Teleportation of a Vacuum–One-Photon Qubit

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We report the experimental realization of teleporting a one-particle entangled qubit. The qubit is physically implemented by a two-dimensional subspace of states of a mode of the electromagnetic field, specifically, the space spanned by the vacuum and the one-photon state. Our experiment follows the line suggested by Lee and Kim [Phys. Rev. A **63**, 012305 (2000)] and Knill, Laflamme, and Milburn [Nature (London) **409**, 46 (2001)]. An unprecedented large value of the teleportation "fidelity" has been attained: $F = (95.3 \pm 0.6)\%$.

In their pioneering paper Bennett, Brassard, Crepeau, Jozsa, Peres, and Wootters introduced the concept of teleportation of a quantum state [1]. Since then, teleportation came to be recognized as one of the basic methods of quantum communication and, more generally, as one of the basic ideas of the whole field of quantum information. Following the original teleportation paper and its continuous-variables version [2], an intensive experimental effort started for the practical realization of teleportation. The quantum state teleportation (QST) has been realized in a number of experiments [3–6]. In a beautiful example of ingenuity, although starting from a common theme, each of these experiments followed a completely different route and principle. In the present paper, we report a new teleportation experiment following yet another different idea. In our experiment, we consider a qubit which is physically realized not by a particle but by a *mode* of the electromagnetic (e.m.) field, and whose orthogonal base states $|0\rangle$, $|1\rangle$ are the vacuum state and the one-photon state, respectively. We designed our scheme by adapting a method proposed by Knill, Laflamme, and Milburn [7] to make it experimentally easily feasible. We later learned that our method is identical to that proposed by Lee and Kim [8] and also closely related to [9]. In our experiment, the role of the two particles in a singlet state which constitute the nonlocal communication channel in the original teleportation scheme [1] is played by a photon in an equal superposition of being at Alice and Bob $|\Psi\rangle = 2^{-1/2}(|\text{Alice}\rangle + |\text{Bob}\rangle)$ where $|\text{Alice}\rangle$ and $|\text{Bob}\rangle$ represent the photon located at Alice and Bob, respectively. The scheme seems puzzling. Indeed entanglement is considered the basis of teleportation, and here we do not even have two particles, let alone two particles in an entangled state. The puzzle is solved, however, by noting that in second quantization the state of the nonlocal channel reads as $|\Phi\rangle_{\text{singlet}} = 2^{-1/2}(|1\rangle_A|0\rangle_B - |0\rangle_A|1\rangle_B)$, where the labels A and B represent two different modes of the e.m. field, with wave vectors (wv) k_A and k_B, one directed towards Alice and the other towards Bob. The mode indexes 0

and 1 denote the Fock state population by zero (vacuum) and one photon, respectively. In effect, the role of the two entangled quantum systems which form the nonlocal channel is played by the e.m. fields of Alice and Bob. In other words, the *field's modes* rather than the photons associated with them should be properly taken as the information and entanglement carriers, i.e., *qubits*. The nonlocality of a single photon, first introduced by Albert Einstein [10], has been discussed by [11–13].

Of course, in order to make use of the entanglement present in this picture, we need to use the second quantization procedure of creation and annihilation of particles and/or use states which are superpositions of states with different numbers of particles. Another puzzling aspect of this second quantized picture is the need to define and measure the relative phase between states with a different number of photons, such as the relative phase between the vacuum and one-photon state in Eq. (1) below. That we can associate a relative phase between the *vacuum* and anything else seems most surprising, but it is less so if we recall the more familiar case of a coherent state, where the relative phase between the different photon number states in the superposition is reflected physically in the phase of the classical electric field. To be able to control these relative phases we need, in general and in analogy with classical computers, to supply all gates and all sender/receiving stations of a quantum information network with a common *clock* signal, e.g., provided by an ancillary photon or by a multiphoton, Fourier transformed coherent e.m. pulse [14]. In simple cases an *ad hoc* clock generator is not needed as the phase information can be retrieved by a linear superposition of two optical modes in a beam splitter. This will be demonstrated by the present experiment.

The quantum system whose state we want to teleport is physically represented by another mode of the e.m. field, one with wv k_S. Again we consider only a two-dimensional Hilbert space of this mode, i.e., spanned by $|0\rangle_S$ and $|1\rangle_S$. Thus, the *mode* k_S can be considered the

qubit to be teleported. Suppose now that the qubit k_S is in an arbitrary *pure* state,

$$\alpha|0\rangle_S + \beta|1\rangle_S . \tag{1}$$

The overall state of the system and the nonlocal channel is then

$$
\begin{aligned}
|\Phi_{\text{total}}\rangle &= 2^{-1/2}(\alpha|0\rangle_S + \beta|1\rangle_S)(|1\rangle_A|0\rangle_B - |0\rangle_A|1\rangle_B) \\
&= 2^{-1/2}\alpha|\Psi^1\rangle_{SA}|1\rangle_B + 2^{-1/2}\beta|\Psi^2\rangle_{SA}|0\rangle_B \\
&\quad + \tfrac{1}{2}|\Psi^3\rangle_{SA}(\alpha|0\rangle_B + \beta|1\rangle_B) \\
&\quad + \tfrac{1}{2}|\Psi^4\rangle_{SA}(\alpha|0\rangle_B - \beta|1\rangle_B) , \tag{2}
\end{aligned}
$$

where the states $|\Psi^j\rangle_{SA}$, $j = 1, 2, 3, 4$ are defined below in Eq. (3). The teleportation proceeds with Alice performing a partial Bell measurement. She combines the modes k_S and k_A on a symmetric (i.e., 50:50) beam splitter BS_A whose output modes k_1 and k_2 are coupled to two detectors D_1 and D_2, respectively (see Fig. 1). The action of BS_A on the field operators is expressed by $\hat{a}_S^\dagger = 2^{-1/2}(\hat{a}_2^\dagger - \hat{a}_1^\dagger)$; $\hat{a}_A^\dagger = 2^{-1/2}(\hat{a}_2^\dagger + \hat{a}_1^\dagger)$, where labels 1, 2 refer to modes k_1, k_2. As a consequence, we obtain

$$
\begin{aligned}
|\Psi^1\rangle_{SA} &= |0\rangle_S|0\rangle_A = |0\rangle_1|0\rangle_2 , \\
|\Psi^2\rangle_{SA} &= |1\rangle_S|1\rangle_A = 2^{-1/2}(|0\rangle_1|2\rangle_2 - |2\rangle_1|0\rangle_2) , \\
|\Psi^3\rangle_{SA} &= 2^{-1/2}(|0\rangle_S|1\rangle_A - |1\rangle_S|0\rangle_A) = |1\rangle_1|0\rangle_2 , \\
|\Psi^4\rangle_{SA} &= 2^{-1/2}(|0\rangle_S|1\rangle_A + |1\rangle_S|0\rangle_A) = |0\rangle_1|1\rangle_2 ,
\end{aligned}
\tag{3}
$$

The state $|\Psi^3\rangle_{SA}$ is a Bell-type state [1]. From Eq. (3) we see that its realization implies a single photon arriving at the detector D_1 and no photons at D_2. Similarly, $|\Psi^4\rangle_{SA}$ is a Bell-type state and it implies a single photon arriving at the detector D_2 and no photons at D_1. In both of these cases the teleportation is successful. Indeed, when Alice finds $|\Psi^3\rangle_{SA}$, Bob's e.m. field ends up in the state $|\Phi\rangle = (\alpha|0\rangle_B + \beta|1\rangle_B)$ which is identical to the state to be teleported, while when Alice finds $|\Psi^4\rangle_{SA}$, Bob ends up with the state $|\Phi_\pi\rangle = (\alpha|0\rangle_B - \beta|1\rangle_B) = \sigma_z|\Phi\rangle$ which is identical to the state to be teleported up to a phase shift $\Delta = \pm\pi$. The states $|\Phi\rangle$ and $|\Phi_\pi\rangle$ are connected by a unitary transformation expressed by the Pauli spin operator σ_z. Bob can easily correct the phase shift Δ upon finding out Alice's result. In practice, this phase correction
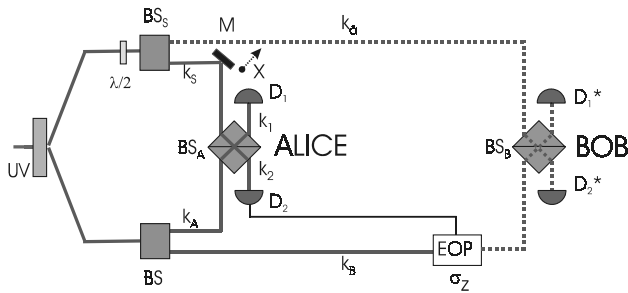

FIG. 1. Experimental apparatus realizing the "active" and "passive" QST.

procedure, generally referred to as "active teleportation" [3] is carried out automatically by means of a fast electro-optic Pockels cell (EOP) inserted in mode k_B and triggered by D_2. On the other hand, when Alice finds $|\Psi^1\rangle_{SA}$ or $|\Psi^2\rangle_{SA}$ the teleportation fails. From Eq. (3) we see that teleportation is successful in 50% of the cases.

A major technical difficulty in the above teleportation scheme is the preparation and manipulation of the pure states to be teleported. Indeed, they are superpositions of the vacuum and one-photon states of the mode k_S. Manipulating such states and, in particular having control about the relative phase between the vacuum and one-photon states is quite problematic. This can be realized in principle, for example, by homodyning techniques as described in [15]. Here, however, we avoid the problem altogether, by teleporting appropriate *entangled* states instead of pure ones. The states we consider are of the form

$$|\Psi\rangle_{S\tilde{a}} = (\alpha|0\rangle_S|1\rangle_{\tilde{a}} + \beta|1\rangle_S|0\rangle_{\tilde{a}}) , \tag{4}$$

where $k_{\tilde{a}}$ is an "ancilla" mode. These states are in fact simple single-photon states and can be easily obtained by, for example, letting a single photon impinge on a beam splitter (BS_S in Fig. 1) with reflectivity r_S and transmissivity t_S, $k_{\tilde{a}}$ being the reflected mode and k_S the transmitted one. For the sake of simplicity and without loss of generality, we assume that α and β are *real* numbers.

In summary, in our experiment we have four qubits: k_A and k_B which constitute the nonlocal communication channel, k_S which represents the system, i.e., the qubit to be teleported, and $k_{\tilde{a}}$ the ancilla. The special states of these four qubits which are used in the experiment are physically implemented by exactly two photons. The state of the qubit k_S is teleported to Bob into the state of the qubit k_B, thus the overall state $|\Psi\rangle_{S\tilde{a}}$ will now be transferred into the state of the qubits k_B and $k_{\tilde{a}}$. To verify that the state has been teleported, we transmit the qubit $k_{\tilde{a}}$ to Bob. The QST verification consists simply by mixing the modes k_B and $k_{\tilde{a}}$ at a beam splitter (BS_B) similar to the one which was used to produce the state to be teleported $|\Psi\rangle_{S\tilde{a}}$. We shall see that the optimum QST verification, viz. implying the maximum *visibility V* of the corresponding interferometric patterns, is obtained by adopting equal optical parameters for both BS_S and BS_B, i.e., $|r_S| = |r_B| = \alpha$ and $|t_S| = |t_B| = \beta$. This verification procedure is generally referred to as "passive teleportation" [3]. Finally, note that the "ancillary" single photon emitted on mode $k_{\tilde{a}}$ indeed provides the "clock" pulse that is needed to retrieve at Bob's side the *full information* content of the vacuum state $|0\rangle_B$ entangled within the nonlocal teleportation channel, i.e., with the *singlet* state $|\Phi\rangle_{\text{singlet}}$ [14].

The experimental setup is shown in Fig. 1. A nonlinear LiIO$_3$ crystal slab, 1.5 mm thick with parallel antireflection coated faces, cut for Type I phase matching is pumped by a single mode UV cw argon laser with wavelength (wl) $\lambda_p = 363.8$ nm and with an average power $\simeq 100$ mW. The UV laser beam was focused close to the crystal by

a lens with focal length $= 2$ m in order to maximize the collection efficiency by the Alice's detector system of the spontaneous parametric down-conversion (SPDC) fluorescence [16]. The two SPDC emitted photons have equal wl $\lambda = 727, 6$ nm and are spatially selected by two pinholes with equal apertures with diameter 0.5 mm placed at a distance of 50 cm from the crystal. One of the photons generates on the two output modes k_A and k_B of a 50:50 beam splitter (BS) the singlet state $|\Phi\rangle_{singlet}$ providing the nonlocal teleportation channel. The other photon generates the state $|\Psi\rangle_{S\tilde{a}}$, i.e., the quantum superposition of the state to be teleported and the one of the ancilla at the output of a *variable* beam splitter BS_S consisting of the combination of a $\lambda/2$ polarization rotator and of a calcite crystal. Furthermore, micrometric changes of the mutual phase φ of the k_S and k_A modes interfering on BS_A were obtained by a piezoelectrically driven mirror M. All detectors were Si-avalanche EG&G-SPCM200 counting modules having nearly equal quantum efficiencies $QE \approx 0.45$. Before detection two equal interference filters selected the frequency of the beams within a 20 nm bandwidth. In Fig. 1, the complete scheme for "active" teleportation is shown, including the high-voltage Pockels cell (EOP) inserted on the mode k_B. In the same figure is reported the interferometric scheme for "passive teleportation" which is also adopted for the verification of the correct implementation of the active protocol, as we shall see.

We have realized experimentally the *passive* teleportation protocol. By this we mean that Bob does not modify his state according to the results obtained by Alice. Instead Bob passes his state unmodified to the verification stage. The verification stage consists of combining the mode k_B (which now contains the teleported state) with the ancilla mode $k_{\tilde{a}}$ at a beam splitter BS_B, as said. In order to check the overall mode alignment, we first checked at Alice's site the two-photon Ou-Mandel interference across the beam splitter BS_A between the modes k_S and k_A that are coupled to detectors D_1 and D_2, respectively. We obtained a two-photon interference pattern with a visibility $V_A \approx 0.96$. In a similar way, we checked at Bob's site the Ou-Mandel interference across BS_B between the modes k_B and $k_{\tilde{a}}$ coupled to the respective detectors D_1^*, D_2^*, obtaining $V_B \approx 0.92$. The QST verification experiment has been carried out first with a 50:50 beam splitter BS_S, i.e., with optical parameters $|r_S| = |t_S| = 2^{-1/2}$. The maximum visibility of the verification fringe pattern is obtained by selecting the same values of the parameters for the test beam splitter BS_B, as said. Then we measured the coincidence counts between D_1, D_2 and D_1^*, D_2^*. By a straightforward calculation, we expect

$$D_1 - D_1^* = D_2 - D_2^* = \frac{1}{2}\sin^2\frac{\varphi}{2}, \tag{5}$$

$$D_1 - D_2^* = D_2 - D_1^* = \frac{1}{2}\cos^2\frac{\varphi}{2},$$

where $(D_i - D_j^*)$ expresses the probability of a coincidence detected by the pair D_i, D_j^* in correspondence with the realization of either one of the states: $|\Psi^3\rangle_{SA}$, $|\Psi^4\rangle_{SA}$. The experimental plots shown in Fig. 2, obtained by varying the position $X = (2)^{-3/2}\lambda\varphi/\pi$ of the mirror M, are in agreement with the theory [Eq. (5)]. This agreement is further substantiated by the data reported in Fig. 3 corresponding to a similar verification experiment carried out with different optical parameters for BS_B: $|r_B|^2 = 0.20$, $|t_B|^2 = 0.80$. There it is shown that the maximum visibility V is attained for values of $\alpha^2 = 1 - \beta^2$ that are equal to $|r_B|^2$ or to $|t_B|^2$ depending on which pair of detectors are excited. In the same figure is also reported a single value $V \simeq 0.91$ related to the fully symmetric case: $|r_B|^2 = |t_B|^2 = \alpha^2 = \beta^2 = \frac{1}{2}$.

Note that by assuming perfect detectors, i.e., with $QE = 1$, the above QST verification procedure involving the ancilla mode $k_{\tilde{a}}$ enables a fully *noise-free* teleportation procedure. Indeed, if no photons are detected at Alice's site, i.e., by D_1 and/or D_2, while photons are detected at Bob's site by D_1^* and/or D_2^*, we can safely conclude that the "idle" Bell state $|\Psi^1\rangle_{SA}$ has been created. If on the contrary no photons are detected at Bob's site while photons are detected at Alice's site, we must conclude that the other idle Bell state $|\Psi^2\rangle_{SA}$ has been realized. The data collected in correspondence with these idle events can automatically be discarded by the electronic coincidence circuit. In addition to that, note that the effect of the above verification procedure involving the ancilla mode $k_{\tilde{a}}$ keeps holding within the *active teleportation* scheme. Indeed, if the D_2-driven electro-optic (EOP) phase modulator works correctly within the *active* scheme, the detector D_2^* should be found to be *always* inactive. Note that by the present method an unprecedented large value of the QST "fidelity" has been attained: $F = \langle\Phi_{in}|\rho_{out}|\Phi_{in}\rangle = (1 + V)/2 = (95.3 \pm 0.6)\%$.
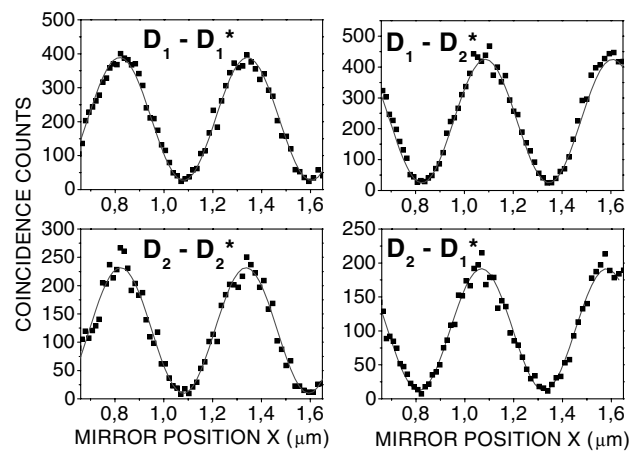

FIG. 2. Interferometric fringe patterns obtained by coincidence experiments involving different pairs of detectors within a *passive* QST verification procedure. The different scales for the coincidence rates account for the overall quantum efficiencies of the corresponding detection channels.
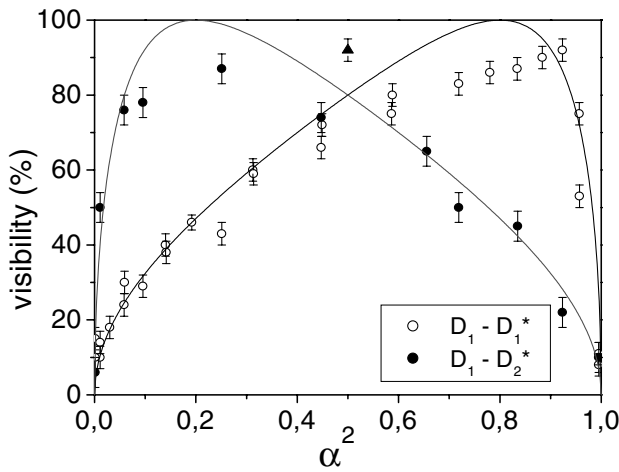
FIG. 3. Visibility V of the coincidence fringe patterns vs the superposition parameter α^2 obtained by two different pairs of detectors within a *passive* QST experiment and for an unbalanced beam splitter BS_B with optical parameters $|r_B|^2 = 0.20$, $|t_B|^2 = 0.80$. A single value of V for the symmetric case $|r_B|^2 = |t_B|^2 = \alpha^2 = 0.50$ is also reported.

Our present effort is directed towards the completion of the teleportation picture by the realization of the active scheme. The main technical problem resides in the relatively large time needed to activate a high-voltage EOP device by a single-photon detection. The best result we have attained thus far for the 1 kV switching time across an EOP modulator is about 10 ns. This figure would enable us to achieve the goal in the near future by the adoption of small $\lambda/2$-voltage EOP devices possibly in conjunction with the use of optical fibers.

In conclusion, we have given the experimental all optical demonstration of a key quantum information protocol in a new conceptual framework. This one is based on the nonlocal properties of a single photon and on the adoption of the Fock states of any single optical mode as basis states of the relevant Hilbert space. We have also shown that these states can be entangled. This new approach, which necessarily involves the adoption of synchronizing clock states, is expected to be far reaching as it can be generalized to all protocols of quantum information and quantum computation. A theoretical analysis of the new perspectives is reported elsewhere.

[1] C. Bennett, G. Brassard, C. Crepeau, R. Jozsa, A. Peres, and W. Wootters, Phys. Rev. Lett. **70**, 1895 (1993).
[2] L. Vaidman, Phys. Rev. A **49**, 1473 (1994).
[3] D. Boschi, S. Branca, F. De Martini, L. Hardy, and S. Popescu, Phys. Rev. Lett. **80**, 1121 (1998).
[4] D. Bouwmeester, J. M. Pan, K. Mattle, M. Eibl, H. Weinfurter, and A. Zeilinger, Nature (London) **390**, 575 (1997).
[5] A. Furusawa, J. L. Sorensen, S. L. Braunstein, C. A. Fuchs, H. J. Kimble, and E. S. Polzik, Science **282**, 706 (1998).
[6] M. Nielsen, E. Knill, and R. Laflamme, Nature (London) **396**, 52 (1998).
[7] E. Knill, R. Laflamme, and G. Milburn, Nature (London) **409**, 46 (2001).
[8] Hai-Wong Lee and J. Kim, Phys. Rev. A **63**, 012305 (2001).
[9] Hai-Wong Lee, Phys. Rev. A **64**, 014302 (2001).
[10] N. Bohr, in *Albert Einstein Philosopher Scientist,* edited by P. A. Schlipp (Cambridge University Press, London, 1949).
[11] B. Oliver and C. Stroud, Jr., Phys. Lett. A **135**, 407 (1989).
[12] S. Tan, D. Walls, and M. Collet, Phys. Rev. Lett. **66**, 252 (1991).
[13] L. Hardy, Phys. Rev. Lett. **73**, 2279 (1994).
[14] F. De Martini, in *Experimental Quantum Computation and Information,* edited by C. Monroe and F. De Martini (IOS, Amsterdam, 2001).
[15] D. F. Walls and G. Milburn, *Quantum Optics* (Springer-Verlag, Berlin, 1995), Chap. 3.
[16] C. Monken, P. Souto Ribeiro, and S. Padua, Phys. Rev. A **57**, R2267 (1998); **57**, 3123 (1998).