

Temperature stabilization of instabilities in force-free magnetospheres

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Received 1976 September 10; in original form 1976 August 4

Summary. Relative streaming between particle species may arise naturally in magnetospheres, to maintain a force-free charge density. Such streaming causes instabilities which are stabilized by temperature. We calculate relativistic stability conditions and show that an energy spread $\Delta\gamma \sim 1.5\gamma$ can stabilize pulsar electron–positron plasmas, for a reasonable parameter range. Compact radio sources without strong beaming may be relatively immune to temperature stabilization, however.

1 Introduction

For a magnetosphere to maintain corotation with its source, in general a specific array of electric fields must fill it. A plasma in this magnetosphere will carry current $J(x)$ and charge density $\rho(x)$ which must in steady state give the force-free values. Where $\mathbf{E} \cdot \mathbf{B} = 0$ on average, a local charge density cannot be sustained except by a relative streaming between components of the plasma, whenever $J/\rho > 1$. (The $J/\rho > 1$ condition insures that J can never be transformed away totally, but in real physical conditions with more than one species present, this is probably too strong a condition, and relative streaming may be inevitable.)

Corotating magnetospheres may arise in the compact centres of quasars (Blandford 1976; Blandford & Znajek 1977, and references therein). Extraction of energy from these magnetospheres can be efficient and may hinge upon the production of electron–positron pairs by vacuum breakdown above a black hole (Blandford & Znajek 1977). Such an electron–positron plasma streaming outward must provide the net ρ required in the magnetosphere, by an appropriate differential streaming. This streaming can give rise to plasma instabilities, which in turn may cause coherent radiation. In quasars this may help explain rapid variations in low-frequency radio flux.

Similar statements hold for pulsar magnetospheres. Recently Cheng & Ruderman (1976) have suggested that electron–positron relative streaming occurs above the pulsar polar caps. This system is unstable to an electrostatic mode that bunches plasma particles which can

then emit coherent microwave radiation because they are moving on curved magnetic field lines.

In this paper we study the stabilizing influence of momentum spreading in the electron–positron distributions, i.e. ‘thermal’ effects. We shall refer mostly to the pulsar case, since more is known (or thought to be known) about pulsar (dipolar) magnetospheres. However, our treatment can be carried over to any magnetosphere for which the charge density is known.

For pulsars we first show that the relative streaming speed $\Delta v/c$ can be smaller than 10^{-4} (as suggested by Cheng & Ruderman) over much of the microwave-emitting region. Hence thermal quenching of the instability can be important. Then we give a relativistic generalization of the familiar Penrose criterion and use it to set limits on the thermal spread. Over much of the near magnetosphere it seems that a relatively mild ‘temperature’ quenches instability.

2 Relative electron–positron streaming in pulsars

For field lines not close to the light cylinder one can evaluate the relative streaming speed of the electron–positron secondaries, for aligned magnetization and rotation axes. Since the required charge density is essentially the Goldreich–Julian charge density, which is the high-energy beam density, modified by angular factors (Cheng & Ruderman 1976)

$$\frac{\Delta v}{c} = \left(\frac{\rho_b}{\rho_{\pm 0}} \right) \left[\frac{\Omega \cdot \mathbf{B} f}{(\Omega \cdot \mathbf{B} f)_0} - 1 \right] \quad (1)$$

where Ω is the angular velocity of the pulsar,

$$f = \left(1 - \frac{\Omega^2 r^2}{c^2} \sin^2 \theta \right)^{-1},$$

and r, θ are conventional spherical polar coordinates. Here ρ is the particle density and γ the relativistic energy factor. Subscript b refers to a positron beam with $\gamma_b \sim 10^6$; subscript \pm indicates a dense electron (–) and positron (+) plasma, $\gamma_{\pm} \sim 10^3$, through which the beam streams. Also, subscript 0 refers to a region near the pulsar surface where there is very little electron–positron velocity separation, $\gamma_+ = \gamma_- \equiv (\gamma_{\pm})_0$. Both the velocity separation $\Delta v/c$ and energy separation $\Delta\gamma = \gamma_+ - \gamma_-$ are as seen in the *laboratory* frame. If we assume a simple dipolar geometry, use the Ruderman & Sutherland (1975) value $(\rho_b/\rho_{\pm})_0 \approx \gamma_{\pm}/\gamma_b$ and assume $r < 5 \times 10^9 P$ (P the pulsar period in s, r in cm) then equation (1) becomes

$$\left| \frac{\Delta v}{c} \right| = \frac{\gamma_{\pm}}{\gamma_b} \epsilon, \quad (2)$$

where

$$\epsilon = \frac{9}{8} \frac{r \theta_0^2}{r_0} = \frac{9}{8} \sin^2 \theta \ll 1.$$

For $P \geq 1$ s, the regime $r < 5 \times 10^9 P$ includes all the radio-emitting region (Benford & Buschauer 1977). Furthermore, even if the field lines are not purely dipolar, we expect $\theta \ll 1$ because of the strong beaming required for the short-duty cycle. Given (2) we find from simple relativistic formulae

$$\Delta\gamma = \frac{2(v_c/c) \gamma_c [\sqrt{1 + (\Delta v/c)^2 \gamma_c^2 (\gamma_c^2 - 1)} - 1]}{[(\Delta v/c)^2 (\gamma_c^2 - 1)^2 - [\sqrt{1 + (\Delta v/c)^2 \gamma_c^2 (\gamma_c^2 - 1)} - 1]^2]^{1/2}} \quad (3)$$

where γ_c denotes the centre of momentum and equals the original $(\gamma_{\pm})_0$. Two limiting forms of equation (3) are

$$\Delta\gamma \cong \frac{v_c}{c} \left(\frac{\Delta v}{c} \right) \gamma_c^3, \quad (4)$$

if

$$\frac{\Delta v}{c} \gamma_c \sqrt{\gamma_c^2 - 1} \ll 1;$$

$$\Delta\gamma \cong \sqrt{2} \gamma_c^2 \sqrt{\frac{\Delta v}{c}}, \quad (5)$$

if

$$\frac{\Delta v}{c} \gamma_c \sqrt{\gamma_c^2 - 1} \gg 1, \gamma_c \gg 1 \text{ and } \frac{\Delta v}{c} \ll 1.$$

Note also that if we denote quantities seen in the centre of momentum frame with a prime,

$$\Delta\gamma = 2(\gamma_c^2 - 1)^{1/2} \sqrt{(\gamma'_+)^2 - 1}. \quad (6)$$

The ‘temperature’ effects in the γ_c frame must be treated relativistically when $\gamma'_+ \gtrsim 2$, which implies $\Delta\gamma/2\gamma_c > 2$. Using equation (4) for example, this means

$$\frac{\Delta v}{c} \frac{\gamma_c^2}{4} \approx \frac{\gamma_c^3 \sin^2 \theta}{4\gamma_b} > 1. \quad (7)$$

For Ruderman–Sutherland parameters the problem becomes relativistic for dipolar angles $\theta \gtrsim 0.06$, i.e. over almost all the emitting region. Further useful relativistic formulas yield

$$\gamma_c \approx \sqrt{\gamma_+ \gamma_-} \quad \text{for } \gamma_+, \gamma_- \gg 1;$$

$$\gamma_+ = \frac{2\gamma_c \sqrt{\gamma_c^2 - 1}}{([2\gamma_c^2 - 1 - \sqrt{1+d}] [\sqrt{1+d} - d])}$$

where

$$d \equiv \left(\frac{\Delta v}{c} \right)^2 \gamma_c^2 (\gamma_c^2 - 1).$$

3 Relativistic Penrose criterion and thermal quenching

The relativistic dispersion relation for one-dimensional electrostatic modes propagating along an external magnetic field is (Montgomery & Tidman 1964)

$$D(k, \omega) = 1 + \sum_{\alpha} \frac{\omega_{p\alpha}^2 m_{0\alpha}}{k} \int_{-\infty}^{\infty} \frac{\partial F_{\alpha 0}(p)}{\partial p} \frac{dp}{\omega - kv_{\alpha}(p)} \quad (8)$$

with normalization

$$\int_{-\infty}^{\infty} F_{\alpha 0}(p) dp = 1$$

and

$$\omega_{p\alpha}^2 = 4\pi n_{\alpha 0} q_{\alpha}^2 / m_{0\alpha}.$$

After rewriting (8) as a velocity integral and applying the Nyquist technique familiar from nonrelativistic plasma theory (Krall & Trivelpiece 1973) we find a relativistically valid form; if

$$\int_{v=-c}^{v=c} \frac{dv}{v-u_0} \frac{\partial f(v)}{\partial v} > 0 \quad (9)$$

then the dispersion relation $D(k, \omega) = 0$ admits temporally growing solutions ($\text{Im } \omega > 0$) over some range of k . We have used

$$f \equiv \sum_{\alpha} \left[\frac{n_{\alpha 0}}{n_0} \frac{q_{\alpha}^2}{e^2} m \right] F_{\alpha 0}[p_{\alpha}(v)]$$

with m the electron mass and

$$n_0 \equiv \sum_{\alpha} n_{\alpha 0}.$$

Equation (9) is valid for all double-humped distribution functions with bounded first and second derivatives. Furthermore, $v = u_0$ is a local minimum of f so the integral in (9) exists. If the plasma is pure electron–positron, we can rewrite (9) conveniently as a momentum integral

$$\int_{-\infty}^{\infty} dp \frac{\partial/\partial p \left[\sum_{\alpha} F_{\alpha 0}(p) \right]}{v(p) - u_0} > 0 \quad (10)$$

for instability. For example, assume a Lorentzian form,

$$\sum_{\alpha} F_{\alpha 0}(p) = \frac{p_t}{2\pi} \left(\left[(p - p_1)^2 + \left(\frac{p_t}{2} \right)^2 \right]^{-1} + \left[(p - p_2)^2 + \frac{p_t^2}{2} \right]^{-1} \right). \quad (11)$$

Here p_t is a ‘thermal’ momentum spread. To avoid a significant number of particles with negative p , we assume the difference $\Delta p (\equiv p_2 - p_1)$ as well as p_t are all $\ll p_1, p_2$. Application of (10) shows that the system is unstable electrostatically if

$$\Delta p > p_t. \quad (12)$$

4 Discussion

To apply the simple condition (12) to pulsars we must estimate ϵ . Ruderman & Cheng take $\epsilon = 1$, i.e. $\theta \sim 1$, which corresponds to a field line which has curved substantially away from the pole. It is difficult to see how this can be so, in view of narrow beaming over the entire radio spectrum. Taking $\epsilon = 1$ and $\gamma_{\pm} \approx 200$, then $\Delta v/c \approx 10^{-4}$, $\Delta\gamma \approx \gamma_c^3 \Delta v/c \approx 800$, so by (7) the streaming is relativistic in the γ_c frame and instability will be suppressed when $p_t > \Delta p = \Delta\gamma mc \approx 800 mc$. This is a high thermal spread. (Incidentally, the values $\gamma_+ \approx 330$ and $\gamma_- \approx 70$ given by Cheng & Ruderman are incorrect; they do not give a centre of momentum frame of $\gamma_c = 200$. Correct values are $\gamma_+ \approx 570$, $\gamma_- \approx 70$.)

However, to explain radio frequencies $\nu \sim 10^9$ Hz requires emission from regions at a distance $L \approx 3 \times 10^{14} \nu^{-2/3} P^{-8/21}$ cm (Benford & Buschauer 1977). With $P = 1$ s, $L \approx 3 \times 10^8$ cm. A field line based at the edge of a polar cap which subtends an angle $\sim 10^{-2}$ rad describes the curve $r \sin^2 \theta = \text{constant}$, so taking a pulsar radius of 10^6 cm, $\theta \approx 3 \times 10^{-2}$ at a distance L from the surface. Then using $\gamma_c \approx 200$ gives $\Delta v/c \approx 2 \times 10^{-7}$ and a spread $p_t/mc \geq 2$ will suppress instability. Taking $\gamma_c \approx 200$ as Cheng & Ruderman do is somewhat unmotivated,

however, since the Ruderman–Sutherland model gives $\gamma_c \approx 800$. If we relax our constraint on θ to $\theta \geq 5 \times 10^{-2}$, with $\gamma_c = 800$, then $\Delta v/c \approx 2 \times 10^{-6}$ and $p_t/mc \approx 1200$ will stabilize all modes, i.e. an energy spread $\approx 1.5\gamma_c$. (The particles are mildly relativistic, $\gamma'_+ \approx 1.2$ in the γ_c frame.)

5 Conclusions

The condition for stability, $p_t > \Delta p$, implies that rather well-peaked distributions must exist to drive the streaming instability. At present there is no firm estimate of p_t in pulsar magnetospheres, so (12) awaits a more detailed model. Certainly, though, any deviation from a strongly peaked distribution will be stabilizing, e.g. a power law. As γ_c increases, however, the p_t required for stability rises,

$$\frac{p_t}{mc} < \Delta\gamma \approx \sqrt{\frac{2\gamma_c}{\gamma_b}} \theta \gamma_c^2 \quad (\text{unstable})$$

so models with higher γ_c are favoured for this mechanism.

The steepening slope of some radio spectra at high frequencies could be explained by the θ dependence of (2) and (12). At small θ the instability is weak (slow growth rate) so coherent amplitudes are small. As r and θ increase, coherence increases as the emitted frequency falls. This may also help explain the often-observed double-humped radio luminosity patterns. The weak portion between the humps is emitted from near $\theta = 0$ as the observer's line of sight sweeps across the polar cap. Here $\Delta v/c$ is small, thermal quenching is important, and radio emission is low. Similarly, the humps arise from field lines with larger θ where Δp is larger. For alternate explanations of double-humped structures see Ruderman & Sutherland (1975) and Benford & Buschauer (1977).

The compact radio sources, if they employ this mechanism, may not be affected by the θ dependence of (12). The entire magnetosphere may comprise the emitting region, so large angles are appropriate and $\Delta\gamma$ may be much larger than a pulsar case. Coherent radiation may be much easier to produce.

Acknowledgments

This work was supported in part by NSF Grant Number MPS75-14625. GB thanks the Institute of Astronomy, Cambridge University for hospitality while a portion of this work was done, and R. Blandford for illuminating discussions.

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