TECHNOLOGY LICENSE SHARING STRATEGY FOR REMANUFACTURING INDUSTRIES UNDER A CLOSED-LOOP SUPPLY CHAIN MANAGEMENT BONDING

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Abstract. Remanufacturing is getting attention nowadays due to increasing waste and corresponding emissions. One of the important factors of remanufacturing is the quality of the remanufactured products. The collection and distribution of used products require proper management. Based on this situation, this study discusses a hybrid closed-loop supply chain management in cooperation with a hybrid production system. The vendor comes up with the policy of sharing remanufacturing responsibility by sharing the technology license with other supply chain players. The carbon cap restricts emissions from the entire hybrid production system of the vendor. Other factors of this proposed study are service by the retailer and quality, gift policy, and customer awareness by the vendor. This study examines the scenario under random market demand. Classical optimization provides the solution under the Stackelberg game policy where the vendor acts as leader and the retailer & third party act as followers. This paper considers two scenarios: Scenario A for a continuous distribution and Scenario B for no specific distribution. A comparison is drawn between various motivating factors-based policies to control supply chain management.

1. Introduction

Nowadays, industries are sharing responsibilities for environmental improvement along with their supply chain participants. Carbon emissions from the production sector are one of the sources of pollution. Different policies are adopted by governments [2] as well as industries. But, the interesting fact is that environmental

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improvement is contributing to the economy though. Mentioning that, responsibility sharing policy is explored in this study. With consumer satisfaction, industries are leaning towards a customer-oriented green policy that helps to increase customer demand.

Various motivating factors have been discussed in the closed-loop supply chain management (CLSCM) for the growing market demand as the carbon tax, gift policy, and remanufacturing. Remanufacturing has a big role in the CLSCM that deals with remanufacturing of used products. The vendor allows the other supply chain players to remanufacturer the collected used products. This responsibility sharing happens in only one condition the supply chain player should maintain the quality of products as same as the vendor. This cooperation between supply chain management (SCM) players for remanufacturing has a huge impact on both economy and the environment [54]. Whenever there is a competition among SCM players, a game starts for decision making on behalf of the SCM.

Remanufacturing requires the repair or replacement of obsolete components of the used product. Spare parts subject to quality affect the performance of the product or the expected lifetime of the product changes. Here, the responsibility sharing for remanufacturing works. The vendor gives licenses to other supply chain players for remanufacturing. One of the benefits of sharing technology licenses is that it reduces the use of transportation. For example, suppose Company A situates at place A and Company B situates at place B. Company A does not have to carry all used products from place B to place A. Already Company B situates at place B. Then, Company A just can share the license with Company B and Company B can remanufacture used products with quality as good as Company A. As a result, extensive transportation can be saved as well as emissions. This research works helps to survey the following research quarries:

– How cooperation between all SCM players works for responsibility sharing? How customer awareness is effective under the circumstances of remanufacturing and responsibility sharing?
– How does gift policy affect the SCM when the vendor gives license to other players for remanufacturing used products by leveling the vendor's license?
– How does stochastic demand affect the management when the demand depends on customer awareness, gift policy, and quality of remanufactured products?

The rest of the paper is structured as Section 2 consists of a literature review, Section 3 consists description of the problem, assumptions, and notation; after that, Section 4 introduces the proposed model under two scenarios, Section 5 introduces solution methodology, Section 6 explores numerical examples and managerial insights, and finally, Section 7 gives conclusions, respectively.

2. LITERATURE REVIEW

Some previous authors have focused on product quality and suggested coordination agreements, but by considering different collecting and remodelling propositions for greening. This research concentrates on quality improvemeffortsfort with sharing remanufacturing responsibility, gift policy, selling price dependent market demand, and carbon tax for carbon discharge declination to generate more market demand. Some surveys on this essence are assessed in this section.

2.1. Distribution-free approach

Moon and Gallego [32] established some important applications of distribution-free approach on various inventory models. Pinto *et al.* [37] expanded a distribution-free model for managing supplier delivery risk under limited information. They showed the distribution-free approach was very effective when the available data did not follow any specific distribution. Pal *et al.* [35] analyzed a newsvendor problem with distribution-free approach and non-linear holding cost. Moon and Choi [31] developed a continuous review inventory model with lead time as one of the decision variables under service level constraints.

2.2. Selling price dependent demand

For the first time, Kotler and Zaltman [23] introduced the idea of a business strategy for decision making. They introduced the relationship between price and the economic order quantity (EOQ) inventory model. An EOQ inventory model with variations in the effect of selling price had been investigated by Ladany and Sternlieb [26]. Bhuniya *et al.* [4] introduced an EOQ model with the variable selling price. Again, Sepehri *et al.* [49] introduced a model, where the selling price influenced customers for purchasing items. Choosing an optimal pricing strategy for new and renewed products can help the manufacturer to achieve maximum profit. The retailer sells new products and collects returnable products, which are shipped for recycling and used as raw materials for other products. Retailers need to invest in branding and pricing for new and remanufactured products. Studies had shown that increasing unit purchasing and remanufacturing costs of used products increased the quality and decreases the brand investment. Kumar *et al.* [24] confirmed that the increasing cost of advertising provided a continuous increasing value of the total cost.

2.3. Production model

The study of defective production systems and the quality of products with unreliable machines had been considered in many papers so far. Sarkar *et al.* [42] proved that SCM was effective for variable production and maximized the profit of the supply chain. Sarkar *et al.* [47] simultaneously considered environmental risks associated with innovative green product manufacturing, while both the return rate and the demand were random with insufficient information on the distribution function. Rosenblatt and Lee [39] considered an economic production cycle and found the effect of the degradation of the traditional economic manufacturing quantity (EMQ) model on the imperfect production process. Porteus [38] discussed optimal lot-sizing in an imperfect production process, improving process quality, and reducing setup costs. Taleizadeh *et al.* [54] discussed that the SCM was effective for improving product quality and maximizing profits of the supply chain. Kang *et al.* [21] considered the effects of imperfect production for work-in-process inventory.

2.4. Remanufacturing

It is very difficult to make 100% perfect products by maintaining a sustainable manufacturing system [2]. Garai and Sarkar [13] proposed a model that simultaneously considered the production of second-generation biofuel from used products in the reverse-chain and the re-manufacturing of casing soil as returnable items in the manufacturing-chain to address the environmental concerns. That ensured long-term sustainability within the SCM. Every manufacturer should be responsible to save the environment when they made renewable items, namely biofuel. Sarkar *et al.* [46] discussed a model for biofuel manufacturing from renewable biomass through a smart manufacturing system. The biofuel was an important alternative to fossil fuels, which helped to decrease dependability on fossils fuel and decreased carbon emissions. Sarkar *et al.* [45] proposed a model that aimed to reduce waste by reworking defective products and maximizing profit. The uncertain situation within a production-inventory model was discussed by Mahapatra *et al.* [29]. Minner and Lindner [30] introduced a model that collectively optimized the production order of new and remanufactured batches, as well as the size and number of those batches. They allowed multiple setups within the same production cycle for both new and remanufactured items. Taleizadeh *et al.* [52] discussed a multi-item production system to redefine defective item policy. Considering the level of service and budget constraints, they found the total cost which was globally minimum. Shi *et al.* [50] introduced maximum profit for uncertain income. They reworked the items and sold them in the market as brand new. Another noteworthy matter was that the introduction of a smart manufacturing framework rose the percentage of green investment utilization [40].

2.5. Inventory management

A sustainable inventory management model under controllable carbon emissions from by reducing pollution was developed by Yadav *et al.* [56]. An animal fat-based renewable supply chain model was recently designed by Habib *et al.* [17], where the SCM was solved by robust possible programming. Still, the authors did not

take the optimum selling price of biofuel and variable production rate. In the make-to-order policy, the vendor produced items in case of insufficient inventory in storage and sent them to the retailer, which was discussed by Mahajan *et al.* [28]. Sarkar *et al.* [41] discussed a model where inventory holding costs consisted mainly of two components, financial and operational. In the traditional system, the total inventory holding cost was paid by the retailer. in consignment stock, the inventory holding cost of the retailer was paid by the vendor. Adida and Ratinsoontorn [1] analyzed how competition among retailers affects supply chain decisions and profits under different price agreements. Again, Corbet and DeCroix [7] created a supply chain model with asymmetric information including consignment stock, cycle time, and safety stock. Gerchak and Khmelnitsky [14] had tested a consignment policy, where there was no opportunity to verify the retailer's sales report. Braglia and Zavanella [5] worked on an industry strategy for managing supply chains and inventory, including consignment stock. Yi *et al.* [57] had developed an integrated inventory model with retailer's space constraints and controllable lead time under the consignment stock policy. Yu *et al.* [58] considered a single-vendor multi-retailer consignment model and solved those models with a uniform and normal distribution. As'ad *et al.* [3] discussed a model on the consignment agreement and the CLSC was adopted due to its economic and environmental benefits. Murmu *et al.* [34] discussed a production-inventory model for perishable items by considering a manufacturing facility and warehouse.

2.6. Closed-loop supply chain management

In commerce, SCM is the management of the flow of items and services between SCM players. This included the movement and storage of raw materials, work-in-process inventory, finished goods as well as end to end order fulfillment from point of origin to point of consumption. Interconnected, interrelated, or interlinked networks, channels, and node businesses combine for supplying products and services required by end customers in a supply chain. The study of Chung *et al.* [11] had shown a multi-unit structure, where the industry was restored to a reconditioning facility and then given back to the retailer. They analyzed an integrated system consisting of a single supplier, a manufacturer, a retailer, and a third party. Lee *et al.* [27] discussed the benefit of carbon emissions during manufacturing and transportation. Multi-objective optimization in logistics problem for multi-product supply chain network model was introduced by Gupta *et al.* [16]. But the authors did not consider the rate of manufacturing and the impact of power for renewable items.

Jaber *et al.* [20] first considered a two-echelon vendor-retailer CLSC with a consignment stock strategy, where a batch was delivered to the retailer as soon as the production was completed. Although they had created mathematical models for equal and unequal batch sizes, the sequencing of batches was considered a priority where m remanufactured batches were produced first and then n batches were produced. The management should take care of a sustainable production system with variable reliability [33]. The transportation of biofuel for a smart manufacturing system played an essential role in a bioenergy supply chain network [18]. By utilizing a smart manufacturing [45], the defective products could be reduced. A partial backorder was studied by Park [36] while backordering was purchasing-price depended. Sarkar *et al.* [44] discussed a artificial neural network with multithreading as a solution policy. Sarkar *et al.* [43] discussed a CLSCM for waste reduction in a single-stage supply chain. They discussed the effectiveness of reducing carbon emissions and the total cost of the two-stage supply chain. Choi *et al.* [10] described a model where the selling price of the product varied with different channels. That helped to determine the demand for the product for the entire supply chain. Vandana *et al.* [51] discussed a green SCM for agile manufacturing. They used a trade-credit policy for maintaining the relationship between both SCM players.

2.7. Customer awareness and gift policy

In this paper, customer awareness is the more important part to buy that product. By awareness, customers are easily able to know about their consumer rights. There are no such disadvantages to customer awareness. In this present situation, awareness is necessary for production planning. Here, in this paper, the market demand for products is depending on customer awareness. Several researcher developed models where demand depends on

TABLE 1. Author(s) contributions table based on literature.

Author(s)	DF approach	Production rate	Customer awareness	Remanufacturing	Gift policy
Hariga *et al.* [19]	−	−	−	√	−
Sarkar *et al.* [41]	−	−	−	√	−
Khouja *et al.* [22]	−	−	−	−	√
Kugele *et al.* [25]	−	√	−	−	−
As'ad *et al.* [3]	−	−	−	√	−
Bhuniya *et al.* [4]	−	√	−	√	−
Sarkar and Bhuniya [40]	−	√	−	√	−
Yadav *et al.* [56]	−	√	−	√	−
Sarkar and Bhuniya [40]	−	√	−	√	−
Moon *et al.* [33]	−	√	−	√	−
This model	√	√	√	√	√

Notes. DF-Distribution-free.

customer awareness. Sen and Bhattacharya [48] analyzed a model where they focused on the customer channel and examined the role of customer awareness in promoting the firm's sales and its financial performance. Cárdenas-Barrón [6] introduced sustainable practices for maximum profit.

A gift policy is an effective approach to inventory management. Here, the vendor invests for the purpose of an additional gift policy for promoting products through the SCM. Generally, through this policy, the market demand for the product increases. In this policy, the vendor gives an additional gift voucher for a certain amount of products sold to the retailer. This policy provides information and guidelines for employees and the company's expectations when those vouchers are accepted by retailers. Gifts can be used to build goodwill between the vendor and retailer. In this policy, the vendor acts as a company that does business with the SCM players. Khouja *et al.* [22] analyzed that the gift policy was more effective in SCM to increase profit.

2.8. Research gap

Different researchers considered many models, where demand was depended on selling price and demand was depended on customer awareness. Here, this study considers a comparison between the continuous distribution and the distribution-free approach, where the vendor shares the technology license with the SCM players. Table 1 gives a glimpse of recent researches and defines the research gap. Here, this model distinguishes the optimal decision procedure of supply chain and inspect the results in two scenarios based on stochastic approach and uses a competitive scenario under game policy. Based of authors knowledge, this is the very first time in the literature that discusses two different stochastic scenarios under technology license.

3. PROBLEM DESCRIPTION, NOTATION, AND ASSUMPTIONS

The problem description is explained in this section along with notation, and assumptions to understand the research problem.

3.1. Problem description

A vendor, retailer, and 3PL form a hybrid CLSCM. The 3PL collects used products from the market at a unit price j (Fig. 1). 3PL remanufactures x portion of the total collected used product $R(j, k)$ and sends remaining $(1 - x)$ portion to the vendor at a unit price B. The vendor gives the license to the 3PL to remanufacturer products on behalf of maintaining the same quality as the vendor. The vendor remanufactures the remaining $(1-x)$ portion of the collected used products. Now, the retailer purchases remanufactured products from the 3PL and vendor. That is, the retailer buys new and remanufactured products from the vendor. Investments for quality

FIGURE 1. Proposed CLSCM under technology license sharing policy among SCM players.

improvement and gift policy are used by the vendor. All CLSCM players are environment conscious and they share the remanufacturing responsibility subject to the condition that the quality of remanufactured products will be the same as the vendor's product quality. This remanufacturing-cooperation between SCM players has an effect on their decentralized decision. This complex concept of cooperative-decentralized decision-making is explored in this study. A portion of the market demand of the retailer is fulfilled by the remanufactured products. Along with the hybrid CLSCM, the vendor operates a hybrid production system for the production process of new products and remanufacturing process of used products. The vendor has the carbon cap G_v for carbon emissions from the hybrid production system. Thus, the carbon tax is only payable for emitting more carbon into the environment than the carbon cap.

3.2. Notation

Following notation is used to understand the mathematical model.

3.3. Assumptions

To construct the mathematical model, the following assumptions are used.

(1) A single-period hybrid CLSCM consists of a single-retailer, single-3PL, and single-vendor. The vendor produces a single type of product using a hybrid production system consisting of both production and remanufacturing facilities. The vendor assembles and transports products without adequate inventory to meet the retailer's demand [28].
(2) The following condition $c_m \leq w \leq p$ should hold in order to ensure the convergence and profit of the system. The retailer's demand is fulfilled by both new and remanufactured products. To ensure the convergence of the reverse chain, $c_1, c_2 < c_m$ should follow. The vendor's unit production cost is a function of production rate of new products p_r as $c_m = \left(\frac{b_1}{p_r} + b_2 p_r \right)$.
(3) The 3PL collects all used products from the market. Only a fraction (x) of collected used products are remanufactured by the 3PL. Rest fraction $(1 - x)$ is remanufactured by the vendor. $x = 0$ indicates that 3PL does not participate in remanufacturing and $x = 1$ implies that 3PL remanufacturers all collected used products [55].

Parameters	
h_r	Retailer's holding cost (\$/unit/unit time)
s_r	Retailer's shortage cost (\$/unit)
$p_{\max}$	Maximum selling price of the product (\$/unit)
$p_{\min}$	Minimum selling price of the product (\$/unit)
w	Unit wholesale price the vendor (purchasing cost of retailer) (\$/unit)
b	Scaling parameter of demand for customer awareness (>0)
ρ	Shape parameter of demand for customer awareness (>0)
c_m	Unit production cost of vendor (\$/unit)
c_1	Unit remanufacturing cost of vendor (>0)
c_3	Unit remanufacturing cost of 3PL (>0)
T_1	Savings by the vendor from unit remanufactured product (\$/unit)
T_3	Savings by the third party from unit remanufactured product (\$/unit)
x	Portion of used products remanufactured by the retailer and 3PL (%)
a_0	Potential market size (units)
a_1	Scaling parameter of demand associated with production cost (>0)
a_2, a_3	Scaling parameters of demand for carbon emission reduction rate (>0)
a_4, a_5	Scaling parameters of demand for purchasing cost of used product (>0)
a_6, a_7	Scaling parameters of demand for quality improvement (>0)
a_8, a_9	Scaling parameters of demand for gift policy (>0)
a	Scaling parameter of demand for service (>0)
γ	Shape parameter of demand for service (>0)
l	Cost of technology license sharing (\$/unit)
c	Scaling parameter of purchasing cost for collected used products (>0)
μ	Constant portion of collected used products (units)
y	Scaling parameter for quality improvement (>0)
G_v	Carbon cap permit (gallons)
P_v	Carbon tax for unit carbon emission (\$/gallon)
M	Ordered quantity of retailer (units)
ϵ	Random market demand (units)
X	Random variable associated with the random demand ϵ
ζ	Mean of X (units)
σ_1	Standard deviation of X
b_1, b_2	Unit cost of vendor for production of new product (\$/unit)
α	Service investment coefficient of retailer (\$)
θ	Carbon emissions investment coefficient reduction of vendor (\$)
ϕ	Quality improvement investment coefficient reduction of vendor (\$)
η	Scaling parameter of carbon tax for vendor
β	Customer awareness investment coefficient of vendor (\$)
σ	Gift policy investment coefficient of vendor (\$)
Decision	**Variables**
v	Reducing carbon emission rate (>0) (vendor)
z	Customer awareness of the (>0) (vendor)
k	Quality improvement of remanufactured product (>0) (vendor)
p_r	Production rate of new product of the (unit/unit time) (vendor)
q	Gift policy (>0) (vendor)
j	Unit purchasing cost used product of 3PL (\$/unit) (3PL)
I	Inventory level (units) (retailer)
p	Selling price of product (\$/unit) (retailer)
s	Service for customer (>0) (retailer)
Others	
$R(j, k)$	Amount of collected used products (units)
$D(v, p, j, k, s, z, q)$	Market demand of product (units)
$E[M]$	Expected ordered quantity (units)
$f(\cdot)$	Probability distribution function
$F(\cdot)$	Cumulative distribution function
$(\cdot)^+$	Positive quantity among 0 and $(\cdot)$
π_r^U	Retailer's profit for uniform distribution function (\$)
π_t^U	3PL's profit (\$)
π_m^U	Vendor's profit for uniform distribution function (\$)
π_r^{DF}	Retailer's profit for distribution-free approach (\$)
π_m^{DF}	Vendor's profit for distribution-free approach (\$)

(4) The Stackelberg game policy is used to maximize the profit of each CLSCM player individually. The vendor plays the dominant role as Stackelberg leader and retailer & 3PL act as Stackelberg followers. Meanwhile, the vendor has cooperation with 3PL by sharing a technology license [54].

(5) Carbon emissions from the hybrid production system are controlled by a carbon cap. The vendor pays carbon tax when the emissions exceed the projected carbon cap. Investment for carbon reductions is used to reduce the carbon tax [9].

(6) The vendor uses additional investments for quality improvement, customer awareness, and gift policy whereas the retailer invests for additional service to customers.

The next section gives a detailed explanation of the mathematical model based on these assumptions.

4. MODEL FORMULATION

Market demand is stochastic and it has two segments: one is variable and another one is random. Variable demand segment is dependent upon decision variables as the selling price of the product (p), quality improvement (k), carbon emissions reduction (v), purchasing price of collected used products (j), service (s), customer awareness (z), and gift policy (q). The random demand is denoted as ϵ. The demand is expressed as

$$D(v, j, k, q, p, s, z) = a_0 + a_1 \frac{p_{\max} - p}{p - p_{\min}} + a_2 v + a_3 v^2 + a_4 j + a_5 j^2 + a_6 k + a_7 k^2$$
$$+ a s^\gamma + b z^\rho + a_8 q + a_9 q^2 + \epsilon. \tag{4.1}$$

The random demand has the random variable X that follows uniform distribution, *i.e.*, $X \sim U[k_2, k_1]$. For random demand ϵ, an additional inventory level of the retailer is added as I such that either $E[X] > I$ or $E[X] < I$.

Due to remanufacturing and $c_1, c_2 < c_m$, savings of the vendor from per unit remanufacturing cost is

$$T_1 = \frac{b_1}{p_r} + b_2 p_r - c_1 \tag{4.2}$$

and savings of the retailer (T_3) from per unit remanufacturing cost is

$$T_3 = \frac{b_1}{p_r} + b_2 p_r - c_2. \tag{4.3}$$

Now, the collected used products $(R(j, k))$ is variable and is a function of unit purchasing cost of used products (j) and the quality of the product (k) [54]. It has a fixed market size μ. $R(j, k)$ is shown in Equation (4.4) as

$$R(j, k) = \mu + cj - yk. \tag{4.4}$$

The scaling parameter y has a negative impact on the collection of used products. More good quality products imply less collection of used products.

4.1. Scenario A: Uniform distribution model

In the traditional policy, the cost of carrying the total inventory is borne by the retailer. The retailer buys the total lot from the seller at the wholesale price, retains ownership of it, and sells the item to the customer at the sale price. The seller takes no responsibility for the product and does not incur any cost to retain the item after delivery at the retailer's end.

4.1.1. Retailer

In this case, the total account holding amount is obtained by the retailer. The retailer buys the total lot from the seller at the wholesale price, retains ownership of it, and sells the item to the customer at the sale price. The seller assumes no responsibility for the product and does not incur any costs for retention after the item is delivered to the retailer's end. The total cost of the retailer is given below: The distributor takes major roles in the supply chain to product selling and helps to maximize profit in the supply chain. The distributor always gives holding costs and shortage costs and receives retrieve the value. In two scenarios, the retailer remanufactures the used products as new by his technical ability along with the vendor and sells them in the market to earn a profit margin along with new products. To increase the demand for products in the market the distributor expands customer awareness investment costs. Here distributor's optimal profit is calculated by subtracting all his costs from his revenue.

Revenue of the retailer (RN)

$E(M)$ is the expected amount of sold out products, *i.e.*, the retailer sells this amount of products in market. This can be calculated by the following equation.

$$E[M] = \int_0^M Df(D)\mathrm{d}D + [1 - F(M)]M = M - \int_0^M F(D)\mathrm{d}D. \tag{4.5}$$

Now, the demand $D(v, p, j, k, s, z, q)$ has two segments: one is variable in nature and another is random in nature (ϵ). During the use of probability theory, the variable segment of the demand has no effect. Only random demand segments will change. That is why M can be replaced by inventory level I of the retailer for integration to avoid the complexity [54]. The random demand segment is calculated with respect to this inventory level I. Then Equation (4.5) can be written as

$$E[M] = M - \int_0^I F(\epsilon)\mathrm{d}\epsilon. \tag{4.6}$$

Hence, the expected revenue of the retailer is $\mathrm{RN} = pE[M]$.

Purchasing cost (PC)

M is the total ordered quantity of the retailer. This M quantity is fulfilled from both the new products and remanufactured products. The retailer purchases new products from the vendor and remanufactured products from both the vendor and 3PL. But, as all the remanufactured products have similar quality as new products, the unit purchasing cost for all products is the same. If w is the unit purchasing cost, then the total purchasing cost is $\mathrm{PC} = wM$.

Holding cost (HC)

Retailer holds products if there are some products that are still in inventory system. h_r is the unit holding cost of those products which are not sold yet in market [47, 54]. The expected leftover inventory $L[M]$ can be calculated as

$$L[M] = (M - D)^+. \tag{4.7}$$

Thus, the expected holding cost is $\mathrm{HC} = h_r L[M]$.

Shortage cost (SC)

The retailer pays a shortage cost only when a shortage occurs. Unit shortage cost of the retailer is s_r [47, 54]. The expected amount of shortage quantity is $S[M]$ is

$$S[M] = (D - M)^+. \tag{4.8}$$

Thus, expected shortage cost of the retailer is $s_r S[M]$.

Investment for service to customers (SI)

The retailer provides a service to customers for increasing market demand of the product. It helps to improve the goodwill of the retailer among customers. It has a direct impact on market demand. The service investment is

$$\mathrm{SI} = \frac{\alpha s^2}{2}. \tag{4.9}$$

Expected total profit of the retailer under uniform distribution $\left(\pi_r^U\right)$

Thus, the ordered quantity M of the retailer can be expressed as

$$M = I + a_0 + a_1 \frac{p_{\max} - p}{p - p_{\min}} + a_2 v + a_3 v^2 + a_4 j + a_5 j^2 + a_6 k + a_7 k^2 + a_8 q + a_9 q^2 + a s^\gamma + b z^\rho \tag{4.10}$$

and the expected total profit of the retailer is given by

$$\pi_r^U (I, s, p) = \mathrm{RN} - \mathrm{PC} - \mathrm{SC} - \mathrm{SI} - \mathrm{HC}$$

$$= (p - w)M + s_r(I - \zeta) - \frac{\alpha s^2}{2} - (s_r + p + h_r) \int_0^I F(\epsilon) \mathrm{d}\epsilon. \tag{4.11}$$

4.1.2. Third-party logistics (3PL)

The vendor ties a not with the 3PL and it formulates a hybrid CLSCM. The 3PL collects used products $R(j, k)$ from the market with a unit purchasing cost j. Now, the vendor shares a technology license with the 3PL such that the 3PL is able to remanufacture x portion of collected products $R(j, k)$. The remaining portion of the collected products is sent to the vendor.

Revenue of 3PL from the retailer ($\mathrm{RN_{3PL}}$)

After remanufacturing, the remanufactured products are labeled as new products. Thus, the price of these remanufactured products are same as the new products. The 3PL sells $xR(j, k)$ amount of remanufactured products to the retailer. Then, the revenue of the 3PL is

$$\mathrm{RN_{3PL}} = wxR(j, k). \tag{4.12}$$

Technology license fees (TL)

The 3PL gives l as unit license fees to the vendor for getting the permit of remanufacturing from the vendor. Then, the total technology license fees [54] of the 3PL is

$$\mathrm{TL} = lxR(j, k). \tag{4.13}$$

Remanufacturing cost of 3PL ($\mathrm{RC_{3PL}}$)

3PL remanufactures $xR(j, k)$ amount of used products. If c_3 is the unit remanufacturing cost of the 3PL, then the remanufacturing cost is

$$\mathrm{RC_{3PL}} = c_3 xR(j, k). \tag{4.14}$$

Now, if these amount of products are produced as new products, then the unit production is $c_m = \frac{b_1}{p_r} + b_2 p_r$ and $c_m > c_3$, then unit savings is given by Equation (4.3). Thus, the savings from the remanufacturing is $\mathrm{RC_{3PL}} = T_3 xR(j, k)$.

Selling price remaining collected used products (SCP)

The 3PL sells remaining $(1 - x)$ portion of the collected products $R(j, k)$ to the vendor at a unit price B. Then, total selling price of used products is

$$\mathrm{SCP} = B(1 - x)R(j, k). \tag{4.15}$$

Total profit of 3PL $\pi_t(j)$

Total profit of the 3PL is given by

$$\begin{aligned}
\pi_t(j) &= \text{RN}_{3\text{PL}} + \text{SCP} - \text{TL} - \text{RC}_{3\text{PL}} \\
&= [(w - c_m + T_3 - B - l)x + B - j](\mu + cj - yk).
\end{aligned} \tag{4.16}$$

4.1.3. Vendor

The vendor uses a cooperation policy for remanufacturing by sharing licenses. Along with this hybrid CLSCM, the manufacturer uses a hybrid production system that produces new products as well as remanufactured products. For the convergence, the price and costs follows the relation $c_m \leq w \leq p$. Carbons are emitted from the hybrid production system of the vendor at a rate $v, 0 \leq v \leq 1$. Other intriguing factors are described below.

Revenue (RN_m)

The vendor makes the revenue by selling the new products and remanufactured products. As both types of products have the same quality, the unit wholesale price is the same. If w is the unit wholesale price, the revenue of the vendor is

$$\text{RN}_m = w(M - xR(j, k)). \tag{4.17}$$

Investment for carbon emission reduction ($C(v)$)

As the vendor has a carbon cap, the vendor tries to reduce the emissions rate such that the total emissions can be restricted within the carbon cap. To reduce the emissions rate, the vendor invests the following amount.

$$C(v) = \frac{\theta v^2}{2}. \tag{4.18}$$

Carbon tax (CT)

If emissions from the system does not follow the reduced emissions rate v, then total emissions may exceed the total carbon cap G_v [15]. For that excess emissions, the vendor has to pay a carbon tax. If the unit carbon tax is P_v, then total carbon tax is

$$\text{CT} = P_v[\eta(1 - v)M - G_v]. \tag{4.19}$$

This tax gives a positive effect on reducing CO_2 emissions.

Investment for quality improvement ($C(k)$)

The vendor invests on quality improvement (k) for products. This investment has an inverse effect on remanufactured products as the quantity of collected used products depends on k. The investment [8] for quality improvement is

$$C(k) = \phi k^2. \tag{4.20}$$

Purchasing cost of used products from 3PL (PC_m)

The vendor buys $(1 - x)$ portion of used products from the 3PL for remanufacturing. If the unit purchasing cost of used products is B, then total purchasing cost is

$$\text{PC}_m = B(1 - x)R(j, k). \tag{4.21}$$

Remanufacturing cost of vendor (RC_m)

Now, the vendor remanufactures all purchased used products. But the quality of these remanufactured products remains as good as new. If the unit remanufacturing cost of the vendor is c_1, then $c_m < c_1$. This implies that the vendor saves T_1 unit cost per unit product (Equation (4.1)) by using remanufacturing. Total remanufacturing cost is

$$\text{RC}_m = c_1(1 - x)R(j, k)). \tag{4.22}$$

Production cost of new products (NPC)

Vendor produces $(M - R(j, k))$ amount of new products for retailer. Rest order is fulfilled by the remanufactured products. Unit production cost of new products is $c_m = \frac{b_1}{p_r} + b_2 p_r$. Unit production cost depends upon the production rate. Then, the total production cost of new product is

$$\text{NPC} = \left(\frac{b_1}{p_r} + b_2 p_r\right)(M - R(j, k)). \tag{4.23}$$

Investment for customer awareness (AC)

Customer awareness is one of the CLSCM driving factors that help to achieve green and sustainable practice. Vendors actively take part in customer awareness programs such that the process of reuse through remanufacturing goes well. Then the investment for customer awareness is

$$\text{AC} = \frac{\beta z^2}{2}. \tag{4.24}$$

Investment for gift policy (GP)

The vendor uses gift policy to boosts up the market demand. This helps to attract more customers. Then, the investment for gift policy is

$$\text{GP} = \frac{\sigma q^2}{2}. \tag{4.25}$$

Total profit of vendor under uniform distribution $\left(\pi_m^U\right)$

Thus, the total profit of the vendor is given by

$$\begin{aligned}
\pi_m^U(k, v, p_r, z, q) &= \text{RN}_m - \text{CT} - C(v) - \text{PC}_m - \text{RC}_m - \text{NPC} - C(k) - \text{AC} - \text{GP} \\
&= \left(w - \frac{b_1}{p_r} - b_2 p_r - P_v \eta + P_v \eta v\right)M + P_v G_v - \frac{\theta v^2}{2} - \phi k^2 \\
&\quad + R(j, k)[(l + c_m - w)x + (T_1 - B)(1 - x)] - \frac{bz^2}{2} - \frac{\sigma q^2}{2}.
\end{aligned} \tag{4.26}$$

4.2. Scenario B: Distribution-free model

Scenario B considers that the random variable X of the market demand ϵ does not have any known distribution function. Thus, the expected shortage quantity $E_1(M)$ of the retailer can not be calculated using the distribution function. Then, a minimum–maximum distribution-free approach is used [12, 15] to find the expected shortage quantity. ζ and σ_1 are the mean and standard deviation of X, respectively. Then, $E_1(M)$ is found from the following expression as

$$E_1[M] \leq M - \frac{1}{2}\left(\sqrt{\sigma_1^2 + (I - \zeta)^2} - (I - \zeta)\right). \tag{4.27}$$

Then, $Z_1(M)$ is the of the retailer's leftover for holding, which can be expressed as

$$L_1[M] = (M + R(j,k)x - D)^+ = R(j,k)x + E_1[M] - E[D]. \tag{4.28}$$

Thus, the expected holding cost of retailer becomes $\frac{h_r}{2}\left(\sqrt{\sigma_1^2 + (I - \zeta)^2} - (I - \zeta)\right)$. Similarly, the expected shortage cost $S_1[M]$ can be expressed as

$$S_1[M] = (D - R(j,k)x - M)^+ = E[D] - R(j,k)x - E_1[M]. \tag{4.29}$$

Thus, expected shortage cost of retailer is

$$s_r\left[(I - \zeta) - \frac{1}{2}\left(\sqrt{\sigma_1^2 + (I - \zeta)^2} - (I - \zeta)\right)\right].$$

Further, the upper limit of the upper inequality is tight [12]. Hence, the expected total profit of the retailer is

$$\pi_r^{\mathrm{DF}}(I, s, p) = (p - w)M + s_r(I - \zeta) - (s_r + p + h_r)\frac{1}{2}\left(\sqrt{\sigma_1^2(I - \zeta)^2} - (I - \zeta)\right) - \frac{\alpha s^2}{2}. \tag{4.30}$$

The ordered quantity is the same as in equation (4.10). Profit functions of the 3PL (π_t^{DF}) and vendor (π_m^{DF}) will be same as in equations (4.16) and (4.26), respectively. That is,

$$\pi_t^{\mathrm{DF}} = \pi_t^U \tag{4.31}$$

$$\pi_m^{\mathrm{DF}} = \pi_m^U. \tag{4.32}$$

Solutions to the objective functions are given in the next section.

5. Solution methodology

Both the models of Scenario A and B are solved for the decentralized case and it is solved by the Stackelberg game policy. The vendor acts as Stackelberg leader and retailer-3PL acts as Stackelberg follower. A backward Stackelberg policy is used where the vendor gives an opportunity to the followers to optimize their decisions. Based on those decisions, the vendor optimizes his decisions.

5.1. Scenario A

Optimum values of decision variables under uniform distribution are given in Propositions 5.1 and 5.4. Retailer and 3PL's decisions are given in Proposition 5.1.

Proposition 5.1. *Applying equilibrium condition on retailer and 3PL's profit function, the optimum values of decision variables I^*, j^*, s^*, and p^* are given in Equations (5.1), (5.2), (5.3), and (5.4), respectively.*

$$I^* = \frac{k_1(p^* - w + s_r)}{p^* + h_r + s_r}, \tag{5.1}$$

$$j^* = \frac{-\mu + yk^* + c\left(-B - l + w - \frac{b_1}{p_r}\right)}{2c} - \frac{c(b_2 p_r + T_3)x + Bc}{2c}, \tag{5.2}$$

$$s^* = \left(\frac{\alpha}{(p^* - w)a\gamma}\right)^{\frac{1}{\gamma - 2}}, \tag{5.3}$$

$$p^* = \frac{-2H_3 + \sqrt{4H_3^2 - 4H_2 H_4}}{2H_2}. \tag{5.4}$$

[See Appendix C]. The obtained optimal solutions are for the retailer and 3PL in Scenario A (Equations (4.11) and (4.16)) based on the Theorems 5.2 and 5.3.

Theorem 5.2. *Profit of the retailer for uniform distribution becomes global maximum at I^*, s^* and p^*, if all the principal minors H_{11}, H_{22}, and H_{33} of the third order Hessian matrix are in alternative in sign.*

Proof. See Appendix B.   □

Theorem 5.3. *Profit of the 3PL for uniform distribution becomes global maximum at j^*, if the second order principal minors is negative.*

Proof. See Appendix B. Vendor's optimal decision variables v^*, k^*, p_r^*, q^*, and z^* are given in Proposition 5.4 by applying equilibrium conditions.   □

Proposition 5.4. *The vendor's optimal decision variables v^*, k^*, p_r^*, q^*, and z^* are given in Equations (5.5)–(5.9).*

$$v^* = \frac{-H_6 + \sqrt{H_6^2 - 12a_3 H_7 p_v \eta}}{6a_3 p_v \eta}, \tag{5.5}$$

$$k^* = \frac{H_8}{H_9}, \tag{5.6}$$

$$p_r^* = \frac{b_1 x(\mu + cj - yk) - b_1 H_{10}}{b_2 x(\mu + cj - yk) - b_2 H_{10}}, \tag{5.7}$$

$$q^* = \frac{H_{11} a_8}{2(\sigma - H_{11} a_9)}, \tag{5.8}$$

$$z^* = \left(\frac{\beta}{b\rho(w - p_v \eta - \frac{b_1}{p_r} - b_2 p_r + p_v v \eta)} \right)^{\frac{1}{\rho - 2}}. \tag{5.9}$$

[See Appendix C] The obtained optimal solutions are for the vendor in Scenario A (Equation (4.26)) based on the Theorem 5.5.

Theorem 5.5. *Profit of the vendor for uniform distribution becomes global maximum at v^*, k^*, p_r^*, q^*, and z^*, if all the principal minors H_{11}, H_{22}, H_{33}, H_{44}, and H_{55} of the fifth order Hessian matrix are in alternative in sign.*

Proof. See Appendix B.   □

5.2. Scenario B

Optimum values of decision variables for distribution-free approach are given in Propositions 5.6 and 5.9. Retailer and 3PL's decisions are given in Proposition 5.6.

Proposition 5.6. *Applying equilibrium conditions on retailer and vendor's profit function of the CLSCM, the optimum values of decision variables I^*, j^*, s^*, and p^* are given in Equations (5.10), (5.11), (5.12), and (5.13), respectively.*

$$I^* = \zeta + \frac{\sigma_1 A}{\sqrt{1 - A^2}}, \tag{5.10}$$

$$j^* = \frac{-\mu + yk^* + c\left(-B - l + w - \frac{b_1}{p_r}\right)}{2c} - \frac{c(b_2 p_r + T_3)x + Bc}{2c}, \tag{5.11}$$

$$s^* = \left(\frac{\alpha}{(p^* - w)a\gamma}\right)^{\frac{1}{\gamma - 2}}, \tag{5.12}$$

$$p^* = \frac{-H_4 + \sqrt{H_4^2 - 4H_3 A_1}}{2H_3}. \tag{5.13}$$

[See Appendices A and C]. The obtained optimal solutions are for the retailer and 3PL in Scenario A (Equations (4.30) and (4.31)) based on the Theorems 5.7 and 5.8.

Theorem 5.7. *Profit of the retailer for uniform distribution becomes global maximum at I^*, s^* and p^*, if all the principal minors H_{11}, H_{22}, and H_{33} of the third order Hessian matrix are in alternative in sign.*

Proof. See Appendix B. □

Theorem 5.8. *Profit of the 3PL for distribution-free approach becomes global maximum at j^*, if the second order principal minors is negative.*

Proof. See Appendix B. Vendor's optimal decision variables $v^*, k^*, p_r^*, q^*,$ and z^* are given in Proposition 5.9 by applying equilibrium conditions on vendor's profit function. □

Proposition 5.9. *The vendor's decision variables $v^*, k^*, p_r^*, q^*,$ and z^* are defined in equations (5.14)–(5.18).*

$$v^* = \frac{-H_6 + \sqrt{H_6^2 - 12a_3 H_7 p_v \eta}}{6a_3 p_v \eta}, \tag{5.14}$$

$$k^* = \frac{H_8}{H_9}, \tag{5.15}$$

$$p_r^* = \frac{b_1 x(\mu + cj - yk) - b_1 H_{10}}{b_2 x(\mu + cj - yk) - b_2 H_{10}}, \tag{5.16}$$

$$q^* = \frac{H_{11} a_8}{2(\sigma - H_{11} a_9)}, \tag{5.17}$$

$$z^* = \left(\frac{\beta}{b\rho(w - p_v\eta - \frac{b_1}{p_r} - b_2 p_r + p_v v\eta)}\right)^{\frac{1}{\rho - 2}}. \tag{5.18}$$

[See Appendix C]. Global optimal conditions of the solutions of the vendor for Scenario B (Eqs. (5.14)–(5.18)) are given in the Theorem 5.10.

Theorem 5.10. *Profit of the vendor for distribution-free approach becomes global maximum at $v* = \delta_1$, $k* = \delta_2$, $p_r* = \delta_3$, $z^* = \delta_4$, and $q^* = \delta_5$, if all the principal minors H_{11}, H_{22}, H_{33}, H_{44}, and H_{55} of the 5×5 Hessian matrix are in alternative in sign.*

Proof. See Appendix B. □

6. NUMERICAL EXPERIMENT

This investigation uses data from Sarkar *et al.* [41], Taleizadeh *et al.* [54], and Bhuniya *et al.* [4]. Two examples are provided two test the analytic model of Scenario A and Scenario B, respectively.

6.1. Example 1

Example 1 gives numerical results for Scenario A. Table 2 gives input values of parameters. Total profit and optimum values of decision variables for Scenario A are shown in Table 3.

Table 3 shows that the profit of the vendor is maximum, which is \$98 309.23, followed by the 3PL \$43 161.94, and the retailer \$887.15. Each CLSCM player optimizes their decisions individually as a decentralized case. Thus, the sum of these three profits is \$142 358.32, which is treated as total profit for giving an overall aspect of the CLSCM profit.

TABLE 2. Parametric values for Scenario A and Scenario B.

Parameters	Value	Parameters	Value
$[k_2, k_1]$	$[0, 90]$	w	\$70/unit
s_r	\$0.7/unit	h_r	\$0.7/unit/unit time
σ	700	B	\$40/unit
c_1	\$5/unit	c_3	\$5/unit
P_v	\$0.2/gallons	G_v	100 gallons
η	2.99	c	1.22
μ	30 units	y	0.6
$p_{\max}$	\$200/unit	$p_{\min}$	\$90/unit
γ	3.65	a	30
a_0	2000 units	a_1	0.7
a_2	0.2	a_3	0.2
a_4	0.5	a_5	0.4
a_6	5	a_7	0.3
ζ	3.2 units	θ	\$2000
ϕ	\$1000	α	\$1000
x	0.4	l	\$50/unit
b	40	ρ	0.2
b_1	\$30/unit	b_2	\$5/unit
β	20	a_8	3
σ_1	5	a_9	0.1

TABLE 3. Optimal solutions and maximum profit of Scenario A.

I units	s	\$$j$/unit	\$$p$/unit	v
20.39	0.62	6.77	90.00	0.66

k	\$$p_r$/unit time	z	q	
0.12	2.45	5.00	15	

\$$\pi_r^U$	\$ π_t^U	\$$\pi_m^U$	Total profit (\$)	
887.15	43 161.94	98 309.23	142 358.32	

TABLE 4. Results of special cases for Scenario A.

Special cases	\$$\pi_r^U$	\$$\pi_t^U$	\$$\pi_m^U$	Total profit (\$)
No customer awareness	1498.11	42 057.50	96 058.81	139 614.42
No gift policy	887.15	41 811.12	95 251.14	137 949.41
No customer awareness & gift policy	1498.11	40 706.72	93 001.40	135 206.23

6.1.1. Special cases

Three observations are made from Scenario A as no customer awareness, no gift policy, and no customer & gift policy. Table 4 shows the result for above scenario.

Profit of the supply chain continuously decreasing in the sequence of no customer awareness, no gift policy and no customer awareness & gift policy, respectively. This implies that the proposed study with both the policies helps to generate more market demand when the CLSCM uses remanufactured products as a brand new products.

TABLE 5. Optimal solutions and maximum profit for Scenario B.

I units	s	$\$j$/unit	$\$p$/unit	v
3.26	0.62	6.77	90	0.65
k	p_r units/unit time	z	q	
0.118	2.45	5	15	
$\$\pi_r^{\mathrm{DF}}$	$\$\pi_t^{\mathrm{DF}}$	$\$\pi_m^{\mathrm{DF}}$	Total profit ($)	
887.15	42 789.35	97 209.09	140 885.60	

6.2. Example 2

Example 2 gives numerical results for Scenario B. Table 2 gives the parametric values for Example 2. Maximum profit and optimal values of decision variables are given in Table 5.

Table 5 shows that the vendor has the maximum profit $97 209.09 followed by the 3PL $42 789.35, and the retailer $42 789.35. A centralized case is not discussed here. Thus, the total profit of the CLSCM is just the sum of the profits of three CLSCM players as 140 885.60. Results show that Scenario A has more profit than Scenario B. This happens because Scenario A has a known distribution function. Thus, all information about demand is known to the management and thus, it is easy to optimize the objective. But, in reality, there are a lot of risks and uncertainties that evolve along with information. Then, the management needs to justify that information more than the known case, as Scenario A.

A comparison between Scenario A and Scenario B is given to clarify the difference between the two Scenarios.

6.3. Comparative study between Scenario A and Scenario B

Expected value of additional information (EVAI)

EVAI is found to compare two discussed scenarios in this study. If the optimal values of Scenario B are used as an input for Scenario A, then the total profit of Scenario A becomes $140 904.46, *i.e.*, the difference is $(140 904.46−140 885.60) = $(18.86). This implies that Scenario B with a distribution-free approach has $(18.86) less profit. This amount of money is needed for gathering information in the case of no distribution. This amount of money is used for EVAI purposes, which is less than 1%, *i.e.*, EVAI < 1%.

6.4. Sensitivity analysis

A sensitivity analysis of Example 1 for Scenario A are conducted with respect to the cost parameters. Table 6 gives the changes of sensitivity analysis. As game policy is used, cost parameters of one CLSCM player has effect on the the profit of other players.

(1) Minimum selling price of the product $p_{\min}$ is the most sensitive parameter for the retailer. 50% increment of minimum selling price increases the profit 226.36%. This implies that the retailer can earn more profit if the minimum selling price increases. This happens because the remanufacturing cost of used products is less than the production cost of new products. Thus, it gives a huge profit range.
(2) Production cost of new products is the second most sensitive cost parameter for the 3PL. It has a direct impact on the profit of 3PL. 50% increment of production cost increases 90.97% profit of the 3PL. Other parameters are less sensitive.

6.5. Managerial insights

After analyzing the results and hypotheses, the following insights are drawn from this study.

(1) Production managers should focus on the hybrid production process, where both 3PL & vendor can reman-ufacturer used products and the retailer can focus on selling new and remanufactured products. Thus, the retailer does not have to make decisions on used products collection and send those to the vendor. Instead,

TABLE 6. Sensitivity analysis of Example 1 for Scenario A with respect to parameters.

Parameters	Change (%)	π_r^U (%)	π_t^U (%)	π_m^U (%)
	-50%	-0.007		-0.008
	-25%	-0.004		-0.004
s_r	$+25\%$	$+0.004$		$+0.004$
	$+50\%$	$+0.018$		$+0.020$
	-50%	$+\,0.0013$		$+0.0032$
	-25%	$+0.007$		$+\,0.0016$
h_r	$+25\%$	-0.0007		-0.0016
	$+50\%$	-0.003		-0.007
	-50%	-0.98	-67.88	-0.70
	-25%	-0.66	-36.82	-0.49
b_1	$+25\%$	$+0.99$	$+42.60$	$+0.78$
	$+50\%$	$+2.31$	$+90.97$	$+1.85$
	-50%	-0.000076	$+0.00059$	$+0.058$
	-25%	-0.000038	$+0.00030$	$+0.029$
c_1	$+25\%$	$+0.000038$	-0.00030	-0.029
	$+50\%$	$+0.000077$	-0.00059	-0.058
	-50%	$+0.30$	$+3.16$	$+0.32$
	-25%	$+0.15$	$+1.58$	$+0.15$
c_3	$+25\%$	-0.14	-1.58	-0.14
	$+50\%$	-0.26	-3.17	-0.28
	-50%	-0.006	-0.000012	$+0.33$
	-25%	-0.003	$+0.000076$	$+0.14$
P_v	$+25\%$	$+0.004$	-0.00024	-0.083
	$+50\%$	$+0.009$	-0.00064	-0.11
$p_{\max}$	-50%	-0.32	$+0.000002$	-0.13
	-25%	-0.16	$+0.000001$	$+0.065$
	$+25\%$	$+0.16$	-0.000001	$+0.065$
	$+50\%$	$+0.32$	-0.000002	$+0.13$
$p_{\min}$	-50%	-185.16	$+0.0004$	-33.71
	-25%	-110.04	-0.0003	$+20.17$
	$+25\%$	$+113.02$	-0.000004	$+0.33$
	$+50\%$	$+226.36$	-0.000009	$+0.70$
θ	-50%	$+0.02$	-0.001	$+0.44$
	-25%	$+0.006$	-0.004	$+0.15$
	$+25\%$	-0.003	$+0.0003$	-0.09
	$+50\%$	-0.005	$+0.0004$	-0.15
	-50%	$+0.02$	-0.16	0.004
	-25%	$+0.007$	-0.05	$+0.001$
ϕ	$+25\%$	-0.004	$+0.03$	-0.0008
	$+50\%$	-0.007	$+0.05$	-0.001

the retailer can promote the product by using additional services, which helps the vendor to receive more ordered quantities from the retailer. This responsibility sharing policy through cooperation is an effective management strategy.

(2) As the demand varies with several factors, the vendor receives clear information about market demand and how customers react to the ongoing policy. Even though all remanufactured products have the highest quality as new products as well as gift policy, there is always a risk. Thus, industry managers focus on customer awareness. This only can help to sell more remanufactured and to save more production costs.

(3) Thus, this hybrid production process with hybrid CLSCM is a smart management policy. This can promote both environments as well as the economic purpose of sustainable development. Besides, the effort for emissions rate reduction with a carbon cap is another positive decision for management toward emissions reduction.

Finally, the conclusions of this study if describes in the next section.

7. CONCLUSIONS

Customer awareness and the gift policy had positive effects on the ordered quantity of the retailer. In addition, quality management positively affects the profitability of the CLSCM. This investigation helped to gain optimal policies for a hybrid production system. 0.0134% of the total profit of uniform distribution requires more to gather information if there is a lack of proper distribution functions for information. Thus, 0.0134% profit is less in the distribution-free approach. Even the $U[k_2, k_1]$ provided more profit, but it was almost impossible that all information always followed uniform distribution or any other known distribution. It was found that the proposed hybrid CLSCM helped to execute the purpose of the hybrid production system very well. Few limitations of this study can be overcome by extending it. Lead time issues and monetary policy are two of them. In the future study, a monthly installment (EMI) policy can be used as a monetary policy such that the 3PL can pay with a credit period. Other motivation factors like an advertisement, and credit policy for customers can be used in a future study. In addition, while a single-channel was used in this study, a dual-channel policy can be used for further investigation. A comparison between traditional and consignment policies can be incorporated into future research. Sustainability issues based on this context [2] can be an interesting future extension.

APPENDIX A.

Proof of Proposition 5.1. The optimum value of I, s, and p are obtained from the first order partial derivatives of Equation (4.11) with respect to I, s, and p. Then, the optimum values are found by equating the first order derivative with zero as

$$\frac{\partial \pi_r^U}{\partial I} = 0, \qquad i.e., \quad I^* = \frac{k_1(p^* - w + s_r)}{p^* + h_r + s_r},$$

$$\frac{\partial \pi_r^U}{\partial s} = 0, \qquad i.e., \quad s^* = \left(\frac{\alpha}{(p^* - w)a\gamma},\right)^{\frac{1}{\gamma-2}}$$

$$\frac{\partial \pi_r^U}{\partial p} = 0, \qquad i.e., \quad p^* = \frac{-2H_3 + \sqrt{4H_3^2 - 4H_2 H_4}}{2H_2}.$$

Equating the first order partial derivative of Equation (4.16) with zero, the optimum value of j is obtained as

$$j^* = \frac{-\mu + yk^* + c\left(-B - l + w - \frac{b_1}{p_r}\right)}{2c} - \frac{c(b_2 p_r + T_3)x + Bc}{2c}.$$

$\square$

Proof of Proposition 5.4. The optimum values of decision variables are obtained from the first order partial derivatives of Equation (4.26) with respect to k, v, p_r, z, and q. Then, the optimum values are found by equating the first order derivative with zero as

$$\frac{\partial \pi_m^U}{\partial v} = 0, \qquad i.e., \quad v^* = \frac{-H_6 + \sqrt{H_6^2 - 12a_3 H_7 p_v \eta}}{6a_3 p_v \eta},$$

$$\frac{\partial \pi_m^U}{\partial k} = 0, \qquad i.e., \quad k^* = \frac{H_8}{H_9},$$

$$\frac{\partial \pi_m^U}{\partial p_r} = 0, \qquad i.e., \quad p_r^* = \frac{b_1 x(\mu + cj - yk) - b_1 H_{10}}{b_2 x(\mu + cj - yk) - b_2 H_{10}},$$

$$\frac{\partial \pi_m^U}{\partial z} = 0, \qquad i.e., \quad z^* = \left(\frac{\beta}{b\rho(w - p_v \eta - \frac{b_1}{p_r} - b_2 p_r + p_v v\eta)} \right)^{\frac{1}{\rho - 2}},$$

$$\frac{\partial \pi_m^U}{\partial q} = 0, \qquad i.e., \quad q^* = \frac{H_{11} a_8}{2(\sigma - H_{11} a_9)}.$$

$\square$

Proof of Proposition 5.6. The optimum value of I, s, and p are obtained from the first order partial derivatives of Equation (4.30) with respect to I, s, and p. Then, the optimum values are found by equating the first order derivative with zero as

$$\frac{\partial \pi_r^{\mathrm{DF}}}{\partial I} = 0, \qquad i.e., \quad I^* = \zeta + \frac{\sigma_1 A}{\sqrt{1 - A^2}},$$

$$\frac{\partial \pi_r^{\mathrm{DF}}}{\partial s} = 0, \qquad i.e., \quad s^* = \left(\frac{\alpha}{(p^* - w)a\gamma} \right)^{\frac{1}{\gamma - 2}},$$

$$\frac{\partial \pi_r^{\mathrm{DF}}}{\partial p} = 0, \qquad i.e., \quad p^* = \frac{-H_4 + \sqrt{H_4^2 - 4H_3 A_1}}{2H_3}.$$

Equating the first order partial derivative of Equation (4.31) with zero, the optimum value of j is obtained as

$$j^* = \frac{-\mu + yk^* + c(-B - l + w - \frac{b_1}{p_r})}{2c} - \frac{c(b_2 p_r + T_3)x + Bc}{2c}.$$

These are the optimum solutions for the retailer and 3PL for the distribution-free approach. $\square$

Proof of Proposition 5.9. The optimum values of decision variables are obtained from the first order partial derivatives of Equation (4.26) with respect to v, k, p_r, z, and q, respectively. Then, the optimum values are found by equating the first order partial derivatives with zero as

$$\frac{\partial \pi_m^{\mathrm{DF}}}{\partial v} = 0, \qquad (i.e.), \quad v^* = \frac{-H_6 + \sqrt{H_6^2 - 12a_3 H_7 p_v \eta}}{6a_3 p_v \eta},$$

$$\frac{\partial \pi_m^{\mathrm{DF}}}{\partial k} = 0, \qquad (i.e.), \quad k^* = \frac{H_8}{H_9},$$

$$\frac{\partial \pi_m^{\mathrm{DF}}}{\partial p_r} = 0, \qquad (i.e.), \quad p_r^* = \frac{b_1 x(\mu + cj - yk) - b_1 H_{10}}{b_2 x(\mu + cj - yk) - b_2 H_{10}},$$

$$\frac{\partial \pi_m^{\mathrm{DF}}}{\partial z} = 0, \qquad (i.e.), \quad z^* = \left(\frac{\beta}{b\rho(w - p_v \eta - \frac{b_1}{p_r} - b_2 p_r + p_v v\eta)} \right)^{\frac{1}{\rho - 2}},$$

$$\frac{\partial \pi_m^{\mathrm{DF}}}{\partial q} = 0, \qquad (i.e.), \quad q^* = \frac{H_{11} a_8}{2(\sigma - H_{11} a_9)}.$$

Hence, these are the optimal values of decision variables of the vendor for the distribution-free approach. $\square$

Second order partial derivatives

Second order partial derivatives of Equation (4.11) are given below:

$$\frac{\partial^2 \pi_r^U}{\partial I^2} = -(s_r + p + h_r)I,$$

$$\frac{\partial^2 \pi_r^U}{\partial s^2} = (p - w)a\gamma(\gamma - 1)s^{\gamma - 2} - \alpha,$$

$$\frac{\partial^2 \pi_r^U}{\partial p^2} = \frac{2a_1}{(p - p_{\min})^3}\big(p - p_{\max} + pp_{\max} + wp_{\min} - wp_{\max} - p^2 - p_{\max}p_{\min}\big),$$

$$\frac{\partial^2 \pi_r^U}{\partial I \partial p} = \frac{2 + I^2}{2},$$

$$\frac{\partial^2 \pi_r^U}{\partial I \partial s} = \frac{\partial^2 \pi_r^{US}}{\partial p \partial s} = \frac{\partial^2 \pi_r^U}{\partial s \partial I} = 0, \frac{\partial^2 \pi_r^U}{\partial s \partial p} = a\gamma s^{\gamma - 1},$$

$$\frac{\partial^2 \pi_r^U}{\partial p \partial I} = -I.$$

Second order partial derivative of Equation (4.16) is

$$\frac{\partial^2 \pi_t^U}{\partial j^2} = -2c.$$

Second order partial derivatives of Equation (4.26) are given below:

$$\frac{\partial^2 \pi_m^U}{\partial v^2} = 2P_v\eta(a_2 + 3a_3v) - \theta + \left(w - \frac{b_1}{p_r} - b_2p_r - P_v\eta + P_v\eta v\right)2a_3,$$

$$\frac{\partial^2 \pi_m^U}{\partial k^2} = 2a_7\left(w - \frac{b_1}{p_r} - b_2p_r - P_v\eta + P_v\eta v\right) - 2\phi,$$

$$\frac{\partial^2 \pi_m^U}{\partial p_r^2} = -\frac{2Mb_1}{p_r^3},$$

$$\frac{\partial^2 \pi_m^U}{\partial z^2} = \left(w - \frac{b_1}{p_r} - b_2p_r - P_v\eta + vP_v\eta\right)bp(p - 1)z^{p-2} - b,$$

$$\frac{\partial^2 \pi_m^U}{\partial q^2} = \left(w - \frac{b_1}{p_r} - b_2p_r - P_v\eta + vP_v\eta\right)2a_9 - \sigma,$$

$$\frac{\partial^2 \pi_m^U}{\partial v \partial k} = P_v\eta(a_6 + 2ka_7), \frac{\partial^2 \pi_m^U}{\partial v \partial z} = P_v + bpz^{p-1}, \frac{\partial^2 \pi_m^U}{\partial v \partial q} = P_v\eta(a_8 + 2a_9q),$$

$$\frac{\partial^2 \pi_m^U}{\partial k \partial v} = P_v\eta(a_6 + 2ka_7), \frac{\partial^2 \pi_m^U}{\partial k \partial p_r} = (a_6 + 2ka_7)\left(\frac{b_1}{p_r^2} - b_2\right), \frac{\partial^2 \pi_m^U}{\partial k \partial z} = \frac{\partial^2 \pi_m^U}{\partial k \partial q} = 0,$$

$$\frac{\partial^2 \pi_m^U}{\partial p_r \partial v} = \left(\frac{b_1}{p_r^2} - b_2\right)(a_2 + 2va_3), \frac{\partial^2 \pi_m^U}{\partial p_r \partial k} = (a_6 + 2ka_7)\left(\frac{b_1}{p_r^2} - b_2\right),$$

$$\frac{\partial^2 \pi_m^U}{\partial p_r \partial z} = \left(\frac{b_1}{p_r^2} - b_2\right)bpz^{p-1}, \frac{\partial^2 \pi_m^U}{\partial p_r \partial q} = \left(\frac{b_1}{p_r^2} - b_2\right)(a_8 + 2qa_9), \frac{\partial^2 \pi_m^U}{\partial z \partial v} = P_v\eta bpz^{p-1},$$

$$\frac{\partial^2 \pi_m^U}{\partial z \partial p_r} = \left(\frac{b_1}{p_r^2} - b_2\right)bpz^{p-1}, \frac{\partial^2 \pi_m^U}{\partial z \partial q} = \frac{\partial^2 \pi_m^U}{\partial v \partial p_r} = \frac{\partial^2 \pi_m^U}{\partial z \partial k} = \frac{\partial^2 \pi_m^U}{\partial q \partial k} = \frac{\partial^2 \pi_m^U}{\partial v \partial q} = 0,$$

$$\frac{\partial^2 \pi_m^U}{\partial q \partial v} = P_v\eta(a_8 + 2qa_9), \frac{\partial^2 \pi_m^U}{\partial q \partial p_r} = \left(\frac{b_1}{p_r^2} - b_2\right)(a_8 + 2qa_9).$$

Second order partial derivatives of Equation (4.30) are given below:

$$\frac{\partial^2 \pi_r^{\mathrm{DF}}}{\partial I^2} = \frac{(s_r + p + h_r)\big(k^2 - I^2 + \sigma^2\big)}{2\Big(\sqrt{\sigma_1^2 + (I - \zeta)^2}\Big)^3},$$

$$\frac{\partial^2 \pi_r^{\mathrm{DF}}}{\partial s^2} = (p - w)a\gamma(\gamma - 1)s^{\gamma - 2} - \alpha,$$

$$\frac{\partial^2 \pi_r^{\mathrm{DF}}}{\partial p^2} = \frac{a_1}{(p - p_{\min})^4}\big[(p - p_{\max})^2 w(p_{\max} - 2p - p_{\min})$$
$$- (p_{\min} - p_{\max}) + w(p - p_{\min})(p_{\max} - p) - 2w(p - p_{\min})(p_{\min} - p_{\max})\big],$$

$$\frac{\partial^2 \pi_r^{\mathrm{DF}}}{\partial I \partial p} = \frac{3 - I + \zeta}{2k}, \frac{\partial^2 \pi_r^{\mathrm{DF}}}{\partial s \partial p} = a\gamma s^{\gamma - 1}, \frac{\partial^2 \pi_r^{\mathrm{DF}}}{\partial p \partial I} = \frac{\partial^2 \pi_r^{\mathrm{DF}}}{\partial p \partial s} = \frac{\partial^2 \pi_r^{\mathrm{DF}}}{\partial s \partial I} = \frac{\partial^2 \pi_r^{\mathrm{DF}}}{\partial I \partial s} = 0.$$

Second order partial derivative of Equation (4.31) is given below:

$$\frac{\partial^2 \pi_t^{\mathrm{DF}}}{\partial j^2} = -2c.$$

Second order partial derivatives of Equation (4.32) are given below:

$$\frac{\partial^2 \pi_m^{\mathrm{DF}}}{\partial v^2} = 2P_v\eta(a_2 + 3a_3v),$$

$$\frac{\partial^2 \pi_m^{\mathrm{DF}}}{\partial k^2} = 2a_7\left(w - \frac{b_1}{p_r} - b_2 p_r - P_v\eta + P_v\eta v\right) - 2\phi,$$

$$\frac{\partial^2 \pi_m^{\mathrm{DF}}}{\partial p_r^2} = -\frac{2Mb_1}{p_r^3},$$

$$\frac{\partial^2 \pi_m^{\mathrm{DF}}}{\partial z^2} = \left(w - \frac{b_1}{p_r} - b_2 p_r - P_v\eta + vP_v\eta\right)bp(p - 1)z^{p-2} - b,$$

$$\frac{\partial^2 \pi_m^{\mathrm{DF}}}{\partial q^2} = \left(w - \frac{b_1}{p_r} - b_2 p_r - P_v\eta + vP_v\eta\right)2a_9 - \sigma,$$

$$\frac{\partial^2 \pi_m^{\mathrm{DF}}}{\partial v \partial k} = P_v\eta(a_6 + 2ka_7), \frac{\partial^2 \pi_m^{\mathrm{DF}}}{\partial v \partial z} = P_v + bpz^{p-1}, \frac{\partial^2 \pi_m^{\mathrm{DF}}}{\partial v \partial q} = P_v\eta(a_8 + 2a_9q),$$

$$\frac{\partial^2 \pi_m^{\mathrm{DF}}}{\partial k \partial v} = P_v\eta(a_6 + 2ka_7), \frac{\partial^2 \pi_m^{\mathrm{DF}}}{\partial k \partial p_r} = (a_6 + 2ka_7)\left(\frac{b_1}{p_r^2} - b_2\right), \frac{\partial^2 \pi_m^{\mathrm{DF}}}{\partial k \partial z} = \frac{\partial^2 \pi_m^{\mathrm{DF}}}{\partial k \partial q} = 0,$$

$$\frac{\partial^2 \pi_m^{\mathrm{DF}}}{\partial p_r \partial v} = \left(\frac{b_1}{p_r^2} - b_2\right)(a_2 + 2va_3), \frac{\partial^2 \pi_m^{\mathrm{DF}}}{\partial p_r \partial k} = (a_6 + 2ka_7)\left(\frac{b_1}{p_r^2} - b_2\right),$$

$$\frac{\partial^2 \pi_m^{\mathrm{DF}}}{\partial p_r \partial z} = \left(\frac{b_1}{p_r^2} - b_2\right)bpz^{p-1}, \frac{\partial^2 \pi_m^{\mathrm{DF}}}{\partial p_r \partial q} = \left(\frac{b_1}{p_r^2} - b_2\right)(a_8 + 2qa_9), \frac{\partial^2 \pi_m^{\mathrm{DF}}}{\partial v \partial p_r} = 0,$$

$$\frac{\partial^2 \pi_m^{\mathrm{DF}}}{\partial z \partial v} = P_v\eta bpz^{p-1}, \frac{\partial^2 \pi_m^{\mathrm{DF}}}{\partial z \partial p_r} = \left(\frac{b_1}{p_r^2} - b_2\right)bpz^{p-1}, \frac{\partial^2 \pi_m^{\mathrm{DF}}}{\partial z \partial q} = \frac{\partial^2 \pi_m^{\mathrm{DF}}}{\partial z \partial k} = 0,$$

$$\frac{\partial^2 \pi_m^{\mathrm{DF}}}{\partial q \partial v} = P_v\eta(a_8 + 2qa_9), \frac{\partial^2 \pi_m^{\mathrm{DF}}}{\partial q \partial p_r} = \left(\frac{b_1}{p_r^2} - b_2\right)(a_8 + 2qa_9), \frac{\partial^2 \pi_m^{\mathrm{DF}}}{\partial v \partial q} = \frac{\partial^2 \pi_m^{\mathrm{DF}}}{\partial q \partial k} = 0.$$

Appendix B.

Proof of Theorem 5.2. The third order Hessian matrix can be written as

$$H = \det \begin{pmatrix} \frac{\partial^2 \pi_r^U}{\partial I^{*2}} & \frac{\partial^2 \pi_r^U}{\partial I^* \partial s^*} & \frac{\partial^2 \pi_r^U}{\partial I^* \partial p^*} \\ \frac{\partial^2 \pi_r^U}{\partial s^* \partial I^*} & \frac{\partial^2 \pi_r^U}{\partial s^{*2}} & \frac{\partial^2 \pi_r^U}{\partial s^* \partial p^*} \\ \frac{\partial^2 \pi_r^U}{\partial p^* \partial I^*} & \frac{\partial^2 \pi_r^U}{\partial p^* \partial s^*} & \frac{\partial^2 \pi_r^U}{\partial p^{*2}} \end{pmatrix}.$$

The first order principal minor is

$$\det(H_{11}) = \frac{\partial^2 \pi_r^U}{\partial I^{*2}} = -(s_r + p + h_r)I < 0.$$

Hence, the first order principal minor is negative. The second order principal minor is

$$\det(H_{22}) = \frac{\partial^2 \pi_r^U}{\partial I^{*2}} \frac{\partial^2 \pi_r^U}{\partial p^{*2}} - \left(\frac{\partial^2 \pi_r^U}{\partial I^* \partial s^*} \right)^2$$
$$= \frac{2a_1}{(p - p_{\min})^3} \left(p_{\max} + wp_{\max} + p^2 + p_{\max}p_{\min} - pp_{\max} - wp_{\max} \right)(s_r + p + h_r)I > 0.$$

The aforementioned principal minor of the second order is positive. The third order principal minor is

$$\det(H_{33}) = \frac{\partial^2 \pi_r^U}{\partial s^{*2}} \det(H_{22}) - \left(\frac{\partial^2 \pi_r^U}{\partial I^* \partial p^*} \right)^2 \left(\frac{\partial^2 \pi_r^U}{\partial p^{*2}} \right)$$
$$= -\left[\alpha - (p - w)a\gamma(\gamma - 1)s^{\gamma-2} \right] \det(H_{22}) < 0,$$

as $\det(H_{22}) > 0$. The calculated third order principal minor is less than zero. (See Appendix A for all second order partial derivatives). The first and third minors are negative, and the second order principal minor is positive. Thus, it can be concluded that the total profit, obtained by the CLSCM, has the global maximum profit. $\qquad \square$

Proof of Theorem 5.3. The second order partial derivative of π_t^U with respect to j is

$$\frac{\partial^2 \pi_t^U}{\partial j^2} = -2c < 0$$

as $c > 0$. Thus, the total profit of the 3PL is global maximum. $\qquad \square$

Proof of Theorem 5.5. The Hessian matrix can be written as

$$H = \det \begin{pmatrix} \frac{\partial^2 \pi_m^U}{\partial v^{*2}} & \frac{\partial^2 \pi_m^U}{\partial v^* \partial k^*} & \frac{\partial^2 \pi_m^U}{\partial v^* \partial p_r^*} & \frac{\partial^2 \pi_m^U}{\partial v^* \partial z^*} & \frac{\partial^2 \pi_m^U}{\partial v^* \partial q^*} \\ \frac{\partial^2 \pi_m^U}{\partial k^* \partial v^*} & \frac{\partial^2 \pi_m^U}{\partial k^{*2}} & \frac{\partial^2 \pi_m^U}{\partial k^* \partial p_r^*} & \frac{\partial^2 \pi_m^U}{\partial k^* \partial z^*} & \frac{\partial^2 \pi_m^U}{\partial k^* \partial q^*} \\ \frac{\partial^2 \pi_m^U}{\partial p_r^* \partial v^*} & \frac{\partial^2 \pi_m^U}{\partial p_r^* \partial k^*} & \frac{\partial^2 \pi_m^U}{\partial p_r^{*2}} & \frac{\partial^2 \pi_m^U}{\partial p_r^* \partial z^*} & \frac{\partial^2 \pi_m^U}{\partial p_r^* \partial q^*} \\ \frac{\partial^2 \pi_m^U}{\partial z^* \partial v^*} & \frac{\partial^2 \pi_m^U}{\partial z^* \partial k^*} & \frac{\partial^2 \pi_m^U}{\partial z^* \partial p_r^*} & \frac{\partial^2 \pi_m^U}{\partial z^{*2}} & \frac{\partial^2 \pi_m^U}{\partial z^* \partial q^*} \\ \frac{\partial^2 \pi_m^U}{\partial q^* \partial v^*} & \frac{\partial^2 \pi_m^U}{\partial q^* \partial k^*} & \frac{\partial^2 \pi_m^U}{\partial q^* \partial p_r^*} & \frac{\partial^2 \pi_m^U}{\partial q^* \partial z^*} & \frac{\partial^2 \pi_m^U}{\partial q^{*2}} \end{pmatrix}.$$

The first order principal minor is

$$\det(H_{11}) = \frac{\partial^2 \pi_m^U}{\partial v^{*2}}$$

$$= -\left[\theta + \left(P_v\eta + b_2p_r + \frac{b_1}{p_r} - w - P_v\eta v\right)2a_3 - 2a_2P_vx + 6a_3vP_v\eta\right] < 0,$$

if $\left[\theta + \left(P_v\eta + b_2p_r + \frac{b_1}{p_r} - w - P_v\eta v\right)2a_3\right] > [2a_2P_vx + 6a_3vP_v\eta]$. Here, the first order principal minor is negative. The second order principal minor is

$$\det(H_{22}) = \frac{\partial^2\pi_m^U}{\partial k^{*2}}\det(H_{11}) - \left(\frac{\partial^2\pi_m^U}{\partial v^*\partial k^*}\right)^2$$

$$= 2\left[a_7\left(P_v\eta + b_2p_r + \frac{b_1}{p_r} - w - P_v\eta v\right) + \phi\right]$$

$$\left[\theta + \left(P_v\eta + b_2p_r + \frac{b_1}{p_r} - w - P_v\eta v\right)2a_3 - 2a_2P_vx + 6a_3vP_v\eta\right] > 0,$$

as $\det(H_{11}) < 0$. The aforementioned principal minor of the second order is positive. The third order principal minor is

$$\det(H_{33}) = \frac{\partial^2\pi_m^U}{\partial p_r^{*2}}\det(H_{22}) - \left(\frac{\partial^2\pi_m^U}{\partial v^*\partial p_r^*}\right)^2\left(\frac{\partial^2\pi_m^U}{\partial k^{*2}}\right)$$

$$= -\left(\frac{2b_1M}{p_r^3}\right)2\left[a_7\left(P_v\eta + b_2p_r + \frac{b_1}{p_r} - w - P_v\eta v\right) + \phi\right]$$

$$\left[\theta + \left(P_v\eta + b_2p_r + \frac{b_1}{p_r} - w - P_v\eta v\right)2a_3 - 2a_2P_vx + 6a_3vP_v\eta\right] < 0,$$

as $\det(H_{22}) > 0$. The calculated third order principal minor is less than zero. The fourth order principal minor is

$$\det(H_{44}) = \frac{\partial^2\pi_m^U}{\partial z^{*2}}\det(H_{33}) - \left(\frac{\partial^2\pi_m^U}{\partial v^*\partial z^*}\right)^2\left(\frac{\partial^2\pi_m^U}{\partial k^{*2}}\right)\left(\frac{\partial^2\pi_m^U}{\partial p_r^{*2}}\right)$$

$$= \left(\left(\frac{b_1}{p_r} + b_2p_r + P_v\eta - w - vP_v\eta\right)bp(p-1)z^{p-2} + b\right)$$

$$\left\{\frac{2b_1M}{p_r^3}2\left[a_7\left(P_v\eta + b_2p_r + \frac{b_1}{p_r} - w - P_v\eta v\right) + \phi\right]\right.$$

$$\left.\left[\theta + \left(P_v\eta + b_2p_r + \frac{b_1}{p_r} - w - P_v\eta v\right)2a_3 - 2a_2P_vx + 6a_3vP_v\eta\right]\right\} > 0,$$

as $\det(H_{33}) < 0$. The calculated fourth order principal minor is greater than zero. The fifth order principal minor is

$$\det(H_{55}) = \frac{\partial^2\pi_m^U}{\partial q^{*2}}\det(H_{44}) - \left(\frac{\partial^2\pi_m^U}{\partial v^*\partial q^*}\right)^2\left(\frac{\partial^2\pi_m^U}{\partial k^{*2}}\right)\left(\frac{\partial^2\pi_m^U}{\partial p_r^{*2}}\right)\left(\frac{\partial^2\pi_m^U}{\partial z^{*2}}\right)$$

$$= -\left(\left(\frac{b_1}{p_r} + b_2p_r + P_v\eta - w - vP_v\eta\right)2a_9 + \sigma\right)\det(H_{44}) < 0,$$

as $\det(H_{44}) < 0$. The calculated fifth order principal minor is less than zero. (See Appendix A for second order derivatives and calculations). Thus the first, third, and fifth order minors are negative and the second and fourth principals are positive. Thus, it can be concluded that the total profit is global maximum. □

Proof of Theorem 5.7. The Hessian matrix of third order is

$$
H = \det \begin{pmatrix}
\frac{\partial^2 \pi_r^{\mathrm{DF}}}{\partial I^{*2}} & \frac{\partial^2 \pi_r^{\mathrm{DF}}}{\partial I^* \partial s^*} & \frac{\partial^2 \pi_r^{\mathrm{DF}}}{\partial I^* \partial p^*} \\[2mm]
\frac{\partial^2 \pi_r^{\mathrm{DF}}}{\partial s^* \partial I^*} & \frac{\partial^2 \pi_r^{\mathrm{DF}}}{\partial s^{*2}} & \frac{\partial^2 \pi_r^{\mathrm{DF}}}{\partial s^* \partial p^*} \\[2mm]
\frac{\partial^2 \pi_r^{\mathrm{DF}}}{\partial p^* \partial I^*} & \frac{\partial^2 \pi_r^{\mathrm{DF}}}{\partial p^* \partial s^*} & \frac{\partial^2 \pi_r^{\mathrm{DF}}}{\partial p^{*2}}
\end{pmatrix}.
$$

Then the first order principal minor is

$$
\det(H_{11}) = \frac{\partial^2 \pi_r^{\mathrm{DF}}}{\partial I^{*2}} = -\frac{(s_r + p + h_r)\left(I^2 - k^2 - \sigma^2\right)}{2\left(\sqrt{\sigma_1^2 + (I - \zeta)^2}\right)} < 0,
$$

if $I^2 > k^2 + \sigma^2$. Here, the first order principal minor is negative. The second order principal minor is

$$
\begin{aligned}
\det(H_{22}) &= \frac{\partial^2 \pi_r^{\mathrm{DF}}}{\partial p^{*2}} \det(H_{11}) - \left(\frac{\partial^2 \pi_r^{\mathrm{DF}}}{\partial I^* \partial s^*}\right)^2 \\
&= \left[\frac{2a_1(p_{\max} - p_{\min})}{(p - p_{\min})^3} + \frac{a_1(p_{\max} - p_{\min})}{(p - p_{\min})^2} + p\frac{2a_1(p_{\max} - p_{\min})}{(p - p_{\min})^3} \right. \\
&\quad \left. - 2pbz^{p-1} - bp^2(p-1)z^{p-2} - bpz^{p-1} \right] \frac{(s_r + p + h_r)\left(I^2 - k^2 - \sigma^2\right)}{2\left(\sqrt{\sigma_1^2 + (I - \zeta)^2}\right)} > 0,
\end{aligned}
$$

if $\frac{a_1(p_{\max} - p_{\min})(2 + 2p - p_{\min})}{(p - p_{\min})^3} > bpz^{p-2}[z + p(p-1)]$. The aforementioned principal minor of second order is positive. The third order principal minor is

$$
\begin{aligned}
\det(H_{33}) &= \frac{\partial^2 \pi_r^{\mathrm{DF}}}{\partial s^{*2}} \det(H_{22}) - \left(\frac{\partial^2 \pi_r^{\mathrm{DF}}}{\partial I^* \partial p^*}\right)^2 \left(\frac{\partial^2 \pi_r^{\mathrm{DF}}}{\partial p^{*2}}\right) \\
&= -\left[\alpha - (p - w)a\gamma(\gamma - 1)s^{\gamma - 2}\right] \det(H_{22}) < 0,
\end{aligned}
$$

if $\alpha > (p - w)a\gamma(\gamma - 1)s^{\gamma - 2}$. The calculated third order principal minor is less than zero. (See Appendix A for all second-order derivatives and calculations). Thus the first and third minors are negative, and the second order principal minor is negative. Thus, it can be concluded that the total profit is global maximum. □

Proof of Theorem 5.8. Second order derivative of π_t^{DF} with respect to j is

$$
\frac{\partial^2 \pi_t^{\mathrm{DF}}}{\partial j^2} = -2c < 0 \tag{B.1}
$$

as $c > 0$. Thus, the total profit of the 3PL is global maximum. □

Proof of Theorem 5.10. Using the Hessian matrix, it can be proved that the vendor has its global maximum profit. The Hessian matrix is

$$
H = \det \begin{pmatrix}
\frac{\partial^2 \pi_m^{\mathrm{DF}}}{\partial v^{*2}} & \frac{\partial^2 \pi_m^{\mathrm{DF}}}{\partial v^* \partial k^*} & \frac{\partial^2 \pi_m^{\mathrm{DF}}}{\partial v^* \partial p_r^*} & \frac{\partial^2 \pi_m^{\mathrm{DF}}}{\partial v^* \partial z^*} & \frac{\partial^2 \pi_m^{\mathrm{DF}}}{\partial v^* \partial q^*} \\[2mm]
\frac{\partial^2 \pi_m^{\mathrm{DF}}}{\partial k^* \partial v^*} & \frac{\partial^2 \pi_m^{\mathrm{DF}}}{\partial k^{*2}} & \frac{\partial^2 \pi_m^{\mathrm{DF}}}{\partial k^* \partial p_r^*} & \frac{\partial^2 \pi_m^{\mathrm{DF}}}{\partial k^* \partial z^*} & \frac{\partial^2 \pi_m^{\mathrm{DF}}}{\partial k^* \partial q^*} \\[2mm]
\frac{\partial^2 \pi_m^{\mathrm{DF}}}{\partial p_r^* \partial v^*} & \frac{\partial^2 \pi_m^{\mathrm{DF}}}{\partial p_r^* \partial k^*} & \frac{\partial^2 \pi_m^{\mathrm{DF}}}{\partial p_r^{*2}} & \frac{\partial^2 \pi_m^{\mathrm{DF}}}{\partial p_r^* \partial z^*} & \frac{\partial^2 \pi_m^{\mathrm{DF}}}{\partial p_r^* \partial q^*} \\[2mm]
\frac{\partial^2 \pi_m^{\mathrm{DF}}}{\partial z^* \partial v^*} & \frac{\partial^2 \pi_m^{\mathrm{DF}}}{\partial z^* \partial k^*} & \frac{\partial^2 \pi_m^{\mathrm{DF}}}{\partial z^* \partial p_r^*} & \frac{\partial^2 \pi_m^{\mathrm{DF}}}{\partial z^{*2}} & \frac{\partial^2 \pi_m^{\mathrm{DF}}}{\partial z^* \partial q^*} \\[2mm]
\frac{\partial^2 \pi_m^{\mathrm{DF}}}{\partial q^* \partial v^*} & \frac{\partial^2 \pi_m^{\mathrm{DF}}}{\partial q^* \partial k^*} & \frac{\partial^2 \pi_m^{\mathrm{DF}}}{\partial q^* \partial p_r^*} & \frac{\partial^2 \pi_m^{\mathrm{DF}}}{\partial q^* \partial z^*} & \frac{\partial^2 \pi_m^{\mathrm{DF}}}{\partial q^{*2}}
\end{pmatrix}.
$$

The first order principal minor is

$$\det(H_{11}) = \frac{\partial^2 \pi_m^{\mathrm{DF}}}{\partial v^{*2}} = -\theta < 0.$$

Here, the first order principal minor is negative. The second order principal minor is

$$\det(H_{22}) = \frac{\partial^2 \pi_m^{\mathrm{DF}}}{\partial k^{*2}} \det(H_{11}) - \left(\frac{\partial^2 \pi_m^{\mathrm{DF}}}{\partial v^* \partial k^*}\right)^2$$

$$= \left[2\phi + 2a_7\left(\frac{b_1}{p_r} + b_2 p_r + P_v\eta - w - P_v\eta v\right)\right]\theta > 0,$$

if $\left(\frac{b_1}{p_r} + b_2 p_r + P_v\eta\right) > (w + P_v\eta v)$. The principal minor of the second order is positive. The third order principal minor is

$$\det(H_{33}) = \frac{\partial^2 \pi_m^{\mathrm{DF}}}{\partial p_r^{*2}} \det(H_{22}) - \left(\frac{\partial^2 \pi_m^{\mathrm{DF}}}{\partial v^* \partial p_r^*}\right)^2 \left(\frac{\partial^2 \pi_m^{\mathrm{DF}}}{\partial k^{*2}}\right) = -\frac{2Mb_1}{p_r^3}\det(H_{22}) < 0,$$

as $\det(H_{22}) > 0$. The calculated third principal minor is less than zero. The fourth order principal minor is

$$\det(H_{44}) = \frac{\partial^2 \pi_m^{\mathrm{DF}}}{\partial z^{*2}} \det(H_{33}) - \left(\frac{\partial^2 \pi_m^{\mathrm{DF}}}{\partial v^* \partial z^*}\right)^2 \left(\frac{\partial^2 \pi_m^{\mathrm{DF}}}{\partial k^{*2}}\right)\left(\frac{\partial^2 \pi_m^{\mathrm{DF}}}{\partial p_r^{*2}}\right)$$

$$= \left[\left(\frac{b_1}{p_r} + b_2 p_r + P_v\eta - w - vP_v\eta\right)bp(p-1)z^{p-2} + b\right]\det(H_{33}) > 0,$$

as $\det(H_{33}) < 0$. The calculated fourth principal minor is greater than zero. Fifth order principal minor is The fifth principal minor is

$$\det(H_{55}) = \frac{\partial^2 \pi_m^{\mathrm{DF}}}{\partial q^{*2}} \det(H_{44}) - \left(\frac{\partial^2 \pi_m^{\mathrm{DF}}}{\partial v^* \partial q^*}\right)^2 \left(\frac{\partial^2 \pi_m^{\mathrm{DF}}}{\partial k^{*2}}\right)\left(\frac{\partial^2 \pi_m^{\mathrm{DF}}}{\partial p_r^{*2}}\right)\left(\frac{\partial^2 \pi_m^{\mathrm{DF}}}{\partial z^{*2}}\right)$$

$$= -\left[\left(\frac{b_1}{p_r} + b_2 p_r + P_v\eta - w - vP_v\eta\right)2a_9 + \sigma\right]\det(H_{44}) < 0,$$

as $\det(H_{44}) > 0$. See Appendix A for all second order partial derivatives. Thus, all the principal minors are in alternative in sign. Thus, it can be concluded that the expected total profit of the CLSCM is the global maximum at the optimum values. $\qquad\square$

Appendix C.

$$H_1 = I + a_0 + a_2 v^* + a_3 v^{*2} + a_4 j^* + a_5 j^{*2} + a_6 k^* + a_7 k^{*2} + a_8 q + a_9 q^2 + as^{*\gamma} + bz^\rho,$$

$$H_2 = \left(H_1 - \frac{I^2}{2k_1} - a_1\right),$$

$$H_3 = a_1 p_{\min} - p_{\min}\left(H_1 - \frac{I^2}{2k_1}\right),$$

$$H_4 = p_{\min}^2\left(H_1 - \frac{I^2}{2k_1}\right) + a_1(-p_{\max}p_{\min} - wp_{\min} + wp_{\max}),$$

$$H_5 = I + a_0 + a_1\frac{p_{\max} - p}{p - p_{\min}} + a_4 j + a_5 j^2 + a_6 k + a_7 k^2 + as^\gamma + bz^\rho,$$

$$H_6 = 2a_3\left(w - p_v\eta - \frac{b_1}{p_r} - b_2 p_r\right) + 2a_2 p_v\eta - \theta,$$

$$H_7 = H_5 p_v\eta + a_2\left(w - \frac{b_1}{p_r} - b_2 p_r - p_v\eta\right),$$

$$H_8 = y\left[\left(l - w + a_2 p_r + \frac{a_1}{p_r}\right)x + (T_1 - B)(1 - x)\right] - a_6\left(w - p_v\eta - \frac{b_1}{p_r} - b_2 p_r + p_v v\eta\right),$$

$$H_9 = 2a_7\left(w - p_v\eta - \frac{b_1}{p_r} - b_2 p_r + p_v v\eta\right) - 2\phi,$$

$$H_{10} = \left(I + a_0 + a_1\frac{p_{\max} - p}{p - p_{\min}} + a_2 v + a_3 v^2 + a_4 j + a_5 j^2 + a_6 k + a_7 k^2 + a_8 q + a_9 q^2 + a s^\gamma + b z^\rho\right),$$

$$A = \frac{3p - 2w + 3s_r + h_r}{p + h_r + s_r},$$

$$A_1 = p_{\min}^2 H_2 + 2H_1 p_{\min}^2 - 2a_1 p_{\max} p_{\min} + 2a_1 w p_{\min} - 2a_1 w p_{\max},$$

$$H_{11} = w - \frac{b_1}{p_r} - b_2 p_r - P_v\eta + P_v\eta v.$$

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