Temperature dependence of bulk viscosity in water using acoustic spectroscopy

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1. Introduction

The parameter of bulk or volume viscosity μ receives little attention, despite being defined in the Navier-Stokes equation for a compressible liquid. It is relevant in the field of acoustic propagation where it plays the equivalent role in viscosity to the bulk modulus K in solids. Specifically, the total viscosity for fluids $\eta_{total} = \mu + 4\eta/3$ is comparable with the longitudinal modulus $M = K + 4G/3$ in solids where G is the shear modulus and η is shear viscosity using the time dependence relation $i\omega\eta_{total} \rightarrow M$. In the absence of scattering, the classical theory of acoustic attenuation predicts a frequency squared dependence on the viscosity, which is governed by both shear and bulk viscosity. We demonstrate that this technique can be readily applied to fluids and accurately determine temperature dependent bulk viscosities.

2. Governing Equations (Navier-Stokes and Energy equations)

Considering first order periodic states enables the equations to be simplified by replacing partial time derivatives with the factor $-i\omega$ (where angular frequency $\omega = 2\pi f$ *and f* is the frequency of the acoustic wave) to yield the following equations (1) and (2).

$$\omega^2\mathbf{v} + \left(\frac{v^2}{\gamma} - i\omega\frac{N\eta}{\rho}\right)\nabla(\nabla\cdot\mathbf{v}) + i\omega\frac{\eta}{\rho}\nabla\times\nabla\times\mathbf{v} = \frac{i\omega\beta v^2}{\gamma}\nabla T, \qquad N = \left(\frac{4}{3}+\frac{\mu}{\eta}\right) \qquad (1)$$

$$\gamma\frac{\tau}{\rho C_p}\nabla^2 T + i\omega T = \frac{(\gamma-1)}{\beta}\nabla\cdot\mathbf{v} \qquad (2)$$

where **v** is the velocity vector of the fluid, T is temperature, γ is ratio of specific heats, τ is thermal conductivity, ρ is density, C_p the specific heat at constant pressure, β is bulk compressibility, η is shear viscosity, μ is bulk viscosity and v is the velocity of sound. This system can be further simplified by representing the velocity vector **v** in terms of the scalar potentials which describe the longitudinal compressional field φ and the thermal field ψ together with a transverse shear vector potential $\boldsymbol{A}$. This allows equations (1) and (2) to be combined into a biharmonic type equation which may be then decoupled into independent Helmholtz equations for each given field as shown in equations (3) with respective acoustic, thermal and shear wavenumbers K, L and M, see [2], [3], [4].

$$(\nabla^2 + K^2)\,\varphi = 0 \qquad (\nabla^2 + L^2)\,\psi = 0 \qquad (\nabla^2 + M^2)\mathbf{A} = 0 \qquad (3)$$

In particular, the wavenumbers K, L and M are given in equations (4).

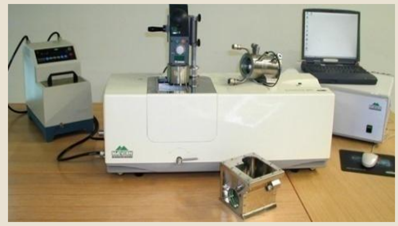
$$K = \frac{\omega}{v} + i\alpha = \frac{\omega}{v} + i\frac{\eta\omega^2}{2\rho v^3}\left[\frac{4}{3}+\frac{\mu}{\eta}+\frac{(\gamma-1)\tau}{\eta C_p}\right], \qquad L = \left(\frac{\omega\rho C_p}{2\tau}\right)^{1/2}(1+i), \qquad M = \left(\frac{\omega\rho}{2\eta}\right)^{1/2}(1+i) \qquad (4)$$

From the wavenumber K in equation (4) the acoustic attenuation coefficient α is frequency squared dependent and is related to the bulk viscosity μ by the following,

$$\mu = \left[\frac{2\alpha\rho v^3}{\omega^2} + \frac{4\eta}{3} + \frac{(\gamma-1)\tau}{C_p}\right] \qquad (5)$$

By populating equation (5) with the relevant parameter values and measured acoustic attenuation values, using a Malvern Ultrasizer for example, enables temperature dependent bulk viscosities to be determined.

3. Attenuation measurement



The Malvern Ultrasizer™ The Ultrasizer was used to perform 30 repeat attenuation measurements across 50 frequency values in the range 1-100MHz for 6 selected temperatures for millipore water.

Selected References

[1] A S Dukhin and P J Goetz (2009). *Bulk viscosity and compressibility measurement using acoustic spectroscopy.* The Journal of Chemical Physics 130, 124519,1-13.

[2] M J Povey (1997). *Ultrasonic Techniques For Fluids Characterization* Academic Press.

[3] P S Epstein and R R Carhart (1953). *The Absorption of Sound in Suspensions and Emulsions I Water Fog in Air.* The Journal of the Acoustical Society of America 25, 553-565.

[4] J R Allegra and S A Hawley (1972). *Attenuation of Sound in Suspensions and Emulsions: Theory and Experiments* The Journal of the Acoustical Society of America 51, 1545-1564.

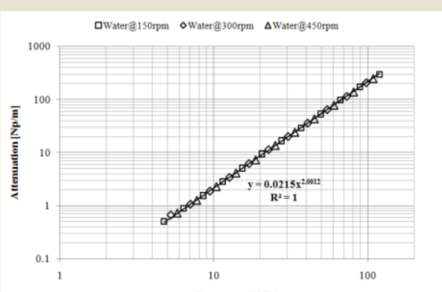
[5] T A Litovitz and C M Davis (1964). *Physical Acoustics*, edited by W P Mason Academic, New York, Vol. 2, Chap 5.

[6] J Xu, X Ren, W Gong, R Dai and D Liu (2003). Measurement of the bulk viscosity of liquid by Brillouin scattering *Applied Optics*, Vol 42(33), 6704-6709.

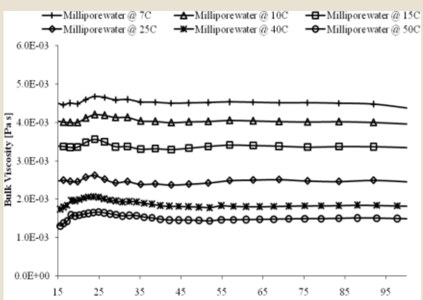
4. Results. Bulk viscosity temperature dependence for millipore water

Temperature	Density	Shear Viscosity	Thermal Conductivity	Specific Heat	Velocity	Mean Bulk Viscosity	Standard error
T	ρ	η	τ	C_p	v	μ	
°C	kg/m^{-3}	Pa.s	W/m K	J /kg m^3	m/s	Pa.s	Pa.s
7	999.81	1.43E-03	0.5747	4198.1	1434.92	4.50E-03	1.20E-05
10	999.70	1.31E-03	0.5800	4192.1	1447.29	4.03E-03	1.17E-05
15	998.97	1.14E-03	0.5900	4187.0	1465.96	3.38E-03	1.10E-05
25	997.00	8.88E-04	0.6075	4180.1	1496.73	2.47E-03	1.08E-05
40	992.22	6.53E-04	0.6305	4178.5	1528.89	1.84E-03	2.70E-05
50	988.03	5.47E-04	0.6435	4180.6	1542.57	1.48E-03	2.76E-05

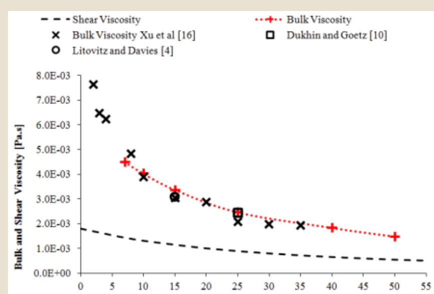
Table 1. Parameter values for water for 6 selected temperatures. Bulk viscosity values are computed by averaging the 30 measured attenuation values at each frequency. Each average value is then used to calculate the bulk viscosity using equation (5) at each frequency. A total mean bulk viscosity with standard error are then computed by averaging over all frequencies.



Attenuation-frequency plot for 0.22μm Millipore water at 25°C with agitation speeds 150, 300 and 450 rpm. Note the frequency squared dependence and that varying agitation speed does not affect the attenuation results



Bulk viscosity [Pa.s] for 0.22μm Millipore water against frequency [MHz] for the temperatures 7, 10, 15, 25, 40 and 50°C. For Newtonian fluids bulk viscosities are approximately constant.



Bulk viscosity (··+··) [Pa.s] for 0.22μm Millipore water against temperature [°C] using acoustic spectroscopy. Shear viscosity (- - -) taken from [2]. Bulk viscosity data (×) by Xu et al [6] using Brillioun scattering, (o) Litovitz and Davies [5] and (□) Dukhin and Goetz [1]. Bulk viscosity is a factor a 3 times higher than the shear viscosity at low temperature.

5. Discussion.

Our results show that the bulk velocity is approximately constant over the measured frequency range for fixed temperature which is expected for Newtonian fluids. Deviations from this behaviour would indicated a breakdown of the Newtonian assumption and could be used as a test to determine non-Newtonian fluids. In [1] Dukhin and Goetz reported a bulk viscosity value of 2.43E-03 Pa.s at 25°C for distilled water in good agreement with our value of 2.47E-03 [2.46E-03, 2.48E-03]. Similarly, they reported a ratio of bulk to shear viscosity as 2.73 whereas we obtained a value of 2.78. Another example cited in [1] is the result obtained by Litovitz and Davis [5] who reported a value of 3.09E-03 for the bulk viscosity of water at 15°C and viscosity ratio of 2.81, in our case we recorded a value of 3.38E-03 with ratio 2.96. Additionally, Xu *et al* [6] used Brillioun scattering to determine the bulk viscosity of water over a similar temperature range to this work and determined comparable results.

We have successfully employed acoustic spectroscopy to determine the bulk viscosity of 0.22μm Millipore water across the temperature range 7-50°C and shown that we may represent the data using a cubic approximation which exhibits the temperature dependence. Using a Malvern Ultrasizer provides a robust consistent measurement technique that be readily applied to other fluids. This work provides further validation and results for the use of acoustic spectroscopy to determine bulk viscosity values for fluids and is in agreement with the work conducted by Dukhin and Goetz (2009) [1].