

# Teacher Training and Historical Studies on Professional Knowledge: Mathematics to Teach and Mathematics for Teaching

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## ABSTRACT

This text aims to present new research perspectives on teacher training. In particular, it is dedicated to analyzing the training of teachers who teach mathematics from a historical perspective. To this end, it uses the concepts of mathematics to teach and mathematics for teaching, indicating research results, characterizing the professional knowledge of the teacher who teaches mathematics and explaining some elements that constitute this knowledge.

**Keywords:** mathematics to teach, mathematics for teaching, professional knowledge, teacher training

## INTRODUCTION

Why consider the teacher's professional knowledge as a research theme? The answer to the question has been given, it seems, since the 1980s. One of the first theorists to emphasize the need for research on the subject is Lee Shulman (Fernandez, 2015). From his studies, the need to characterize specific knowledge for teaching is emphasized. This knowledge distinguishes the teaching profession from other professions.

In fact, Lee Shulman, with his studies, developed a paradigm for research on the professional knowledge of the teacher. It is proved that, after him, a multiplicity of authors has been building various typologies, as initially proposed by Shulman (Hofstetter & Schneuwly, 2017).

But, we must not forget the past. It is reasonable to think that teachers, at different times, used knowledge for teaching. And: this more current way of considering such knowledge is elaborated from previously accumulated knowledge. Thus, considering that the paradigm of knowledge typologies guides current research, we could ask how to characterize the teacher's professional knowledge historically. How did teaching establish parameters about knowledge, for its characterization as an ingredient of the profession over time? Such a broad question, in this text will be limited to teaching in mathematics. In an even more specific way, we will analyze the teaching of mathematics in primary school. We ask: How to characterize historically the professional knowledge of the teacher who teaches mathematics?

Somewhat different from the paradigm established by Shulman, we will retain the characterization given by Hofstetter and Valente (2017). These researchers consider that the professional knowledge of teaching is composed of two types: *the knowledge to teach* and the *knowledge for teaching*. Such characterization, even though it is contemporary, can be used to analyze past times and we can say that regardless of time, the themes of teaching and teacher training are under discussion. And one of the central aspects of this discussion concerns knowledge. The knowledge that gives reference to teaching and which integrates the training of the future teacher.

Thus, it is possible to consider, at any time, school knowledge as a knowledge to teach. Such knowledge refers to knowledge as an object of teaching: what should the teacher teach to his or hers students. But this is not the sole kind of knowledge involved in teacher education as we must also consider teacher training knowledge: knowledge for teaching. This knowledge concerns the teacher's tools to carry out his or her work. Thus, the object of teaching and tools for teaching are elements that can be used in the analysis of the past. They allow to characterize teaching as a job different from others. And even more: it is possible to consider the articulation of these two knowledges, at each time, theoretical as the representation of the teacher's professional knowledge. Emphasizing: professional knowledge is given by the relationship, at a given time, between knowledge to teach and knowledge for teaching.

## THE PROFESSIONAL KNOWLEDGE OF THE TEACHER WHO TEACHES MATHEMATICS

In our studies on the history of mathematical education we have been treating teacher training based on the knowledge present in his training analysing the relationships that are established in different historical times between the educational sciences and the disciplinary fields mathematics, in our case. More precisely, we are interested in the relations between Education and the mathematical disciplinary field - academic and university mathematics, that is, the relations between pedagogy and mathematics.

In this perspective we can consider the existence of a mathematics to teach, object of the teacher's teaching; and a mathematics for teaching, a tool that the teacher should use to teach mathematics. Also, the articulation between the teaching object and the teaching tool will theoretically characterize the professional knowledge of the teacher who teaches mathematics. That is, the relationship established between mathematics to teach and mathematics for teaching will reveal the professional knowledge of the teacher who teaches mathematics, in our case, in primary school, at a certain historical time (Bertini, Morais, & Valente, 2017). Mathematics to teach originated from the mathematical disciplinary field, from complex transformations to being present as a teaching object; mathematics for teaching was shaped as a teacher training element, a tool that the teacher should use in the exercise of the teaching profession.

Some important studies have already been carried out under this new perspective. A first one deals with the analysis of the transformations of arithmetic in the period 1870 to 1920 (Oliveira, 2017). Using mainly textbooks, the author shows how the educational transformations of the late 19th century changed school arithmetic. The detailed study, shows that there is a new organization of arithmetic, providing teachers with a different teaching object, modified by the pedagogical wave of intuitive teaching. The author characterizes in his thesis a new arithmetic, an intuitive arithmetic. It is a new school knowledge, a new school mathematics, an intuitive arithmetic, in short, a new arithmetic to teach.

With the arrival of a new pedagogical current - scientific pedagogy - mathematics as a teaching object in primary school changes. The teaching of mathematics must consider the field of education, now influenced by experimental psychology. In this case, Pinheiro's study (2017) explained the constitution of a new arithmetic, which took place in the teaching of mathematics in primary school in the period between 1920 and 1950. A new organization for school arithmetic appears. Textbooks begin to systematize an arithmetic to teach tested by the rules of experimental psychology. There is a new teaching object, a new mathematics to teach. The analysis of the research sources revealed the constitution of a *tailor-made arithmetic*, prepared with the aim of following a psychological order, adjusted to children's maturity, replacing the logical order of arithmetic itself.

These two works focused attention on the processes of constituting a mathematics to teach and analyzed the complex movements of elaborating school knowledge. They revealed the existing dialogues, from the end of the 19th century to the middle of the 20th century, between the field of educational sciences and the mathematical disciplinary field.

In relation to mathematics for teaching, mathematics present in teacher education, some recent studies can also be cited. Maciel (2019) dedicated himself to the analysis of pedagogical manuals, used in the late 19th century until the first decades of the 20th century. The manuals were true guides for teacher training. These sources allowed the author to characterize an arithmetic for teaching. This knowledge was constituted in times of the pedagogical wave of intuitive teaching. Specifically, an "intuitive arithmetic for teaching", considering an intuitive arithmetic present in teaching, Maciel (2019) shows the relationships established between mathematics to teach with changes in teacher training. The study indicates what tools the teacher would need to teach intuitive arithmetic.

These mentioned researches and many others are being developed at GHEMAT Brasil - Associated Group of Studies and Research in the History of Mathematics Education ([ghemat-brasil.com.br](http://ghemat-brasil.com.br)) through collective research projects.

## HOW TO CONDUCT HISTORICAL RESEARCH ON THE PROFESSIONAL KNOWLEDGE OF THE TEACHER WHO TEACHES MATHEMATICS? METHODOLOGICAL ASPECTS

So far we have mentioned the bases of the research on the professional knowledge of the teacher who teaches mathematics, revealing some of the results of these researches, now it is necessary to analyze the methodological aspects of this modality of research.

A first point that seems to us of great importance refers to the epistemological perspective to be adopted in the investigation of the teacher's professional knowledge. Historical research on the professional knowledge of the teacher who teaches mathematics in primary school must be understood as a theoretical construction derived from the systematization of teaching experiences carried out in a given period. Professional knowledge does not represent an empirical fact, something to be just and only named, taken from the phenomenological world. In the investigation of professional knowledge, our epistemological perspective treats reality, what one wants to know, as an object to be theoretically constructed. "That is, the real to be known is not the real in its fullness of appearance, but it is the real that appears theoretically, which is constructed in thought" (Borba; Valdamarin, 2010, p. 26). Specifically, it is the historian's construction of a historical object, "since the past is never an object that is already there" (Chartier, 2009, p. 16).

Under this perspective of analysis, since professional knowledge is taken as an object of knowledge, the process of its construction must promote an abstraction from the teaching experiences, trying, at a given time, to verify how to treat them as initial knowledge and, later, to verify the possibility of this knowledge being considered as systematized knowledge.

Thus, it is necessary to clarify what is meant by initial knowledge and by systematized knowledge. The first term - initial knowledge - is directly linked to the experiences accumulated by the individual, the teacher, and encompasses knowledge of his or her actions in the world, the practices of daily life in the classroom, for example. The second term - systematized knowledge - refers to a systematized discourse, ready to be mobilized, with the capacity to circulate. Systematized knowledge is communicable so that it can be used and appropriated in different contexts (Valente, 2019).

Thus initial knowledge and systematized knowledge involve discourses that are distinguished by the degree of systematization in which they are presented. The practice of a craft is considered to produce knowledge; in the case of teaching, such knowledge constantly permeates the work of the teacher, situated amid his or her experiences with the teaching and learning of his or her students (Molano & Romero, 2019). The knowledge involved in teaching may be able to be systematized through the analysis of teaching experiences.

It is now necessary to discuss what is being called "teaching experience". And, based on that, to have in view the way of treating the process of systematizing these experiences with the intention of elaborating knowledge.

Let us start by considering the teaching experience. Promptly, teaching experience represents any and all actions of the teacher in his or her daily work related to teaching and monitoring the students' learning.

In historical studies, we do not consider the practical actions of teachers, their teaching experiences, the instant they occur. The analysis should be made by means of traces of the past of these experiences, which were left in the present, and which have become subject to study. This is the first phase of a historical production methodology (Valente, 2007). At the limit, even current investigations regarding teaching actions, when analyzed, show themselves as study activities of the past, even if very recent...

The teaching experiences, carried out in past times, can be analyzed through a variety of documentation that contains records of discourses about the teachers' practices (Chartier, 1990, p. 7). These data read directly in the documents can be taken as information. In fact, they constitute initial information about a given teaching experience. How can information dispersed in different documents be converted into consolidated knowledge at a given historical moment? (Burke, 2016). This is the fundamental question for the initial analysis of the documentation in order to characterize the teacher's professional knowledge. And this task involves a process of systematization, understood as

(...) a type of investigation that can be called reconstructive; that is, a work that a posteriori converts an experience into an object of knowledge production, so that the experience is reconstructed according to the actions taken by the actors who participated in it (Rodríguez, 2019, p. 20).

As mentioned, teaching experiences, considered for the historical analysis of the teacher's professional knowledge are found in several kinds of documentation. For example: the analysis of students' notebooks may reveal the teacher's experiences in the courses that are present in those notebooks and that were noted by the students. The analysis of a mathematics textbook, too, can be treated as a document of teaching experience. In fact, such material fixes, at a given time, teaching content, ways to teach, school purposes, etc. The studied phenomenon of the school vulgate gives an idea of knowledge shared by different authors, in the search to fix knowledge for teaching practice (Chervel, 1990).

Notebooks with math lessons, math textbooks, journal articles on math teaching, references on the use of teaching materials for teaching mathematics etc. all constitute a wide range of documents that can be analyzed from the perspective of representing teaching experiences that can be systematized towards the characterization of the professional knowledge of the teacher who teaches mathematics at a given time.

At first, the analysis of the documents, which record teaching experiences, must be considered as a work of reconstruction of the knowledge mobilized for its production, the elaboration of the document by a given author or authors. It is a process that deals with information contained in the document for the passage to initial knowledge, a form still extremely linked to the teacher who elaborates the document that is considered as a source for the study. At a later stage, by comparing various documents from the same period, the possibility of having a systematized knowledge is analyzed.

In the study of teaching experiences we come across those related to teaching or teacher training. In the analysis of teaching experiences, knowledge to teach is investigated, mathematics to teach. Similarly, in the study of teacher training experiences, the systematization of knowledge for teaching, mathematics for teaching is investigated. However, the analysis of teaching experiences can reveal knowledge for teaching; nor, those of teacher training, show knowledge to teach. It all depends on the original intentions that guide the analysis.

## FINAL CONSIDERATIONS

We must at this point return to the original question of our text: How to characterize historically the professional knowledge of the teacher who teaches mathematics?

Considering the focus on mathematics present in the teaching and training of teachers, as well as the study of the relationships between these mathematics, seen as the professional knowledge of the teacher who teaches mathematics, it is possible to enumerate some characteristics that integrate this professional knowledge.

A first characteristic concerns the *graduation* of knowledge. Since the modern pedagogy of intuitive method, installed at the end of the 19th century, the way of organizing professional knowledge has changed. If, in a previous era, when it is generally possible to identify a "traditional pedagogy", there is a degree of knowledge given by the internal logic of the contents, the implementation of the graduated school will imply a different organization of knowledge in terms of school grades. Therefore,

each school year must have specific mathematics for that level of education. If, before the intuitive method, disciplinary training was considered sufficient to train teachers, the end of the 19th century will bring about new knowledge, changing the relationship between training and teaching. The training will have to consider that the teaching should be graduated, in view of a new conception of how the student learns: intuitively. Thus, the teacher will have to organize his course, his classes, before a new graduation established for teaching.

Graduation is an important research topic. Analyzing the graduation allows to make intelligible the historical options considered for each school year. And the knowledge to teach needs to attend a given graduation. The understanding of the constitution of the graduation for mathematics to teach, combined with the study of the relations maintained by this graduation with mathematics for teaching, is an important element of characterization of the professional knowledge of the teacher who teaches mathematics at a time.

The graduation of mathematics to teach is linked to another characteristic to be taken into account in the discussion of professional knowledge: the teaching program. The program will be the place for the sequences of the contents, for the election of what are the purposes for teaching new content.

The programming may be considered as somewhat different from the program. It is more general, determining how knowledge should be distributed over the different school years: an itinerary. What mathematics should be present in the early years? In secondary education? Programming changes involve changes in the macro level of school knowledge. And, of course, they reorient the teacher's training knowledge.

Graduation, Program, Programming seem to be important elements for the analysis of the relations between mathematics to teach and mathematics for teaching. As research progresses, new elements reveal themselves as important constituents of the teacher's professional knowledge.

## REFERENCES

- Bertini, L. de F., Morais, R. dos S., & Valente, W. R. (2017). *A matemática a ensinar e a matemática para ensinar – novos estudos sobre a formação de professores*. São Paulo: L F Editorial.
- Borba, S., & Valdemarin, V. (2010). A construção teórica do real: uma questão para a produção do conhecimento em educação. *Currículo sem Fronteiras*, 10(2), 23-37.
- Burke, P. (2016). *O que é história do conhecimento?* São Paulo, Editora UNESP.
- Chartier, R. (1990). *A história cultural: entre práticas e representações*. Lisboa: Difel; Rio de Janeiro: Bertrand Brasil S.A.
- Chervel, A. (1990). História das disciplinas escolares: reflexões sobre um campo de pesquisa. *Teoria & Educação. Porto Alegre, RS*, (2), 177-229.
- Fernandez, C. (2015). Revisitando a base de conhecimentos e o conhecimento pedagógico do conteúdo (PCK) de professores de ciências. *Revista Ensaio. Belo Horizonte*, 17(2), 500-528. <https://doi.org/10.1590/1983-21172015170211>
- Hofstetter, R., & Schneuwly, B. (2017). Saberes: um tema central para as profissões do ensino e da formação. In: R. Hofstetter & W. R. Valente (Org.). *Saberes em (trans)formação: tema central da formação de professores*. São Paulo: Livraria da Física, 113-172.
- Hofstetter, R., & Valente, W. R. (Orgs.) (2017). *Saberes em (trans)formação: tema central da formação de professores*. São Paulo: Livraria da Física.
- Maciel, V. B. (2017). *Elementos do saber profissional do professor que ensina matemática: uma aritmética para ensinar nos manuais pedagógicos (1880-1920)*. Tese (Doutor em Ciências). Programa de Pós-Graduação em Educação e Saúde na Infância e na Adolescência. Universidade Federal de São Paulo: Guarulhos.
- Oliveira, M. A. de (2017). *A Aritmética Escolar e o Método Intuitivo: um novo saber para o curso primário, 1870-1920*. Tese (Doutorado em Ciências). Programa de Pós-Graduação em Educação e Saúde na Infância e na Adolescência. Universidade Federal de São Paulo.
- Pinheiro, N. V. L. (2017). *A Aritmética sob medida: a matemática em tempos da pedagogia científica*. Tese (Doutor em Ciências). Programa de Pós-Graduação em Educação e Saúde na Infância e na Adolescência. Universidade Federal de São Paulo: Guarulhos.
- Rodriguez, J. C. G. (2019). Aspectos críticos em la formación de maestros a través de la sistematización de experiencias. In: Molano, Alejandra Dalila Rico; Romero, Carlos Enrique Cogollo (Eds.). *La sistematización de experiencias*. Bogotá: Universidad Santo Tomás, p. 19-35. <https://doi.org/10.2307/j.ctvr33d8f.4>
- Valente, W. R. (2019). Saber objetivado e formação de professores: reflexões pedagógico-epistemológicas. *Revista de História da Educação (Online)*, 23, e77747. <https://doi.org/10.1590/2236-3459/77747>