TEACHING COMPLEX FIELDS OF SOFT MATER, PROPOSAL OF A NEW LIQUID CRYSTAL ANALOGY

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Abstract

The key reason behind the richness of different structures and patterns displayed in nature is the universal mechanism of symmetry breaking. It shapes configurations at all length scales encountered in universe. Structures reached via symmetry breaking transitions are commonly described in terms of order parameter fields. One of the simplest systems where symmetry breaking concepts have already been explored in detail, are various liquid crystal (LC) phases. The reason behind this is rich variety of structures exhibited by LCs and their convenient experimental accessibility. Consequently, a wide spectrum of different theoretical tools have been developed in LCs. In this contribution the orientational ordering of housing block in San Francisco, which we choose as a typical large-city representative, was studied. Following nematic LC analogy we determine the local degree of ordering. The structural pattern of the city displays a domain-type pattern. The average degree of ordering within a domain strongly correlates with crime rate within it. Therefore, the results confirm an intuitive expectation that structures define properties. This model can be used as a helpful tool in education as it provides a way of understanding complex topics with the help of well-known every day phenomena.
Key words: *education, liquid crystal analogy, order parameter, city structure, crime rate, soft mater education*.

Introduction

Symmetry breaking plays an important role in nature. It is the reason behind complexity surrounding us (Zurek, 1985). In particular various liquid crystalline (LC) phases and structures reached via continuous symmetry breaking transitions demonstrate significant impact of attained orientational, translational ordering or combination thereof on LC macroscopic and also mesoscopic properties (Bradač, et al., 2011). LCs are often exploited as a testing ground for basic physics (Repnik, et al., 2010). This is due to their extraordinary experimental accessibility and rich variety of phases and structures displayed in which almost all physical phenomena and mechanisms could be observed as shown by Bradač et al. (2011). Various LC properties could be relatively easily probed experimentally mostly due to their softness, optical transparency and anisotropy, relatively accessible range of order parameter relaxation times and correlation length values, and liquid character (De Gennes, Prost, 1993). LC phases &structures demonstrate that order parameters, describing their average degree of ordering at mesoscopic level, have dominant impact on several mesoscopic or even macroscopic material or physical properties (Repnik, 2005). Order parameters are defined via local ensemble averaging procedure where most microscopic details are averaged out. The most essential property resulting from this averaging is their structural anisotropy (De Gennes, Prost, 1993).

The correlation between orientational ordering of housing blocks in large cities and local crime rate is studied. For example, maps of favelas, which are parts of the city with large population with low income on a small area, in Sao Paolo clearly reveal absence of architectural order and in these areas crime rate is relatively high. The relationship within the degree of ordering and crime rate in a general city is studied. In the analysis the city of San Francisco is chosen because of the accessibility of data, its unique geometry and age of the city. The latter is important since we are studying the orientational ordering of housing blocks within the city which was in San Francisco determined in a narrow time period and has not been changed multiple times across the history. Note that large cities are typical examples of complex systems, where diverse elements constituting it are strongly nonlinearly coupled. Consequently, microscopic details are averaged out which gives opportunity for universal laws to be observed. Often few robust key parameters well describe relevant properties of a system of interest.

In order to study the degree of ordering in cities the focus is on the average local orientational ordering of housing blocks. Ordering fields are introduced in a similar way as in nematic liquid crystals (De Gennes, Prost., 1993). The local degree of orientational ordering and the crime rate are strongly correlated.

Typically new knowledge is achieved due to learning processes which are based on previous gained knowledge. Therefore it is common to use these models which are commonly well understood or easier to grasp, to describe different more complex systems. For example Niels Bohr used the well-known model of the solar system to describe the jet to understand the model of the atom. The mathematical similarities between the movement of the planets around the Sun and the classical representation of the movement of the electrons around the nucleus and the fact that most of us are familiar with the model of the solar system form an analogy which has been used in classrooms all around the world for decades. Although there are some differences between analogous systems like the fact that the gravitational force between planets is attractive whereas the electromagnetic force between electrons is repulsive. But in both systems the attractive force between the central object and the orbiting objects is dominant. Anyway the teacher has to be careful since there are numerous traps in the teaching process with using analogies as shown by Coll, France and Taylor (2005).

The aim of the research is to provide a novel way to teach complex processes in soft matter. The latter is a complex field of physics research which is usually taught in college with the help of complex mathematics, computer simulations and experiments and therefore a base knowledge in those fields is required. But in later years the field of soft matter is crossing the borders into elementary and high schools where it uses analogies as a way to present the students complex phenomena with the help of already known processes. The most interesting process is orientational ordering which provides most of the macroscopic characteristics of soft matter especially liquid crystals. Despite of its richness the understanding and ability to apply the knowledge of orientational ordering is a demanding task for children which are not familiar with the mathematic description in the background. Therefore analogies are used which provide a unique opportunity to help complex forms of knowledge to step into the classroom sooner and the students can apply the gathered knowledge to other phenomena.

Nematic Orientational Order

The uniaxial nematic phase represents the simplest LC phase which is characterised by uniaxial orientational ordering (De Gennes, Prost, 1993). In three spatial dimensions (3D) this ordering is commonly described in terms of the tensor order parameter

$$\underline{Q}(\vec{r}) = \frac{S(\vec{r})}{2}\left(3\vec{n}(\vec{r}) \otimes \vec{n}(\vec{r}) - \underline{I}\right). \quad (1)$$

Here $\vec{n}$ stands for the nematic director field, describing the average local uniaxial orientation of anisotropic LC molecules within a mesoscopic volume at a site determined by the spatial vector $\vec{r}$, presented in Figure 1, where θ describes the angle between one molecule and the orientation of the director. The director field exhibits the so called head-to-tail invariance $\pm\vec{n}$. Amount of fluctuations about that direction is measured by the uniaxial scalar order parameter S defined as shown by De Gennes and Prost (2013)

$$S(\vec{r}) = \frac{1}{2}\left\langle 3\left(\vec{n}.\vec{e}\right)^2 - 1\right\rangle, \quad (2)$$

$S \in \left[-0.5, 1\right]$ Here <…> stands for the local ensemble averaging and the unit vector $\vec{e}$ determines temporal local average orientation of an anisotropic LC molecule within the mesoscopic volume. Rigid alignment of LC molecules along $\vec{n}$ yields S=1, and isotropic distribution of $\vec{e}$ corresponds to S=0.

In two dimensions (2D) the tensor order nematic order is defined as

$$\underline{Q}(\vec{r}) = S(\vec{r})\left(\vec{n}(\vec{r}) \otimes \vec{n}(\vec{r}) - \vec{n}_{\perp}(\vec{r}) \otimes \vec{n}_{\perp}(\vec{r})\right), \quad (3)$$

where $\vec{n}$ (the nematic director field), $\vec{n}_{\perp} = \vec{v} \times \vec{n}$ are unite vectors and $\vec{v}$ is the normal of a local surface area at $\vec{r}$. The corresponding scalar order parameter is given by

$$S(\vec{r}) = \left\langle 2\left(\vec{n}.\vec{e}\right)^2 - 1\right\rangle, \quad (4)$$

$S \in \left[-1, 1\right]$. As in the 3D case values S=1 and S=0 reveal rigid and isotropic alignment, respectively.

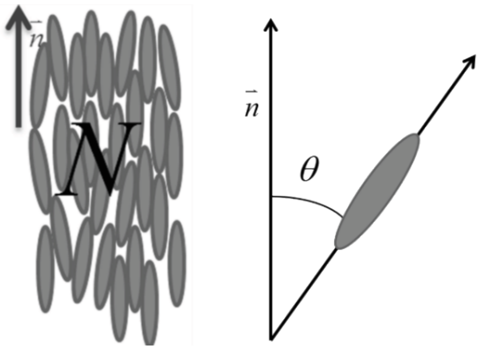

Figure 1: Nematic liquid crystal molecules are in average oriented along the nematic director, whereas on the right side the angle between the nematic director and an individual molecule is shown.

Image Analyses Algorithm

Online available maps are used (Mapquest, 2013), where the resolution was large enough to determine *housing blocks*. Pictures of local areas were taken and converted into greyscale to determine the borders between housing blocks. The latter were defined as an area of buildings enclosed by streets. In figure 2, a map of San Francisco and the corresponding extracted *housing* blocks is shown.

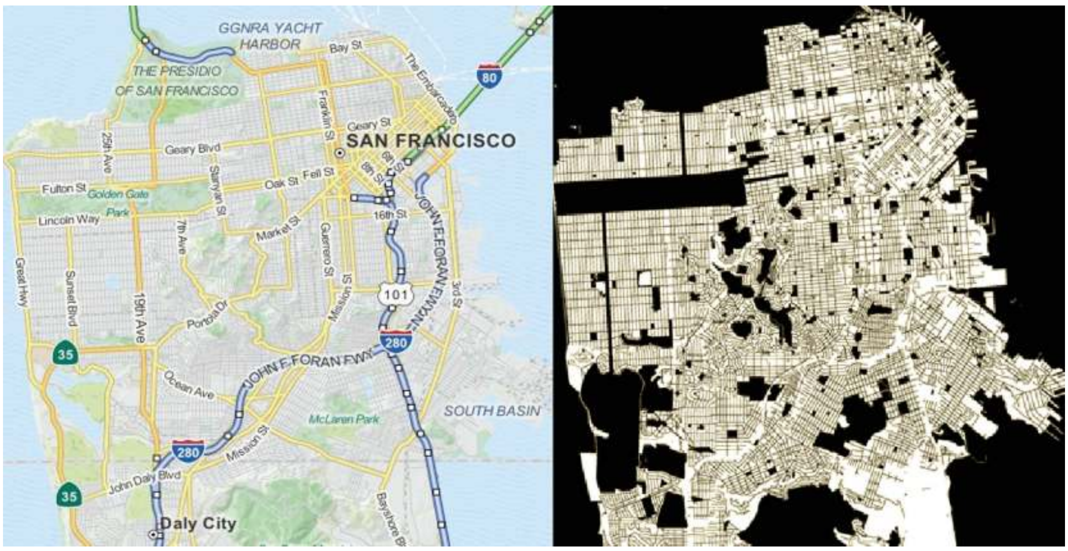

Figure 2: The map of San Francisco as provided by Mapquest, (2013). After the algorithm only a skeleton of the city remains which exposes the orientation or individual housing blocks.

The pictures were edited with means of a photo editing software, where parts of the city for which no data could be gathered were removed. Areas with high areas as parks and streets were removed because of the confinement to the mesoscopic scale. Next the desired data was extracted with means of an algorithm. The algorithm used to determine *housing blocks* is as follows. The map is introduced to the Cartesian (x, y) coordinate system. Regions like city parks, large car parks or similar elements were deleted. For each *housing block,* the (x, y) coordinates of its centre and its approximate surface area were determined. The average area of *blocks* within the city was calculated. All blocks with area (relatively small objects) and (relatively large objects) are discarded in further analysis. To each remaining *block* the local director field was assigned, which estimates its anisotropic shape. For this purpose a *block* was represented by a rod like object, where points along its longest axis. The head-to-tail symmetry is assumed in which states are equivalent. Therefore, each *housing block* is represented as an uniaxial *molecule*, whose orientation is determined by .

Domains are introduced as a region within which orientation of neighbouring *molecules* exhibits relatively weak spatial dependence. Two neighbouring *molecules* were set, where the orientations of them is determined by and are within the same *domain* providing

$$\vec{n}_i.\vec{n}_j > 0.8 \,. \tag{5}$$

To each such domain the average director field $\overline{N}_d = (\sin q_d, \cos q_d)$ was assigned. Within each domain the average domain order parameter was calculated as

$$S_d = \left\langle 2\left(\vec{N}_d.\vec{n}\right)^2 - 1 \right\rangle_d \tag{6}$$

where $\langle ... \rangle_d$ stands for the spatial averaging within a domain.

The degree of domain orientational order was correlated with local crime rate. For this purpose *crime* event data of San Francisco within the fall of 2013 was gathered. Available data contain x and y coordinates (x, y) of the *crime* event and the description of the *crime* being committed. As a *crime* event each activity being reported to the police was taken into account.

Results of Crime Rate Correlation

The correlation between the average *domain* order parameter and the corresponding local crime rate is as follows. In Figure 3 the average orientational distribution of domains using colour map is presented. In it each angle $q_d \in [0, p]$ is represented by different colour.

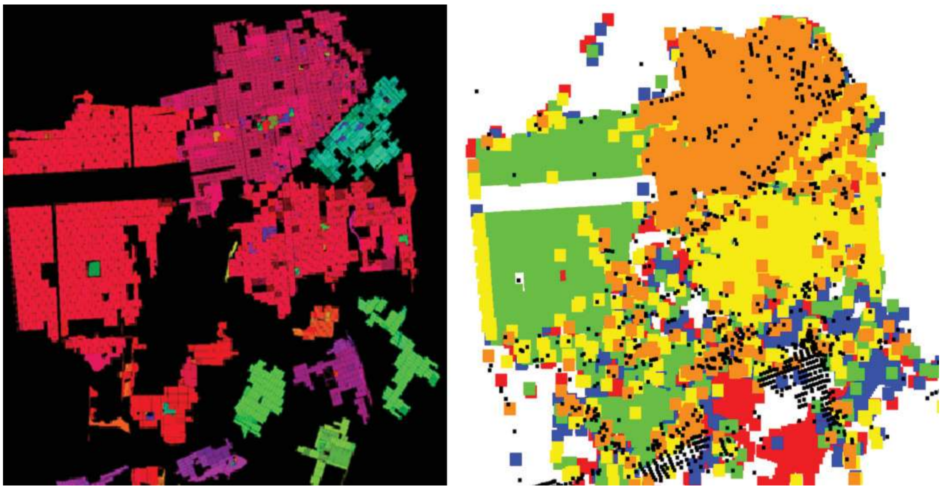

Figure 3: Domain pattern of San Francisco in the director field (left) and in order parameter (right). In both images greyscale coloring is used only to distinguish between individual domains.

Within each *domain* S_d was calculated using Eq. (5). The corresponding *crime* rate C_r within domains is shown in Fig. 5. For presentation purpose the normalized crime rate density is introduced as

$$c_r = \frac{C_r}{A_d}, \quad (7)$$

where stands for the area corresponding to an individual domain.

Figure 4: The crime rate map of San Francisco (left) and domains in orientational order of housing blocks (right).

In figure 5 a plot of the degree of ordering within each domain as a function of c_r is presented. One sees that S_d exhibits pronounced decreasing on increasing crime rate. Therefore, the result suggests that average local city architecture is apparently correlated with local crime rate.

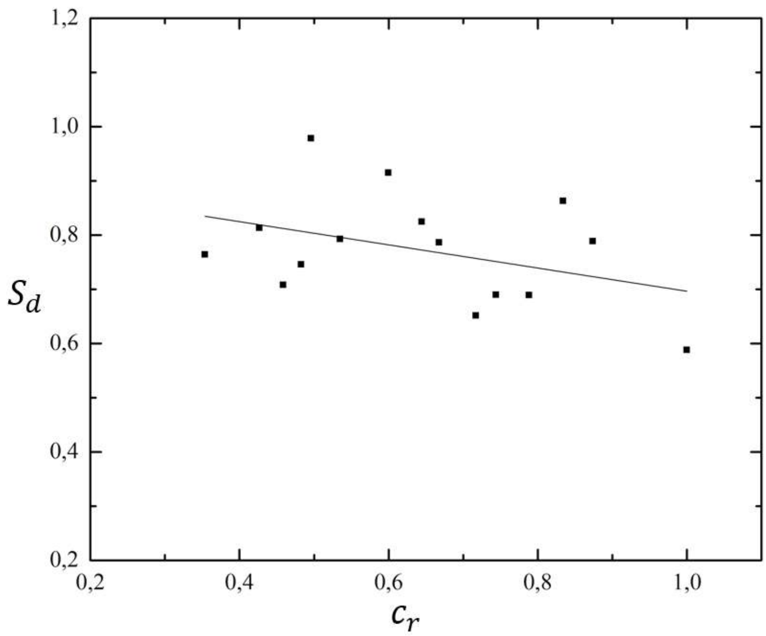

Figure 5: Plot of domain order parameter (S_d) as a function of crime density (c_r) in individual domains.

Discussion

In all fields of education it is necessary to create analogies which can provide an efficient way of understanding complex topics in natural sciences and connect them to everyday life. Therefore, a novel approach of understanding basic properties of systems with broken symmetry is presented. As Coll et al. (2005) stated there are several ways how an analogy can be misunderstood by or even counterproductive for students. For the analogy to be appropriate in our case mathematical similarities between LCs in two dimensions and cities had to be provided. The city of San Francisco was chosen to be the analogous system because of the online accessibility of satellite maps and the appropriate properties of the city. The influence of technology and the understanding of the use of it is being a pillar of modern education. Most students are nowadays almost daily surrounded by maps and city structures with the easiness and availability of internet use. Especially of students who were introduced to technology in the early age, the capability of individual work is astonishing. Therefore, one can expect that students can relate to complex mathematical models as in this case and visualise them with ease with the help of individual computer work. One factor which provides the necessary effectiveness of the proposed analogy is the relative ease with which students who are used to individual computer work can apply gathered knowledge of one field and apply it to a whole range of similar fields. This is the key step which sets the effectiveness of the analogy. Another interesting fact which arises from individual work with students, who are well thought in computer work, is the desire to investigate universality of gathered knowledge. All the facts have to be satisfied so that the analogy can be said to be effective and a useful teaching tool. Therefore, it has to be tested in classrooms and then judged in its effectiveness.

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Conclusions

In the study structural properties of San Francisco were investigated, where the focus was on orientational ordering of housing blocks. A degree of local ordering following analogy with nematic liquid crystals was calculated. For this purpose an algorithm which extracts data from maps of cities was developed. Out from the data the average orientational pattern of San Francisco was established, which exhibits a domain-type pattern. As a "domain" a region exhibiting orientation along a similar symmetry breaking direction was defined. Within each domain the average degree of orientational ordering was calculated, which measures the extent of spatial fluctuations about a domain average symmetry breaking direction. In addition crime rate within individual domains, using public available data on internet, was established. The analysis reveals strong and robust correlation between degree of orientational order and local crime rate. This mechanism shows a strong correlation between processes which are well understood, knowledge and the data is available easily online, therefore a use of this process in the education system is proposed. The latter needs a way to present complex knowledge to students with the use of well-established knowledge. This analogy can be especially useful with the universality of its application and the unlimited range of new field it can be applied to. In the future this analogy needs to be tested on its effectiveness on students and further refined, so a compact form of the algorithm can be available for students, with the interest in soft matter, easily online.

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