# Technological Change and the Stock Market*

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**Abstract**

Tobin's average $q$ has usually been well above 1, but fell below 1 during 1974-1984. Our model explains this pattern and reconciles it with unchanging aggregate investment. The stock market value in the numerator of $q$ reflects ownership of physical capital and knowledge, but the denominator measures just physical capital. Therefore, $q$ is usually above 1. Periodic arrivals of important new technologies, such as the microprocessor in the 70's, suddenly render old knowledge and capital obsolete, causing the stock market to drop. National accounts measures of physical capital miss this rapid obsolescence. Then $q$ appears to drop below 1.

JEL E44, O3, O41

## 1 Introduction

If one compares the aggregate market value of U.S. businesses with the replacement cost of their capital stock, a surprising outcome is that over the last 45 years the market value has more often than not been larger than the second measurement, with the ratio climbing to 1.4 or more.[1] As Figure 1 shows, the ratio, which we call $q^*$, rose from about 1 to 1.4 from the early 1950s to the early 1970s; then it fell abruptly and remained below 1 from the mid 1970s to mid 1980s; subsequently it rose, passing 1.4 in 1995.[2] This paper attempts to provide a simple, yet comprehensive model which we can use to explain and interpret the average and time series features of the data. Conversely, the framework which we propose allows us to integrate financial market values into the study of economic growth.

Three basic elements underpin our analysis. (i) We assume that the frontier technology evolves over time in an exogenous and uneven fashion. (ii) We assume that each unit of capital, tangible or intangible, embodies a particular technology. And, (iii) we assume that firms produce with physical capital, labor, and (applied) knowledge. We treat the last as intangible capital. We show that the first two elements can cause data in the national accounts to mismeasure depreciation severely at certain times.

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[1]We compute the market value of the U.S. businesses from the Federal Reserve's Flow of Funds. For the replacement cost, we use the current-price value of the physical capital stock computed by the Bureau of Economic Analysis from National Income and Products Account physical investment and perpetual inventory equations. See Appendix 1.

[2]Although data sources vary somewhat, note that our Figure 1 resembles, for example, Laitner (2000, fig.1) and Hall (2001, fig.13). See also McGrattan and Prescott (2000, p.21) and Smithers and Wright [2000, chart 2.1].

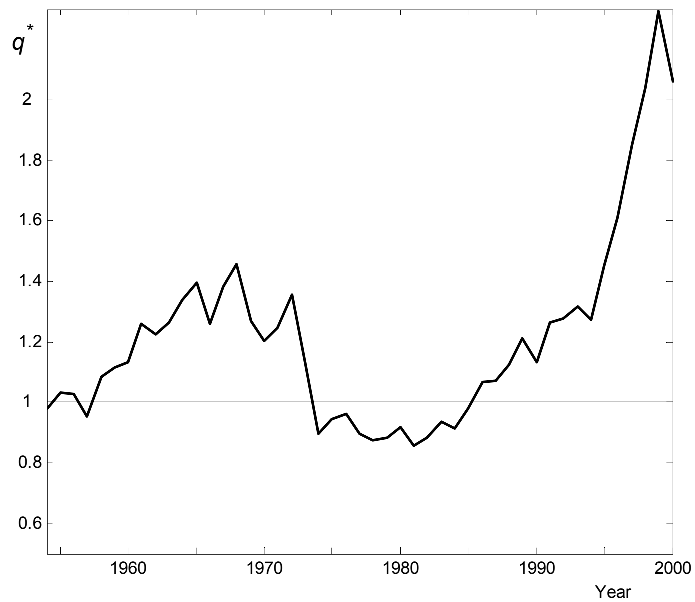


Figure 1: Tobin's average $q$, 1953-2000.

The idea that technological progress may be discontinuous comes from the literature of economic history. Mokyr (1990a, 1990b) argues that seminal inventions, occurring at intervals of several decades or more, are the ultimate source of growth. In the same spirit, Wicksell (1953, p.67) wrote, "It is in the nature of things that new, great discoveries and inventions must occur sporadically, and that the resulting increase in output cannot take the form of an evenly growing stream ...."[3] According to Cohen et. al. (2000, p. 30), for example, electricity, internal combustion engines and chemicals were the leading technologies in the early 1900s — whereas microelectronics and information and communication technologies may have been principal after 1970. In our framework, transforming inventions emerge at randomly spaced times, and our empirical section assumes that historians and other commentators can identify for us the key arrival dates.

The embodiment assumption is familiar from Solow [1960]. Recent studies find a significant role for embodied change in growth. Greenwood et al [1997] estimate that embodied technical progress in equipment alone contributed 60% to overall TFP change in the U.S. from 1954 to 1990; Gort et al [1999] argue that embodied progress is very important for structures too.

The embodiment framework fits particularly well with inventions that transform the whole economy, because in the model each new technology inaugurates a new aggregate production function. For example, David [1990] describes how dedicated electric motors replaced central steam engines as power sources for manufacturing. Beyond the steam engine itself, the old system of power delivery used overhead shafts and drive belts, which, in turn, required multi-story factories with linear production layouts. When electrification made possible delivery of power through wires, many of the existing components and structures became obsolete.

The recent microelectronics/information technology revolution seems to have attributes similar to seminal inventions of the past. Progress in microelectronics has brought spectacular advances in

[3]See Evans (1969, p.334–335) for a brief discussion of similar concepts in Schumpeter and Wicksell.

computers and telecommunications equipment. However, that progress, in turn, has led to broader, structural changes in other industries. Mowrey and Rosenberg (1998) write,

> Like electricity, the postwar electronics revolution has derived much of its economic impact from a complex and lengthy process of interindustry diffusion and adoption. The products of these high–technology industries have transformed the structure of mature industries (e.g., retailing) as well as newer ones (e.g., commercial aircraft design). (p.164)

Hall [2001, p.1198] makes similar observations. Brynjolfsson and Hitt [2000] describe how decentralization of information processing has led away from hierarchical forms of organization, changing business practices in health care, automotives, and build–to–order retailing. Machine tools offer another illustration. While microchips may be a small component of their cost, numerically controlled machine tools became much more practical and their use greatly expanded after microcomputer controls replaced paper tapes in the 1970s (e.g., Ray [1984, chart 7.3–7.4]). Although computerized numerically controlled tools are expensive and require programming, they work faster than corresponding manual equipment and eliminate old–fashioned set–up time, offering substantial gains in flexibility.[4]

In our framework, a firm's intangible capital includes applied knowledge such as product designs and firm–specific human capital. Our distinction between seminal inventions and applied knowledge is as follows. Seminal inventions emerge exogenously, and randomly, and they define the economy's basic technology. They are too general for private agents to own. In contrast, individual firms develop knowledge of how to apply a basic technology, and their understanding remains proprietary. Applied knowledge is our "intangible capital." For example, while the concept of a laser would be a basic idea, the blueprint for a CD player that uses laser technology would be applied knowledge. The difference is analogous to Mokyr's (1990a, pp 12-13) distinction between macroinventions and microinventions. He writes (1990b, p. 7), "Microinventions are more or less understandable with the help of the standard economic concepts. They result from search and inventive effort, and respond to prices and incentives. ... Macroinventions, on the other hand, do not seem to obey obvious laws, do not necessarily respond to incentives, and defy most attempts to relate them to exogenous economic variables."[5]

Our model offers an explanation of the dynamic pattern on Figure 1: the denominator of $q^*$ is mismeasured, especially at certain times. We believe there was a fairly abrupt, transforming advance in technology in the early 1970s. As Hobijn and Jovanovic (2001) and others argue, we think development of the microprocessor was the crucial change. The advent of a revolutionary invention decreases the market value of existing intangible and tangible capital, which embody previous technologies. Since revolutionary inventions do not create new ownership rights, no corresponding capitalized rents arise for the new technology itself. Thus, the numerator of $q^*$, reflecting market valuations, falls. The denominator, on the other hand, has much more inertia because national accountants tend to construct physical capital stock series using depreciation rates that are constant over time. If the numerator drops steeply after a seminal invention but the denominator reacts slowly, $q^*$ will plummet.

Because of the measurement problem, investment does not need to collapse when $q^*$ falls below 1. In fact, our analysis below shows that an upward step in technology initiates an interval of capital deepening, because the economy finds itself below its steady–state capital intensity. Flows

[4] E.g., Ray [1984] and Kelley [1994].

[5] Note that our approach is almost the opposite of Young [1993]. In his model, wholly new lines of production emerge from intentional R&D, but gradual improvements in existing technologies occur exogenously (through learning by doing).

of new investment, tangible and intangible, subsequently rebuild the market value of the capital stock, causing $q^*$ to rise. Provided agents learn about seminal inventions rapidly, the long–run pattern of stock market swings will tend to be one of abrupt declines followed by more protracted "bull advances."

Intangible capital alone explains how the value of $q^*$ can exceed 1 for long periods of time: the market value of firms, the numerator of $q^*$, reflects both physical capital and knowledge; the denominator measures just physical capital.

Our story builds on ideas in the recent literature. Hall [2001] shows how one can use investment theory to identify separately the price and quantity of capital from the value of securities. He finds evidence suggesting that firms have accumulated large amounts of intangible capital, and that adjustment costs alone have difficulty explaining observed values of $q^*$. In our empirical application, we follow Hall in estimating the quantity of intangible capital from the market value of businesses. Our analysis differs from Hall in that we have endogenous obsolescence and accumulation of intangible capital.

As stated, our explanation for the drop in the stock market during the 1970s relies on obsolescence of capital. Here we draw on insights from Greenwood and Jovanovic [1999] and Hobijn and Jovanovic [2001]. They show how the expected arrival of a new type of capital can lead to a large fall in the stock market's capitalization in an economy with a fixed capital stock and no investment. The Hobijn and Jovanovic paper also relates the magnitude of the fall to the productivity of the new technology. Our framework is different because it features capital accumulation.

The model we propose is related to several papers from the general purpose technology (GPT) literature on transitional slowdowns. Greenwood and Yorukoglu [1997] analyze a vintage capital model where the IT revolution increases the rate of progress in equipment, but it also raises adoption costs for new technologies. As in our analysis, the economy can benefit from new technology only gradually, through investment in new knowledge. Unlike our formulation, the technology frontier moves continuously, and the IT revolution increases the share of unmeasured knowledge investment, causing a slowdown in measured output growth. Andolfatto and Macdonald [1998] have a model in which technology is embodied in human capital and technological revolutions come in waves associated with mass diffusions of new knowledge. The timing of the waves is endogenous and depends upon technology–specific learning parameters. Differences between technologies lead to uneven diffusion and uneven output growth over time. Helpman and Trajtenberg [1998] analyze a model in which the periodic arrival of new GPTs leads to cycles in output growth and stock market values. Reallocations of labor to the R&D sector generate the output cycles. Also closely related to our work is Howitt's [1998] model in which a new technology increases the economy's rate of invention and causes obsolescence of capital and a transitional slowdown. These papers focus primarily on explaining the link between technological change and variations in output growth. We present a general equilibrium system which can tie together output growth, NIPA measured depreciation, and stock market valuations.

Although two of the elements of our model, intangible capital and the level of the economy's technology, are not directly observable, Sections 4–5 illustrate how a quantitative analysis is feasible. We can identify the size and importance of intangible capital through comparisons of tangible investment and market values of total national wealth from the U.S. Flow of Funds. Our model actually provides two avenues for measuring TFP: (i) as is conventional, we can compare changes over time in output with changes in inputs, and (ii) we can study the time path of the market value of (all) capital, using the facts that at the advent of a seminal invention, the value should drop, and that our model explicitly relates the degree of the fall to the degree of improvement in TFP.

Our theoretical steps yield a straightforward system of dynamic equations for an economy's aggregative variables, and Section 4 employs them to estimate our model's parameters from U.S.

post–WWII data. Section 5 suggests desirable changes in national accounts. Since national accounts usually treat knowledge-creating activities as intermediate goods, whereas our analysis implies they are investments, our framework implies a downward bias in measured GDP, and we can estimate its magnitude.[6] We also show that conventional accounting will tend to overstate the average rate of return on capital, and we provide a new interpretation of the Solow residual, warning that inputs conventionally employed in its calculation omit intangible capital and misstate obsolescence.

# 2 Model

This section presents our model. The model incorporates the three basic elements outlined in the introduction. We show that it yields a very simple aggregative equation of motion, but one featuring the aggregative market value of national wealth rather than the physical capital stock.

## 2.1 Elements

Suppose all firms behave competitively and have identical production functions with constant returns in labor, physical capital, and intangible capital. Then aggregate output obeys the same function. Let aggregate output be $Y_t$, the aggregate stock of applied knowledge (or intangible capital) be $A_t$, the aggregate stock of physical capital be $K_t$, and labor be $L_t$, and let the production function be[7]

$$Y_t = Z \cdot [A_t]^{\alpha} \cdot [K_t]^{\beta} \cdot [L_t]^{1-\alpha-\beta}, \quad \text{with} \quad \alpha, \beta > 0, \quad \text{and} \quad \alpha + \beta < 1, \tag{1}$$

where $Z$ registers the economy-wide level of technology. We assume that individual firms purchase inputs of $A$, $K$, and $L$, but that $Z$ is freely available to all. Suppose that knowledge and physical capital deteriorate (from normal obsolescence and physical wear) at the same constant rate $\delta$. Output is homogeneously divisible into consumption, $C_t$, and the two types of investment, so that

$$Y_t = C_t + \dot{A}_t + \delta \cdot A_t + \dot{K}_t + \delta \cdot K_t$$

Treat GDP as the economy's numeraire. Assume that the economy saves a constant fraction $\sigma$ of its income, implying

$$\dot{A}_t + \delta \cdot A_t + \dot{K}_t + \delta \cdot K_t = \sigma \cdot Y_t, \quad \sigma \in (0, 1). \tag{2}$$

Assume that investment is irreversible:

$$\dot{A}_t + \delta \cdot A_t \geq 0 \text{ and } \dot{K}_t + \delta \cdot K_t \geq 0. \tag{3}$$

Irreversibility will be important when we discuss technological change: when $Z$ rises abruptly, firms would like to disinvest, exchanging their old capital and know–how for new; however, (3) rules that out. Let labor supply grow exogenously at a constant rate $n$:

$$L_t = L_0 e^{nt} .$$

Assume, without loss of generality, that initially[8]

$$\frac{A_0}{K_0} = \frac{\alpha}{\beta}. \tag{4}$$

[6] See also Howitt (1996). The National Accounts classify organizational investments, including new business processes, new production systems, hiring consultants and training workers as intermediate goods (Brynjolfsson and Hitt, 2000).

[7] Laitner and Stolyarov (2002) extend this framework to allow increasing returns to scale.

[8] If (4) is violated, investment in one of the capital stocks ceases until the economy reaches ratio (4) at a finite date, which we label $t = 0$.

When $Z$ stays constant, irreversibility constraints do not bind. Then we can analyze the model as follows. Competitive behavior implies that the marginal products of knowledge and physical capital both equal the rental fee; hence, they equal each other:

$$\alpha \cdot \frac{Y_t}{A_t} = \beta \cdot \frac{Y_t}{K_t} \iff A_t = \frac{\alpha}{\beta} \cdot K_t. \tag{5}$$

Let the market value of tangible plus intangible capital be

$$M_t = A_t + K_t \tag{6}$$

Then from (1)–(6),

$$\dot{M}_t = \sigma \cdot Z \cdot \frac{[\frac{\alpha}{\beta}]^{\alpha}}{[\frac{\alpha}{\beta} + 1]^{\alpha+\beta}} \cdot [M_t]^{\alpha+\beta} \cdot [L_t]^{1-\alpha-\beta} - \delta \cdot M_t \ . \tag{7}$$

This is mathematically identical to the familiar Solow (1956) model; hence, from any $M_0 > 0$, we can see that the market value per worker monotonically converges to a stationary level.

We actually want $Z$ to evolve over time through a series of discrete upward steps. Think of an exogenous Poisson process as determining the timing of steps, and think of the relative size of each step as an independent draw from an exogenously given distribution.[9]

At initial time $t = 0$ the prevailing technology is $Z_0$. Let $\{t_i\}_{t=1}^{\infty}$ with

$$0 < t_1 < t_2 < ...$$

be the dates of technological revolutions corresponding to a sequence of realizations $t_i - t_{t-1}$ of a Poisson random variable. Let the corresponding sequence of draws from the TFP distribution be $\{Z_i\}_{i=1}^{\infty}$ with

$$Z_0 < Z_1 < Z_2 < ....$$

Let $\iota(t)$ be the index of the frontier technology at time $t$:

$$\iota(t) \equiv i, \, t \in [t_i, t_{i+1}) \, , \, i \geq 0.$$

With a Poisson process, as time passes after any $t_i$, agents need not become more and more reluctant to invest. As we will see below, the independence of $\frac{Z_i}{Z_{i-1}}$ means that knowing $Z_i$ gives the agents no new information about the expected future prices of capital goods.

We assume that tangible and intangible capital embody the technology they are used with and cannot be transferred or recycled for use with a new technology when it arrives. Let $K_{it}$ and $A_{it}$ be the date $t$ stocks of tangible and intangible capital that embody technology $Z_i$. We assume that labor is not technology-specific. Let $L_{it}$ denote the amount of labor that works with technology $Z_i$ at date $t$. Firms can produce with all technologies discovered so far:

$$Y_t = \sum_{i=0}^{\iota(t)} Z_i \cdot [A_{it}]^{\alpha} \cdot [K_{it}]^{\beta} \cdot [L_{it}]^{1-\alpha-\beta}.$$

[9] Strictly speaking, these two assumptions are not needed for the present paper. However, if the intervals between technological revolutions and the magnitudes of TFP steps are unpredictable, our results will carry over in a straightforward manner to a framework with intertemporal utility maximization by a representative consumer. See the discussion in section 3.

As a preview of results to follow, we first describe the model's reaction to a single change in $Z$. For $0 \leq t < t_1$, suppose the maximum technology-specific TFP level is $Z = Z_0$; for $t \geq t_1$, on the other hand, let $Z = Z_1$. After date $t_1$, businesses can invest in capital stocks that embody the unambiguously more productive technology $Z_1$. New investment goods always have price 1. For full employment of capital, the resale price of old capital must drop below 1 at date $t_1$. As we will demonstrate, the price of $A_{0t}$ or $K_{0t}$ relative to $A_{1t}$ or $K_{1t}$ equals $P_0 \equiv [Z_0/Z_1]^{1/(\alpha+\beta)}$ from $t_1$ onward. The magnitude of $P_0$ reflects the degree of inferiority of the old technology. Although new investments embodying the old technology remain feasible after $t_1$, they entail an immediate capital loss of $1 - P_0$. Agents therefore choose to invest only in capital embodying the frontier technology $Z_1$. Of course, this causes irreversibility constraints (3) to bind for $K_{0t}$ and $A_{0t}$ subsequent to $t_1$.

As in Solow (1960), it is also the case that the aggregate production function from $t < t_1$ remains valid after $t_1$ provided we substitute $Z = Z_1$ for $Z = Z_0$, aggregate physical capital with $K_t = P_0 \cdot K_{0t} + K_{1t}$, and aggregate intangible capital similarly. The proof of Proposition 1 establishes this formally.

To extend this reasoning to an endless series of changes in $Z$, we need a formal definition of equilibrium. We look for an equilibrium where $A_{it}$ and $K_{it}$ always have the same relative price. In particular, if $p(X,t)$ is the resale price of a unit of capital $X$ at time $t$, we want

$$p(A_{it}, t) = p(K_{it}, t) = P_{it}, \text{ for all } i,\ t. \tag{8}$$

Equilibria of this kind seem empirically relevant, since investment in $A$ or $K$ ceases if (8) is violated, yet both types of are positive in our data — see Section 3.

Let $W_t$ be the wage rate, $R_t$ be the rental rate on new physical and intangible capital, and $I_{it}^K$ and $I_{it}^A$ be date $t$ gross investment in the stocks of $A_i$ and $K_i$. As above, let output be the numeraire. Then

**Definition:** *An equilibrium is a sequence of functions of time denoting factor prices*

$$\{W_t, R_t, P_{it}\}\,, \text{ all } i \geq 0,\ t \geq 0$$

*and quantities*

$$\left\{A_{it}, I_{it}^A, K_{it}, I_{it}^K, L_{it}\right\}$$

*such that*

*1. The price of capital embodying the frontier technology equals* 1*:*

$$P_{\iota(t),t} = 1, \text{ all } t$$

*2. Firms maximize profits, and all existing capital is employed:*

$$(A_{it}, K_{it}, L_{it}) = \arg\max_{(a,k,l)} \left( Z_i \cdot a^\alpha \cdot k^\beta \cdot l^{1-\alpha-\beta} - P_{it} \cdot R_t \cdot a - P_{it} \cdot R_t \cdot k - W_t \cdot l \right),$$

$$\text{all } i \geq 0, t \geq t_i$$

*3. Investment seeks the highest return:*

$$\text{For all } t, \text{ we have } I_{it}^A, I_{it}^K > 0 \Longleftrightarrow i \in \arg\max_j \{P_{jt}\}$$

*4. Labor and goods markets clear*

$$\sum_{i=0}^{\iota(t)} L_{it} = L_t, \text{ all } t$$

$$\sum_{i=0}^{\iota(t)} \left(I_{it}^{A}+I_{it}^{K}\right)=\sigma Y_t, \text{ all } t$$

*5. Capital stocks follow their laws of motion*

$$\dot{A}_{it}=I_{it}^{A}-\delta A_{it},\ i\geq 0,\ t\geq t_i$$
$$\dot{K}_{it}=I_{it}^{K}-\delta K_{it}\ i\geq 0,\ t\geq t_i$$

*given initial conditions*

$$\begin{aligned}\frac{A_{0,0}}{K_{0,0}} &= \frac{\alpha}{\beta},\\ A_{i,t_i} &= 0,\ K_{i,t_i}=0,\ \text{all } i>0.\end{aligned}$$

Proposition 1 establishes existence and characterizes the equilibrium. The price of an old vintage of capital falls over time in a series of discrete downward steps that coincide with changes in technology. There exists a useful aggregate for the capital stock: current output depends on capital stocks of different vintages only through their aggregate market value.

**Proposition 1:**

(i) *Characterization of equilibrium:*

$$P_{it}=\left(\frac{Z_i}{Z_{\iota(t)}}\right)^{\frac{1}{\alpha+\beta}},\ i\geq 0,\ t\geq t_i \tag{9}$$

$$A_{it}=\begin{cases}\int_{t_i}^{t} e^{\delta(s-t)}\theta\sigma Y_s ds, & t\in[t_i,t_{i+1})\\ A_{i,t_{i+1}}\cdot e^{-\delta(t-t_{i+1})}, & t\geq t_{i+1}\end{cases} \tag{10}$$

$$K_{it}=\begin{cases}\int_{t_i}^{t} e^{\delta(s-t)}\left(1-\theta\right)\sigma Y_s ds, & t\in[t_i,t_{i+1})\\ K_{i,t_{i+1}}\cdot e^{-\delta(t-t_{i+1})}, & t\geq t_{i+1}\end{cases} \tag{11}$$

*where*

$$\theta\equiv\frac{\alpha}{\alpha+\beta}.$$

(ii) *Aggregation: letting*

$$M_t=A_t+K_t,$$

*where*

$$A_t=\sum_{i=0}^{\iota(t)} P_{it}A_{it} \text{ and } K_t=\sum_{i=0}^{\iota(t)} P_{it}K_{it}, \tag{12}$$

*we have*

$$A_t=\theta M_t;\ K_t=\left(1-\theta\right)M_t; \tag{13}$$
$$R_t=\left(\alpha+\beta\right)\frac{Y_t}{M_t};\ W_t=\left(1-\alpha-\beta\right)\frac{Y_t}{L_t};$$

*and, aggregate output can be expressed as*

$$Y_t=Z_{\iota(t)}\cdot A_t^{\alpha}\cdot K_t^{\beta}\cdot L_t^{1-\alpha-\beta}=Z_{\iota(t)}\cdot\frac{[\frac{\alpha}{\beta}]^{\alpha}}{[\frac{\alpha}{\beta}+1]^{\alpha+\beta}}\cdot[M_t]^{\alpha+\beta}\cdot[L_t]^{1-\alpha-\beta},\ \text{all } t\geq 0. \tag{14}$$

**Proof:** See Appendix.

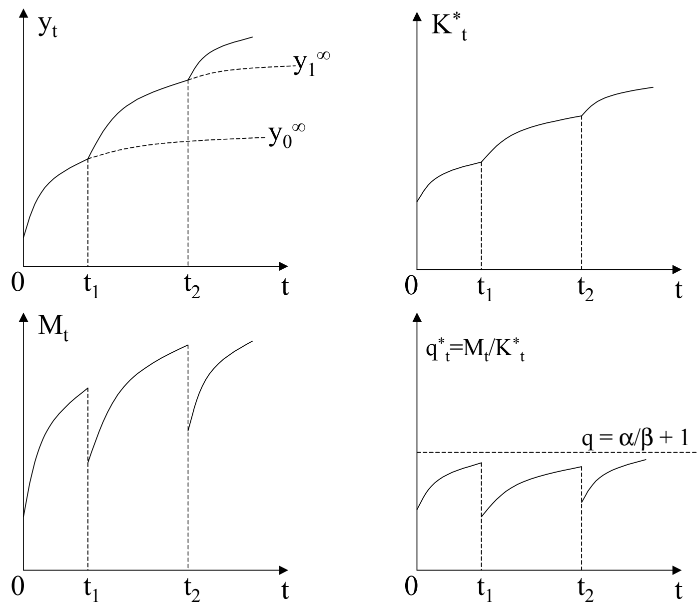


Figure 2: Time series outcomes of the model

Aggregation result (14) implies that even with a series of technological revolutions, the aggregate dynamics of the model follow (7) between changes in $Z$. Specifically, starting from given $M_0$, we can solve

$$\dot{M}_t = \sigma \cdot \bar{Z}_t \cdot [M_t]^{\alpha+\beta} \cdot [L_t]^{1-\alpha-\beta} - \delta \cdot M_t \ , \tag{15}$$

where

$$\bar{Z}_t \equiv Z_{\iota(t)} \cdot \frac{[\frac{\alpha}{\beta}]^{\alpha}}{[\frac{\alpha}{\beta}+1]^{\alpha+\beta}} \ ,$$

on each $(t_{i-1}, t_i)$, using the terminal value $M_{t_i}$ from one interval as the initial condition for $M_t$ on the next.

Figure 2 illustrates time series outcomes. For times between $t_i$ and $t_{i+1}$, (15) determines growth. GDP per worker, $y_t = \frac{Y_t}{L_t}$, will converge toward its stationary–state level, say, $y_i^{\infty}$. However, $Z$ takes a discrete upward step at $t_{i+1}$. The old dynamic path is interrupted, and we follow a new one, toward a higher stationary–state $y_{i+1}^{\infty}$. There is no drop in output at $t_{i+1}$; production as before remains feasible. Conversely, there is no upward leap in $y_t$ at $t_{i+1}$, since society can only take advantage of $Z_{i+1}$ through production with new capital.

Since $M_t$ is a market value, it abruptly drops at $t = t_{i+1}$, as existing capital loses its resale potential. Subsequently $M_t$ rises, because its balanced growth path is higher with $Z_{i+1}$ than with $Z_i$ (see the lower left panel on Figure 2). In our framework, both $A_t$ and $K_t$ are strictly proportional to $M_t$ at all dates.

Figure 2's $K_t^*$ is a conventional measure of the physical capital stock — corresponding to the denominator of our $q^*$. The conventional physical capital stock $K_t^*$ evolves according to a differential equation

$$\dot{K}_t^* = I_t - \bar{\delta} \cdot K_t^*, \tag{16}$$

where $I_t$ is NIPA real physical investment (i.e., our $I_{\iota(t),t}^K$) and $\bar{\delta}$ is the average rate of depreciation. The NIPA aggregate capital stock is constructed from different asset subcategories using a separate perpetual inventory equation with a constant depreciation rate $\bar{\delta}_j$ for each asset $j$. For each asset category, $\bar{\delta}_j$ reflects data on service lives and resale prices. Then $\bar{\delta}$ in equation (16) is the weighted sum of $\bar{\delta}_j$, with weights equal to shares of asset $j$ in the aggregate capital stock. An important point is that each $\bar{\delta}_j$ is constant — we can think of it as reflecting average physical depreciation and obsolescence through (many) complete cycles $[t_i, t_{t+1})$ in Figure 2. For example, the U.S. Department of Commerce writes, "The depreciation rates used to derive the estimates [of $K_t^*$ ] reflect the effects of normal obsolescence over time. They are not adjusted to take account of obsolescence that is unusually or unexpectedly larger than the amounts built in to the depreciation schedules ...." [10]

Our model's $\delta$ reflects only normal depreciation between revolutions (i.e., within intervals $(t_i, t_{i+1})$), whereas $\bar{\delta}$ reflects this plus the discontinuous obsolescence at dates $t_i$. According to the pricing formula (9), the average depreciation rate on an arbitrarily chosen interval $(0, T)$ equals

$$\bar{\delta} = \delta + \frac{1}{T} \frac{1}{\alpha + \beta} \ln \left( \frac{Z_{\iota(T)}}{Z_0} \right). \tag{17}$$

On the one hand, because $\bar{\delta}$ exceeds $\delta$, physical capital $K_t^*$ will grow slower than $M_t$ (and $K_t$) within intervals $(t_i, t_{i+1})$. On the other hand, $\bar{\delta}$ only captures abrupt obsolescence at each $t_i$ in a long–run average sense, and $K_t^*$ will not discontinuously drop as $M_t$ does at times $t_i$.

### 2.2 Correspondence with Figure 1

We now have the basic elements of our proposed explanation of Figure 1.

We believe the data show two intervals, say, $[t_{i-1}, t_i)$ and $[t_i, t_{t+1})$. As stated, the ratio on Figure 1 is measured Tobin's $q$, defined as

$$q_t^* = \frac{M_t}{K_t^*}.$$

While the numerator will move sharply to reflect market valuations, the denominator will adjust with inertia — as discussed above, $K_t^*$ is roughly a moving average of undepreciated past physical investment. At the beginning of each interval, abrupt obsolescence lowers the value of tangible and intangible capital stocks. Over the course of the interval, however, values of these stocks rise faster than $K_t^*$, because $\delta$ is less than $\bar{\delta}$. Thus, $q^*$ should dip at $t_i$; then it should rise with $t$ for $t \in (t_i, t_{i+1})$. That is what Figure 1 shows.

The numerator of $q_t^*$ is $A_t + K_t$. The denominator, $K_t^*$, best approximates $K_t$ towards the end of each interval $[t_i, t_{i+1})$. Thus, we expect $q_t^*$ to converge to its theoretical value

$$q_t = \frac{M_t}{K_t} = \frac{\alpha}{\beta} + 1$$

for $t \in (t_i, t_{i+1})$. This covergence is interrupted at date $t_{i+1}$, when $q_t^*$ drops again. This is consistent with Figure 1 (also see Table A1 in Appendix 1).

## 3 Discussion

This section discusses several of our key assumptions and their implications.

[10] U.S. Department of Commerce, Bureau of Economic Analysis, "Fixed Reproducible Tangible Wealth in the United States, 1925–94," August 1999, http://www.bea.doc.gov/bea/Articles/National/NIPAREL/Meth/wlth2594.pdf. See also Fraumeni (1997).

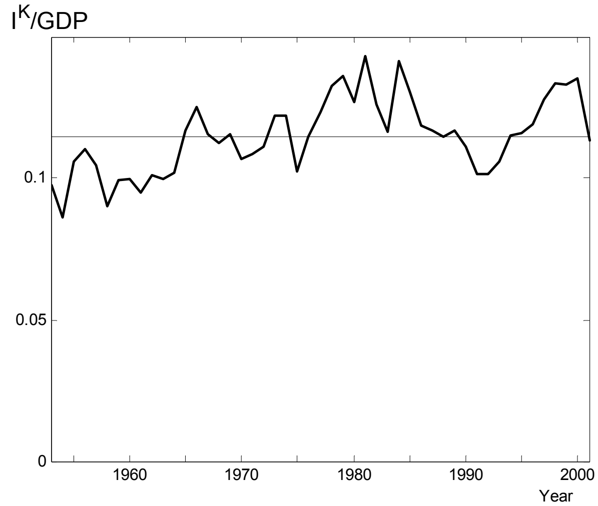


Figure 3: Share of physical investment in measured GDP, 1953-2001.

*Average labor productivity.* As figure 2 shows, a technological revolution does not cause a slowdown in average labor productivity (and the real wage). We do not view this as a limitation of our model. For simplicity, the model assumes that workers are homogenous and that new technology can be operated at full efficiency by anyone. Relaxing these assumptions can produce a slowdown in productivity and wages at $t_i$. For example, in Greenwood and Yorukoglu (1997) workers need to learn to realize the full potential of the new technology, and productivity grows slowly during the learning phase. Caselli (1999) shows that when a new technology arrives, workers whose learning cost is high stay with the old technology, and their wage drops as capital flows away from old equipment.

*Tobin's q and aggregate investment.* Figure 3 shows that although $q^*$ remained below one from 1974 to 1984, investment rate during this period was not much different from its historical average. This is contrary to investment theories (e.g. Hayashi, 1982) where $q^*$ is a proxy for the marginal benefit from investment. Our analysis offers a way to reconcile the time series for $q^*$ and investment: in our model, the "theoretical" value of $q$ that determines firms' investment decisions is always constant and equal $\frac{\alpha}{\beta} + 1$; $q^*$ differs from $q$ because of mismeasurement.

*Constant saving rate.* If the saving rate is not assumed to be constant, agents' expectations can, in principle, affect the time paths for variables. For example, an expectation that the next change in technology is close or that it is drastic will make consumers more reluctant to save. However, the assumed processes for $t_i$ and $Z_i$ are free of these expectational effects. Knowing $t_i$ and $Z_i$ does not give any additional information about $t_{i+1}$ and $\frac{Z_{i+1}}{Z_i}$. Therefore, expected capital losses due to changes in $Z$ are independent of the current state and depend only on exogenous variables: the rate of arrival of the new technologies and the distribution of $\frac{Z_{i+1}}{Z_i}$.

A more general model with intertemporal utility maximization will have different adjustment dynamics, but the basic elements of our explanation for the pattern of Figure 1 will not be affected. Even in a more general model, the market will still drop when a new technology arrives and will rise eventually when the new capital is accumulated. Besides, Figure 3 shows an approximately constant investment rate, so our assumption of constant saving rate looks empirically plausible. One distinct implication of a variable saving rate is noteworthy. Proposition 1 implies that the share of unmeasured knowledge investment is a *constant* fraction $\sigma\theta$ of national income. If instead

$I^A$ and $I^K$ both respond to changes in the interest rate, the fraction of national income that goes unmeasured will change over time. One can then use the estimation procedure proposed in the next section to identify the mismeasurement in GDP growth year by year, but we leave this for future research.

*Equal depreciation rates of physical capital and applied knowledge.* This paper assumes that depreciation rates for $A$ and $K$ equal one another. If depreciation rates were different, investment in one of the capital stocks might cease periodically. The arrival of new technology might leave the stocks of $A$ and $K$ imbalanced, and their relative price would no longer be the same. Then all new investment would go to the capital stock with the higher relative price.[11] Data do not support such a scenario in the time period Section 4 studies. Physical gross investment (nonresidential fixed capital investment plus change in private inventories) as a percentage of GDP is always positive, and the same is true for private R&D spending, which corresponds to a fraction of our variable $A$.

Alternatively, measured depreciation rates do not seem too different in practice. Jones and Williams (2000) suggest an average rate .10 for the depreciation of applied knowledge.[12] Our estimate below of NIPA annual depreciation of nonresidential fixed (physical) capital yields an average rate of 0.0752. (Roughly equal rates are presumably not a mere coincidence: if TFP is embodied in physical capital, producers would logically design capital goods to last as long as a technology typically does.)

# 4 Estimation/Calibration

To illustrate our model's potential usefulness in Section 5, we set parameter values using U.S. data from 1953–2001.[13] The time period begins after the Korean War. As above, we assume Figure 1 has two intervals, say, $[t_0, t_1)$ and $[t_1, t_2)$, with separate technology levels $Z_0$ and $Z_1$, respectively. The first episode began, we assume, prior to 1953, and it ended, say, in 1973. The second was still underway in 2001. In our notation, $t_0 < 1953$, $t_1 = 1973$, and $t_2 > 2001$. As Hobijn and Jovanovic (2001) argue, the underlying crucial invention at $t_1$ might have been the microprocessor.[14] Since many commentators believe that the U.S. stock market's very recent performance manifested "irrational exuberance," this section actually only uses net worth data for 1953–95 — leaving Section 5 to consider whether or not subsequent years display a pattern distinct from earlier dates.

[11] Formally, full employment conditions for $A$ and $K$ imply that

$$\frac{A_t}{K_t} = \frac{\alpha}{\beta} \cdot \frac{p(K,t)}{p(A,t)} \cdot \frac{R_K(t)}{R_A(t)},$$

where $R_A$ ($R_K$) is the rental rate on $A$ ($K$) expressed in units of output and $p(\cdot)$ is the price of capital (relative to output). If depreciation rates are equal,

$$\frac{R_K(t)}{R_A(t)} = 1 \text{ and } p(K,t) = p(A,t), \text{ all } t.$$

With different depreciation rates, rental rates on $A$ and $K$ are no longer equal. Moreover, when a new technology arrives, the ratio $\frac{R_K(t)}{R_A(t)}$ will change, because the interest rate will now impact the rental rates differently. Then full employment of factors implies that the relative price $\frac{p(K,t)}{p(A,t)}$ must adjust. When the ratio $\frac{p(K,t)}{p(A,t)}$ is different from 1, there is no investment in the capital stock with lower relative price — see Shell and Stiglitz (1967).

[12] Estimates of the depreciation of monopoly profits from patents range from 4 to 25 years. See Pakes and Schankerman (1984), Mansfield et al. (1981) and Caballero and Jaffe (1993).

[13] The BEA data on $K^*$ ends in 2000, but it plays no role in the estimation. Our financial–return data (described below) ends in 1999, with this section only using returns for 1953–1995.

[14] See also Andolfatto and MacDonald [1998]. With a model of the origins of dates $t_i$, they detect a number of changes during the post–WWII era. The two largest occur in the early 1950s and the early 1970s. The authors associate the first with chemicals and synthetic materials, and the second with electronics.

We derive a 6 equation statistical model from Section 2's economic framework. We estimate the following 6 parameters: $\alpha$, the output elasticity of applied knowledge; $\beta$, the output elasticity of physical capital; $\sigma$, the aggregate average propensity to save out of gross domestic product; $Z_0$, the TFP level for the interval $[t_0, t_1)$; $Z_1$, the TFP level for $[t_1, t_2)$; and, $\delta$, the rate of physical depreciation for $t \neq t_i$ any $i$. Call the vector of parameters $\vec{u}$. We estimate $\vec{u}$ using a method of moments approach. This section first derives each equation of the statistical model; then it describes our instruments; finally, it presents our parameter estimates. Appendix 3 explains our data sources.

For the estimation, we switch to a discrete–time version of our model, which matches annual data. Note that the variable $M_t$ measures end–of–year net worth, hence the level at the start of year $t+1$. The new version of our basic equation of motion is

$$M_t = m\left(M_{t-1}, t\right) \equiv \sigma \cdot \bar{Z}_t \cdot \left[M_{t-1}\right]^{\alpha+\beta} \cdot \left[L_t\right]^{1-\alpha-\beta} + (1-\delta) \cdot M_{t-1}. \tag{18}$$

Although Section 2 has, for simplicity, a constant rate of labor force growth, the present section allows the labor supply $L_t$ to vary from year to year — see Appendix 3.

The equations of our statistic model have the form

$$f_t^i = \epsilon_t^i, \quad i = 1, ..., 6 \ .$$

In each case, $f_t^i$ gives, as described below, the discrepancy between a current dependent variable and a function, which the theoretical model determines, of exogenous variables, past dependent variables, and parameters. Thus, each $f_t^i$ is itself a function of current and past dependent variables, exogenous variables, and $\vec{u}$. $\epsilon_t^i$ is a regression error.

Equation 1 We assume that the instant after the revolution of $t_1$, the resale value of existing capital falls according to (9). Using that formula, define

$$\tilde{M}_t = \begin{cases} M_t \left(\frac{Z_0}{Z_1}\right)^{\frac{1}{\alpha+\beta}} & t = t_1 \\ M_t & t \neq t_1 \end{cases} .$$

Then we set

$$f_t^1 \equiv \ln(M_t) - \ln\left(m\left(\tilde{M}_{t-1}, t\right)\right), \quad t = 1953, ..., 1996. \tag{19}$$

As stated, on the basis of external information, we are suspicious of the connection over the years 1996–2001 between measured financial net worth and capital as an input to production. Thus, we never employ $M_{t-1}$ for $t-1 > 1995$ as a predetermined variable. Instead, we set

$$f_t^1 \equiv \ln(M_t) - \ln\left(m\left(M_{t-1}^*, t\right)\right), \quad t = 1997, ..., 2001 \ , \tag{20}$$

where the following equations inductively define $M^*$:

$$M_{1996}^* \equiv m\left(M_{1995}, 1996\right)$$

$$M_t^* \equiv m\left(M_{t-1}^*, t\right), \quad t = 1997, ..., 2001 \ .$$

Notice that since $f_t^1$ is a difference of logarithms, there is no reason to think of $\epsilon_t^1$ as having a time trend — it equals the log of the ratio of actual to predicted $M$. The same logic applies for the second, third, and fifth equations below.

Equation 2 The second equation of our statistical model compares data on GDP with $Y$ determined from our model's aggregate production function.

We need two adjustments. First, because residential capital plays no role in our analysis, we subtract housing services from NIPA GDP, calling the difference $GDP^*$.[15] Second, our model's aggregate output, $Y$, treats gross investment in applied knowledge as a final good; thus,

$$Y_t = GDP_t^* + \dot{A}_t + \delta \cdot A_t.$$

Although the national income and product accounts provide no data on investment in intangible capital, we can use Proposition 1, and then equation (2), to write

$$Y_t = GDP_t^* + \theta \cdot (\dot{M}_t + \delta \cdot M_t) = GDP_t^* + \theta \cdot \sigma \cdot Y_t.$$

So,

$$GDP_t^* = (1 - \theta \cdot \sigma) \cdot Y_t.$$

The second equation of our statistical model is

$$f_t^2 \equiv \ln(GDP_t^*) - \ln\Big((1 - \theta \cdot \sigma) \cdot \bar{Z}_t \cdot [\tilde{M}_{t-1}]^{\alpha+\beta} \cdot [L_t]^{1-\alpha-\beta}\Big), \quad t = 1953, ..., 1996. \tag{21}$$

For $t$ beyond 1996 the formula remains the same but we replace $\tilde{M}_{t-1}$ on the right with $M_{t-1}^*$ as defined in equation (20).

Equation 3 Let $I_t^K$ be NIPA gross physical nonresidential investment plus change in inventories for year $t$. If $I_t$ is total investment from our model — including intangible as well as tangible capital — Section 2 shows

$$I_t^K = (1 - \theta) \cdot I_t \ .$$

The third equation of our statistical model sets

$$f_t^3 \equiv \ln(I_t^K) - \ln((1 - \theta) \cdot \sigma \cdot \bar{Z}_t \cdot [\tilde{M}_{t-1}]^{\alpha+\beta} \cdot [L_t]^{1-\alpha-\beta}), \quad t = 1953, ..., 1996. \tag{22}$$

Again, for times past 1996, we use the same formula but replace $\tilde{M}_{t-1}$ on the right with $M_{t-1}^*$.

Equation 4 We can relate the factor share of labor to the parameters of our aggregate production function. Before doing so, we introduce indirect business taxes, which affect the marginal revenue products of inputs.

Let $\tau_t^*$ be the measured indirect tax rate.[16] We set our model's tax rate $\tau_t$ to make tax collections from $Y_t$ match NIPA data: using the relationship of $GDP^*$ and $Y$ derived above,

$$\begin{aligned} \tau_t \cdot Y_t &= \tau_t^* \cdot GDP_t^* \text{ iff} \\ \tau_t &= \tau_t^* \cdot (1 - \theta \cdot \sigma) \end{aligned}$$

(Thus, $\tau_t$ is a function of $\tau_t^*$, $\theta$, and $\sigma$.)

Setting the marginal revenue product of labor equal to the wage rate, we have

$$\frac{W_t \cdot L_t}{Y_t} = (1 - \tau_t) \cdot (1 - \alpha - \beta) \ .$$

Using the relationship of $GDP^*$ and $Y$ again, our statistical model's fourth equation is

$$f_t^4 \equiv \frac{W_t \cdot L_t}{GDP_t^*} - (1 - \tau_t) \cdot \frac{1 - \alpha - \beta}{1 - \theta \cdot \sigma} \text{ all } t. \tag{23}$$

[15] In our data 1953–2001 the average ratio of $GDP^*$, measured $GDP$ less housing services, to $GDP$ is .9067. The coefficient of variation of the ratio is .0064.

[16] The average indirect tax rate for 1953–2001 is .0817, with coefficient of variation .0657.

Equation 5   Section 2's analysis shows that physical capital obeys

$$K_t = (1-\theta)\cdot M_t \ .$$

The flow of depreciation on physical capital, reflecting wear and tear, is $\delta \cdot K_t$. Section 2 implies there will also be discontinuous obsolescence at $t_1$, causing a fall in the resale price of existing units of capital from 1 to $[Z_0/Z_1]^{1/(\alpha+\beta)}$. NIPA measured depreciation reflects both continuous and discontinuous depreciation (recall Section 3), and, since accountants construe the latter from formulas based on average past experience, the NIPA flow treats the combined rate as a constant which we call $\bar{\delta}$. Recall its definition in (17). Letting $D_t$ be the NIPA flow of depreciation of physical capital during year $t$, the fifth equation of our statistical model sets

$$f_t^5 \equiv \ln(D_t) - \ln(\bar{\delta}\cdot(1-\theta)\cdot \tilde{M}_{t-1}), \quad t = 1953, ..., 1996, \tag{24}$$

with, for years after $t = 1996$, $M^*_{t-1}$ in place of $\tilde{M}_{t-1}$.

There remains the task of specifying the average rate of obsolescence (in effect, $T$ for (17)). We cannot deduce the average span between revolutions from our time interval, because the latter has only a single seminal invention. Turning to longer term data, the diagram of leading sectors in Cohen *et al.* [2000, p.30] seems to imply that there were 5–7 distinct revolutionary inventions over the last 200 years. That suggests an average periodicity of 30 to 40 years. The long series for $q^*$ of Smithers and Wright [2000, chart 2.1] is similar to our Figure 1.[17] It shows three declines during the twentieth century, and the magnitudes of the declines are all roughly the same. The two complete episodes roughly run from 1910 to 1973, suggesting $T = 30$ and

$$\bar{\delta} = \delta + \frac{1}{30}\cdot\frac{1}{\alpha+\beta}\cdot\ln\left(\frac{Z_1}{Z_0}\right). \tag{25}$$

This is our baseline case.

Equation 6   Let $r_t$ be the average real interest rate for year $t$. We measure $r_t$ from Robert Shiller's data on *ex post* returns on financial investments — see Appendix 3. In our theoretical model, the interest rate should equal the marginal revenue product of $M_t$ less depreciation and obsolescence; thus, we set

$$f_t^6 \equiv r_t - [(1-\tau_t)\cdot(\alpha+\beta)\cdot \bar{Z}_t \cdot [\tilde{M}_{t-1})]^{\alpha+\beta-1}\cdot [L_t]^{1-\alpha-\beta} - \Delta_t], \quad t = 1953, ..., 1996, \tag{26}$$

where

$$\Delta_t \equiv \begin{cases} \delta + \left[1 - \left(\frac{Z_0}{Z_1}\right)^{\frac{1}{\alpha+\beta}}\right] & t = t_1 \\ \delta & t \neq t_1 \end{cases} .$$

As above, for $t > 1996$ we employ $M^*_{t-1}$ in the place of $\tilde{M}_{t-1}$.

Instruments   We estimate our six parameters using a method of moments approach. Nonlinear least squares would have a consistency problem from the lagged dependent variable in, say, our first equation if $\epsilon_t^1$ were autocorrelated, and business cycles make autocorrelation plausible. There is also a potential simultaneity problem from, say, $L_t$, in equations (19)–(21) — recall Prescott [1986] and others. Pure calibration would not yield standard errors for parameter estimates, would not

[17] Smithers and Wright's data refers only to nonfarm, nonfinancial corporations; they have used historical data to extend their series back before 1950; and, they have processed their numerator somewhat differently from our Figure 1. After 1950, their chart displays qualitatively the same pattern as our Figure 1, though their average $q^*$ is lower.

determine parameters simultaneously across our six equations, and presumably would not be able to employ the details of the shapes in Figure 2.

Our moments are

$$m(\vec{u}) = \frac{1}{101-52} \cdot \sum_{t=1953}^{2001} \begin{pmatrix} \underset{6\times 1}{f_t^1 \cdot v_t^1} \\ \underset{6\times 1}{f_t^2 \cdot v_t^2} \\ \underset{1\times 1}{f_t^3 \cdot v_t^3} \\ \underset{1\times 1}{f_t^4 \cdot v_t^4} \\ \underset{1\times 1}{f_t^5 \cdot v_t^5} \\ \underset{6\times 1}{f_t^6 \cdot v_t^6} \end{pmatrix},$$

where each $v_t^i$ is a vector of instruments. For equations (22)–(24), vector $v_t^i$ has a single element, always equal to 1. For equations (19), (21) and (26), the vector has the following six elements:

$$v_t^i = \begin{pmatrix} \begin{cases} 1, & t < t_1 \\ 0, & \text{otherwise} \end{cases} \\ \begin{cases} 1, & t \geq t_1 \\ 0, & \text{otherwise} \end{cases} \\ \begin{cases} 1, & t \leq t_0 + 5 \text{ or } t_1 - 5 \leq t < t_1 \\ 0, & \text{otherwise} \end{cases} \\ \begin{cases} 1, & t \leq t_0 + 5 \\ -1, & t_1 - 5 \leq t < t_1 \\ 0, & \text{otherwise} \end{cases} \\ \begin{cases} 1, & t_1 \leq t \leq t_1 + 6 \text{ or } t_1 \geq 1989 \\ 0, & \text{otherwise} \end{cases} \\ \begin{cases} 1, & t_1 \leq t \leq t_1 + 6 \\ -1, & t_1 \geq 1989 \\ 0, & \text{otherwise} \end{cases} \end{pmatrix}.$$

As stated, there are external reasons for doubting the quality of the net worth data after 1995; thus, in this section we set $v_t^1$ and $v_t^6 = 0$ all $t > 1995$.

The rationale for the components of $v_t^i$, $i = 1, 2, 6$, is as follows. We think of history as having a panel of episodes $[t_i, t_{i+1})$, $i = 0, 1,...,$ of which we have data on two. Econometrically, we think of the length of each episode as long. Each episode has a "fixed effect" $Z_i$, and there are extensive cross–episode parameter constraints. The first two components of $v_t^i$ embody the idea that the mean of $\epsilon_t^i$, $i = 1, 2, 6$, should be 0 over $[t_0, t_1)$ and, separately, over $[t_1, t_2)$. Beyond this, Section 2 provides a precise model, and we want to enforce consistency between the data and specific patterns which the model generates. To illustrate, think about the interval $[t_0, t_1)$ from the graph of $M_t$ in Figure 2. If data follows the solid curve, the first component of $v_t^1$ alone would allow a "perfect fit" from a model generating a horizontal line through the middle of the curve. In contrast, instruments 3–4 and 5–6 require our model to follow the more detailed implications of the figure.

Equations (22)–(24) merely match data with our constant average propensity to save, factor share of labor, and average rate of depreciation.

Our estimation steps follow Gallant (1987, ch.6). Define

$$S(\vec{u}, V) \equiv [(101-52) \cdot m(\vec{u})]' \, V^{-1} \, [(101-52) \cdot m(\vec{u})] \, .$$

We choose parameter values to minimize $S(.)$. There are two stages. In the first, $V$ has sub matrices

$$\sum_{t=1953}^{2001} [\overrightarrow{v_t^i}][\overrightarrow{v_t^i}]' ,$$

for $i = 1, ..., 6$, along its principal diagonal. Minimizing $S(.)$ with respect to $\vec{u}$ yields consistent estimates. Using these to evaluate each $f_t^i$, we form an improved estimate $\widehat{V}$ of $V$, which is consistent even with autocorrelated and heteroscedastic errors. Specifically, for each $t$, form the vector $\overrightarrow{w_t} \equiv (\widehat{f_t^1} \cdot v_t^1, ..., \widehat{f_t^6} \cdot v_t^6)$ where $\widehat{f_t^i}$ means $f_t^i$ evaluated using first–stage estimates of $\vec{u}$. Determine a matrix $\widehat{V_0}$ from

$$\widehat{V_0} = \sum_{t=1953}^{2001} [\overrightarrow{w_t}][\overrightarrow{w_t}]' ;$$

a second matrix, $\widehat{V_1}$, using products $[\overrightarrow{w_t}][\overrightarrow{w_{t-1}}]'$; and a third, $\widehat{V_{-1}}$, from $[\overrightarrow{w_{t+1}}][\overrightarrow{w_t}]'$. Then $\widehat{V}$ sums $\widehat{V_0}$, $\widehat{V_1}$, and $\widehat{V_{-1}}$, using Parzen weights. The second stage minimizes $S(., \widehat{V})$, yielding our second–stage estimator $\widehat{u}$.

Parameter Estimates Table 1 presents our preferred parameter estimates. The starting date is 1953; the revolution date, $t_1$, is 1973; and, the ending date is 2001. All coefficients are significantly different from 0 at the 5% level. The bottom row shows that a test of the over–identifying restrictions accepts at the same significance level.

Our estimated average rate of depreciation (inclusive of obsolescence), 7.52% per year, seems conventional. The combined output elasticity of intangible and physical capital is $\alpha + \beta = .30$, with 95% confidence interval (.26, .34). This lies on the upper edge of the range of conventional estimates. Since the estimated ratio $\alpha/\beta$ is about .50, our model implies that the economy's stock of intangible capital is half as large as physical capital. As we expect, Table 1 implies $Z_1 > Z_0$. Section 5 discusses various aspects of Table 1's outcomes in more detail.

Figure 4 presents dynamic simulations forward and backward from $M_{1972}$, the level of $M$ at the start of 1973. The simulations use the coefficients from Table 1. When computing the ratios for $q^* = M/K^*$ on Figure 4, we do not simulate the value of $K^*$ since we do not model the process by which the BEA computes $K^*$; instead, we use the same denominator, the BEA's $K^*$, to obtain both simulated and empirical $q^*$. For the years 1953–95, the model seems to track the data quite well, including replicating the patterns of Figure 1. Section 5 separately considers the time period 1996–2001.

The model does overpredict the drop in $M$ at revolution date $t_1$. The empirical decline in Flow of Funds net worth from the beginning of 1973 to the start of 1975 is 22 percent of the first figure; the simulated decline is 55 percent. Data quality may explain part of the difference: although flow of funds net worth includes both corporate and non corporate businesses, presumably accurate market valuations exist in the short run only for corporations. Indeed, Hall [2001, fig.13] and Hobijn and Jovanovic [2001, fig.1], which are both based on corporate shares alone, show net worth drops in the early 1970s at least as large as our simulation. Long–term debt creates another problem: account ledgers typically do not make adjustments for capital gains and losses on long–term bonds stemming from changes in market interest rates. As the simulations show, our model predicts temporarily higher interest rates following a technology revolution. After $t_1$, rising interest rates cause capital losses for households on their bond holdings — and empirical $M$ will not capture these losses. Businesses which had issued the bonds garner corresponding capital gains. The latter prop up, to some extent, equity valuations, which, at least in the case of corporations, $M$ does register. This creates another upward bias in measured $M$ in the aftermath of $t_1$.

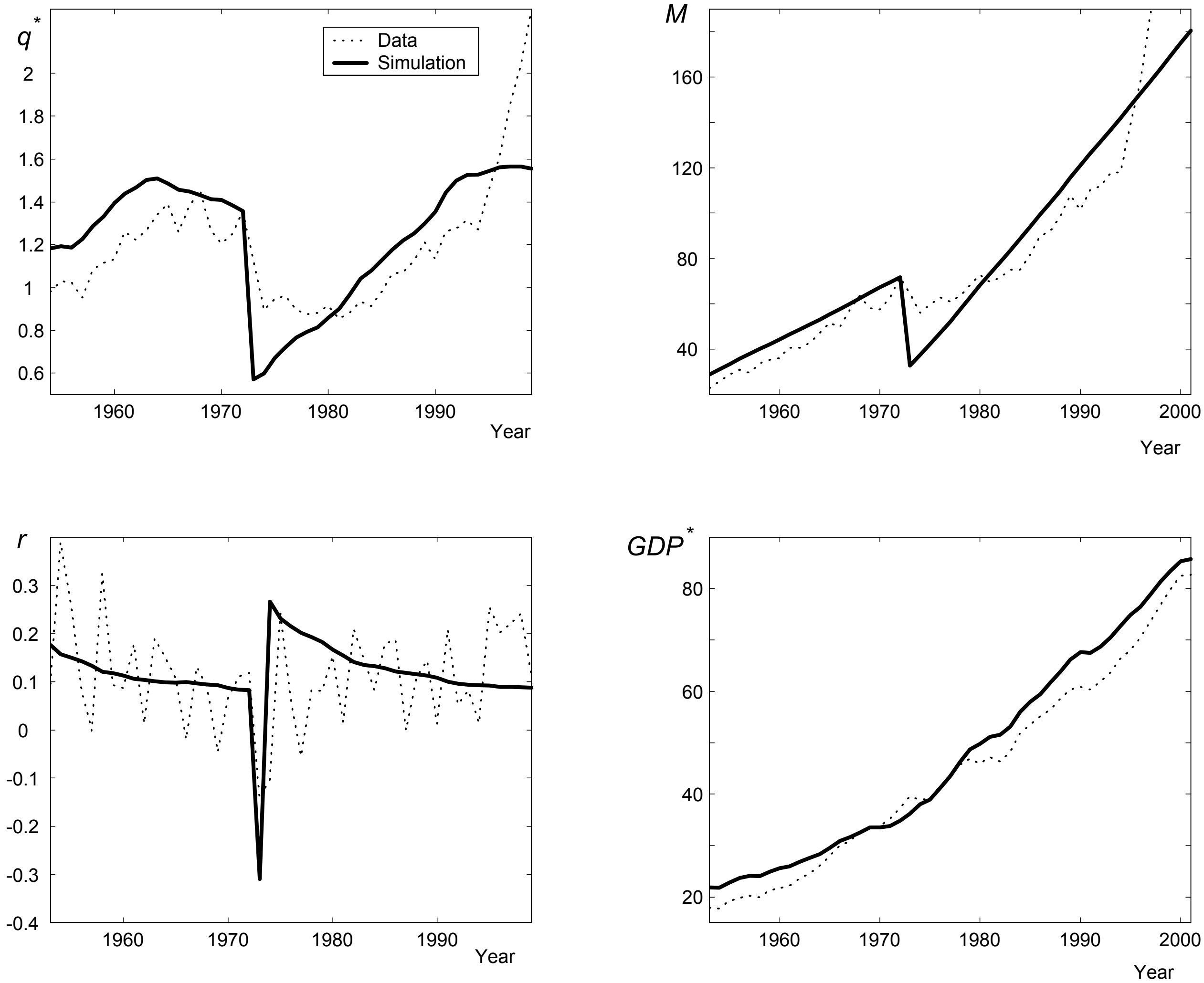


Figure 4: Simulation results

Although $r$, the *ex post* return on financial investments depends exclusively on corporate securities, the problem with capital gains and losses on debt persists. Nevertheless, our simulation does quite well: the measured return for 1973–1974 combined is -26 percent; the simulated return for our model's abrupt, single–period change in 1973 is -31 percent.

We experimented with different revolution periodicities for equation (25). Setting a period of 25 years (e.g., replacing 1/30 with 1/25 in (25)) produced inferior simulation results; periods of 35 or 40 years, on the other hand, yielded outcomes virtually identical to Table 1.

Our model does not predict revolution date $t_1$ — the latter is exogenous. Table 2 repeats our method of moment steps for $t_1 = 1974$. Although the parameter estimates are qualitatively similar to Table 1, corresponding simulations (not shown) are much less satisfactory. In particular, with the 1974 parameters, by 1953 simulated $M$ under predicts empirical net worth by over two thirds. (Setting $t_1 = 1972$ also leads to poor simulation results.)

The remainder of this paper uses the parameter estimates of Table 1.

# 5 Results

We turn now to accounting implications of our model. We finish with an assessment of the behavior of the U.S. stock market in the late 1990s.

GDP bias The U.S. national accounts omit investment in knowledge from the set of final goods. Thus, our analysis implies that measured GDP is understated. Parameter estimates from Table 1 imply that the bias is rather substantial:

$$\frac{Y_t - GDP_t^*}{GDP_t^*} = \frac{\theta \cdot \sigma}{1 - \theta \cdot \sigma} \approx 6.07\% \ .$$

Measurement of TFP The traditional approach to TFP measurement is to compute the difference

$$\frac{d\ln(GDP_t)}{dt} - \gamma^* \cdot \frac{d\ln(K_t^*)}{dt} - (1-\gamma^*) \cdot \frac{d\ln(L_t)}{dt} \equiv s_t \tag{27}$$

where $\gamma^*$ is the factor share of capital, and to define "technological progress" as the (Solow) residual $s_t$. Although measured $GDP_t$ differs from $Y_t$ in not counting business investments in applied knowledge as final output, if the relative discrepancy is constant, as it is in our model, GDP growth is still measured correctly. Similarly, although we think that $K_t^*$ omits intangible capital, our analysis makes physical capital strictly proportional to comprehensive capital $M_t$.

Nevertheless, according to our model there remain two sources of mismeasurement that will affect one's computation of the Solow residual. One is the growth rate of capital stock. Although in our model obsolescence is uneven, the construction of $K^*$ assumes the rate of depreciation is constant. The growth rate in $q^*$ equals the difference in growth rates of actual and measured physical capital:

$$\frac{d\ln(q_t^*)}{dt} = \frac{d\ln(K_t)}{dt} - \frac{d\ln(K_t^*)}{dt} \ .$$

This implies that the traditional Solow residual understates (overstates) the rate of output growth from technological progress when $q_t^*$ is falling (rising). A second problem is capital's share, $\gamma^*$ in (27). If GDP omits investment in applied knowledge as a final good, and if one estimates $1-\gamma^*$ from wage and salary payments divided by measured GDP, $1-\gamma^*$ will be overstated and $\gamma^*$ understated.

In the end, our analysis suggests that (27) may not provide a useful way of thinking about technological progress. In our model, exogenous technological progress is episodic, abrupt, and infrequent. The effects of such change appear in national output only after investment in new physical capital and in applied knowledge, and we need a general equilibrium approach to interpret the manifestations. As a bonus, the equilibrium approach yields a second way of measuring exogenous improvements in technology — namely, through changes in the market value of existing capital. The two ways are evident in our equations (21) and (19), respectively.[18]

Measurement of Depreciation and Embodied Technical Change As stated in Section 4, average depreciation, $\bar{\delta}$, is the sum of physical wear and tear and economic obsolescence. According to Table 1, the rate of the former is 5.48% per year, and the latter is 2.04% per year. Equation (24) in our statistical model relates $\bar{\delta}$ to NIPA measured depreciation. In addition, pricing formula (9) shows that obsolescence is proportional to embodied technical progress — a link which equation (19) exploits.[19] Gort *et al.* [1999] employ similar ideas to study structures. They use a vintage capital

[18] Sakellaris and Wilson (2001) follow an alternative course. Assuming embodied technical progress at a continuous rate, they use plant–level time series of investment and output to estimate the rate of technological improvement on equipment of varying vintages.

[19] Of course, our analysis presupposes that firms do not retire assets which are still productive. In a model where capital goods have finite economic life spans, depreciation schedules are no longer geometric. See Whelan [2002].

framework, with continuous technical progress, to derive an analog of equation (17); measure $\bar{\delta}$ with data on building rents by age; derive $\delta$ from maintenance expenditures; and then back out a rate of technical change. Greenwood *et al.* [1997] use a continuous–time version of (17) to relate the rate of embodied technical change to the rate of decline in the quality–adjusted price of new investment goods relative to consumption.[20]

Rate of return In our framework, since NIPA GDP is understated, measured aggregate factor payments are too small. If one measures wages and salaries and then computes payments to capital as a residual, all of the understatement falls on the latter. If we compute the return to capital by dividing residual payments by the total capital stock, both the numerator, as just discussed, and the denominator, which omits the stock of knowledge, should be larger.

Letting $R$ be the rental fee on capital in our model, we have

$$Y = GDP^* + I^A = R \cdot (K + A) + W \cdot L \ .$$

In the national accounts, measured output, $GDP^*$, is the sum of payments to capital and labor. Letting $R^*$ be the conventionally measured rental fee on physical capital,

$$GDP^* = R^* \cdot K + W \cdot L \ .$$

Combining the equations,

$$R^* \cdot K + W \cdot L + I^A = R \cdot K + R \cdot A + W \cdot L \text{ iff}$$

$$R^* = R + \frac{R \cdot A - I^A}{K}$$

In conventional growth models, the long–run equilibrium condition for an economy not accumulating capital beyond the so–called "golden rule" level implies

$$R \cdot A > I^A$$

(e.g., Abel *et al.* [1989]).[21] Since less than golden rule accumulation is generally taken to be the empirically relevant case (e.g., Abel *et al.*), we then expect

$$R^* > R \ .$$

Our data from Shiller on *ex post* financial returns from corporate investment shows an average return for 1953–95 of 10.35 percent. Calculating $R^*$ for the same years from

$$\frac{(1 - \tau_T) \cdot GDP^*_T - W_T \cdot L_T}{(1 - \theta) \cdot M_T} \ ,$$

where the numerator is our model's physical capital stock, and then subtracting our Table 1 estimate of $\bar{\delta}$, the average return is indeed larger, 13.21 percent. (Both rates of return are gross of income taxes. If we subtract corporate income taxes, Shiller's return, for instance, drops to 6.77 percent.)

Relative importance of intangible capital How can one measure the stocks of tangible and intangible capital from market data? Our model implies that after a major technological change, the

[20] It follows from aggregation results (12) and (14) in Proposition 1 that one unit of capital of vintage 0 is the equivalent of $P_0 < 1$ units of vintage $\iota$, since both have an equal contribution to output. Then the quality adjusted price of capital good of vintage $i$ (i.e., the amount of consumption that needs to be sold in order to buy the equivalent of one unit of, say, $K_0$ or $A_0$) is $P_{0t}/P_{it}$. See also Laitner and Stolyarov [2002].

[21] Note that in our model, $A = \theta \cdot M$ and $K = (1 - \theta) \cdot M$. Hence, $R \cdot A > I^A$ if and only if $R \cdot K > I^K$.

market value of existing capital precipitously declines. The obsolescence which leads to the decline leaves our model's (quality–adjusted) physical capital stock $K$ below its "book value" — the latter being, for example, Figure 1's $K^*$. Under this interpretation, we can separately determine $K$ and $A$ from Figure 1 only when the stock market is near its peak, because that is when $K^*$ approximates $K$, so that $q^* \approx (A+K)/K$. Figure 1 then suggests that in the U.S., the stock of applied knowledge is 30–50 percent as large as the physical capital stock.

Calibrating our model provides another way of assessing the magnitudes of $K$ and $A$. According to the model, $A/K = \alpha/\beta$. Table 1's estimate of the ratio $\alpha/\beta$ is .48 — implying the stock of intangible capital is 48 percent as large the physical capital stock.[22]

<u>Valuing markets by "historical standards"</u> Because changes in technology interrupt convergence to the steady state (see Figure 2), our model predicts that the long–run average value of $q^*$ always understates $q$. This means it may be treacherous to compare the current $q^*$ to its long–term average in predicting whether the stock market is likely to rise or fall (see, for instance, Smithers and Wright [2000]). For example, if a long time has passed since the last major technology change, stock market values will seem too high by "historical standards."

<u>Is the stock market overvalued?</u> Many commentators suggest that stock prices rose beyond levels of "rational" valuation in the late 1990s. The graphs on Figure 4 show our simulations severely under predict both the level of net worth and rate of return on financial investment after 1995.

We can perform a statistical test of the significance of the discrepancy of data and our model's simulation 1996–2001 in the case of $M$, and 1996–1999 in the case of $r$. As Section 4 explains, our basic instruments disregard net worth and rate of return data after 1995, and our calibrations never use initial–condition values of $M_t$ with $t > 1995$. We now add one additional element to our instrument vector for the first equation, $v_t^1$, and for the sixth equation, $v_t^6$. The new element is 1 for $t > 1995$ and 0 elsewhere. We add a new parameter $\mu^M$, affecting only our first equation, and $\mu^R$, affecting only our sixth equation. Specifically, the new versions of $f_t^1$ and $f_t^6$ are

$$f_t^{1*} \equiv \begin{cases} f_t^1 - \mu^M & t \geq 1996 \\ f_t^1 & \text{otherwise} \end{cases} ,$$

$$f_t^{6*} \equiv \begin{cases} f_t^6 - \mu^R & t \geq 1996 \\ f_t^6 & \text{otherwise} \end{cases} .$$

We then reestimate the specification of Table 1 with 8 parameters, the original 6 plus $\mu^M$ and $\mu^R$. We continue Section 4's procedure of always simulating values of $M_t$, $t > 1996$, from $m(.)$ with initial condition $M_{1995}$. Notice that $\mu^M$ counterbalances the new element for instrument vector $v_t^1$: the new element imposes a new first–moment condition, but the maximization process can adjust $\mu^M$ to make

$$\frac{1}{101-95} \cdot \sum_{t=1996}^{2001} f_t^{1*} \cdot 1 = 0 \ .$$

On the other hand, if one were to impose $\mu^M = 0$, the model would have to fit data 1996–2001 without new help. The same is true for $\mu^R$.

Table 3 presents the estimates for $\mu^M$ and $\mu^R$. In each case, the T–statistic is a test of the hypothesis that the new data points for $M$ and $r$, respectively, are consistent with the model. Evidently, the hypothesis is rejected in both cases. A joint Wald test of $\mu^M = \mu^R = 0$ also strongly rejects.

Table 4 shows the ratio of actual and simulated (from the parameters of Table 1) values of net worth. We can see that the actual value exceeded the simulation by 48 percent in 1999.

[22] The second approach has the advantage of not relying on a constructed time series for $K^*$.

The discrepancy was down to 18 percent by 2001. If indeed market values experienced a bubble, according to our model the necessary correction had run two–thirds of its course by the end of 2001.

In the end, there is an implicit warning in Table 4 as follows: if actual $M$ includes a substantial noise component, simply replacing a conventional measure of capital ($K_t^*$ here) with, say, $(1-\theta)\cdot M_t$, is not without peril. This paper advocates utilizing market data — but in combination with external information.

# 6 Conclusion

We offer a new model of technological progress and economic growth. The model has three fundamental elements. First, inventions that transform production occur sporadically and exogenously. Second, production requires labor, physical capital, and applied knowledge. Third, a seminal invention gives society a new production function with a higher scaling constant (as in traditional treatments), but it also requires new inputs — in particular, new physical capital and applied knowledge.

The first element allows us to incorporate information about seminal inventions from the literature of economic history. Our focus on episodes, initiated by transforming inventions, immediately provides a reason for why the measured rate of technological change varies over different time periods.

The second element makes the stock market value of businesses higher than the value of their physical capital alone. We attempt to measure the volume of intangible capital, and it appears to be about 50% as large as physical capital.

The third element means that society can only realize standard of living gains from a seminal invention after physical and (applied) knowledge investments build a new capital stock embodying the new ideas. Thus even discontinuous changes in technology will tend to lead to continuous advances in living standards. To put it differently, while standard neoclassical models assume finite rates of technological change, our model derives the flow rate of change from an economy's flow rate of investment.

We advocate using a market-based measure of national net worth in studying technological change and growth. Section 5 warns that market valuations can be misleading in some periods. Nevertheless, we argue that uneven technological advance leads to widely and irregularly spaced dates at which existing capital depreciates very rapidly due to obsolescence. Traditional measures of the aggregate capital stock, based on perpetual inventory methods, will miss these dates, but financial–market valuations almost surely will not. Somewhat surprisingly, in our framework the advent of a transforming invention will tend to lead to a precipitous stock market decline. Rapid obsolescence will leave the economy's capital stock below its steady–state level (though output is, at first, unaffected). Convergence toward the new steady–state will follow, with the stock market rising, savers garnering favorable rates of return, and output per person growing. An analyst is left with two data sources for assessing changes in the economy's underlying technology: changes in market valuations of existing capital, and changes in the rate of growth of output.

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# Appendix 1: Data

| Year | Business Fixed Capital and Inventories | Market value of businesses | Ratio of Column 2÷Column 1 |
|---|---|---|---|
| 1953 | 505.10 | 445.04 | 0.88 |
| 1954 | 516.80 | 507.28 | 0.98 |
| 1955 | 552.70 | 569.47 | 1.03 |
| 1956 | 606.80 | 622.66 | 1.03 |
| 1957 | 642.50 | 613.12 | 0.95 |
| 1958 | 662.90 | 719.87 | 1.09 |
| 1959 | 686.30 | 765.00 | 1.11 |
| 1960 | 700.90 | 793.80 | 1.13 |
| 1961 | 719.60 | 907.79 | 1.26 |
| 1962 | 746.80 | 913.76 | 1.22 |
| 1963 | 770.50 | 975.55 | 1.27 |
| 1964 | 811.50 | 1087.03 | 1.34 |
| 1965 | 871.70 | 1214.71 | 1.39 |
| 1966 | 951.70 | 1197.77 | 1.26 |
| 1967 | 1022.10 | 1413.42 | 1.38 |
| 1968 | 1116.00 | 1627.46 | 1.46 |
| 1969 | 1229.00 | 1557.52 | 1.27 |
| 1970 | 1336.80 | 1609.94 | 1.20 |
| 1971 | 1466.40 | 1828.84 | 1.25 |
| 1972 | 1601.30 | 2171.23 | 1.36 |
| 1973 | 1823.10 | 2059.21 | 1.13 |
| 1974 | 2204.00 | 1972.42 | 0.89 |
| 1975 | 2397.40 | 2269.53 | 0.95 |
| 1976 | 2623.30 | 2523.77 | 0.96 |
| 1977 | 2911.70 | 2611.47 | 0.90 |
| 1978 | 3319.50 | 2906.77 | 0.88 |
| 1979 | 3847.30 | 3395.33 | 0.88 |
| 1980 | 4386.60 | 4025.26 | 0.92 |
| 1981 | 4900.70 | 4189.16 | 0.85 |
| 1982 | 5154.80 | 4549.70 | 0.88 |
| 1983 | 5302.40 | 4962.73 | 0.94 |
| 1984 | 5640.10 | 5156.01 | 0.91 |
| 1985 | 5912.90 | 5801.03 | 0.98 |
| 1986 | 6138.20 | 6547.70 | 1.07 |
| 1987 | 6475.00 | 6942.10 | 1.07 |
| 1988 | 6903.60 | 7751.84 | 1.12 |
| 1989 | 7301.40 | 8842.95 | 1.21 |

| Year | Business Fixed Capital and Inventories | Market value of businesses | Ratio of Column 2÷Column 1 |
|---|---|---|---|
| 1990 | 7677.20 | 8696.71 | 1.13 |
| 1991 | 7788.50 | 9833.48 | 1.26 |
| 1992 | 8038.80 | 10255.04 | 1.28 |
| 1993 | 8409.90 | 11060.55 | 1.32 |
| 1994 | 8894.50 | 11308.31 | 1.27 |
| 1995 | 9346.60 | 13592.80 | 1.45 |
| 1996 | 9778.70 | 15742.74 | 1.61 |
| 1997 | 10303.30 | 19089.90 | 1.85 |
| 1998 | 10783.40 | 22015.44 | 2.04 |
| 1999 | 11410.30 | 26184.50 | 2.29 |
| 2000 | 12218.40 | 25183.08 | 2.06 |
| 2001 | NA | 23275.62 | NA |

Table A1: Market Value of Businesses and the Stock of Reproducible Capital (billions US dollars).

Source: Column 1: Herman [2001, tab.1, Nonresidential Private Fixed Assets] plus National Income and Product Accounts current dollar business inventories. Column 2: U.S. Flow of Funds

(http://www.federalreserve.gov/releases/z1/Current/data.htm) Table L.100, row 1; minus L.100, row 25; minus L.106, row 15, and L.105, row 18; plus L.105, row 7, and row 10; plus L.108, row 10; minus L.106, row 14, and L108, row 15; plus L.107, row 1; minus L.107, row 23.

# Appendix 2: Proofs

**Proof of Proposition 1:**

Take any technology $j < \iota(t)$ and any date $t \geq t_j$. The time subscript will be dropped for more compact notation. Let

$$Y_j = Z_j A_j^{\alpha} K_j^{\beta} L_j^{1-\alpha-\beta}, \text{ all } j \leq \iota(t) \tag{28}$$

The first order conditions for profit maximization read

$$\alpha \frac{Y_\iota}{A_\iota} = R \tag{29}$$

$$\beta \frac{Y_\iota}{K_\iota} = R \tag{30}$$

$$\alpha \frac{Y_j}{A_j} = P_j R \tag{31}$$

$$\beta \frac{Y_j}{K_j} = P_j R \tag{32}$$

$$(1-\alpha-\beta)\frac{Y_j}{L_j} = (1-\alpha-\beta)\frac{Y_\iota}{L_\iota} = W \tag{33}$$

Using the expression for the production function (28), first order condition (33), and from (**??**)–(32) the fact that

$$\frac{A_j}{K_j} = \frac{A_\iota}{K_\iota} = \frac{\alpha}{\beta},$$

we obtain the following relationship between the capital labor ratios $\frac{K_j}{L_j}$ and $\frac{K_\iota}{L_\iota}$:

$$1 = \frac{Y_j/L_j}{Y_\iota/L_\iota} = \frac{Z_j \left(\frac{A_j}{K_j}\right)^\alpha \left(\frac{K_j}{L_j}\right)^{\alpha+\beta}}{Z_\iota \left(\frac{A_\iota}{K_\iota}\right)^\alpha \left(\frac{K_\iota}{L_\iota}\right)^{\alpha+\beta}} = \frac{Z_j \left(\frac{K_j}{L_j}\right)^{\alpha+\beta}}{Z_\iota \left(\frac{K_\iota}{L_\iota}\right)^{\alpha+\beta}}. \tag{34}$$

Dividing (32) by (30), and using (33) and (34),

$$P_j = \frac{Y_j/K_j}{Y_\iota/K_\iota} = \frac{Y_j/L_j}{Y_\iota/L_\iota}\frac{K_\iota/L_\iota}{K_j/L_j} = \left(\frac{Z_j}{Z_\iota}\right)^{\frac{1}{\alpha+\beta}} < 1.$$

This immediately implies

$$I_{jt}^A = 0,\ I_{jt}^K = 0 \text{ for all } j < \iota(t)$$

Since

$$\frac{A_j}{K_j} = \frac{\alpha}{\beta}$$

at all times, the laws of motion for capital imply the same ratio for investments

$$\frac{I_j^A}{I_j^K} = \frac{\alpha}{\beta}.$$

Therefore, from the market clearing condition,

$$I_j^A = \begin{cases} \frac{\alpha}{\alpha+\beta}\sigma Y, & j = \iota \\ 0, & j < \iota \end{cases}, \qquad I_j^K = \begin{cases} \frac{\beta}{\alpha+\beta}\sigma Y, & j = \iota \\ 0, & j < \iota. \end{cases}$$

Integrating investment with respect to time yields (10) and (11). This also implies (13).

We now turn to aggregating the output. From (34),

$$P_j \frac{K_j}{L_j} = \frac{K_\iota}{L_\iota}.$$

Similarly,

$$P_j \frac{A_j}{L_j} = \frac{A_\iota}{L_\iota}$$

Therefore,

$$\sum_{j=0}^{\iota} \frac{P_j K_j}{L} = \sum_{j=0}^{\iota} \frac{P_j K_j}{L_j}\frac{L_j}{L} = \frac{K_\iota}{L_\iota}$$

and

$$\sum_{j=0}^{\iota} \frac{P_j A_j}{L} = \sum_{j=0}^{\iota} \frac{P_j A_j}{L_j}\frac{L_j}{L} = \frac{A_\iota}{L_\iota}\ .$$

Using (9), (12), (28), and the above expressions,

$$Y = \sum_{j=0}^{\iota} Y_j = \sum_{j=0}^{\iota} Z_\iota P_j^{\alpha+\beta} A_j^\alpha K_j^\beta L_j^{1-\alpha-\beta} = \sum_{j=0}^{\iota} Z_\iota \left(\frac{P_j A_j}{L_j}\right)^\alpha \left(\frac{P_j K_j}{L_j}\right)^\beta L_j$$

$$= Z_\iota \left(\frac{A_\iota}{L_\iota}\right)^\alpha \left(\frac{K_\iota}{L_\iota}\right)^\beta L = Z_\iota \left(\sum_{j=0}^{\iota} \frac{P_j A_j}{L}\right)^\alpha \left(\sum_{j=0}^{\iota} \frac{P_j K_j}{L}\right)^\beta L$$

$$= Z_\iota \left(\sum_{j=0}^{\iota} P_j A_j\right)^\alpha \left(\sum_{j=0}^{\iota} P_j K_j\right)^\beta L^{1-\alpha-\beta} = Z_\iota A^\alpha K^\beta L^{1-\alpha-\beta}.$$

From (31) and (32)

$$\alpha \sum_{j=0}^{\iota} Y_j = R \sum_{j=0}^{\iota} P_j A_j, \qquad \beta \sum_{j=0}^{\iota} Y_j = R \sum_{j=0}^{\iota} P_j K_j \ ,$$

so that

$$(\alpha + \beta) Y = R (A + K) \ .$$

Finally, from (33),

$$(1 - \alpha - \beta) \cdot Y = (1 - \alpha - \beta) \cdot \sum_{j=0}^{\iota} Y_j = W \cdot \sum_{j=0}^{\iota} L_j = W \cdot L \ .$$

■

# Appendix 3: Data Sources for Section 4

This appendix presents the data sources for Section 4's calculations.

Equation 1. $M_t$ is private, nonresidential net worth from the U.S. Flow of Funds — see Appendix 1 — divided by the NIPA personal consumption chain price index (NIPA table 7.1, row 7). All NIPA data comes from

$$http: ///www.bea.doc.gov/bea/dn/nipaweb/SelectedTables.asp$$

$L_t$ is millions hours worked by full and part–time employees of domestic industries (NIPA table 6.9B & C, row 2).

Equation 2. GPD* is nominal GDP (NIPA table 1.1, row 1); less housing services (NIPA table 2.2, row 14); divided by personal consumption chain price index (NIPA table 7.1, row 7). See text.

Equation 3. $I^K$ is fixed nonresidential investment (NIPA table 1.1, row 8); plus change in private inventories (NIPA table 1.1, row 12); divided by personal consumption chain price index (NIPA table 7.1, row 7).

Equation 4. $\tau^*$ is indirect business taxes (NIPA Table 1.9, row 13), divided by final sales to domestic purchasers (NIPA table 1.5, row 6). We use compensation of employees (NIPA table 1.14, row 2), proprietor's income (NIPA table 1.14, row 9), and compensation of employees of proprietorships (NIPA table 1.15, row 13). We construct $NI_t^*$ from $GDP_t^*$ (see above) less consumption of nonresidential fixed capital (NIPA table 5.2, row 8), less indirect business taxes (NIPA Table 1.9, row 13). Then

$$\frac{W_t \cdot L_t}{GDP_t^*} = \frac{\text{compensation of employees}_t - \text{compensation of employees of proprietorships}_t}{NI_t^* - \text{compensation of employees of proprietorships}_t - \text{proprietor's inocme}_t} \cdot \frac{NI_t^*}{GDP_t^*}$$

Equation 5. $D$ is consumption of nonresidential fixed capital (NIPA table 5.2, row 8), divided by personal consumption chain price index (NIPA table 7.1, row 7).

Equation 6. $r$ is as follows. For each $t$, let $d_1$ be U.S. Flow of Funds

$$http://www.federalreserve.gov/releases/z1/Current/data.htm$$

nonfarm, nonfinancial corporate credit market instruments (L.102, row 21), divided by nonfarm, nonfinancial corporate market value equity (L.102, row 41). Setting

$$d_2 \equiv \frac{d_1}{d_1 + 1} \; ,$$

$d_2$ is debt as a fraction of debt plus equity. Then at each time $t$,

$$r = d_2 \cdot \text{interest} + (1 - d_2) \cdot \text{equity} + (1 - d_2) \cdot \text{corp. tax} - \text{inflation} \, ,$$

where "interest" is the 6–month nominal interest rate on prime commmercial paper, series 4 from

$$http://www.econ.yale.edu/\tilde{}shiller/data/chapt26.html$$

"equity" is percent appreciation in average share price plus dividend divided by share price (series 1–2 same source); "corp tax" is the same dividend, times the NIPA corporate profits tax liability (NIPA table 1.14, row 23), divided by NIPA aggregate dividends (NIPA table 1.14, row 25); and, "inflation" is percent rate of inflation for the NIPA personal consumption chain price index (NIPA table 7.1, row 7).

**Table 1. Method of Moments:**

**Starting Date 1953; Revolution Date 1973; Ending Date 2001**

| Parameter | Stage 1 Value | Stage 2 Value | Stage 2 Std. Error | Stage 2 T–Stat |
|---|---|---|---|---|
| Estimated Parameters | | | | |
| $\delta$ | .0393 | .0548 | .0121 | 4.5361 |
| $\alpha$ | .0418 | .0980 | .0361 | 2.7147 |
| $\beta$ | .2358 | .2025 | .0177 | 11.4720 |
| $\sigma$ | .1455 | .1754 | .0210 | 8.3621 |
| $Z_0$ | 12.0435 | 12.1334 | .2216 | 54.7424 |
| $Z_1$ | 15.7357 | 16.1431 | .3450 | 46.7916 |
| Calculated Values | | | | |
| $\overline{\delta}$ | .0599 | .0752 | | |
| $1-\alpha-\beta$ | .7224 | .6995 | | |
| $\theta$ | .1506 | .3261 | | |
| $(Z_0/Z_1)^{1/(\alpha+\beta)}$ | .3817 | .3866 | | |
| Summary Numbers | | | | |
| $S(.)$ | .1855 | 17.9342 | | |
| years | 49 | 49 | | |
| Test of Overidentifying Restrictions | | | | |
| p-value $\chi^2(S(.),15)$ | | .7339 | | |

**Table 2. Method of Moments:**

**Starting Date 1953; Revolution Date 1974; Ending Date 2001**

| | Stage 1 | Stage 2 | | |
|---|---|---|---|---|
| Parameter | Value | Value | Std. Error | T–Stat |
| Estimated Parameters | | | | |
| $\delta$ | .0623 | .0717 | .0102 | 7.0402 |
| $\alpha$ | .1152 | .1442 | .0237 | 6.0925 |
| $\beta$ | .1864 | .1754 | .0095 | 18.4826 |
| $\sigma$ | .1888 | .2061 | .0160 | 12.9143 |
| $Z_0$ | 12.5923 | 12.1265 | .2959 | 40.9810 |
| $Z_1$ | 16.4858 | 15.9759 | .4719 | 33.8538 |
| Calculated Values | | | | |
| $\bar{\delta}$ | .0820 | .0910 | | |
| $1-\alpha-\beta$ | .6984 | .6804 | | |
| $\theta$ | .3819 | .4512 | | |
| $(Z_0/Z_1)^{1/(\alpha+\beta)}$ | .4093 | .4221 | | |
| Summary Numbers | | | | |
| $S(.)$ | .1631 | 17.1294 | | |
| years | 49 | 49 | | |
| Test of Overidentifying Restrictions | | | | |
| p-value $\chi^2(S(.),15)$ | | .6888 | | |

**Table 3. Method of Moments Estimates of $\mu^M$ and $\mu^R$:**

**Starting Date 1953; Revolution Date 1973; Ending Date 2001**

| | Stage 1 | Stage 2 | | |
|---|---|---|---|---|
| Parameter | Value | Value | Std. Error | T–Stat |
| Estimated Parameters | | | | |
| $\mu^M$ | .2617 | .2412 | .0454 | 5.3158 |
| $\mu^R$ | .1044 | .0980 | .0223 | 4.6883 |
| Wald Test of $\mu^M = \mu^R = 0$ | | | | |
| p-value $\chi^2(S(.), 2)$ | 1.0000 | | | |

**Table 4. Actual and Simulated Net Worth and Rate of Return**

| Year | Actual M/Simulated M | Actual r/Simulated r |
|---|---|---|
| 1996 | 1.0310 | 2.2693 |
| 1997 | 1.1840 | 2.4804 |
| 1998 | 1.3047 | 2.7404 |
| 1999 | 1.4756 | 1.3215 |
| 2000 | 1.3391 | NA |
| 2001 | 1.1765 | NA |