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A Note on Cost Estimation and the Optimal Bidding Strategy

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This note determines the optimal bid price for a firm bidding against several other firms for a contract, when each firm knows the expected value of its own cost and a probability distribution of the other firms' cost. The optimal bid price is calculated as a function of the firm's own expected cost, first for the case of two identical firms and then for several firms.

BIDDING THEORY, as described in numerous publications, is firmly based on observed frequency distributions of past bids. But what if no previous experience exists because the product is new?

This note addresses itself to this question. It is based on the hypothesis that enough is known of the technology of producing the product to have an estimate or 'prior' notion of the probability distribution of *production costs* among firms. This is likely to be true when the technology of the new product is similar to that of other known products.

This paper is thus relevant in all situations where little or no information exists about bid frequencies. It is shown how a rational approach to the bid problem can then be developed on the basis of known or estimated probability distributions for production cost.

Consider a firm bidding against several other firms for a contract. Each firm knows its own cost (or at any rate the expected value of its own cost) k, but estimates the costs c of the other firms. We assume that this estimate has the form of a probability distribution $G(c)$ for the cost c of the lowest-cost competitor.

For conceptual simplicity, consider first the case of two bidders. Then $G(c)$ is firm one's estimate of the probability distribution of firm two's cost. Let the two firms be equal in all respects but cost and assume that they use the same optimal strategy. This strategy specifies the bid price p as a function of cost c:

$$p=p(c). \tag{1}$$

In fact, assuming that this is the opponent's strategy, the probability that our own bid q will be successful is

$$1-G[\phi(q)], \tag{2}$$

where ϕ is the inverse function of p in (1). Let the object of firm one be to maximize expected profit, and let its own cost be k. Then consider

$$\max_q\ (q-k)\{1-G[\phi(q)]\}.$$

Setting the derivative with respect to q equal to zero, we have

$$1-G[\phi(q)]=(q-k)G'[\phi(q)]\cdot\phi'(q)=0, \qquad (3)$$

whence

$$1/\phi'(q)=(q-k)G'[\phi(q)]/\{1-G[\phi(q)]\}.$$

If we use the same strategy as the opponent, then by definition of ϕ, $\phi(q)=k$, and

$$dq/dk=(q-k)G'(k)/[1-G(k)]. \qquad (4)$$

To solve this differential equation in $q=q(k)$, let $q-k=y$, $k=x$:

$$dy/dx+1=yG'(x)/[1-G(x)],$$
$$y'-yG'(x)/[1-G(x)]=-1,$$
$$\{y(x)\cdot[1-G(x)]\}'=G(x)-1.$$

Integrating from x to the maximum cost level $x=M$, we obtain

$$y(M)[1-G(M)]-y(x)[1-G(x)]=\int_x^1[G(x)-1]\,dt.$$

If there is no finite M, we assume that the probability distribution converges sufficiently rapidly to $G=1$ that $\lim_{M\to\infty} y(M)[1-G(M)]=0$. Then

$$y(x)=\{1/[1-G(x)]\}\int_x^\infty [1-G(t)]\,dt.$$

In terms of p and c,

$$p(c)=c+\{1/[1-G(c)]\}\int_c^\infty [1-G(x)]\,dx. \qquad (5)$$

Integrating by parts, we have, as an alternative expression,

$$p(c)=\{1/[1-G(c)]\}\int_c^\infty x\,dG(x). \qquad (6)$$

Here

$$M(c)=\{1/[1-G(c)]\}\int_c^\infty [1-G(x)]\,dx$$

is the profit margin that the bidder is seeking.

In particular, if $G(c)$ is the exponential distribution $G(c)=1-e^{-\lambda c}$, then $p(c)=c+1/\lambda$. The profit margin is constant and equal to the expected value of the opponent's cost. In the case of the normal distribution

$$p(c)=n[(c-\mu)/\sigma]/\{1-N[(c-\mu)/\sigma],$$

where n is the standardized normal density and N is the standardized normal distribution. If the assumed probability distribution is normal, then only the mean and variance of the competitor's cost distribution need to be known. The case of Weibull distributions was analyzed by ROTHKOPF.[4]

The symmetry assumption of this analysis that our estimate of the opponent's cost distribution is the same as the opponent's estimate of our cost distribution is essential for the analysis, as developed so far. Suppose, however, that our estimate is $G_2(c)$ and that our estimate of the opponent's estimate of our cost distribution is $G_1(c)$. Then the inverse functions $c_1=\phi_1(p)$ and $c_2=\phi_2(p)$ need to be distinguished. Equation (3) becomes

$$1-G_2[\phi_2(q)]=[q-\phi_1(q)]G_2'[\phi_2(q)]\phi_2'(q),$$

from which

$$\phi_2'(p)=\{[1-G_2(\phi_2)]/G_2'(\phi_2)\}\cdot\{1/[p-\phi_1(p)]\},$$
$$\phi_1'(p)=\{[1-G_1(\phi_1)]/G_1'(\phi_1)\}\cdot\{1/[p-\phi_2(p)]\}.$$

A general solution of this pair of differential equations is difficult to obtain. Consider, however, the exponential distributions

$$G_i(x)=1-e^{-\lambda_i x},$$
$$\phi_2'(p)=(1/\lambda_2)\{1/[p-\phi_1(p)]\},$$
$$\phi_1'(p)=(1/\lambda_1)\{1/[p-\phi_2(p)]\}.$$

A solution is

$$p=\phi_1(p)+1/\lambda_2=c_1+1/\lambda_2, \qquad p=\phi_2(p)+1/\lambda_1=c_2+1/\lambda_1.$$

Thus player one's profit margin should be $1/\lambda_2$, the expected value of the opponent's cost, and player two's profit margin should be the expected value of player one's cost.

Consider now several bidders, but all with the same cost structure. By this we mean that everybody estimates every other firm's cost to have the same probability distribution $G(c)$. The probability of any particular firm's bid p to be the winning low bid is then $\{1-G[\phi(p)]\}^{n-1}$, where n is the total number of bidders. Expected profit is $(p-c)\{1-G[\phi(p)]\}^{n-1}$, and this is maximized for

$$(p-c)\cdot(n-1)\{1-G[\phi(p)]\}^{n-2}G'(\phi)\phi'(p)=\{1-G[\phi(p)]\}^{n-1},$$

from which

$$dp/dc=(n-1)(p-c)G'(c)/[1-G(c)].$$

The solution of this differential equation is

$$p(c)=c+\{1/[1-G(c)]^{n-1}\}\int_c^{\infty}[1-G(x)]^{n-1}\,dx. \qquad (7)$$

Thus the function $1-G(c)$ in the two-bidder case is effectively replaced by $[1-G(c)]^{n-1}$, the probability of being low against $n-1$ random bids.

If costs are exponentially distributed, $1-G(c)=e^{-\lambda c}$, then $p=c+1/(n-1)\lambda$. The profit margin is reduced by a factor $1/(n-1)$.

In conclusion, two points should be made. First, since $p(c)$ is an increasing function of c, this bidding process assures that the lowest-cost firm gets the bid. Thus it is economically efficient. Of course, the bidder receives a profit. This approaches zero only as the number of bidders increases. This profit is a measure of market imperfection.

Second, if repeated bidding occurs for the same type of job having the same cost, then one successful bid, or one known bid, reveals the true cost of the bidder. From then on the game would be changed, since all participants would know each other's costs. The proper interpretation of this bidding game is that costs vary from job to job and that $G(c)$ describes the frequency distribution of costs over the jobs that are bid for in this industry.

Of course, with repeated bidding the probability distribution of bids rather than of costs may be observed directly and the strategy of FRIEDMAN[1] may be applied. What this paper shows is that a stable distribution of costs will generate a stable distribution of bids, and this supports the Friedman solution in the case of repeated bidding. A more sophisticated analysis may be attempted along Bayesian lines, but this goes beyond the purpose of this note.

REFERENCES

1. L. FRIEDMAN, "A Competitive Bidding Strategy," *Opns. Res.* **4,** 104–112 (1956).
2. R. M. STARK AND R. H. MEYER, JR., "Some Multi-Contract Decision-Theoretical Competitive Bidding Models," *Opns. Res.* **19,** 469–483 (1971).
3. ———, "Competitive Bidding: A Comprehensive Bibliography," *Opns. Res.* **19,** 484–490 (1971).
4. M. H. ROTHKOPF, "A Model of Rational Competitive Bidding," *Management Sci.* **15,** 362–373 (1969).

Price and Production Decisions with Random Demand

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This note studies the problem of setting price and production levels simultaneously in a series of *N* periods, where price is a parameter in the probability distribution of demand. It develops a simple policy characterized by two stock levels and an optimal price line in the price-inventory plane, gives an algorithm to compute such a policy and check conditions for its optimality, and presents a counterexample to the simple policy, using a small set of allowable prices, along with a discussion of both the counterexample and some characteristics of the model.

THE STRUCTURE of optimal inventory policies has been a subject of considerable interest in the management-science literature; *see*, e.g., references 1, 2, 6, and 8. For the most part, selling price has not been included in these models, with references 4, 5, 7, 9, and 10 being exceptions that have studied deterministic demand. It is the purpose of this paper to include price as a decision variable in a situation with random demands. Price-inventory adjustments are observed in real situations, e.g., in January sales to clear inventory.

The problem can be characterized as follows: Let

$X=\{x\colon x$ is an allowable stock level$\}$,

$P=\{p\colon p$ is an allowable price$\}$,

$\phi_n(d|p)=$the cumulative probability function of demand, given a particular price p in period n; $d\phi_n(d|p)$ is the density function, if it exists, and we

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