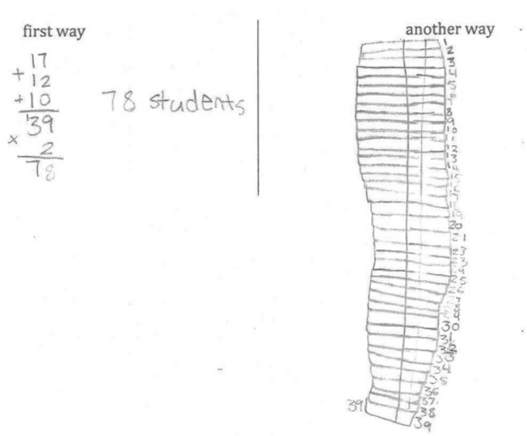
Students were invited to solve the problem in two different ways, which in itself was an innovative practice for this team of teachers: the valuing of eliciting multiple approaches to solving a problem. It is also worth noting that the teachers developed this problem around using as a context the sixth grade camp experience that all students were about to embark upon together. This underscores the teachers' desire for students to use mathematics to describe and model their own experiences as a way of developing "productive disposition" (Kilpatrick, Swafford, & Findell, Eds., 2001) in mathematics.

In listening to students present their solutions to this problem and examining all students' written work during the post-lesson discussion session, a number of teachers realized that they themselves were not entirely clear on what the distributive property means. It was through this discussion that teachers revealed that they expected to "see the distributive property in students' solutions," and by this they meant something resembling Elise's work:

In fact, in the discussion it became clear that teachers understood this precise representation—and only this one—to "*be* the distributive property," that is, this exact form *is* the property. But they were not sure if Fernando's work showed the distributive property:



Even more puzzling was Aric's work:



While these are correct representations of the problem that give correct answers, do they show the distributive property? At one point, just as the group is trying to analyze these examples of student work, one of the teachers says, “Can you use the distributive property for every problem? I don’t really understand the distributive property. The distributive property or the commutative property.” Of course, prior to the research lesson—and prior to this discussion—teachers did not question their understanding of these properties. It was only upon the team’s discussion of which student work constituted use of the distributive property that the teachers began to reconsider their own understanding of what this property really means. At that point the facilitator could see that teachers had been thinking of the distributive property as a formula for computation, rather than a consequence of the underlying structure of the real numbers.

Big Ideas about Mathematics and Learning

Occasionally, teachers offered comments that hint at broader philosophical orientations about mathematics and learning. For example, a teacher who taught one of the research lessons said, “The hardest part at the end was trying not to guide students towards the right answer. I kept having to remind myself to ask students if the representations fit the story problem.” Implicit in her comment is the team’s shared commitment to supporting students’ autonomous problem solving in math class, and her efforts to try to help students in a way that does not spoon-feed answers to them. This orientation towards teaching and learning mathematics is one that is shared, and new, for most members of Team Bass.

Team Bass Conclusion

The analysis offered here is based on the field notes from three post-lesson discussion sessions to give us some insights into what Team Bass teachers may have been learning through their participation in lesson study. These categories are important because they give us a sense of what teachers work on through lesson study and how we can best use this professional development tool to strengthen mathematics instruction in a systemic initiative.

Conclusion

Across these three site-based teams, strong common themes emerge, despite significant differences among the teams' composition, leadership, and content foci. This is particularly surprising because neither the facilitators nor the teacher participants collaborated across teams on the content of the work in lesson study. We attribute this remarkably similar progress across the teams to the lesson study process itself, in conjunction with the shared values of the facilitators. Lesson study groups in this country have modified the structure of lesson study in a number of ways: shortening the time spent in content and curriculum study, videotaping research lessons instead of live observations, skipping the use of knowledgeable others, and abbreviating or eliminating the post-lesson discussion (Yoshida, 2012). We have made a conscious effort to stay faithful to the essential features of the canonical form of lesson study, with minimal adaptations to the local environment. In particular, the lesson study teams mentioned in this paper (and throughout the Noether Project) engage in extended study of content and of student thinking utilizing multiple print and human resources, conduct live research lessons, determine their team goals based on teachers' and their students' needs, invite outside experts to comment on their work, and devote significant amounts of time to face-to-face post-lesson reflection and discussion.

Some of themes that are common across the three teams are:

- Development of teachers' "mathematical care"
- Elicitation and deep analysis of student thinking
- Developing curiosity about mathematics and about student thinking
- Emphasis on students' autonomous problem-solving
- Increased use of multiple representations for solving problems
- A generous and supportive collegial atmosphere of learning

In all three teams we see teachers developing a significant degree of "mathematical care" in their instruction, that is,

> the care with which the teachers consider mathematical choices or options, and with which they attend to children's mathematical thinking and expressions, in the flow of instruction…Mathematical care means that the instructional choices that shape the mathematics in play are treated with heedfulness and attention (Lewis, 2007, p. 144).

As the teachers have engaged in lesson study and are developing "eyes to see students" (Lewis, 2002b), they grow in their ability to question students productively and to devise classroom situations that will reveal student thinking, including students' correct understandings as well as misconceptions. Teachers have become more adept at differentiating conceptual work from rote application of formulas, eliciting student thinking, and analyzing students' ideas about the mathematics.

Teachers' deepening knowledge about mathematics and teaching has stimulated curiosity about their students and mathematical content. Increasingly, we hear teachers express curiosity about mathematical ideas or how their students will react to a particular problem or lesson. With this new perspective, teachers in all three teams are much more likely to widen their instructional efforts to focus on a concept (e.g. area as space, or distributive property as a relationship) rather than a sole focus on algorithms (e.g. length x width for the area of a rectangle, or a particular form of the distributive property).

In all three teams, teachers have moved away from telling students what to do and moved toward developing students' desire and ability to make sense of mathematical situations and to solve problems autonomously. This represents a significant shift for both teachers and students, and progress is slow—but teachers are committed to continuing their work in this direction.

The teams have learned that teaching and learning is not one-size-fits-all. With increased observation of varieties of student representations, and a heightened understanding of concepts, teachers are increasing their interest in representing mathematics in a variety of ways to reach a wider range of students. They now frequently seek multiple representations of mathematical situations, and are becoming adept at devising these on their own.

Underlying the process is a feeling of generosity—teachers being generous with their ideas and their time, gently supporting one another in taking risks, looking for the sense in students' ideas, and sharing successes as a group. Teachers in all three teams are excited about the changes they are making in their learning and teaching. Even small changes the teachers see in their students encourage the teachers to deepen their commitment to continue their own personal growth in facilitating student learning and we anticipate continued exciting developments in the future.

What is the future of lesson study in this district beyond the grant-funded project? In Japan, lesson study is an ongoing, career-long method of improving instruction for all teachers in elementary schools. Unlike our Japanese counterparts, lesson study is not woven into the fabric of teachers' typical work schedules in the U.S. The Noether Project is creating cultural changes in teachers' approach to teaching and patterns of collaboration, and several of the project schools have begun seeking ways to extend the changes throughout the school. Additionally, the district is committed to assuming increasing financial and leadership responsibility for lesson study throughout the five years of the project. During this time, we intend that lesson study will become well-established as a systematic method of enhancing instruction in the district; the positive outcomes that are becoming apparent in the Noether Project give us reason to hope that at all levels (teachers, site and district administrators) lesson study will come to be seen as indispensable to teachers' continuing professional growth and, therefore, to the students' success.

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