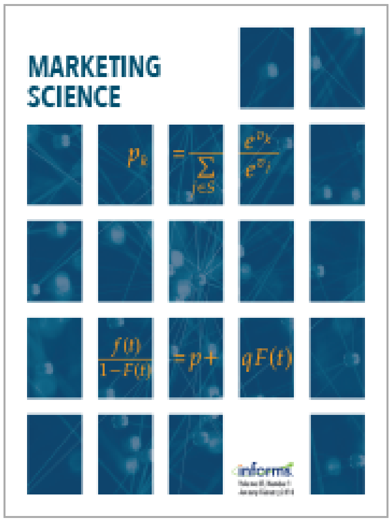

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Technical Note—Price as an Aspect of Choice in EBA

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TECHNICAL Note

PRICE AS AN ASPECT OF CHOICE IN EBA

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Elimination By Aspects (EBA) is a feature-based, psychological processing model of choice whose potential for customer decision modeling has not been exploited. One of several barriers to econometric application of the theory is the lack of an explicit framework for incorporating quantitative variables, such as price. The present study discusses a theoretical treatment of price within EBA which also serves as a guide to the treatment of other quantitative variables. Specifically, it is proposed that prices be represented as a sequence of nested price feature sets, in which the price feature set of an alternative is included in the price feature sets of all lower priced alternatives. The formal consequences of this representation are examined. Some predictions from the theory are tested on customer choice data.

(Choice Models; Elimination by Aspects; Price as an Attribute)

1. Introduction

Elimination By Aspects (EBA) is a feature-based, context-sensitive theory of choice (Tversky 1972a, b; Tversky and Sattath, 1979). McFadden (1981, p. 226) has noted that the EBA functional form has considerable potential for econometric applications. Despite EBA's salutary influence on choice theory, no marketing application of EBA has appeared since its introduction more than a decade ago. One of several obstacles to application of EBA is the problem of how quantitative features of the alternatives, such as price, should enter the model. The small literature on EBA only contains examples involving nominal or unspecified features. The subject of the present study is a theoretical treatment of one quantitative aspect of choice in EBA, the prices of the alternatives. The treatment is intended to serve as a guide to the incorporation of other quantitative characteristics, and to increase the attractiveness of EBA for marketing applications.

In principle, quantitative aspects present no special difficulties for feature theories such as EBA. In his paper on features of similarity Tversky (1977) notes that, though the term *feature* (or aspect) usually denotes the value of a binary or nominal variable, feature representations are also applicable to ordinal or cardinal variables. He gives the example

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of a series of tones that differ only in loudness being represented as a sequence of nested sets where the feature set associated with each tone is included in the feature sets associated with louder tones. Such a representation is isomorphic to a directional unidimensional structure. Tversky's example conveys the general approach to the incorporation of price that will be taken here. Restle's (1959) investigation of set-theoretical representations of qualitative and quantitative dimensions provides some of the theoretical foundations of this approach.

2. EBA Without Explicit Price Features

The theory of EBA begins with the supposition that each alternative in a choice set consists of a set of aspects or features. A choice is made by means of a covert sequential elimination process which is controlled by the decision maker's preference ordering over the aspects. In the individual version of EBA, a person's ordering of the aspects is stochastic. At each stage of the process an aspect is selected from those included in the currently available alternatives with a probability proportional to the scale value of the aspect. Those currently available alternatives that do not contain the selected aspect are then eliminated. The process is repeated on the reduced choice set until only a single alternative remains. The aggregate version of the model is similar, except that each person's ordering of the aspects is assumed to be fixed and the stochastic component of the model is associated with (taste) differences across individuals in aspect ordering (Tversky and Sattath 1979). Both the individual and aggregate versions of the model have the same mathematical form but with slightly different interpretations attached to the parameters. EBA has been shown to be consistent with random utility maximization (see Tversky 1972a).

To formalize the model let T be the total set of alternatives and for each alternative $x \in T$ let x' be the set of aspects associated with x. For any set of alternatives $A \subseteq T$ let A' be the set of aspects belonging to at least one alternative in A and let A^0 be the set of aspects common to all the alternatives in A. For any aspect $\alpha \in T'$ let A_α be those alternatives in A which include α. Finally, let $P(x, A)$ denote the probability of choosing alternative x out of choice set A. The EBA model asserts that there exists a positive scale u on aspects $\alpha \in A' - A^0$ such that for all $x \in A$

$$P(x,A) = \sum_{\alpha \in x' - A^0} \frac{u(\alpha)}{\sum_{\beta \in A' - A^0} u(\beta)} P(x, A_\alpha). \tag{1}$$

The outer summation ranges over all aspects of x except those that are common to all the alternatives in A. The inner summation ranges over all aspects of the alternatives in A except those common to all the alternatives in A. Hence, those aspects common to all the alternatives in A have no effect on choice probabilities. Equation (1) gives the probability of choosing x from A as a weighted sum of probabilities of choosing x from various subsets of A, where the weights, $u(\alpha)/\Sigma\ u(\beta)$, are the probabilities (proportions of individuals in aggregate EBA) of selecting the respective aspects of x.

3. Price Features

The price of an alternative will be treated as an additional collection of features belonging to that alternative. The elimination process which characterizes EBA will act on price features in exactly the same way it acts on any other feature. Inherent in this view is the assumption that price features are separable attributes. Consequently, no price/quality inferences are permitted. Moreover, due to the nature of EBA, price is not a compensatory variable. Rather, prices have a threshold-like character. The effect of any price-based elimination step is to split the current choice set into a set of alternatives whose prices are (temporarily) acceptably low and a complementary set of alternatives whose prices are unacceptably high. All members of the high price set are eliminated and the process continues on the remaining alternatives.

To operationalize this view of price in EBA it is necessary to define price features and a measure of price advantage so that, other things being equal, higher priced alternatives have a higher probability of being eliminated than lower priced alternatives. Following Tversky's approach, this is done by representing prices as nested feature sets, where the set associated with a given price strictly includes the sets associated with all higher prices. Such a nested set representation of price automatically confers on each alternative a price feature advantage over all higher priced alternatives.

The proposed framework requires a feature set for each price. Two questions concerning price feature sets immediately arise. What are the elements of such sets? How are these elements to be interpreted? For a given price p the associated price feature set is defined as the half-open interval of real numbers $[p, \infty)$. Given an increasing sequence of prices the nested set property follows immediately. A price feature set is reasonably interpreted as a collection of price limits. A decision maker (temporarily) adopting price limit t rejects those alternatives in the choice set whose price exceeds t. For a given alternative priced at p, the price feature set $[p, \infty)$ is the set of all price limits that would *not* permit rejection of that alternative.

Consider the consequences of this representation in the case of two alternatives, x and y. Let p_x and p_y denote the prices of x and y, respectively and suppose that $p_x < p_y$. A graphical representation of the structure of the choice set is shown in Figure 1. As noted earlier, features common to all alternatives have no effect on choice probabilities. Since $p_x < p_y$, the price feature set common to x and y is $[p_x, \infty) \cap [p_y, \infty) = [p_y, \infty)$, which is y's price feature set. Hence, all of y's price features are common features. Only x has noncommon price features, namely $[p_x, \infty) - [p_y, \infty) = [p_x, p_y)$. Consequently, there are two general routes leading to the choice of x; one through the selection of a nonprice feature unique to x, and the other through the selection of a price limit eliminating y but not x. The one and only route leading to the choice of y is through the selection of a nonprice feature unique to y.

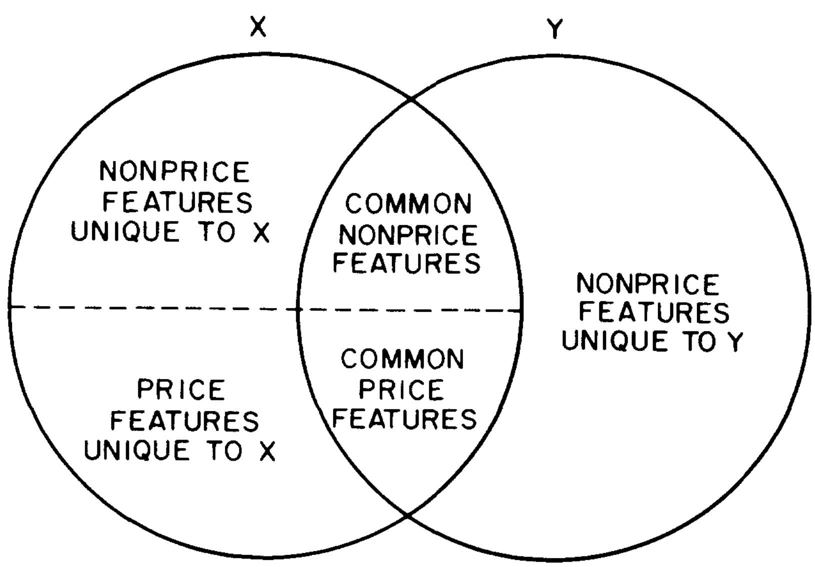

FIGURE 1. Structure of a Two-Alternative Choice Set Where $p_x < p_y$.

4. Measures of Price Advantage

In EBA each noncommon feature, α, is associated with a positive scale value, $u(\alpha)$, which is a measure of the utility of that feature. If we are to incorporate price into the model the same must be true for price features. We explore how to do this in the next two sections.

Consider an n-element choice set in which the prices of the alternatives, arranged in ascending order, are $p_1, p_2, \ldots, p_n$. Since price features common to all alternatives have no effect on choice probabilities, the price feature set of the most expensive alternative, $[p_n, \infty)$, may be excluded from consideration. Scale values need only be assigned to noncommon price features, that is, those belonging to the set $[p_1, p_n)$.

Since the set of noncommon price features constitutes an interval of real numbers, any assignment of positive scale values to these features defines a positive valued function on that interval. It is convenient to assume that such a function is smooth. Accordingly, let v be a positive valued continuous function on $[p_1, p_n)$. The function v will be referred to as a *price scale function.*

The main use of the price scale function is in defining a measure of price advantage. For any two alternatives x and y, if $p_x < p_y$, then the set of price features belonging to x and not to y is $[p_x, p_y)$. Let us define a measure of x's price advantage over y as the "sum" of the price scale values over the set $[p_x, p_y)$, that is, $\int_{p_x}^{p_y} v(\alpha)d\alpha$. One possible formulation for $v(\alpha)$ is a constant price scale function, i.e., $v(\alpha) = c > 0$. This choice of $v(\alpha)$ implies that the measure of x's price advantage over y is $c(p_y - p_x)$, a quantity proportional to the price difference. Another example is $v(\alpha) = c/\alpha$, which yields the price advantage measure $c \log(p_y/p_x)$. Various choices for the price scale function, $v(\alpha)$, will be discussed in a later section.

5. EBA with Price

EBA choice probabilities can be expressed as a function of the scale values of the nonprice features and derived scale values for price advantage.

The following notation will be used:

A an n-element choice set;

$A_{(i)}$ the set containing the i least expensive alternatives in A, $1 \le i \le n$;

$u(i) \equiv \int_{p_i}^{p_{i+1}} v(\alpha)d\alpha$ the price advantage of the ith least expensive alternative in A over the $i+1$th least expensive alternative in A;

x'_{np} the set of nonprice features of alternative x;

A'_{np} the set of all nonprice features belonging to at least one alternative in A;

A^0_{np} the set of nonprice features common to all alternatives in A.

Then, if alternative x is the jth least expensive alternative in A,

$$p(x, A) = \frac{a + \sum_{i=j}^{n-1} u(i)p(x, A_{(i)})}{b + \sum_{i=1}^{n-1} u(i)} \quad \text{where} \tag{2}$$

$$a = \sum_{\alpha \in x'_{np} - A^0_{np}} u(\alpha)p(x, A_\alpha), \quad \text{and} \quad b = \sum_{\alpha \in A'_{np} - A^0_{np}} u(\alpha).$$

The formal derivation of Equation (2) is shown in the Appendix. Since Equation (2) is a special case of Equation (1), introducing price in this manner is consistent with EBA.

In the individual version of EBA the derived scale value $u(i)$ is a measure proportional to the probability of selecting a price feature (limit) which eliminates those alternatives with prices greater than p_i, the price of the ith least expensive alternative in A. The probability of selecting such a price feature from the collection of all available price and nonprice features is $u(i)/(b + \sum_{j=1}^{n-1} u(j))$. Given that some price feature is selected the conditional probability of selecting a feature that eliminates those alternatives with prices greater than p_i is $u(i)/\Sigma_{j=1}^{n-1} u(j)$.

The interpretation of $u(i)$ in the aggregate version of EBA is somewhat different. In that case, $u(i)$ is interpreted as a measure proportional to the percentage of the population who first eliminate alternatives priced higher than p_i when presented with choice set A.

As a partial check on the reasonableness of Equation (2) the following propositions were verified.

PROPOSITION 1. *If all alternatives are identical in price, choice probabilities depend only on the scale values of nonprice features not common to all alternatives.*

PROOF: The proposition follows immediately from the fact that EBA choice probabilities do not depend on features common to all alternatives.

PROPOSITION 2. *If all alternatives are identical except for price the (unique) least expensive alternative is chosen with probability one.*

PROOF: In this case the least expensive alternative strictly includes the features of all other alternatives. A general property of EBA is that any alternative that strictly includes the features of all other alternatives is chosen with probability one.

PROPOSITION 3. *The probability of choosing alternative x is nonincreasing in its price, and is strictly decreasing if x does not strictly include the nonprice features of all other alternatives and no other alternative strictly includes the nonprice features of x.*

PROOF: Available on request.

The above three propositions show that the proposed representation of price meets at least some minimal standards of reasonableness.

In the 2-alternative case Equation (2) reduces to

$$P(x > y) = \begin{cases} \dfrac{a + \int_{p_x}^{p_y} v(\alpha)d\alpha}{b + \int_{p_x}^{p_y} v(\alpha)d\alpha} & \text{if} \quad p_x \le p_y, \\ \dfrac{a}{b + \int_{p_y}^{p_x} v(\alpha)d\alpha} & \text{if} \quad p_x \ge p_y, \end{cases} \tag{3}$$

where a is the sum of scale values of nonprice feature belonging to x but not y, and b is the sum of scale values of all nonprice features not common to x and y.

The next proposition shows how price scale functions compatible with observed binary choice proportions can be identified. When EBA with price holds, there is a simple relationship between the odds of choosing x over y and the measure of x's price advantage over y.

PROPOSITION 4. *If $p_x \le p_y$ and $(b - a) \ne 0$ then $P(x > y)/P(y > x)$ increases linearly in the measure $\int_{p_x}^{p_y} v(\alpha)d\alpha$.*

PROOF: For $p_x \le p_y$ and $(b - a) \ne 0$,

$$\frac{P(x > y)}{P(y > x)} = \frac{a}{(b-a)} + \frac{1}{(b-a)} \int_{p_x}^{p_y} v(\alpha)d\alpha.$$

Proposition 4 is useful for graphically testing assumptions about the form of $v(\alpha)$, the price scale function. If the form of $v(\alpha)$ is correct then a plot of x's odds should be linear in its price advantage. Since Proposition 4 only involves binary choice probabilities the proposition is also true of any theory which is equivalent to EBA in the binary case, e.g. Restle's (1961) model.

6. Price, Separability, and Choice

The proposed treatment of price is based on two assumptions: that price can be treated as an attribute and that it is separable. Price separability means that the values (effects) of the nonprice attributes do not depend on the prices of the alternatives. One kind of violation of separability occurs when price is used as a surrogate measure of quality (see Rao 1984 and Gardner 1977 for review). In that case, the perception of quality, at least partially, depends on prices. If the price/quality interaction is sufficiently strong, the probability of choosing an alternative may increase with price in some price regions. Although research on this issue is far from conclusive, it is generally believed that separability is less severely violated when decision makers are well informed about quality or the cost of becoming so is low. Some recent results suggest that for many product sets the relationship between price and quality is very weak (Gertsner 1985, Geistfeld 1982).

It should be noted that nonseparable forms of EBA are also possible. In this respect, EBA is like other probabilistic choice models, including logit and probit, which allow both separable and nonseparable forms. In comparing choice models, EBA with probit, for example, assumptions concerning price separability should be commensurable. A (non) separable form of one model is best compared with (non) separable forms of another.

A second assumption concerns treating price as an attribute versus some other way, such as a constraint, or through "per dollar" perceptual maps (e.g., Rao and Gautschi 1982, Srinivasan 1982, Hauser and Shugan 1983, Hauser and Gaskin 1984). How price is treated depends to a large extent on how the basic choice process is conceptualized. In fact, the evaluation rules or processes employed in various choice models have characteristic differences that go beyond price separability and related price issues. The larger issue is whether consumers actually use (linear) compensatory rules or lexicographic elimination rules, such as EBA, or conjunctive rules (e.g., Shocker and Srinivasan 1979, Hansen 1976, Wright 1975). The present study does not attempt to further resolve these issues. Rather, it offers a possible treatment of price in the context of a lexicographic decision process.

7. Empirical Comparisons of Price Scale Functions

In the previous sections we showed that price can be incorporated into an EBA formulation. In this section we perform some weak tests of the ability of various scale functions to account for the effect of price variation on aggregate choice data, and compare the parameter-free implications of EBA, with price as separable attribute, to the implications of separable price versions of logit and probit. We do this using a 1980 AT&T survey data set on demand for various types of telephone station sets. Most of the survey was devoted to a comparison of three particular station sets at various prices. Each station set was tested at seven different price levels. Survey subjects were presented with triples of alternatives and asked to rank order the alternatives from most to least preferred. The triples used in the survey were selected so that at least four levels of price *differences* would be available on each pair of station sets. Because of the large number of station set/price combinations, each subjects was only presented with a small subset of all possible triples.

Since Proposition 4 concerns a two alternative choice situation, pairwise choice probabilities for alternative pairs (x, y) were derived from the triple ranking data. This was done by computing the proportion of occasions on which x proceeded y in the rank ordering. Using a theorem due to Block and Marschak (1960), it can be shown that this estimate is consistent and unbiased for pairwise choice probabilities under any random utility model. The number of subjects contributing data to each pairwise estimate differed across pairs, varying from 149 to 301.

Proposition 4 implies that if the choice process is EBA and price is separable then the choice probability ratio $P(x > y)/P(y > x)$ is linear in the measure of x's price advantage over y. The linear relationship implied by the model is tested on the pairwise preference data for three telephone station sets under various hypotheses about the form of the price scale function, $v(\alpha)$. If $v(\alpha)$ is a constant then the measure of x's price advantage is proportional to the price difference, $p_y - p_x$. For $v(\alpha) = c/\alpha$ the measure of advantage is proportional to the log of the price ratio, $\log(p_x/p_y)$. In addition to the above two scale functions, five other scale functions shown in Table 1, are included for comparative purposes.

For each pair of station sets and each hypothesized scale function the correlation between $P(x > y)/P(y > x)$ and $\int_{p_x}^{p_y} v(\alpha)d\alpha$ was computed. For station sets 1 and 2 each correlation was based on 15 estimated binary choice probabilities in which the price of the first set was less than the second. These 15 situations yielded 5 unique price differences. Similarly, for station sets 2 and 3 the correlations were based on 16 estimated probabilities (4 unique price differences) and for sets 1 and 3 on 17 estimates (5 unique price differences). The correlational analysis is parameter-free in the sense that the measure of linearity, the correlation coefficient, can be estimated without knowing or estimating the values of the nonprice features (see Proposition 4).

The results of the correlational analysis are shown in Table 1. The hypothesized scale functions are all of the form α^k, where $k = 1, \frac{1}{2}, 0, -\frac{1}{2}, -1, -\frac{3}{2}, -2$. For each product pair the correlations increase as k approaches 0 from above or below. Thus the constant scale function, whose performance is indexed by the correlations in the third row of the table, is consistently best among the seven competing price scale functions. The results suggest that price difference is an appropriate measure of price advantage for these data. Scatter plots of $P(x > y)/P(y > x)$ against $p_y - p_x$ for each product pair are shown in Figure 2.

A second test of the EBA formulation is to compare it to others choice models. The correlated probit model is comparable to EBA in flexibility and complexity. The implications of a price separable probit model can be tested in the same manner as EBA's. Suppose that the random variable representing the utility of alternative x can be expressed as

$$U_x = f(a_x) - g(p_x) + e_x, \tag{4}$$

where f is a function of the nonprice attributes of alternative x, g is a function of the price of x and e_x is a normal random variable with expectation zero. Suppose, further, that the vector $e = [e_x]$ has a multivariate normal distribution with an arbitrary nonsingular

TABLE 1

Correlations of Choice Probability Ratios with Measures of Price Advantage

Scale Function	Price Advantage	Product Pair		
$v(\alpha)$	$\int_{p_x}^{p_y} v(\alpha)d\alpha$	(1, 2)	(1, 3)	(2, 3)
α	$p_y^2 - p_x^2$	0.77	0.75	0.73
$\alpha^{1/2}$	$p_y^{3/2} - p_x^{3/2}$	0.85	0.81	0.83
1	$p_y - p_x$	0.92	0.86	0.88
$\alpha^{-1/2}$	$p_y^{1/2} - p_x^{1/2}$	0.91	0.85	0.79
α^{-1}	$\log p_y - \log p_x$	0.80	0.78	0.61
$\alpha^{-3/2}$	$p_x^{-1/2} - p_y^{-1/2}$	0.63	0.64	0.43
α^{-2}	$p_x^{-1} - p_y^{-1}$	0.47	0.48	0.30

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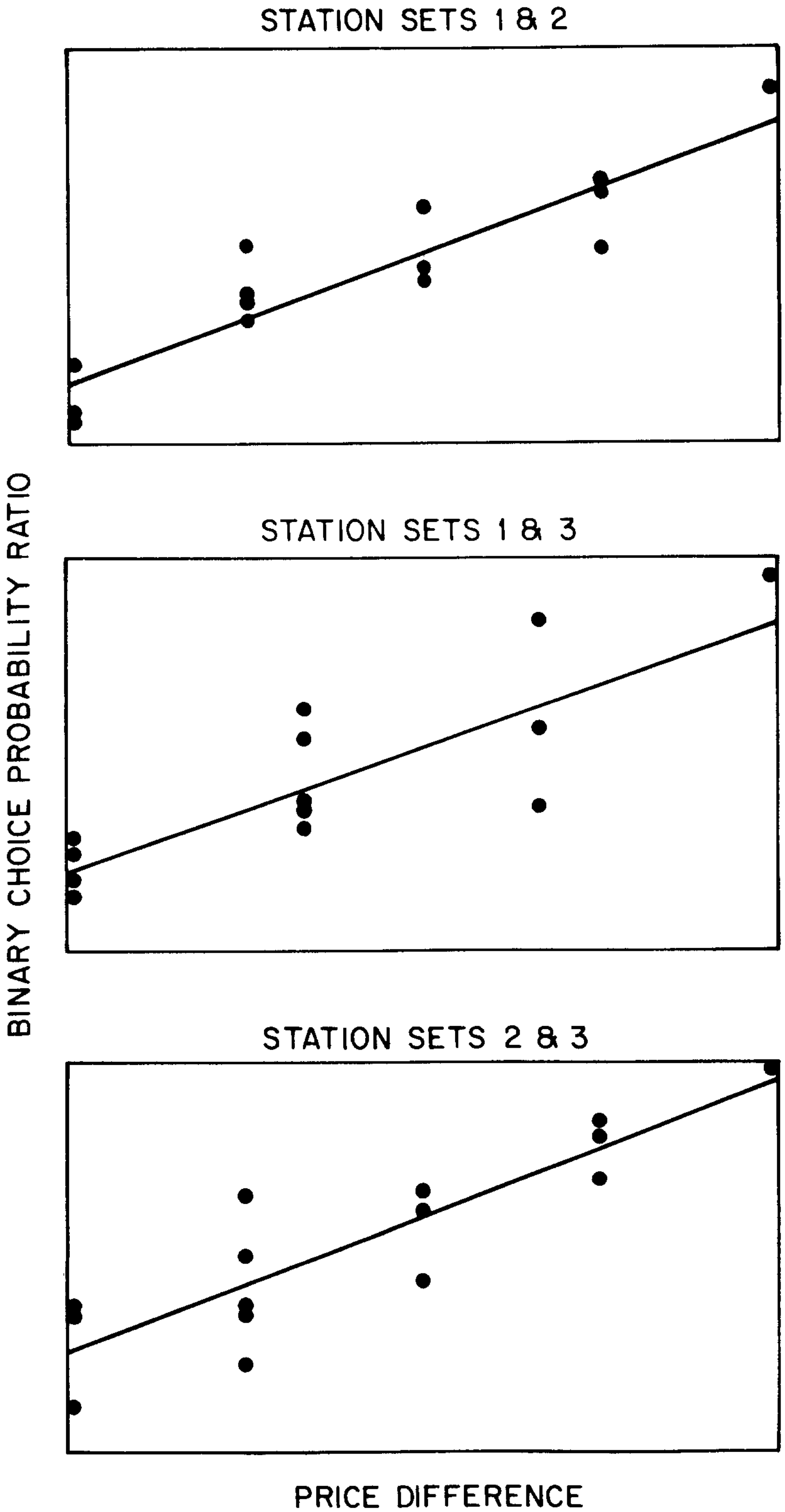

FIGURE 2. Scatterplots of Choice Probability Ratio against Price Difference for Three Pairs of Station Sets.

TABLE 2

Correlations of Inverse Normal Choice Probability with Measures of Price Advantage

Scale Function	Price Advantage	Product Pair		
$g(p)$	$g(p_y) - g(p_x)$	(1, 2)	(1, 3)	(2, 3)
p^2	$p_y^2 - p_x^2$	0.74	0.77	0.70
$p^{3/2}$	$p_y^{3/2} - p_x^{3/2}$	0.82	0.83	0.79
p	$p_y - p_x$	0.87	0.86	0.83
$p^{1/2}$	$p_y^{1/2} - p_x^{1/2}$	0.86	0.85	0.76
$\log p$	$\log p_y - \log p_x$	0.76	0.78	0.59
$-p^{-1/2}$	$p_x^{-1/2} - p_y^{-1/2}$	0.60	0.65	0.43
$-p^{-1}$	$p_x^{-1} - p_y^{-1}$	0.45	0.50	0.31

covariance matrix which does not depend on prices. Then the probability that x is preferred to y is of the form

$$\Phi[(f(a_x) - f(a_y) + g(p_y) - g(p_x))/\sigma_{xy}],$$

where Φ is the standard normal *cdf* and σ_{xy} is the variance of $U_x - U_y$. In turn, this implies that $\Phi^{-1}[p(x > y)]$ linear in the measure of price advantage, $g(p_y) - g(p_x)$. For example, choosing $g(p) = p$ yields $(p_y - p_x)$ as the measure of price advantage, which is identical to the EBA measure of price advantages for $v(\alpha) = 1$.

For each pair of station sets the correlations between $\Phi^{-1}[p(x > y)]$ and $g(p_y) - g(p_x)$ were computed for seven choices of g, as shown in Table 2. Each choice of g yields a measure of price advantage identical to one of the EBA measures of price advantage. The correlational analysis again shows that the straight price difference is best among the seven competing measures of price advantage. Moreover, the entire pattern of results in Table 2 is nearly identical to that in Table 1.

A similar analysis was conducted for the logit model. A price separable version of this model results from assuming that utilities have the form of Equation (4) and the e_x's have independent and identical Type I extreme value distributions. From these assumptions it follows that log $(P(x > y)/P(y > x))$ is linear in the measure price advantage, $g(p_y) - g(p_x)$. The correlations of log odds with the seven measures of price advantage are identical, to two decimal places, to those in Table 2. Overall, the price implications of EBA, probit and logit are nearly identical.

8. Discussion

An interesting but minor finding was that the price difference measure was reasonably consistent with the available data and somewhat more consistent than the small number of other competitors tested. However, no single measure of price advantage is likely to perform satisfactorily across all product sets, prices and customer populations. The more significant finding was that the relationships between price advantage and binary choice probabilities implied by price separable versions of logit, probit and EBA were equally well supported by the data. This result suggests that EBA may be a reasonable alternative to logit or probit when price separability is assumed.

When IIA is satisfied, there is no compelling reason to prefer EBA with price to the more familiar, price separable logit. The only advantage of EBA in such a case is the dominance property, that is, any alternative whose nonprice features are all included in some other lower priced alternative is chosen with probability zero. When differential substitutability among the alternatives causes a violation of order independence, EBA may be preferable to correlated probit or tree structured generalizations of logit. One advantage of EBA over probit is closed form expressions for all choice probabilities. In a moderately restricted EBA model, such as EBT (Elimination by Tree, Tversky and Sattath 1979), these expressions are much more tractable than the corresponding multiple integrals for probit choice probabilities. In particular, the EBT model with price may be a reasonable alternative to other hierarchical models, such as the TEV model (Tree Extreme Value, McFadden 1981). For some choice sets, however, the nonprice structure of the set may not be amenable to a (rooted) tree representation. EBA has an advantage in such cases, since it is not limited to hierarchial structures.

The correlational analysis presented earlier was based entirely on aggregate choice data. Hence, the results only have bearing on the aggregate, price separable versions of EBA, probit and logit. Further research is needed on EBA with price as a model of individual choice behavior, especially if EBA is to be taken seriously as a process model.

Price separability remains an issue for EBA and for discrete choice models in general. The appeal of the assumption stems from the simplicity it confers. It is unlikely that separability is ever exactly satisfied. However, the choice data examined in this study, which may not be atypical of inexpensive durables, suggest that price separability is a reasonable, first order approximation.

9. Conclusion

We examined the problem of introducing price as a separable aspect of choice in EBA, a feature based choice theory. Despite the quantitative nature of price, it can be incorporated as naturally into the theory as nominal features can. Except for separability, the formal consequences of the theory are relatively weak and in accord with intuition. The results suggest that EBA with price is a reasonable alternative to other separable choice models. Although the treatment of price may serve as a guide for incorporating other quantitative variables into EBA, not all such variables can be treated identically. Further research is needed on variables such as color or temperature, whose ideal values exist at nonextreme points of the variable's domain.

The purpose of this research has been to remove an impediment to marketing applications of EBA by providing a partial framework for the treatment of quantitative variables. However, the fact that EBA has remained unexploited for so long must be attributed to more than one factor. Other inhibitors include the lack of packaged software for estimation and testing, an apparent lack of recognition of the tractability of restricted EBA forms such as EBT, the absence of published marketing applications of EBA, and the existence of well entrenched alternative models. These disincentives to further exploration and application of EBA are not inherent to the theory. Rather, they reflect the fact that current levels of model awareness and convenience of use are low.[1]

[1] This paper was received November 1983 and has been with the author for 3 revisions.

Appendix. Formal Derivation of EBA with Price (Equation 2)

Consider x', the set of aspects associated with alternative x. Assuming price separability, we may partition x' into a subset x'_{nr} of nonprice aspects and a subset x'_p of price aspects. Similarly, let A^0_{np} be the set of nonprice aspects common to all the alternatives in A and let A^0_p be the set of price aspects common to the alternatives in A. Finally, let A'_{np} be the set of nonprice aspects belonging to at least one alternative in A, and let A'_{np} be the set of price aspects belonging to at least one alternative in A. Then, assuming x'_{np} and x'_p are countable, Equation (1) may be rewritten as

$$P(x,A)=\frac{a+\Sigma_{\alpha\in x'_p-A^0_p}u(\alpha)P(x,A_\alpha)}{b+\Sigma_{\beta\in A'_p-A^0_p}u(\beta)} \quad \text{where} \tag{A1}$$

$$a=\sum_{\alpha\in x'_{np}-A^0_{np}} u(\alpha)P(x,A_\alpha) \quad \text{and} \quad b=\sum_{\beta\in A'_{np}-A^0_{np}} u(\beta).$$

Equation (A1) separates sums involving price aspects from sums involving nonprice aspects. In subsequent developments it will be desirable to allow $x'_p - A_p^0$ and $A'_p - A_p^0$ to be uncountable sets of price aspects corresponding to intervals of the positive reals, in which case Eq. (A1) is replaced by

$$P(x,A) = \frac{a + \int_{\alpha \in x'_p - A_p^0} v(\alpha) P(x, A_\alpha) d\alpha}{b + \int_{\beta \in A'_p - A_p^0} v(\beta) d\beta} \tag{A2}$$

where $v(\alpha) > 0$ is a continuous function on the positive reals. The fact that replacing Equation (A1) with Equation (A2) is consistent with EBA will be demonstrated.

It is proposed that the price aspects associated with a series of alternatives that increase in price be represented as a sequence of nested sets in which the price aspect set of each alternative is included in the price aspect sets of lower priced alternatives. To formalize this notion let p_x denote the price of alternative x and let $Q = \{[t, \infty) | t > 0\}$ be a collection of price aspect sets indexed by the positive reals. Then say alternative x has price aspect t if and only if $p_x \le t$. Hence, the set of price aspects belonging to alternative x is $x'_p = [p_x, \infty) \in Q$. The nested set property follows immediately: if $p_x \le p_y$ then $x'_p \supseteq y'_p$ with $x'_p = y'_p$ if and only if $p_x = p_y$.

Let p_A be the set of prices of the alternatives in choice set A. Then the set of price aspects common to all alternatives in A is

$$A_p^0 = \bigcap_{\alpha \in pA} [\alpha, \infty) = [\max p_A, \infty).$$

Similarly the set of price aspects belonging to at least one alternative in A is

$$A'_p = \bigcup_{\alpha \in pA} [\alpha, \infty) = [\min p_A, \infty).$$

Substituting for x'_p, A_p^0 and A'_p in Equation (A2) yields

$$P(x,A) = \frac{a + \int_{p_x}^{\max p_A} v(\alpha) P(x, A_\alpha) d\alpha}{b + \int_{\min p_A}^{\max p_A} v(\beta) d\beta}. \tag{A3}$$

Let $p_A^{(1)} = \min p_A, p_A^{(2)}, \ldots, p_A^{(n)} = \max p_A$ be the ordering of prices in A, where p_x is jth in the ordering. Then Equation (A3) may be rewritten as

$$P(x,A) = \frac{a + \sum_{i=j}^{n-1} \int_{p_A^{(i)}}^{p_A^{(j+1)}} v(\alpha) P(x, A_\alpha) d\alpha}{b + \sum_{i=1}^{n-1} \int_{p_A^{(i)}}^{p_A^{(j+1)}} v(\beta) d\beta}. \tag{A4}$$

When there is no risk of confusion we will denote $p_A^{(i)}$ simply as $p^{(i)}$. Since in each interval $[p^{(i)}, p^{(i+1)})$ the choice probability $P(x, A_\alpha) = P(x, A_p^{(i)})$ is constant, it may be factored out of each integral expression, respectively, yielding

$$P(x,A) = \frac{a + \sum_{i=j}^{n-1} \left[\int_{p^{(i)}}^{p(i-1)} v(\alpha) d\alpha \right] P(x, A_p^{(i)})}{b + \sum_{i=1}^{n-1} \int_{p^{(i)}}^{p^{(i+1)}} v(\beta) d\beta}. \tag{A5}$$

Letting $A(i) = A_p^{(i)}$ and $u(i) = \int_{p^{(i)}}^{p^{(i+1)}} v(\alpha) d\alpha$, Equation (A5) becomes

$$P(x,A) = \frac{a + \sum_{i=j}^{n-1} u(i) P(x, A(i))}{b + \sum_{i=1}^{n-1} u(i)} \tag{A6}$$

where $u(i)$ is a derived scale value associated with price aspects in the interval $[p^{(i)}, p^{(i+1)})$. However, Equation (A6) is a special case of Equation (1) (EBA). Hence, introducing price in this manner is consistent with EBA.

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