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Teaching problem solving through cooperative grouping. Part 1: Group versus individual problem solving

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An experiment was conducted to investigate the effects of cooperative group learning on the problem solving performance of college students in a large introductory physics course. An explicit problem solving strategy was taught in the course, and students practiced using the strategy to solve problems in mixed-ability cooperative groups. A technique was developed to evaluate students' problem solving performance and determine the difficulty of context-rich problems. It was found that better problem solutions emerged through collaboration than were achieved by individuals working alone. The instructional approach improved the problem solving performance of students at all ability levels.

I. INTRODUCTION

Problem solving is one of the primary tools of college physics instruction. Unfortunately, many students in introductory courses consider problem solving to be independent of physics concepts and principles being taught (e.g., "I understand the material, but I just can't solve the problems."), or they believe that specific patterns of mathematical solutions *are* the physics to be learned (e.g.,"I can follow the examples in the textbook, but your test problems are too different."). For the past 3 years the University of Minnesota has been developing and testing an instructional approach to help students in a large introductory physics course integrate the conceptual and mathematical aspects of problem solving. This approach combines the explicit teaching of a problem-solving strategy with a supportive environment to help students implement that strategy. The supportive environment is provided by having students practice solving problems in cooperative groups.[1] The results of experiments on the structure and management of well-functioning cooperative groups are described in a companion article.[2]

Cooperative-group problem solving was adopted for two reasons. First, it has been shown to be an effective technique for helping students learn a complex skill.[3–5] We hypothesized that in well-functioning groups, students share their conceptual and procedural knowledge as they solve a problem together. During this joint construction of a solution, individual group members can request explanations and justifications from one another. This mutual critique would clarify all the members' thinking about the physics concepts and principles to be used, and how those concepts and principles should be applied to the particular problem. Moreover, each member can observe others perform the varied thinking strategies that he or she must perform independently and silently on individual problem assignments. The second reason for adopting cooperative grouping is more mundane and practical. Of the recommended teaching techniques for helping students become better problem solvers, cooperative grouping places the least demand on the instructors. With minimal training, graduate student teaching assistants can implement cooperative groups in recitation and laboratory sections.[6]

The purpose of this study was to investigate the effectiveness of our problem-solving instructional approach, particularly the cooperative grouping aspect. Although research in precollege education suggests that all students benefit from group work,[5,7] we are not aware of any studies that have been done on group problem solving in college physics. We assumed that if well-functioning cooperative groups were established, then group problem solutions would surpass those of any single group member on equivalent problems, even those of the highest ability member. In addition, if all students acquired individual problem solving skills, then there would be an increase in the quality of students' individual problem solutions over time, independent of their increasing knowledge of physics. If, on the other hand, the desired collaborative behaviors did not occur in the groups, then the group solutions would simply reflect the performance of the best (highest ability) student in the groups, and little benefit would accrue to anyone from the exercise. In that case, extensive group work might actually hinder the learning of the best students, who would spend their time dealing with other students rather than extending their own knowledge and skills.

This paper reports the results of our investigations to answer the following questions:

1. Are problem solutions worked out in cooperative groups better than the work of the best students in the groups?

2. If group problem solutions are better than individual problem solutions, which aspects of problem solving are accomplished better in the groups?

3. Do students' individual problem-solving performances improve during the course of instruction. If so, does the performance of the best students show the same improvement as that of the other students?

4. Are students taught with our instructional approach better individual problem solvers than students in a traditional course?

Of course, no educational research is independent of the context in which it is done. The first section below outlines the introductory physics course and the problem-solving instructional approach employed. The following sections describe measurement procedures and the investigation methods and results. The last section summarizes our conclusions and discusses some implications of the results and unresolved issues for further research.

II. DESCRIPTION OF THE COURSE

A. Organization and goals

The University of Minnesota offers a two-quarter, algebra based physics course that is required by 24 different departments (e.g., architecture, agricultural engineering, geology, food science and nutrition, pharmacy, veterinary medicine, fishery and wildlife, soil science). The course consists of four 50-min lectures, a 50-min recitation section, and a 2-h laboratory per week. The course traditionally covers about two-thirds of the chapters in an introductory text for this level.[8] The enrollment is about 120 students in the lecture section, with 15–20 students in the recitation sections and laboratories, which are taught by graduate student teaching assistants.

We recently surveyed the faculty in the departments that require this physics course to determine what they want their students to learn. The survey revealed that their important goals are for students to: (1) learn the fundamental principles of physics (e.g., force laws, conservation of energy, conservation of momentum); (2) learn general qualitative and quantitative problems solving skills that they can apply to new situations; and (3) overcome their misconceptions[9] about the behavior of the physical world. To meet these goals, several changes were implemented in one experimental section of the course. These changes were based on recommendations in the physics instructional literature.[10–13] For the sake of brevity, only the problem-solving instructional approach is described here.

B. Problem solving instructional approach

The problem-solving instructional approach had several integrated components. First, students were taught a general problem-solving strategy that is based on a variety of research and instructional literature describing the nature of effective (or "expert") problem solving in physics. Second, a set of context-rich practice and test problems were constructed that reinforce the usefulness of the strategy. Third, during recitation and laboratory sessions students worked in carefully managed cooperative groups to practice using the prescribed strategy to solve context-rich problems. Finally, grading practices were changed to reflect the importance of both cooperative-group problem solving and the use of the prescribed strategy. The following sections describe these four components of the problem-solving instructional approach.

1. The prescribed problem-solving strategy

There is an expanding body of research comparing expert and novice problem-solving strategies in physics.[14–16] Novices typically begin to solve a problem by plunging into the algebraic and numerical solution—they search for and manipulate equations, plugging numbers into the equations until they find a combination that yields an answer.[17] All too often they neither use their conceptual knowledge of physics to qualitatively analyze the problem situation, nor do they systematically plan a solution before they begin numerical and algebraic manipulations of equations.[14,15,18] When they arrive at a numerical answer, they are usually satisfied—they rarely check to see if the answer makes sense.

The instructional literature recommends several strategies to help students integrate the conceptual and procedural aspects of problem solving.[19,20] These strategies are very similar. The basic form of the five-step strategy designed for our students was strongly influenced by the work of Frederick Reif and Joan Heller,[21] but it has many elements in common with Alan Schoenfeld's framework for mathematics problem solving.[22] It requires students to make a systematic series of translations of the problem into different representations, each in more abstract and mathematical detail.

Table I. Examples of context-rich group and individual problems.

Airplane problem (group)
One morning while waiting for class to begin, you are reading a newspaper article about airplane safety. This article emphasizes the role of metal fatigue in recent accidents. Metal fatigue results from the flexing of airframe parts in response to the forces on the plane, especially during takeoff and landing. The reporter uses as an example a plane with a weight of 200,000 lbs and a takeoff speed of 200 mph which climbs at an angle of 30° with a constant acceleration to reach its cruising altitude of 30,000 ft with a speed of 500 mph. The jet engines provide a forward thrust of 240,000 lbs by pushing the air backwards. The article then goes on to explain that a plane can fly because the air exerts an upward force on the wings perpendicular to their surface called "lift." You know the air resistance is also a very important force on the plane and is in the direction opposite to the velocity of the plane. The article tells you this force is called the "drag." Although the reporter writes that some metal fatigue is primarily caused by the lift, and some by the drag, she never tells you their size for her example plane. Luckily, the article contains enough information to calculate them, so you do.

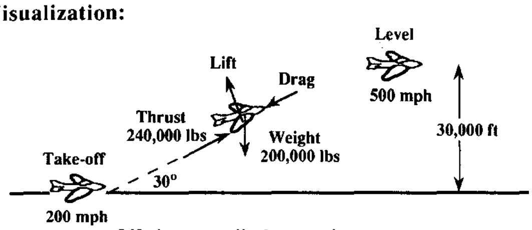
Elevator problem (matched individual)
You have been hired to design the interior of a special express elevator for a new office building. This elevator has all the latest safety features and will stop with an acceleration of $g/3$ in case of any emergency. The management would like a decorative lamp hanging from the unusually high ceiling of the elevator. You design a lamp with three sections that hang one directly below the other. Each section is attached to the previous one by a single thin wire that also carries the electric current. The lamp is also attached to the ceiling by a single wire. Each section of the lamp weighs 7.0 N. Because the idea is to make each section appear as if it is floating on the air without support, you want to use the thinnest wire possible. Unfortunately, the thinner the wire, the weaker it is. To determine the thinnest wire that can be used for each stage of the lamp, calculate the force on each wire in case of an emergency stop.

Moving problem (easier individual)
You are helping a friend move into a new apartment. A box weighing 150 lbs needs to be moved to make room for a couch. You are taller than the box, so you reach down and push it at an angle of 50 deg from the horizontal. The coefficient of static friction between the box and the floor is 0.50 and the coefficient of kinetic friction between the box and the floor is 0.30. If you want to exert the minimum force necessary, how hard would you push to keep the box moving across the floor?

The five steps of the strategy are defined below in the context of the solution to a dynamics problem involving an airplane, which is shown in Table I. This problem gives the take-off angle and speed of a plane, its cruising speed and altitude, its weight, and the thrust of the plane. Students are asked to find the lift and the drag on the plane during take-off. Prior to this test question, the students had not been taught the specific concepts of lift, drag, and thrust. Figure 1 illustrates how a typical group used the following five steps to solve this problem:

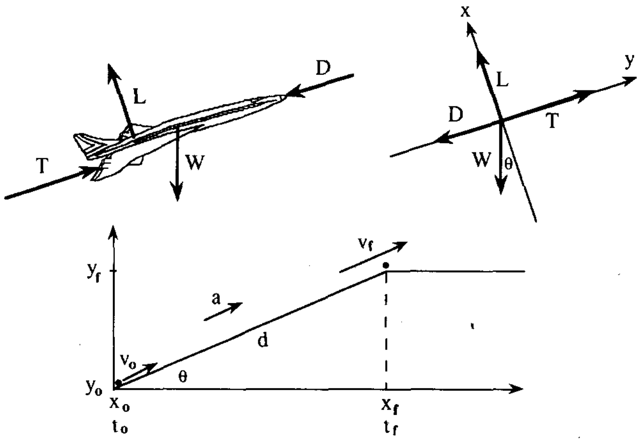
1. Visualize the problem: This step is a translation of the problem statement into a visual and verbal understanding of the problem situation. In the airplane problem, students sketched the plane moving down the runway, taking off,

Visualization:



Lift is perpendicular to wings.
Drag is air resistance -- opposite to velocity.
Acceleration is constant.
Find the lift and drag forces.
Use $\Sigma F_r = ma_r$ to find forces, kinematics definitions to find acceleration.

Physics Description:



D = force of drag (constant)
L = force of lift (constant)
T = force of thrust (240,000 lbs)
W = weight of the plane (200,000 lbs)
θ = angle of incline (30°)
v_o = 200 mph
v_f = 500 mph
a = constant
x_o = initial take-off position (0)
x_f = position when plane levels off
y_o = initial position when plane takes off (0)
y_f = final position when plane levels off (30,000 ft)
t_o = time when plane takes off (0)
t_f = time when plane levels off
d = distance plane needed to travel before leveling off

Find D and L.

Plan:

For Lift	For Drag
$\Sigma F_x = ma_x$	$\Sigma F_y = ma_y$
1) $L - W\cos\theta = 0$	2) $T - D - W\sin\theta = ma_y$

Need to find a_y:

3)	$a_y = \frac{v_f - v_o}{t_f - t_o}$	need t_f
4)	$\bar{v} = \frac{v_f + v_o}{2}$	need $\bar{v}$
5)	$\bar{v} = \frac{d}{t_f - t_o}$	need d
6)	$\sin\theta = \frac{y_f - y_o}{d}$	

We have 6 equations and 6 unknowns (L, D, a_y, t_f, $\bar{v}$, and d)!

Solve (1) for L. Solve (6) for d and substitute into (5). Equate (4) and (5) and solve for t_f. Substitute t_f into (3) and solve for a_y. Substitute a_y into (2) and solve for D.

Execute: (only last steps shown)

$$L = W\cos\theta = 200{,}000 \text{ lbs} \cdot 0.866 = 173{,}205 \text{ lbs}$$

$$D = T + W\sin\theta + \frac{W(v_f - v_o)(v_f + v_o)\sin\theta}{2(y_f - y_o)g}$$

Check: $D = \text{lbs} + \text{lbs} + \text{lbs}\,\frac{(\text{ft/s})(\text{ft/s})}{(\text{ft})(\text{ft/s}^2)} = \text{lbs}$ OK

$$D = 240{,}000 \text{ lbs} + 100{,}000 \text{ lbs} - 23{,}635 \text{ lbs} = 316{,}365 \text{ lbs}$$

Lift seems OK -- it's less than the weight.
Drag is larger than the thrust. We made a mistake somewhere!

Fig. 1. A cooperative group solution of the airplane problem (Table I), illustrating the five steps of the problem solving strategy.

and leveling off at cruising altitude. They determined the relevant information they were given to answer the question, and the general approach to take to the problem.

2. Physics description: This step requires students to use their qualitative understanding of physics concepts and principles to analyze and represent the problem in physics terms (e.g, vector diagrams). In the airplane problem, students drew a free-body and vector force diagram of the plane, and a kinematics diagram showing its position, velocity, and acceleration when it takes off and when it levels off. They then identified symbolically the relevant variables in the problem.

3. Plan a solution: This step involves translating the physics description into an appropriate mathematical representation of the problem (equations of the principles and constraints), determining if there is enough information represented to solve the problem, then specifying the algebraic procedure to extract the unknown variable(s). In the airplane problem, the students used their vector diagrams to generate the correct force and kinematics equations. From the ordering and position of statements in their written solution, we inferred that when they reached Eq. (5), they realized that they did not have enough information to solve the problem.[23] At that point, they went back to their physics description and determined that they could use a trigonometric relation to find the distance the plane traveled while accelerating to cruising altitude.

4. Execute the plan: Students use mathematical rules to obtain an expression with the desired unknown variable on one side of the equation and all the known variables on the other side. Specific values are then substituted into the expression to obtain a numerical solution. In the airplane problem, the students made an algebraic error.

5. Check and evaluate: Finally, the students evaluate the reasonableness of their answer—are the sign and units correct and does the answer match their experience of the world and/or their expectations of how large the numerical answer should be. In the airplane problem, the students first checked for correct force units. They evaluated the reasonableness of their answers by comparing the lift to the given weight, and the drag to the given thrust. They concluded that they had made a mistake somewhere (it was in the algebra) because their drag was larger than the thrust.

An outline of the five-step strategy is shown in Table II. The strategy was presented early in the first quarter and modeled subsequently in all lectures. To encourage the use of the strategy, students were given (1) flow charts that outlined a procedure and major decision points for each step of the strategy, and (2) problem-solving format sheets for solving homework problems. The format sheets reserved a section of the paper for each step of the problem-solving strategy, and each section included brief prompts for the type of information to include in the space provided. For example, the three prompts for planning a solution were: "State relevant general equations. Introduce specific variables into general equations; which specific equations are needed to determine the target quantity? Check—are there as many equations as unknowns?"

2. Context-rich problems

The rationale for constructing context-rich problems is described in detail in a companion article.[2] Briefly, context-rich problems, illustrated in Table I, are designed to focus students' attention on the need to use their concep-

Table II. Outline of the five-step problem solving strategy.

1. Visualize the problem

Translate the words of the problem statement into a visual representation:

- draw a sketch (or series of sketches) of the situation;
- identify the known and unknown quantities and constraints;
- restate the question;
- Identify a general approach to the problem—what physics concepts and principles are appropriate to the situation.

2. Describe the problem in physics terms (physics description)

Translate the sketch(s) into a physical representation of the problem:

- use identified principles to construct idealized diagram(s) with a coordinate system (e.g., vector component diagrams) for each object at each time of interest;
- symbolically specify the relevant known and unknown variables;
- symbolically specify the target variable (e.g., find v_0 such that $h_{max} \geqslant 10$ m).

3. Plan a solution

Translate the physics description into a mathematical representation of the problem:

- start with the identified physics concepts and principles in equation form (e.g., $\bar{a}_x = \Delta v_x / \Delta t$, $\Sigma F_x = m a_x$);
- apply the principles systematically to each object and type of interaction in the physics description (e.g., $N_1 - W_1 \cos\theta = m_1 a_{1x}$ and $W_1 = m_1 g$);
- add equations of constraint that specify the special conditions that restrict some aspect of the problem (e.g., two objects have the same acceleration, $a_1 = a_2$);
- Work backward (from target variable) until you have determined that there is enough information to solve the problem (the same number of independent equations as unknowns);
- specify the mathematical steps to solve the problem (e.g., solve equation #2 for N_1, then substitute into equation #1, etc.).

4. Execute the plan

Translate the plan into a series of appropriate mathematical actions:

- use the rules of algebra to obtain an expression with the desired unknown variable on one side of the equation and all the known variables on the other side;
- substitute specific values into the expression to obtain an arithmetic solution.

5. Check and evaluate

Determine if the answer makes sense:

- check—is the solution complete?
- check—is the sign of the answer correct, and does it have the correct units:
- evaluate—is the magnitude of the answer reasonable?

tual knowledge of physics to qualitatively analyze a problem before beginning to manipulate equations. They are essentially short stories that include a reason for calculating some quantity about a real object or event. In addition, they may have one or more of the following characteristics in common with real-world problems: (1) The problem statement does not always explicitly identify the unknown variable; (2) More information may be available than is needed to solve the problem; or (3) Information may be missing, but can easily be estimated or is "common knowledge;" (4) Reasonable assumptions may need to be made to solve the problem.

3. *Cooperative group environment*

In both the recitation and laboratory sections, students worked in carefully managed cooperative groups to practice the five-step problem solving strategy. In the recitation sections, they practiced using the strategy to solve context-rich problems like the ones shown in Table I. In the laboratories, which were coordinated with the lectures and recitation sections, they practiced using the strategy to solve concrete, experimental problems. Students had the same instructor (graduate teaching assistant) and worked in the same groups for both their recitation and laboratory section. This allowed students more opportunity to work with the same group members.

The experiments which resulted in the structure and management procedures for cooperative groups are described in a companion article.[2] Briefly, students worked in cooperative groups of three (or occasionally four) members. The composition of the groups changed about three times each quarter. The first groups were formed randomly. After the first test, however, students were assigned to groups by ability: a group consisted of a student from the top third, a student from the middle third, and a student from the lower third of the class based on past test performance. The students were assigned "roles" of Manager, Skeptic, and Checker/Recorder, which were rotated each week. These roles reflect the mental planning and monitoring strategies that individuals must perform when solving problems alone.

4. *Testing and grading*

The testing and grading practices were designed to reinforce the use of the five-step problem solving strategy and the importance of cooperation.[2] In addition to individual final exams, the students took four tests the first quarter, and three tests the second quarter. Each test was given in two parts, (1) a context-rich problem to be solved in cooperative groups during the 50-min recitation section, and (2) a short qualitative question and two context-rich problems to be solved individually during the 50-min lecture period the following day. On most tests, one of the context-rich problems was more difficult, and one was easier, as defined below in Sec. III B. An example of the context-rich problems from one test are shown in Table I.

For the first three tests, students were required to solve the problems on the problem format sheets; thereafter, problems were solved on blank paper in the standard "blue books." For the group problem students turned in only one solution for their group, and each member of a group received the same grade. All problems, group and individual, were graded on a 20-point scale, and students received points for following the steps of the problem-solving strategy as well as for a correct solution. To reduce competition and encourage cooperation, letter grades were based on set criteria (above 70% was a grade of A, 50%–70% was a grade of B, etc.) rather than based on a curve.

III. MEASUREMENT PROCEDURES

The comparisons made in this study required the creation of two measurement scales: a valid and reliable measure of students' problem solving performance independent of the grading, and a rating scale of problem difficulty. In 1989, several performance and problem difficulty scales were developed, tested, and refined. These scales were used to analyze the 1990 data reported in this study. The first section below describes the problem-solving performance rating scales. The second section describes the problem difficulty rating scale.

A. Problem-solving performance

The determination of whether one set of problem solutions are better than another requires a definition of "better." Simply scoring the correctness of an algebraic or numerical solution and comparing average scores is not a good determination of better problem-solving performance. Students can make any number of mistakes solving problems, some more serious than others. Consequently, we defined "better" solutions as those which exhibit characteristics similar to the solution an expert produces when faced with a real problem. The scoring scheme adopted for measuring problem-solving performance is similar to, but more rigorous than, the standard grading practice of giving students partial credit for different characteristics of their problem solution. The following six characteristics of expert-like problem solutions were scored.

1. Evidence of conceptual understanding: Does the physics description reveal a clear understanding of physics concepts and relations? For example, does the description indicate curvilinear trajectories for projectiles or incorrect straight-line trajectories: Does the solution employ unbalanced forces for an accelerating object, or incorrect balanced forces?

2. Usefulness of description: Is the essential information needed for a solution present? For example, do the force diagrams include all the relevant forces? For collision problems, are the momentum vectors both before and after an interaction clearly indicated?

3. Match of equations with description: Are the specific equations used consistent with the physics described? For example, are vector equations used to relate vector quantities? Are the described forces appropriately included in specific force equations?

4. Reasonable plan: Does the solution indicate that sufficient equations were assembled before the algebraic manipulations of equations was undertaken? Does the solution include an indication of how to combine equations to obtain an answer?

5. Logical progression: Does the mathematical solution progress logically from general expressions of physics principles to a problem-specific formulation using defined variables? Are numbers substituted for variables only after an algebraic solution for the unknown variable was obtained?

6. Appropriate mathematics: Aside from minor mistakes, is the mathematics used reasonable? Or does the solution employ invalid mathematical claims in order to obtain an answer (e.g., the mass is small, so set $m = 1$)?

The ratings for these six characteristics were equally weighted and normalized to yield an ordinal problem-solving scale with a maximum score of 100.

Several tests of the 1989 data were made to check the validity and reliability of the scoring scheme. First, we checked that the same mistake was not counted more than once. For example, it is impossible to solve a dynamics problem correctly if all relevant forces are not included in the vector diagram. The scoring scheme counted this type of mistake only once (e.g., to judge whether the equations matched the physics description, the equations were compared with the student's description, not the correct description). Second, we verified that the scoring scheme allowed for the increased sparseness of solution as students' expertise increased. That is, we checked that students independently judged to have gained problem-solving expertise, but who wrote less, received progressively higher problem-solving scores. Procedural neatness was not scored, although we noted that as students gained expertise, their solutions also became procedurally neater. Finally, we made sure that our rating criteria were sufficiently detailed for reliable scoring of solutions by different people.

B. Problem difficulty

A detailed comparison of students' performance on pairs of 1989 individual problems indicated that the following six characteristics contribute to the difficulty of context-rich problems:

1. Problem context: Contexts familiar to the majority of introductory students through direct experience, newspapers, television, or solving standard textbook problems are easier than problems with contexts unfamiliar to the students (e.g., ion beam, protons from the Sun, x-ray signals from pulsars).

2. Problem cues: Problems that cue a standard application of a set of related principles to solve the problem are easier than problems that do not explicitly cue a standard approach. For example, force problems that specify a force as the unknown variable (e.g., What is the lift on the airplane?) are easier than force problems that specify a mass as the unknown variable (e.g., What is the mass of the planet?).

3. Given information: Problems with no extraneous information or missing information in the problem statement are easier than problems with irrelevant information or missing information that must be recalled or estimated.

4. Explicitness of question: Problems that specify a particular unknown variable (e.g., What is the muzzle velocity of the bullet?) are easier than problems for which the desired unknown variable must be determined (e.g., Will this design for the lunar lander work?).

5. Number of approaches: Problems that can be solved with one set of related principles (e.g., kinematics or energy conservation) are easier than problems that require the application of more than one set of related principles for a solution (e.g., both kinematics and energy conservation).

6. Memory load: Problems that require the solution of five or less equations are easier than problems that require the solution of more than five equations.

For each problem, each of these six characteristics was scored as 0 (easier) or 1(more difficult). We found that the sum of the six characteristics accurately predicted the relative difficulty of the course problems. That is, students had lower performance scores on problems with a total difficulty rating of 4 than they did on problems with a difficulty rating of 3, and so on. This scale was used to rate the difficulty of all 1990 group and individual problems.

IV. INVESTIGATIONS

A. Are problem solutions worked out in cooperative groups better than the work of the best students in the groups?

It is difficult to design an experiment to determine if group problem solutions are better than the individual solutions of the best problem solvers from each group. Since students have memories and learn from their experiences, we could not give students the same test problem to solve individually that they had solved in groups the day before. Two alternative research designs are possible; either students are matched, or problems are matched. Creating subsets of matched students within an ongoing class is difficult at best, as is matching classes which may have different student populations. For example, one year the experimental section of the course consisted of 50% freshmen students, while the next year only about 25% of the students in the experimental section were freshmen. Consequently, we decided to match problems. Two criteria were used to select an individual problem from each test that matched the group problem: (1) the individual problem must be equal to or less difficult than the group problem, but (2) the individual problem could not be more than two ratings less difficult than the group problem. An individual problem was found that met these criteria for six of the seven class tests.

The best problem solver in each group was defined as the student receiving the highest total grade on the individual test and final exam problems each quarter. If group problem solutions reflect the work of the best problem solvers in the groups, then the problem-solving scores of the group problem solutions should be the same (statistically) as the scores of the "best-in-group" students on the matched individual problem. The null hypothesis, then, is that there is no statistical difference between the group (G) and best-in-group (B) problem-solving scores on matched problems, H_0: $G - B = 0$. The alternative hypothesis is that group problem-solving performance is significantly better than the performance of the best-in-group students on matched problems; $H_a: G - B > 0$.

The median scores of the group problem solutions and the matched best-in-group individual problem solutions for each test are shown in Table III. For all six tests, the median problem-solving score on the group problem is higher than the median score on the matched individual problem. The Wilcoxon Rank-Sum Test[24] for two matched samples were used to test for significant differences. Since the Z statistics shown in Table III are all larger than 1.65, the null hypothesis was rejected at the $p \leqslant 0.05$ level. Group problem solutions are significantly better than the best-in-group individual solutions on matched problems. We concluded that group problem solutions are not simply the work of the best problem solvers in the groups.

There are two potential sources of systematic error in this analysis. The first source of error occurs in the inherent inability to exactly match the problems. There are three design features of this study that counterbalance this error. First, criteria for judging the difficulty of context-rich problems were developed and tested. Using these criteria, all group problems were judged to be more difficult than the matched individual problems. This error works in favor of the individual problem solutions, not the group solutions. Second, all group problems were completed (and solutions posted) before the individual problems, so any learning effect favors the individual problem solutions. Finally, six different sets of problems in different content areas were matched over the two-quarters of the course. For any given content area, it is possible that the problems were not well-matched due to a conceptual difficulty evoked by the individual problem, but not the group problem. It is unlikely, however, that this would occur in every physics context.

The second potential source of error is the time factor. Students had 50 min to complete a group problem, and the same time to complete one qualitative question and two problems on an individual test. It could be argued that the group problem solutions were better than the matched best-in-group individual problem solutions simply because the students had more time to complete the group problem. On the other hand, the three students in each group had to discuss all steps, explain, argue, and finally agree to everything written down. Thus it could be argued that the "effective" time each student had to contribute to the group solution (50 min/3 students = 17 min per student) was probably about the same as the time the student had to solve the matched individual test problem (50 min/3 problems = 17 min per problem). Moreover, the group problems were more difficult than the matched individual problems, so they require more time to solve, reducing any error caused by the time factor.

Since students were given unlimited time to complete the final exams, the significance of the time factor could be tested. The pattern of complete and incomplete problems on the individual tests and finals was compared to check if time were a significant factor in lowering students' problem-solving performance on the individual problems. If students had insufficient time to complete the individual test problems, then there would be a pattern of complete final problems and incomplete test problems. (Of course, other factors could product this pattern, such as student learning or higher motivation to finish the final problems in

Table III. Group medians, best-in-group medians, and the Wilcoxon Z statistic for each class test.

	N	Group problem median	Best-in-group individual problem median	Wilcoxon Z statistic
Winter test 1	33	78	53	4.43[a]
Winter test 3	32	66	60	1.95[b]
Winter test 4	30	85	61	3.36[a]
Spring test 1	30	89	58	4.19[a]
Spring test 2	24	93	53	4.95[a]
Spring test 3	30	78	55	3.50[a]

[a] $p < 0.01$.
[b] $p < 0.05$.

Table IV. Group medians, best-in-group medians, and the Wilcoxon Z statistic of a sample of students for whom time is not a factor in problem completion.

	N	Group problem median	Best-in-group individual problem median	Wilcoxon Z statistic
Winter test 1	25	78	56	3.55[a]
Winter test 3	25	66	62	2.05[b]
Winter test 4	23	85	62	3.30[a]
Spring test 1	25	89	60	3.73[a]
Spring test 2	20	93	53	3.62[a]
Spring test 3	26	76	58	3.00[a]

[a] $p < 0.01$.
[b] $p < 0.05$.

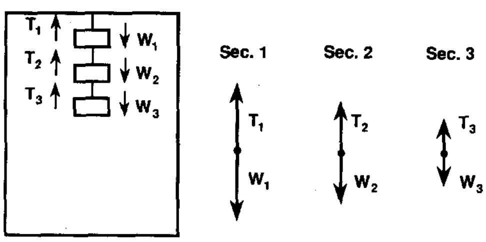
Table V. Difference between percent of groups and percent of best-in-group students receiving the highest problem-solving ratings on matched problems from each test.

	Correct conceptual understanding of physics	Useful physics description	Equations match physics description	Reasonable solution plan	Logical mathematical progression in solution	Appropriate mathematical procedures
Winter test 1	38	44	53	9	15	18
Winter test 3	28	41	44	−7	0	0
Winter test 4	−7	50	37	10	10	10
Spring test 1	73	−1	18	11	51	44
Spring test 2	74	49	63	36	47	44
Spring test 3	73	28	55	21	18	49
Average	46	35	45	13	24	22

order to improve their grade.) If a student completed 50% (or above) more final problems than test problems each quarter, then time was considered a potential factor in lowering a student's individual problem-solving performance. About 20% of the best-in-group students from each test met this criteria and were removed from the sample.

The remaining students completed approximately the same percentage of time-limited test problems as unlimited-time final problems each quarter. For these students, time presumably was not a significant factor influencing problem completion. The Wilcoxon Rank-Sum tests were recalculated for these students. The results are shown in Table IV. The Z statistics were all larger than 1.65, so the null hypothesis was again rejected. For a sample of students for whom time is not a factor influencing problem completion, group problem solutions are significantly better than best-in-group solutions on matched individual problems.

Physics Description:



T_1 = the tension between the ceiling and the 1st section
W_1 = the weight of all three sections
T_2 = the tension between the 1st and 2nd section
W_2 = the weight of the 1st and 2nd section
T_3 = the tension between the 2nd and 3rd section
W_3 = the weight of the last section

Plan: $T_1 + W_1 = ma$ $\quad W = mg = 7N$
$T_2 + W_2 = ma$ $\quad a = g/3$
$T_3 + W_3 = ma$

Want to know T_1, T_2, and T_3.

I. $T_1 + (m_1g) + (m_2g) + (m_3g) = ma$
II. $T_2 + (m_2g) + (m_3g) = ma$
III. $T_3 + (m_3g) = ma$
IV. $m = W/g$

Use I, II, and III to find T_1, T_2, and T_3; use IV to find m.

Execute: (only first step shown)

$$T_1 = \frac{ma}{(m_1g) + (m_2g) + (m_3g)}$$

$$= \frac{\frac{7\ N}{9.8\ m/s^2} \cdot \frac{9.8\ m/s^2}{3}}{21\ N}$$

$$= 0.11\ N$$

Fig. 2. The incorrect solution of the highest-ability student from the cooperative group on the matched elevator problem (Table I).

B. Which components of problem solving are performed better in groups?

To determine which components of problem solving were performed better in groups, we calculated the difference between the percent of group problem solutions and the percent of best-in-group individual problem solutions that received the *highest* rating on each of the six measured characteristics of problem solutions: evidence of conceptual understanding, usefulness of the physics description, match of the equations with the physics description, reasonable solution plan; logical mathematical progression of the solution, and appropriate mathematical procedures. The resulting differences are shown in Table V. As expected, there are variations in the differences that depend on the particular physics concepts and principles covered by each test. Averaged across the content of the two-quarter course, the biggest differences occurred for the three characteristics that measure the qualitative analysis of the problem: conceptual understanding (46%), usefulness of the physics description (35%), and the matching of the physics description with appropriate mathematical expressions of physics concepts and principles (45%). In groups, students generated more useful physics descriptions with fewer conceptual difficulties and a better match to the mathematical expressions of the physics principles than did the best problem solvers from each group on matched individual problems.

The problem solutions in Figs. 1 and 2 illustrate the differences described above in the qualitative analysis of groups and individuals. Figure 1 shows the solution generated by three students to a dynamics problem involving an airplane (see Table I). Their solution was discussed in detail in Sec. II B. Although the solution is not perfect, the group generated a correct and useful description of the problem that they translated into appropriate force and kinematics equations with no major conceptual errors. In contrast, Fig. 2 shows how the best individual from this group solved the matched individual problem involving an elevator (Table I). This problem requires students to calculate the tension in the wires of a three-section lamp hanging in an elevator in the case of an emergency stop. The student started with a correct and rather clever approach

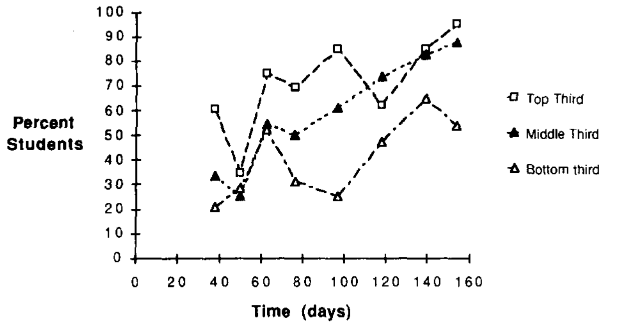


Fig. 3. Percentage of the top third, middle third, and lower third of the class whose solutions followed a logical mathematical progression. The dashed lines are included for ease of reading the graphs.

to the problem. Instead of considering each section of the lamp separately, he considered the sections hanging from each lamp wire as one combined object. However, he was unable to translate his physics description into the correct mathematical expressions of the forces acting on each combined object. In addition to an error in the direction of the forces, he confused the mass of the combined object (m) with the mass of each lamp section (m_1, m_2, and m_3). That is, he did not recognize that the total mass to be accelerated is the sum of the individual lamp sections. He also made an algebraic mistake that resulted in incorrect force units.

This solution is typical of many good students who try to skip steps or carry out steps in their heads. If the student had done a more careful qualitative analysis of the problem (e.g., drawn the free-body diagrams as well as the vector diagrams), he might have caught his error. Our studies of group interactions indicated that although the best problem solver in each group usually provided the leadership in generating approaches to the problem, the medium and lower ability students often provided the monitoring and checking to make sure that conceptual and procedural mistakes were not made.[2] In this way, the group process can give valuable assistance to the best students to help them integrate self-monitoring into their problem solving.

C. Did students' individual problem-solving performance improve over time?

If our problem-solving instructional approach was effective, then the individual performance of *all* students should improve over the two quarters of instruction. One possible outcome of working in cooperative groups, however, is that the problem-solving performance of the best students would either not improve, or not improve as much as the other students in the class, since the best students would spend their time dealing with the other students instead of improving their own skills. To test these possibilities, the class was divided into thirds on the basis of total individual grades on the tests and final exams. We then compared the performance pattern over time of the top third, middle third, and bottom third of the students on the least difficult of the context-rich individual problems. These problems, such as the moving problem shown in Table I, were defined as those with a difficulty rating (Sec. III B) between 0 and 2. Although they were the least difficult problems given in the experimental section, they were more difficult than any problems given in the traditional sections of the course.

The results indicated that the problem solving performance of the top third, middle third, and lower third of the class improved over the two quarters of instruction. For example, Fig. 3 shows the percentage of students whose problem solutions followed a logical mathematical progression. Although there are fluctuations for problems of different difficulties and content areas, the pattern of improvement is roughly the same for all students, including the best students. This improvement pattern was similar for the other five measured characteristics of problem solutions (Sec. III A), with the exception of conceptual understanding. There was no appreciable change over time in the percentage of students in any ability group whose problem solutions indicated a correct conceptual understanding of the physics principles involved in the problems.

We concluded that an instructional approach that combines the explicit teaching of a problem solving strategy with practice implementing the strategy in cooperative groups is effective in improving the problem solving skills of *all* students on the less difficult context-rich problems. The highest-ability students improved at approximately the same rate as the other students in the class, indicating that working in cooperative groups is not detrimental to these students. None of the ability groups showed any significant change over time in their performance on the most difficult individual problems, which were matched to the group problems. (See, for example, the problem solving medians of the best students in Table III.) Although these context-rich problems were solved reasonably well by cooperative groups of students, they appear to be too complex for beginning students to solve individually in a test situation.

D. Are students in the experimental section better individual problem solvers than students in a traditional section?

If our problem-solving instructional approach was effective, then students in our experimental section of the introductory course should become better individual problem solvers than students in a traditional section of the course. Instructors teaching the traditional sections of the course were asked to include some context-rich problems on their final exams. However, the context-rich problems, even the least difficult ones, were judged by those instructors to be too difficult for their students. Consequently, two standard exercises from the first-quarter final of a traditional section were included on the first-quarter final of the experimental section of the course. These exercises, which are shown on Table VI, consist of a series of questions that lead students through the problem. For example, in the first inclined-plane exercise, the normal force on the block (Question a) is needed to find the force of friction (Question b), which is needed to find the acceleration of the block (Question c), which is needed to find how far the block slides in 1 min (Question d). Therefore, unlike context-rich problems, these exercises require minimal planning to solve the problem.

Questionnaire results indicated that the students in the traditional and experimental sections had similar backgrounds and characteristics. The students' solutions to the exercises were scored using the rating scales for the six characteristics of problem solutions described in Sec. III A. The Wilcoxon Rank-Sum Test was used to compare the medians of the total problem-solving scores of the students in the two sections. The results are shown in Table

Table VI. Standard exercises used in comparison of experimental and traditional students.

1. A 60-kg block slides down a plane inclined at an angle of 35° to the horizontal. The coefficient of kinetic friction between the block and the plane is 0.25.
   (a) What is the normal force of the plane on the block?
   (b) What is the force of friction of the plane on the block?
   (c) What is the acceleration of the block?
   (d) If the block starts from rest, how far along the inclined plane does it travel in 1 min?

2. A 450-g ball on a 65-cm string is swung in a VERTICAL circle at a constant rate of 0.85 revolutions per second.
   (a) What is the centripetal acceleration of the ball?
   (b) What is the tension in the string when the ball is at the *highest* point in the circle?
   (c) What is the tension in the string when the ball is at the *lowest* point in the circle?

VII. The students in the experimental section scored significantly higher than the students in the traditional section on both exercises. An examination of the scores on the six characteristics of problem solutions indicated that the biggest difference between the two groups was in the qualitative analysis of the problem. For example, all students in the experimental section who solved the problem drew useful force diagrams, compared to only 57% in the traditional section. In addition, the solutions of students in the experimental section exhibited more logical mathematical progressions than those of the students in the traditional section. We tentatively concluded that students who are taught an explicit problem solving strategy and practice implementing the strategy in cooperative groups solve standard physics exercises better than students who receive traditional instruction. "Better" in this context means that their solutions exhibit more expertlike characteristics.

V. SUMMARY AND IMPLICATIONS

The primary purpose of this study was to test one aspect of our problem-solving instructional approach, cooperative-group problem solving. In well-functioning cooperative groups, students can share conceptual and procedural knowledge and argument roles, and request clarification, justification, and elaboration from one another, so a better solution emerges than could be achieved by individuals working alone. The results of this study suggest that this type of collaboration did occur. Group problem solutions were significantly better than those produced by the best problem solvers from each group on matched individual problems, particularly with respect to the qualitative analysis of the problems. In addition, the individual problem solving performance of students improved over time at approximately the same rate for students of high, medium, and low ability. Comparisons with students taking a traditional section of the same course indicated that students in the experimental section exhibited more expertlike problem solving. We concluded that teaching an explicit problem solving strategy and having students practice using the strategy in cooperative groups is an effective instructional approach.

Table VII. Experimental and traditional medians and Wilcoxon Z statistic for two standard physics exercises.

	Experimental section median ($N=91$)	Traditional section median ($N=118$)	Wilcoxon Z statistic
Problem #1	82	62	8.17[a]
Problem #2	71	50	4.20[a]

[a] $p<0.0001$.

These results suggest that cooperative-group problem solving is a viable alternative to traditional recitation sections. Instead of the instructors answering students' questions or modeling problem solving, the instructor monitors the group work and gives feedback and help only as needed. Many student difficulties can be addressed quickly and efficiently by peers. Moreover, instructor teaching skills need be less well developed to assist groups, since the group process provides a mechanism by which students can clarify the instructor's vague statements or correct simple instructor errors. This is of practical importance when the recitation instructors are inexperienced graduate teaching assistants.

Although the use of cooperative groups in standard recitation sections has no implications for course coverage, the implementation of our entire instructional model required a reduction in the number of topics taught in the course. In 20 weeks the course covered about one-half of the chapters from an introductory text, instead of the traditional two-thirds of the chapters. This reduction in topics allowed the lecturer to teach and model the five-step problem solving strategy, as well as address the other goals of the course (emphasizing the fundamental principles, teaching qualitative reasoning skills, and challenging students' misconceptions). Implementing the entire model with graduate assistants also required more time on the part of the lecturer to organize and coordinate the different components of the course, and to educate the TA's about the goals and structure of the course, common student misconceptions, the five-step problem solving strategy, how to grade problems, and how to form and maintain well-functioning groups.[6]

Finally, one unresolved issue is whether the instructional emphasis on a problem-solving strategy detracts from students' development of conceptual understanding. We have reason to believe that teaching the five-step strategy and having groups practice using the strategy to solve context-rich problems actually enhances students conceptual understanding of the course material. When we observed and videotaped groups, we noticed that peers were often effective "teachers" when they discussed a problem with one another. In the process of justifying statements, clarifying ideas, and elaborating on explanations, students appear to be deepening their understanding of physics concepts and principles. At this time, however, we have only preliminary data indicating that our problem-solving instructional strategy enhance students' conceptual understanding of physics.

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THE BASIC GOAL OF PHYSICS

The basic goal of physics is not mathematical elegance or even the achievement of tenure, but learning the truth about the world around us.

Philip W. Anderson, Phys. Today 43 (2), 9 (1990).