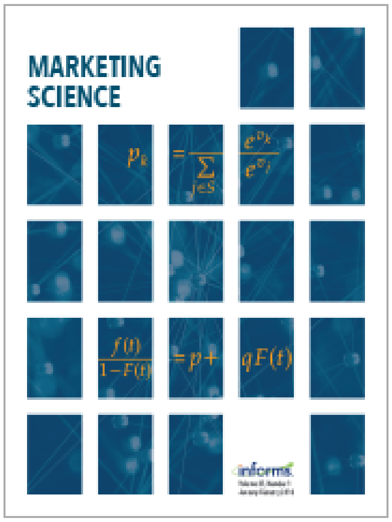

Marketing Science

Publication details, including instructions for authors and subscription information:
http://pubsonline.informs.org

Technical Note—A Note on Optimal Strategic Pricing of Technological Innovations

Frank M. Bass, Alain V. Bultez,

TECHNICAL Note

A NOTE ON OPTIMAL STRATEGIC PRICING OF TECHNOLOGICAL INNOVATIONS*

FRANK M. BASS† AND ALAIN V. BULTEZ‡

The experience curve phenomenon of falling marginal costs associated with accumulated output or production experience has given rise to dynamic pricing models. Optimal pricing policies will depend upon the nature of the dynamic demand and cost functions. In this note we shall show that the demand function employed by Bass (1980) when taken in conjunction with the experience curve cost function leads to a multiperiod pricing strategy which is always less than the myopically optimal price. Further, we present a dynamic programming algorithm for the multiperiod strategy in which we have explored the effects on discounted profits of myopic pricing versus multiperiod pricing. The results of this comparison may, to some, be somewhat surprising and may have managerial significance.

(Pricing-Technological Innovations; Diffusion, Experience Curve)

*Received November 21, 1980. This paper has been with the authors for 3 revisions.

This material is based on research supported by the National Science Foundation under grant PRA 77-15850. Any opinions, findings, and conclusions or recommendations expressed here are ours and do not necessarily reflect the views of the National Science Foundation.

† School of Management, University of Texas at Dallas, Richardson, Texas 75080.

‡ Faculté Universitaire Catholique de Mons (FUCAM, Belgium) and European Institute for Advanced Studies in Management, Brussels, Belgium.

MARKETING SCIENCE
Vol. 1, No. 4, Fall 1982
Printed in U.S.A.

0732-2399/82/0104/0371$01.25

The Bass (1969) new product growth model of the diffusion of innovations has been modified by a number of authors to create a demand function for technological innovations which includes decision variables. Among others there are papers by Glaister (1974), Robinson and Lakhani (1975), Spremann (1975–1978), Horsky and Simon (1981), Dodson and Muller (1978) as well as more recently, by Dolan and Jeuland (1981) and Teng and Thompson (1980a, 1980b). In the Robinson and Lakhani formulation (also used by Dolan and Jeuland) the optimal multiperiod price is one which rises at first and later declines. Moreover, Robinson and Lakhani show a substantial difference in the discounted profit which stems from multiperiod optimization as opposed to myopic optimization. In contrast, however, the demand model employed by Bass (1980) leads to a myopically optimal price which declines monotonically. To complete this work we show in this note that the multiperiod optimal price for the demand function employed by Bass is always less than the myopically optimal price and is therefore also monotonically declining. Moreover, in contrast to the results of Robinson and Lakhani, our studies indicate that there are only small differences in discounted profits for this demand function when price is chosen myopically as opposed to a multiperiod strategy.

1. The Components of the Pricing Model

1.1. *The Experience Curve Cost Function*

The marginal cost function, called the experience curve, is

$$MC[E(t)] = C_1[E(t)]^{-\alpha} \tag{1}$$

where

$MC[E(t)]$ is the cost of producing the Eth unit of output,

$E(t)$ is the accumulative output at time t,

C_1 is a scaling parameter, sometimes referred to as the cost of producing the first unit, and

α is a learning rate parameter, $\alpha > 0$.

The current marginal cost depends not only on current output, but also on earlier output, or experience. In some applications, such as that by Robinson and Lakhani, equation (1) is used in an average cost sense, while Bass used it —more appropriately we believe—in a marginal cost sense. However, the qualitative results of the analysis will not be influenced by this distinction.

1.2. *The Diffusion Model*

The diffusion model developed by Bass (1969) has as its basic premise that the probability of an adoption of a new product at time t given that an adoption has not yet taken place is an increasing linear function of the number of previous adopters. Thus:

$$P(t) = f(t)/(1 - F(t)) = a + bF(t), \tag{2}$$

where:

$P(t)$ is the conditional probability of an adoption at time t given that an adoption has not yet taken place,

$f(t)$ is the density function of time to first purchase (i.e., the fraction of the total population adopting at time t),

$F(t)$ is the proportion of the market penetrated at time t,

a is the coefficient of innovation (i.e., the influence on adoption regardless of the number of previous adopters),

b is the coefficient of imitation (i.e., the impact of previous adopters on the probability of adoption at time t).

If m is the total number of adopters over the life of the product then the sales rate during that period in which sales consist entirely on first purchase demand will be:

$$S(t) = mf(t) = m(a + bF(t))(1 - F(t))$$

$$= am + (b - a)E(t) - b/m(E(T))^2, \tag{3}$$

where $E(t)$ is accumulated sales (output) through period T.

The differential equation shown in (3) has as its solution:

$$E(t) = mF(t) = m\left[1 - e^{-(a+b)t}\right] / \left[(b/a)e^{-(a+b)t} + 1\right]. \tag{4}$$

Therefore:

$$S(t) = mf(t) = m\left[(a + b)^2/a\right]\left[e^{-(a+b)t}/\left((b/a)e^{-(a+b)t} + 1\right)^2\right]. \tag{5}$$

2. Dynamic Demand Models

Robinson and Lakhani and Dolan and Jeuland (for durable goods) have used the demand model:

$$q(p(1), p(2), \ldots, p(t)) = \frac{dE(t)}{dt} = e^{-kp(t)}m(a + bF(t))(1 - F(t)). \tag{6}$$

In this demand model a lower price today will stimulate sales today and will also influence demand in the future because future demand will depend, through the diffusion process, on accumulated sales. This model, then, implies that price will *interact* with the diffusion process.

Bass (1980) proposed the following demand model for technological innovations:

$$\frac{dE(t)}{dt} = q(p(t)) = h(t)\left[p(t)\right]^{-\eta} \tag{7}$$

where:
$h(t)$ is a function of time, and
$p(t)$ is price at time t.
In equation (7) the elasticity of demand, η, is constant, but the demand function itself is being shifted in time by the function h. In the demand function indicated by (7) the diffusion process is exogenous and does not interact with price. Dynamic pricing models continue to evolve and it is not our purpose here to debate the merits or limitations of (6) as compared with (7). Rather, we intend to complete the study of the implications of (7). We will comment, however, that one of the nice things about (7) is that, as shown by Bass (1980), it leads in the myopically optimal case to closed form solutions as functions of time for E, q, and p. The time derivative of price may be expressed as a function of these solutions. It is:

$$-\alpha q(t)p(t)/E(t), \tag{8}$$

indicating the rate of decline in price. Therefore:
the myopically optimal price will decline monotonically.
In the following section we derive the multiperiod optimal price for the demand function indicated in (7).

3. The Multiperiod Optimal Price

Firms facing cost and demand functions behaving according to (1) and (7) would maximize their total discounted profits over the finite horizon of $T + 1$ periods by choosing the sequence of prices: $(P_0, P_1, \ldots, P_T)$ that will maximize discounted profits. Thus the objective is to maximize:

$$\Pi = \sum_{t=0}^{T} \rho^t\left[p_t q_t - C_t\right] \tag{9}$$

with $\rho = 1/(1 + r)$, r being the rate of discount associated with the product's project. The demand function will depend only on the current price and time, but the cost function will depend upon past as well as current prices. Thus to obtain the necessary condition for a maximum of (9) the partial derivative of

Π with respect to p_t is equated to zero:

$$\begin{aligned}\partial\Pi/\partial p_t &= \rho^t[\,p_t\partial q_t/\partial p_t + q_t\,]\\ &= C_1\rho^t\big[\,E_t^{-\alpha} + \rho(E_{t+1}^{-\alpha} - E_t^{-\alpha}) + \rho^2(E_{t+2}^{-\alpha} - E_{t+1}^{-\alpha})\\ &\qquad + \cdots + \rho^{T-t}(E_T^{-\alpha} - E_{T-1}^{-\alpha})\big]\partial q_t/\partial p_t = 0. \qquad (10)\end{aligned}$$

Substituting $-\eta q_t/p_t$ for $\partial q_t/\partial p_t$, dividing by $\rho^t q_t$, and rearranging the terms in the brackets in equation (10) we have:

$$\begin{aligned}&-\eta + 1 + \eta C_1\big[(1-\rho)E_t^{-\alpha} + \rho(1-\rho)E_{t+1}^{-\alpha} + \rho^2(1-\rho)E_{t+2}^{-\alpha} + \cdots\\ &\qquad + \rho^{T-t-1}(1-\rho)E_{T-1}^{-\alpha} + \rho^{T-t}E_T^{-\alpha}\big]/p_t = 0. \qquad (11)\end{aligned}$$

Therefore, the globally optimal price will be:

$$\begin{aligned}p_t^* &= [\eta/(\eta-1)]C_1\big[(1-\rho)E_t^{-\alpha} + \rho(1-\rho)E_{t+1}^{-\alpha} + \cdots\\ &\qquad + \rho^{T-t-1}(1-\rho)E_{T-1}^{-\alpha} + \rho^{T-t}E_T^{-\alpha}\big]. \qquad (12)\end{aligned}$$

Noting that $(1-\rho)\sum_{t=0}^{T-t-1}\rho^t + \rho^{T-t} = 1$, we conclude that the optimal price is proportional to a weighted average of (a convex combination of) current and future marginal costs.

From equation (12) discounted marginal cost is:

$$\mu_t = (1-\rho)\sum_{\tau=t}^{T-1}\rho^{(\tau-t)}MC_\tau + \rho^{(T-t)}MC_T, \qquad (13)$$

for $t \leqslant T-1$. Subtracting $\rho\mu_t$ from the corresponding expression for μ_{t-1}, a recurrence equation can then be obtained for discounted marginal cost, i.e.,

$$\mu_{t-1} = \rho\mu_t + (1-\rho)MC_{t-1}. \qquad (14)$$

From equation (14), it is clear that μ_t declines over time since MC_t declines over time. Now, note that equation (12) can be written as:

$$p_t^* = \frac{\eta}{\eta-1}C_1\mu_t.$$

Since μ_t declines over time and η is a constant, it follows that p_t^* declines over time as well. Hence, we conclude:

that the globally optimal price will always be less than the myopically optimal price ($[\eta/(\eta-1)]C_1E_t^{-\alpha}$), *and it is monotonically decreasing.*

Note that the result does *not* depend on the particular functional form used in this paper to model cost decline. This result is true for *any* marginal cost decline specification.

This result can be explained intuitively in the following manner: the globally optimal price will be less than the myopically optimal price because an additional price reduction from the myopically optimal price will be justified by future cost benefits which accrue with a multiperiod planning horizon. These future benefits assure that the prices and costs in the multiperiod case will be less than those in the myopic case.

4. An Evaluation of Optimal and Myopic Pricing Policies

The discretized version of equation (7) with $f(t)$ in (5) used for $h(t)$ is as follows:

$$q_t = \Theta_t S p_t^{-\eta} \quad \text{and} \quad \Theta_0 = F(1), \qquad \Theta_t = F(t+1) - F(t), \tag{15}$$

where $\Theta_t S$ is the discretized version of $f(t)$ and where $F(t)$ is the solution to the diffusion differential equation (3), i.e.,

$$F(t) = (1-g(t))/(1+(b/a)g(t)) \quad \text{and} \quad g(t) = \exp(-(a+b)t).$$

Substituting (13) and (14) into (12), optimal prices can be determined without any major difficulty, by solving the following system recursively,

$$p_t = \mu_T\eta/(\eta-1), \tag{16}$$

$$p_t = \left[(1-\rho)MC_t + \rho\mu_{t+1}\right]\eta/(\eta-1), \qquad \text{for} \quad t = 0, 1, \ldots, T-1.$$

An algorithm yielding the solution to system (16) is available upon request from the authors. We have employed the algorithm to explore, and compare for different parameter values of the demand and cost functions, the effects on discounted profits of multiperiod versus myopic pricing strategies.

Table 1 shows a comparison of price ranges and discounted profits for the two types of policies for different sets of parameters. The most notable feature of Table 1 is the fact that there are relatively small differences in discounted profits between the two policies. The parameter values shown in Table 1 are, we believe, typical and reasonably plausible. Although other parameter values have been used in simulations, we cannot guarantee insensitivity of discounted profit in every case. For example, if we had used a real discount rate of, say, 10% (i.e., $r = 0.10$ instead of $r = 0.30$, as was used for the results reported in

TABLE 1

Comparative Evaluation of Optimal and Myopic Pricing Rules

Parameters		Optimal Price-Sequence		MyopicBehavior	
	$\eta = 1.5$	$p_0 = 48.4422$	3,464.387	$p_0 = 52.2802$	3,462.091
		$p_{20} = 44.9575$		$p_{20} = 45.0356$	
$\alpha = 0.05$	$\eta = 2.0$	$p_0 = 34.3611$	522.586	$p_0 = 37.1888$	521.611
		$p_{20} = 31.8648$		$p_{20} = 31.9421$	
	$\eta = 2.5$	$p_0 = 30.8421$	83.262	$p_0 = 33.4803$	82.953
		$p_{20} = 28.5783$		$p_{20} = 28.6691$	
	$\eta = 1.5$	$p_0 = 37.5997$	3,899.627	$p_0 = 44.4601$	3,887.813
		$p_{20} = 32.1374$		$p_{20} = 32.3922$	
$\alpha = 0.10$	$\eta = 2.0$	$p_0 = 28.3160$	623.177	$p_0 = 33.9511$	617.510
		$p_{20} = 24.1157$		$p_{20} = 24.3973$	
	$\eta = 2.5$	$p_0 = 27.6441$	95.457	$p_0 = 33.6773$	93.619
		$p_{20} = 23.4535$		$p_{20} = 23.8340$	
	$\eta = 1.5$	$p_0 = 27.7273$	4,494.824	$p_0 = 36.6425$	4,459.426
		$p_{20} = 21.6276$		$p_{20} = 22.0737$	
$\alpha = 0.15$	$\eta = 2.0$	$p_0 = 21.9698$	785.532	$p_0 = 30.1992$	765.583
		$p_{20} = 16.9746$		$p_{20} = 17.5266$	
	$\eta = 2.5$	$p_0 = 23.5021$	117.366	$p_0 = 33.9551$	110.564
		$p_{20} = 17.9704$		$p_{20} = 18.8504$	
	$\eta = 1.5$	$p_0 = 19.1008$	5,347.297	$p_0 = 28.9735$	5,259.699
		$p_{20} = 13.4459$		$p_{20} = 14.0261$	
$\alpha = 0.20$	$\eta = 2.0$	$p_0 = 15.5494$	1,078.194	$p_0 = 25.8335$	1,016.420
		$p_{20} = 10.7283$		$p_{20} = 11.5261$	
	$\eta = 2.5$	$p_0 = 18.1489$	164.245	$p_0 = 34.3761$	140.879
		$p_{20} = 12.2356$		$p_{20} = 13.7780$	

Legend: Parameters: $S = 100{,}000$. $a = 0.05$; $b = 0.40$.
$C_1 = 20$.
$T = 20$; $r = 0.30$.

Table 1), we may well have found the optimal pricing policy to result in greater discounted profits related to the myopic policy. Nevertheless, at least over the range of parameter values we have simulated, we have found only small differences in the discounted profits when comparisons are made between multiperiod and myopic pricing strategies. These results are in sharp contrast to the results of Robinson and Lakhani who found that when demand equation (6) was employed there are large differences in discounted profits between multiperiod and myopic pricing strategies. Clearly, the nature of the demand function may have a bearing on profit differences between multiperiod and myopic strategies.

5. Summary

In this note we have shown that the multiperiod optimal price, when the dynamic demand function indicated in euqation (7) is taken in conjunction with the experience curve cost function, is always less than the myopically optimal price. Both types of prices decline monotonically. In addition, our simulation studies indicate that discounted profits under myopic pricing are only slightly less than those under multiperiod pricing for the particular demand function we have examined, a result that may have managerial significance.

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