

Karin Brodie

Teaching Mathematical Reasoning in Secondary School Classrooms

With Contributions by

Kurt Coetzee

Lorraine Lauf

Stephen Modau

Nico Molefe

Romulus O'Brien

 Springer

Contents

Introduction to Part 1	1
1 Teaching Mathematical Reasoning: A Challenging Task	7
The Centrality of Mathematical Reasoning in Mathematics Education.....	7
Justifying and Generalizing.....	8
The Role of Proof in Mathematical Reasoning.....	9
Creativity and Reasoning.....	10
Theories of Learning and Mathematical Reasoning.....	12
Constructivism.....	12
Socio-Cultural Theories.....	14
Situated Theories.....	16
Teaching Mathematical Reasoning.....	18
Tasks for Mathematical Reasoning.....	19
Classroom Interaction.....	20
The Challenges of Teaching Mathematical Reasoning.....	22
2 Contexts, Resources, and Reform	23
Responses to Reforms.....	23
The South African Context.....	26
Five Schools: Contexts and Resources.....	28
Race and Socio-Economic Status.....	28
School Resources.....	29
Classroom Resources.....	31
Learner Knowledge.....	33
The Tasks.....	35
The Grade 11 Tasks.....	35
The Grade 10 Tasks.....	36

Introduction to Part 2	39
3 Mathematical Reasoning Through Tasks: Learners' Responses	43
Tasks that Support Mathematical Reasoning.....	44
Teaching for Mathematical Reasoning.....	46
The Classroom and the Tasks.....	47
Learners' Responses: An Overview.....	48
Learners' Responses: Detailed Analysis.....	49
Teacher-Learner Interactions.....	52
Encouraging Participation.....	52
Using the Contribution to Move Forward.....	53
Pushing for Explanation of Particular Ideas.....	54
Conclusions and Implications.....	55
4 Learning Mathematical Reasoning in a Collaborative Whole-Class Discussion	57
What Is Mathematical Reasoning?.....	58
Why Teach Mathematical Reasoning?.....	59
Collaborative Learning and Mathematical Reasoning.....	60
Summarizing My Perspective.....	61
My Classroom.....	62
The Analysis.....	62
Winile's Learning.....	63
Making Observations.....	64
Explaining and Justifying Assertions Made.....	64
Connecting Observations with Mathematical Representations.....	65
Reconstructing Conceptual Understanding.....	67
Testing Other Claims.....	68
The Teacher's Role.....	69
Establishing Discourse.....	69
Framing Discussion.....	70
Lesson Flow or Momentum.....	70
Conclusions and Implications.....	71
5 Classroom Practices for Teaching and Learning Mathematical Reasoning	73
Classroom Practices.....	74
Learning Mathematical Reasoning.....	75
Teaching Mathematical Reasoning: Questioning and Listening.....	76
My Classroom.....	78
Teacher Moves and Practices.....	79
Learner Moves and Practices.....	82
Conclusions and Implications.....	84

6 Teaching Mathematical Reasoning with the Five Strands	87
A Social-Constructivist Framework.....	88
Mathematical Practices and Proficiency.....	89
My Classroom and the Tasks.....	90
Initial Analysis.....	94
Classroom Interaction.....	94
Learners' Work.....	95
The Five Strands in the Lesson.....	96
Procedural Fluency.....	96
Conceptual Understanding.....	97
Strategic Competence.....	98
Adaptive Reasoning.....	99
The Five Strands in the Learners' Work.....	99
Conclusion.....	100
7 Teaching the Practices of Justification and Explanation	103
Construction and Practices.....	104
The Practices of Justification and Explanation.....	104
The Importance of Tasks.....	106
The Teacher's Contribution.....	106
My Classroom.....	108
The Learners' Written Responses.....	109
Whole-Class Interaction.....	111
Incorrect Justification.....	112
Partial Justification.....	114
Correct Justification.....	115
Conclusions.....	117
Introduction to Part 3	119
8 Learner Contributions	121
Learner Contributions and Mathematical Reasoning.....	122
Describing Learner Contributions.....	123
Distribution of Learner Contributions.....	124
Accounting for Learner Contributions.....	126
Basic Errors.....	127
Appropriate Errors.....	128
Missing Information.....	130
Partial Insights.....	131
Complete, Correct Contributions.....	132
Going Beyond the Task.....	134
Summary.....	136

9	Teacher Responses to Learner Contributions	139
	Teacher Moves	139
	Distributions of Teacher Moves	142
	Mainly <i>Maintaining</i> : Mr. Nkomo	142
	The Power of <i>Inserting</i> : Ms. King	145
	Strategic Combinations: Mr. Daniels	149
	Supporting Learner Moves: Mr. Mogale	153
	Entertaining Errors: Mr. Peters	157
	Overview: Teacher Responses to Learner Contributions	160
	Trajectories for Working with Learners' Contributions	163
10	Dilemmas of Teaching Mathematical Reasoning	167
	Teaching Dilemmas	167
	Linking Learners with the Subject	168
	Working Simultaneously with Individuals and Groups	169
	The "Press" Move	170
	To Press or Not to Press?	172
	To Take Up or Ignore Learners' Contributions?	176
	Conclusions	179
11	Learner Resistance to Teacher Change	183
	Resistance to Pedagogy	183
	The Context of the Resistance	187
	Learner Resistance	191
	The Teacher's Contributions	193
	Making Sense of the Resistance	196
12	Conclusions and Ways Forward: The "Messy" Middle Ground	199
	Tasks and Mathematical Reasoning	200
	Supporting Learner Contributions	201
	Working with Learner Errors	202
	Classroom Conversations	202
	Maintaining the IRE/F	203
	Supporting all Learners to Participate	204
	Learner Resistance	205
	Conclusions	205
	Appendix	207
	References	213
	Index	223