Teaching Cournot without Derivatives*

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Abstract: A simple technique for teaching the Cournot model to first year students is presented. The approach involves convincing the students that out of all rectangles with a common circumference, the square has the greatest area. No use is made of derivatives. The same approach can be used to understand some other market forms.

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1. INTRODUCTION

Many professors of first year microeconomics avoid the Cournot model on the grounds that the students are uncomfortable using derivatives to find the firms' reaction functions. This is unfortunate in that the Cournot model has a very intuitive and illuminating outcome, which is intermediate to the polar cases of monopoly and perfect competition.

The purpose of this note is to develop an alternative technique for presenting the Cournot model, which does not make use of derivatives. The key idea is to convince the students that out of all rectangles with a common circumference, the square has the greatest area. Once this is recognized, the firms' reaction functions can easily be found. A similar technique can also be used to understand certain other market forms (monopoly, Stackelberg, Bertrand with heterogenous products).[1]

Section 2 explains the main idea in the context of a concrete example which is convenient to use in class. In Section 3 it is explained how one may have to make minor modifications in order to apply the basic idea, and how the approach can be used to understand other market forms.

2. THE RECTANGLE METHOD

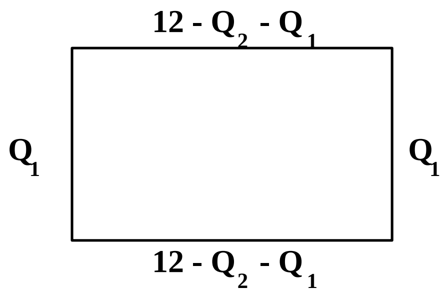
In this section I explain the basic idea. The presentation is phrased in terms of a concrete example, since in my experience that this is what first year students like most.

Problem: Two firms compete in a market for a homogenous good, simultaneously deciding what quantities Q_1 and Q_2 to produce. There are no fixed costs of production, but there is a

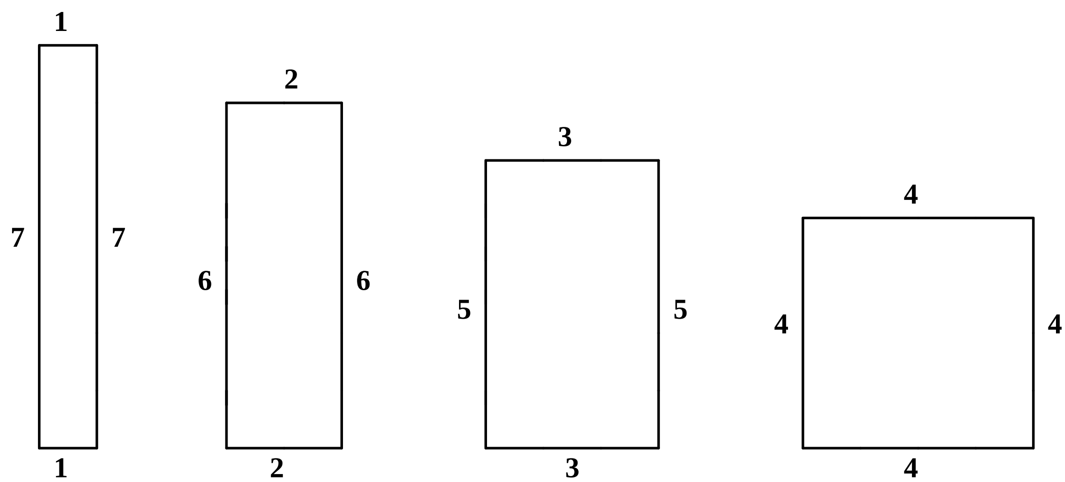
[1] The methods have been successfully tried out (by the author and several assistants) on 400 first semester economics students at Stockholm University in the spring semester of 1999.

given variable cost of 3 per unit produced. The market price P automatically adjusts to clear the market, which happens at price $P=15-Q_1-Q_2$. Find all Nash equilibria of this game!

Answer: Consider first the choice problem of firm 1. In Nash equilibrium this firm will choose a best response to its competitor's choice of quantity. Hence firm 1 will choose Q_1 to maximize its profit $Q_1(12-Q_2-Q_1)$. Note that this profit is the product of two numbers. Hence, the firm's profit corresponds to the area of the following rectangle, where the length of the sides are as indicated:



The problem of firm 1 is to find the value of Q_1 which maximizes the area of the rectangle. Note that if Q_1 is varied the rectangle changes shape, but its circumference is kept constant. Many students no doubt begin to see the answer now. To make it clear, draw them the following set of rectangles, all with a given circumference of 16:



The respective areas are 7, 12, 15, and 16. The *square* has the largest area! My experience is that at this point the students are ready to accept the following geometric truth: *Out of all rectangles with a given circumference, the square has the greatest area*.

It follows that firm 1's optimal choice of Q_1 is such that Q_1=12-Q_2-Q_1, or Q_1=6-Q_2/2. But this is precisely firm 1's reaction function. One finds firm 2's reaction function in an analogous way, and the model is then easily solved for its unique Nash equilibrium. No derivatives are ever used.

3. VARIATIONS

With proper adjustments one can readily handle more general cases of Cournot competition, as well as some other market forms. A successful application of the rectangle method may, however, require some manipulations. To illustrate the typical reason, consider what happens if the above example is changed so that the relationship between price and quantity demanded is P=15-kQ_1-kQ_2, for some is k≠1. *Mutatis mutandis*, firm 1 will choose Q_1 to maximize Q_1(12-kQ_2-kQ_1). Writing up the associated rectangle one detects a problem (do this, and show the students!): as Q_1 changes one does not stay within the class of rectangles with a given circumference. The problem is that the coefficient in front of the Q_1-terms is different for the two factors of the product. However, this is easily fixed by re-writing the firm's profit as kQ_1(12/k-Q_2-Q_1). It is now clear that the firm should choose Q_1 to maximize Q_1(12/k-Q_2-Q_1), or equivalently the size of a rectangle with sides of lengths Q_1 and 12/k-Q_2-Q_1. Note that as Q_1 changes, the circumference of the associated rectangles remains constant.

This manipulation technique comes in handy when one solves the Stackelberg model (which differs from Cournot's model only in that the firms move in sequence—first firm 1 chooses Q_1, then 2 chooses Q_2 after observing 1's choice of Q_1). Using backwards induction one quickly finds that in the Stackelberg version of the market discussed in Section 2 firm 1 should choose Q_1 to maximize $Q_1(12-(6-Q_1/2)-Q_1)=Q_1(6-Q_1/2)$. Again, to apply the rectangle method this must be further rearranged to $\frac{1}{2}Q_1(12-Q_1)$.

An analogous technique can be used to analyze Bertrand competition with heterogenous goods, an exercise which traditionally makes use of derivatives. The details are left for the reader.

Finally, it should be pointed out that the rectangle method may be useful also for analyzing monopoly markets. The usual approach is to let the students find the profit maximizing quantity at the point where marginal revenue curve crosses the marginal cost curve. To many students, this approach is somewhat murky, as finding the marginal revenue curve typically requires taking a derivative. The rectangle method finesses all this, noting instead that the firm's profit can be written as a simple product, et cetera.