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TENTATIVE DESIGN GUIDE

FOR

CALCULATING THE VIBRATION RESPONSE

OF FLEXIBLE CYLINDRICAL ELEMENTS IN

AXIAL FLOW

by

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NOMENCLATURE

Symbol	Description
a	Cylinder radius, in.
a_1	Empirical constant, sec/ft
a_2	Empirical constant, sec^2/ft^2
b	Axial location, in.
c_T	Tangential drag coefficient
d	Cylinder diameter, in.
d_h	Hydraulic diameter, ft
d_i	Tube inside diameter, in.
f	Frequency, Hz
f_o	Fundamental vibration frequency of cylinder in static fluid, Hz
f_1	Fundamental vibration frequency of cylinder, Hz
ℓ	Cylinder length, in.
m	Cylinder mass per unit length, $(lb)(sec^2)/ft^2$
p	Random pressure on cylinder surface, $lb/in.^2$
r	Resultant transverse displacement, in.
t	Time, sec
w	Transverse displacement, in.
x	Axial coordinate, in.
y	Transverse displacement, in.
y_{rms}	Root mean square transverse displacement, in.
C_m	Added mass coefficient
D	Inside diameter of annular flow channel, in.
E	Young's modulus of elasticity for cylinder material, $lb/in.^2$
H	Frequency response (transfer) function for cylinder
I	Area moment of inertia, $in.^4$

Symbol	Description
J_1	Joint acceptance
K	Empirical coefficient, $(lb)(sec^{2.5})/ft^{5.5}$
M	Added mass of fluid per unit length, $(lb)(sec^2)/ft^2$
S	Strouhal number (fd_h/U)
T	Axial tension, lb
U	Mean axial flow velocity, ft/sec
U_c	Mean convection velocity of turbulent eddies ($\approx 0.8U$), ft/sec
β_o	Modal constant
β_1	Modal constant
γ	Strouhal number $(\omega \ell/U_c)$
ζ_o	Damping factor in static fluid
ζ_1	Damping factor
θ	Angle
ν	Strouhal number $(0.275\ \omega d/U_c)$
ν_f	Fluid viscosity, ft^2/sec
η	Dimensionless frequency
ρ	Density of cylinder material, $(lb)(sec^2)/ft^4$
ρ_f	Fluid density, $(lb)(sec^2)/ft^4$
σ	Variance of transverse displacement y, in.
σ_y	Bending stress, $lb/in.^2$
ϕ_1	Fundamental mode
χ	Effective nondimensional diameter
ω	Frequency, rad/sec
ω_o	Fundamental vibration frequency of cylinder in static fluid, rad/sec
ω_1	Fundamental vibration frequency of cylinder, rad/sec

Symbol	Description
Φ_o	Intensity of near-field mean-square spectral density in low-frequency range, $(lb^2)(sec)/in.^4$
Φ_p	Mean-square spectral density of near-field flow noise, $(lb^2)(sec)/in.^4$
Φ_y	Mean-square spectral density of cylinder displacement, $(in.^2)(sec)$

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ABSTRACT

Many reactor and plant equipment components, such as fuel pins, control rods, and heat exchanger tubes, are long, slender, beam-like members which are exposed to nominally axial coolant flow. The flowing coolant represents a source of energy which can induce vibratory motion of these components. This design guide presents a relationship for calculating the root-mean-square (rms) displacement of a flexible rod or tube in axial flow. The relationship is based on the results of a parameter study and is valid for components that can be approximated as beams with either simply-supported or fixed-fixed ends. It is given in terms of beam natural frequency, damping factor, and intensity of the mean-square spectral density of the pressure field in the low-frequency range; all three are functions of mean axial flow velocity. Empirical expressions are developed for damping factor and intensity of the mean-square pressure spectrum. With these, an empirical equation for rms displacement is written which is in terms of known quantities and, therefore, provides a tool which can be used by designers. Since the equation is based on experiments involving a smooth rod in flow with minimal entrance effects, the predicted displacements should be interpreted and used with care. They are not conservative and, at best, will represent the minimum response to be expected.

I. INTRODUCTION

Many reactor and plant equipment components, such as fuel pins, control rods, and heat exchanger tubes, are long, slender, beam-like members which are exposed to nominally axial coolant flow. The flowing coolant represents a source of energy which can induce vibratory motion of these components. Experiments have identified the displacement response of these components as a problem in random vibration [e.g., 1]. The resulting motion is forced, the near-field component of the flow noise (e.g., the random pressure fluctuations generated by turbulent eddies in the flow) being the primary excitation mechanism. Analysis has shown [2] that instabilities caused by fluid-structure coupling are not a problem: for typical reactor system geometries and coolant flow environments, the critical flow velocities associated with the onset of instability are large relative to expected velocities.

Several empirical relationships for component displacement have been proposed [3-5], none of which satisfactorily predicts displacement response for all conditions. One reason is the strong system dependence of the response; turbulence levels, as effected by entrance conditions and flow channel geometries, are important.

This design guide presents a theoretically derived relationship for calculating the root-mean-square (rms) displacement of a flexible rod or tube in nominally axial flow. The basic analysis assumes a pressure field which is homogeneous in space; that is, the mean-square spectral density of the pressure is the same at any point on the rod. The relationship is based on the results of a parameter study; it is valid for component/systems which have parameters in specified ranges and which can be approximated as beams with either simply-supported or fixed-fixed ends. The relationship is given in terms of rod natural frequency, damping factor, and intensity of the near-field spectrum in the low-frequency range. All three are functions of mean axial flow velocity.

The flow velocity dependence of damping and intensity of the near-field spectrum are not very well understood. This precludes direct application of the theoretical equation. However, these features are being studied at Argonne and preliminary experiments tend to verify the derived equation for rms response. Based on this favorable agreement, the equation was solved for the intensity of the near-field mean-square spectrum, and experimental measurements of rms displacement and damping were used to develop an empirical relationship for the flow-velocity-dependence of the intensity of the near-field spectrum. With this relationship, and an empirical expression for the flow-velocity-dependent damping, an equation for rms displacement as a function of mean axial flow velocity and system parameters was obtained.

The equation is in terms of known quantities and, therefore, provides a tool which can be used by designer-analysts to predict rms displacements of flexible rods in annular water flow. However, since the equation is based on experiments involving a smooth rod in flow with minimal entrance effects, the predicted displacements should be interpreted and used with care. They are not conservative and, at best, will represent the minimum response to be expected.

This design guide will be updated as results from the studies of damping and flow-excitation continue to become available.

II. RMS DISPLACEMENT

A flexible cylinder in axial flow and coordinate system is shown schematically in Fig. 1. Since the problem is one of random vibration, the quantities of interest are the rms value of the response and the amplitude distribution. The motion of the cylinder is two-dimensional because it is equally likely to respond in any direction. We will consider motion in the y-direction as defined in Fig. 1.

Mean-square displacement is readily obtained from the mean-square spectral density of the displacement by integration over the frequency:

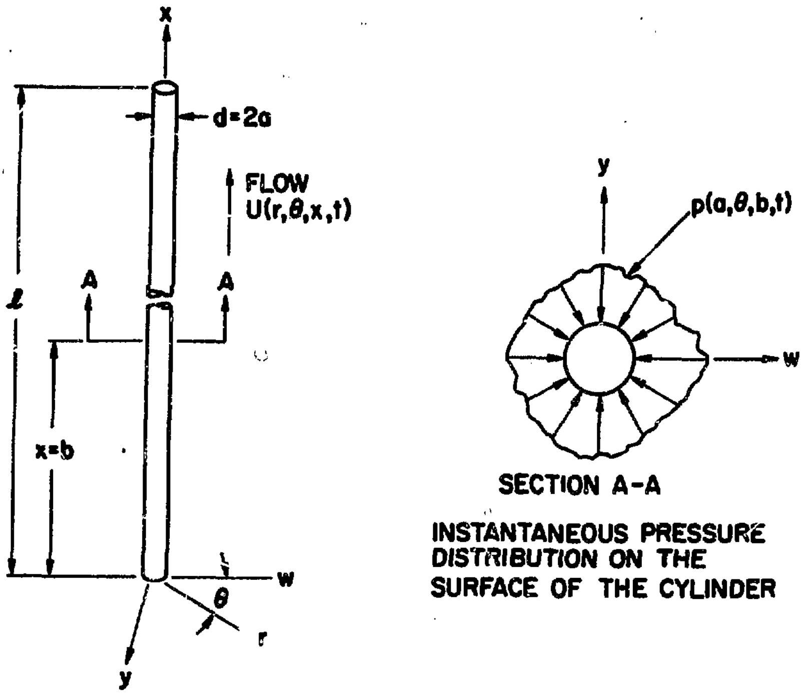

Fig. 1. Flexible cylinder rod in axial flow and coordinate system

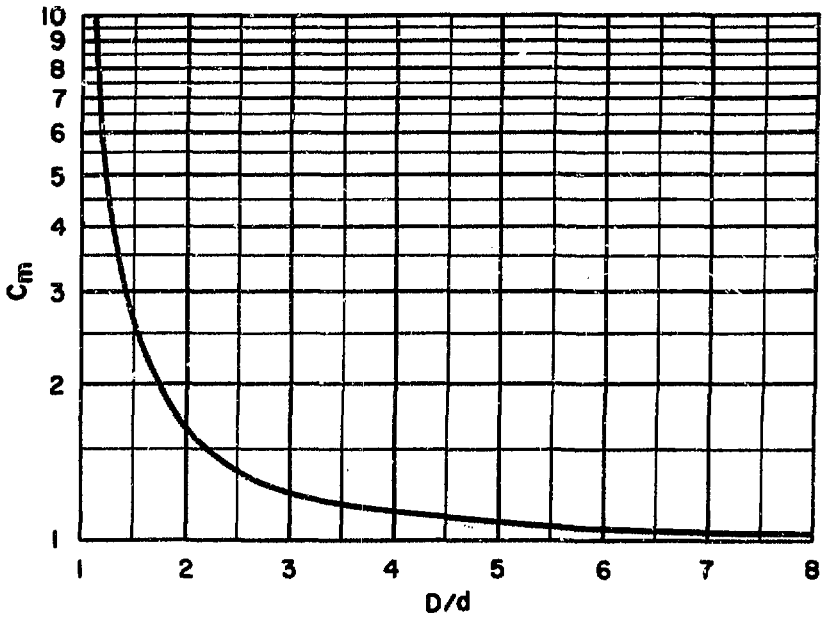

Fig. 2. Added mass coefficient (C_m) as a function of diameter ratio (D/d)

$$\langle y^2(x,t)\rangle = \int_0^\infty \Phi_y(x,\omega)d\omega \quad , \tag{1}$$

since mean-square spectral density is simply the frequency distribution of the mean-square; the symbol < > denotes a time average. On the assumption that the integration implied in Eq. (1) can be performed, prediction of the rms-response then requires mathematical characterization of the mean-square spectral density of the displacement.

A. Equation of Motion

The equation for small lateral motion of a flexible rod in axial flow was formulated by Paidoussis [6]. Damping and a distributed random pressure loading were included in a subsequent analysis [7], where the response was represented in series form as a superposition of normal modes, yielding a series representation of the mean-square spectral density of the displacement. Theory [7] and experiment [1] have shown that less than 5% of the energy is contained in other than the fundamental mode; therefore, a one-mode approximation is sufficient.

We will further assume that the convecting random pressure field is homogeneous in space; that is, its mean-square spectral density is the same at every point on the surface of the cylinder. Using a phenomenological model proposed by Corcos [8], and the best features of the existing data describing the convection and correlation decay of turbulent boundary-layer pressure fluctuations, the mean-square spectral density of the displacement can be written as [7]

$$\Phi_y(x,\omega) = \chi^2(\omega)J_1^2(\omega)\Phi_p(\omega)\,|H(x,\omega)|^2\phi_1^2(x) \quad . \tag{2}$$

The first term in the product on the right side is described as dimensionless "effective diameter;" it embodies the decay of the circumferential correlation of the pressure field. The second term, called the "joint acceptance," is a weighted cross-spectral density of the convecting pressure field; it represents the "effectiveness" of the pressure in exciting a mode of the rod. The third term is the mean-square spectral density of the pressure field measured at a point. The fourth term is the frequency response function (transfer function) for the cylinder and is given by

$$|H(x,\omega)| = [d/(M+m)][(\omega_1^2-\omega^2)^2 + 4\zeta_1^2\omega_1^2\omega^2]^{-1/2} \quad . \tag{3}$$

The last term is the fundamental mode; for fixed-fixed ends, it can be approximated by the classic normal mode

$$\phi_1(x) = \cosh(4.73\,x/\ell) - \cos(4.73\,x/\ell) - 0.983[\sinh(4.73\,x/\ell) - \sin(4.73\,x/\ell)] \quad ; \tag{4a}$$

and for simply-supported ends, it is given by

$$\phi_1(x) = \sqrt{2}\,\sin(\pi x/\ell) \quad . \tag{4b}$$

The added mass of fluid (M in Eq. (3)) is generally assumed to be equal to the mass of fluid displaced by the cylinder; this is valid for a cylinder submerged in an "infinite" fluid. However, for a cylinder vibrating within a bundle, or in close proximity to a wall, boundary effects are important as they influence the added mass. A theoretical analysis of a cylinder vibrating in a compressible fluid annulus has been performed [9]. The results are expressed in the form

$$M = C_m \rho_f (\pi d^2/4) \quad , \tag{5}$$

where C_m is the added mass coefficient given in Fig. 2.

Substitution of Eq. (2) into Eq. (1) yields

$$\langle y^2(x,t)\rangle = \phi_1^2(x) \int_0^\infty \chi^2(\omega) J_1^2(\omega) \Phi_p(\omega) |H(x,\omega)|^2 d\omega \quad . \tag{6}$$

Using the phenomenological model of Corcos [8] and experimental results presented in the literature, the functions $\chi(\omega)$ and $J_1^2(\omega)$ were obtained by integration [7]. They are plotted as functions of the Strouhal numbers $\nu (= 0.275\ \omega d/U_c)$ and $\gamma (= \omega\ell/U_c)$ in Figs. 3 and 4 respectively. In Fig. 4, $J_1^2(\gamma)$ is plotted for fixed-fixed and simply-supported ends.

Mean square spectral densities of the near-field flow noise have been measured, using a method in which the low-frequency acoustic noise is nulled out in the pressure differencing [10]. Representative results [11] are reproduced in Fig. 5, where the frequency has been scaled with the Strouhal number $S(= fd_h/U)$.

Prediction of rms displacement response requires mathematical characterization of χ, J_1, and Φ_p; substitution of these quantities into Eq. (6); and integration over the frequency. Although the integration is not particularly difficult to perform on a computer, it will be shown that from the results of a parameter study a simplified relationship can be developed for rms response of typical reactor components and fluid environments.

B. Parameter Study

Three different Strouhal number relationships have been utilized in the normalization of the mean-square spectra, and in representation of the effective diameter and joint acceptance associated with the random pressure field; they are respectively

$$\left.\begin{aligned} S &= fd_h/U \quad , \\ \nu &= 0.275\ \omega d/U_c \\ \text{and} \quad \gamma &= \omega\ell/U_c \quad . \end{aligned}\right\} \tag{7}$$

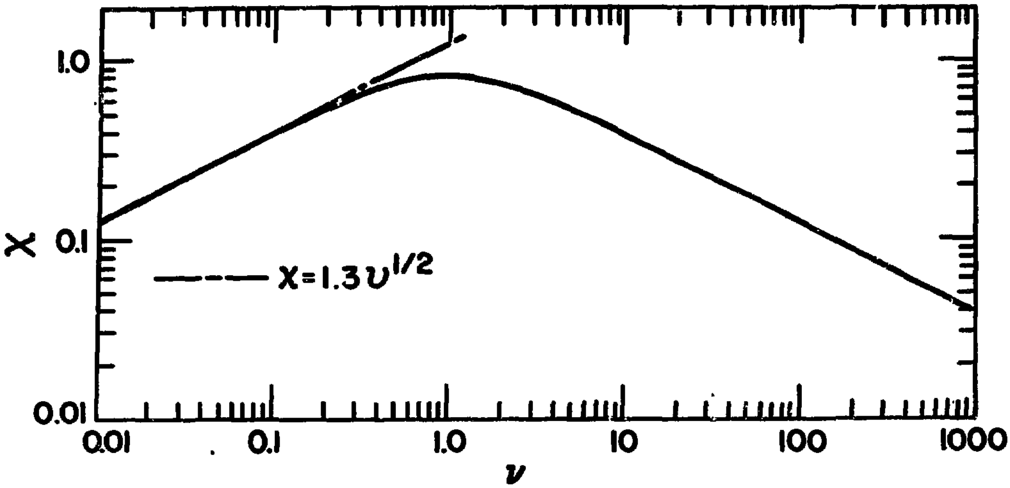

Fig. 3. Effective diameter (χ) as a function of ν

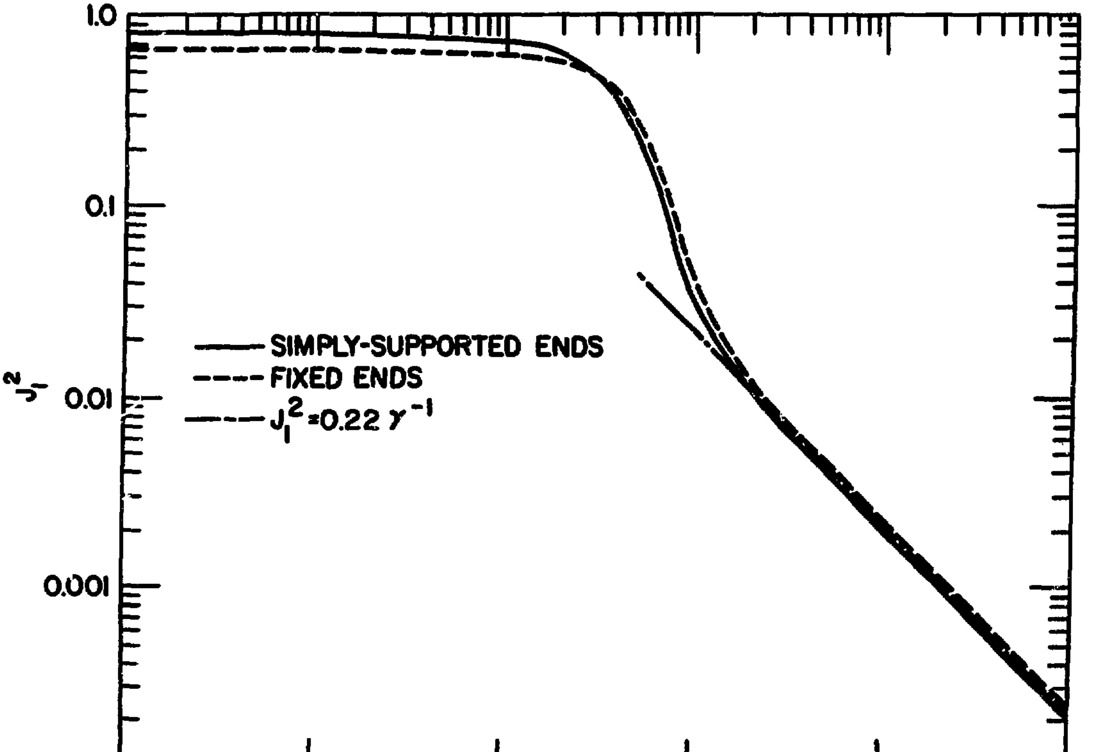

Fig. 4. Joint acceptance (J_1^2) of the fundamental mode as a function of γ

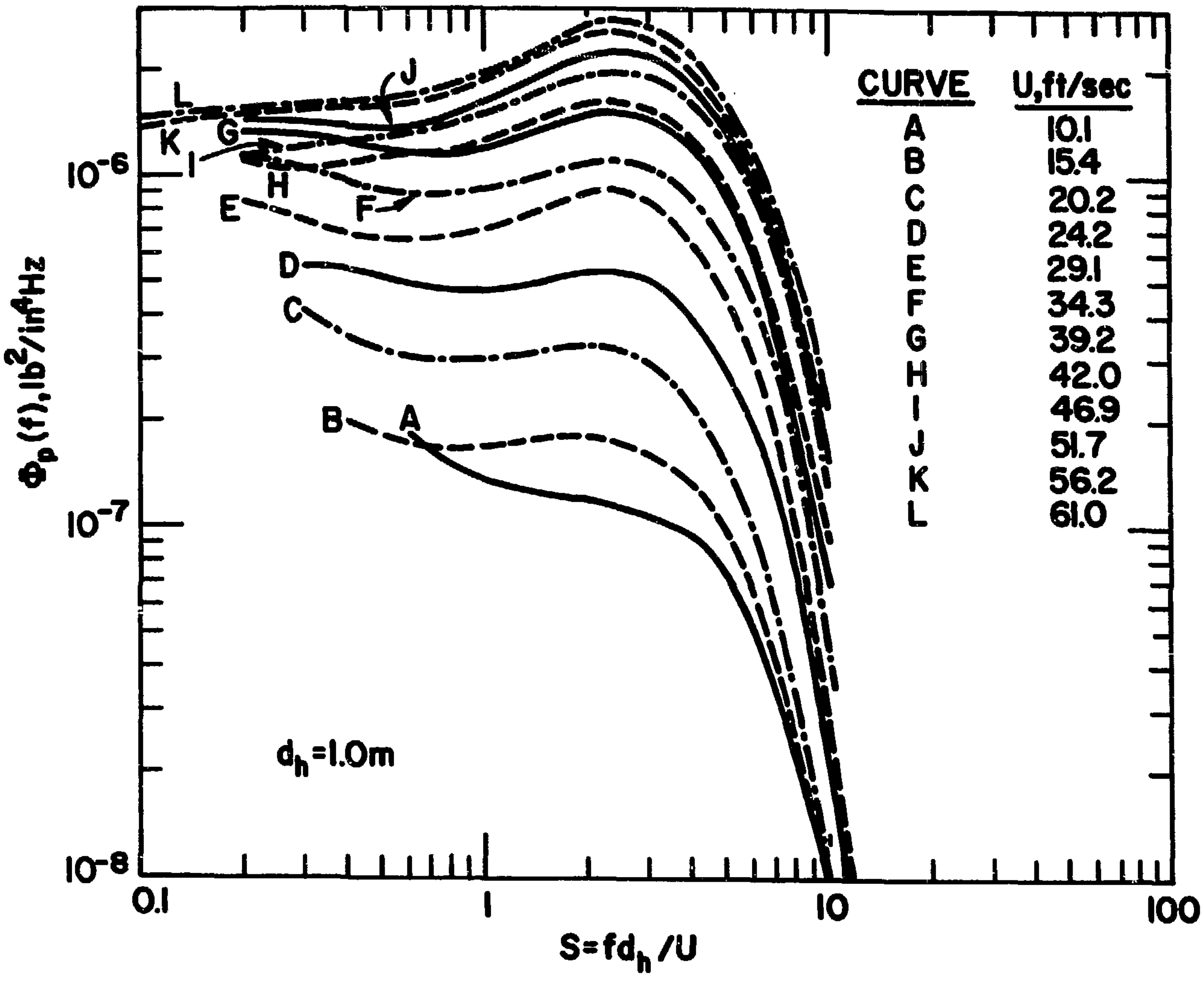

Fig. 5. Family of near-field mean-square spectra (Φ_p) as a function of S

Since the first-mode response is dominant, the cylinder will respond primarily to frequencies in the range of its first natural frequency, hence the Strouhal numbers associated with this frequency locate their respective range of effectiveness.

From Figs. 3 and 4 observe that in the Strouhal number ranges ($0.01 < \nu < 0.6$) and ($10 < \gamma < 1000$), the functions $\chi(\nu)$ and $J_1^2(\gamma)$ can be approximated by straight lines of slopes 0.5 and -1.0 respectively. On log-log plots, these linear relationships imply the approximate power function relationships

$$\chi(\nu) = 1.3\ \nu^{1/2} \quad , \quad 0.01 < \nu < 0.6 \quad , \tag{8}$$

$$J_1^2(\gamma) = 0.22\ \gamma^{-1} \quad , \quad 10 < \gamma < 1000 \quad . \tag{9}$$

From Fig. 5 observe that for a given mean axial flow velocity, the near-field spectrum is relatively flat out to a "break frequency." Therefore, in the Strouhal number range ($S < 2.5$) we can write

$$\Phi(S) = \Phi_o(U) \quad , \quad S < 2.5 \quad , \tag{10}$$

where Φ_o is the intensity, or magnitude, of the near-field mean-square spectrum.

To summarize: Based on these observations, the following approximate relationships can be written which are valid for values of parameters which satisfy the specified ranges,

$$\left.\begin{array}{lll} \Phi_p(\omega) = \Phi_o(U) & , & \dfrac{fd_h}{U} < 2.5 \\ \chi(\omega) = 0.68\ (\omega d/U_c)^{1/2} & , & 0.04 < \dfrac{\omega d}{U_c} < 2.2 \\ J_1(\omega) = 0.47(\omega \ell/U_c)^{-1/2} & , & 10 < \dfrac{\omega \ell}{U_c} < 1000 \end{array}\right\} \tag{11}$$

On substituting Eqs. (11) into Eq. (6), the frequency dependence of the effective diameter (χ) and joint acceptance (J_1) cancel, and the mean-square pressure spectral density, being independent of frequency over the specified range, can be brought outside the integral. If we let $\eta = \omega/\omega_1$, and further substitute Eq. (3) into Eq. (6), the resulting equation becomes

$$\langle y^2(x,t)\rangle = \frac{(0.102)d^3\Phi_o(U)\phi_1^2(x)}{(M+m)^2\ell\omega_1^3}\int_o^\infty \frac{d\eta}{(1-\eta^2)^2 + 4\zeta_1^2\eta^2} \quad . \tag{12}$$

The integral in Eq. (12) can be evaluated as $\pi/(4\zeta_1)$ by introducing a complex variable and employing the residue theorem; whereupon Eq. (12) becomes

$$\langle y^2(x,t)\rangle = \frac{(0.080)d^3\phi_1^2(x)}{(M+m)^2\ell\omega_1^3\zeta_1}\Phi_o(U) \quad . \tag{13}$$

The flowing coolant gives rise to fluid-structure coupling, resulting in an effective axial loading which is flow velocity dependent [7]. This contribution to the resultant axial load (additional contributions arise from initial preload or are caused by differential thermal expansion under operating conditions) effects the natural vibration frequency (ω_1 in Eq. (13)), and we can write $\omega_1 = \omega_1(U)$. Additionally, effective system damping has been shown to be flow-velocity-dependent [12]; consequently, $\zeta_1 = \zeta_1(U)$.

The rms displacement can then be written in the form

$$y_{rms}(x,U) = \frac{(0.0180)d^{1.5}\phi_1(x)\Phi_o^{0.5}(U)}{(M+m)\ell^{0.5}f_1^{1.5}(U)\zeta_1^{0.5}(U)} , \tag{14}$$

where the frequency (f_1), damping factor (ζ_1), and intensity of the near-field flow noise spectrum (Φ_o) are functions of mean axial flow velocity and are discussed below.

C. Fundamental Vibration Frequency

The natural vibration frequency can be obtained from the equation of motion by neglecting damping effects and assuming a fundamental mode shape. For end conditions which give rise to a symmetric fundamental mode, the fundamental frequency of vibration is given by [7]

$$\omega^2 = \frac{EI\int_o^\ell \phi_1\phi_1^{iv}dx + (MU^2 - T)\int_o^\ell \phi_1\phi_1'' dx}{(M+m)\int_o^\ell \phi_1^2 dx} . \tag{15}$$

On using the normal modes given by Eqs. (4a) and (4b), and performing the indicated integrations, one obtains

$$\omega_1^2 = \frac{\beta_o^4 EI}{(M+m)\ell^4}\left[1 + \beta_1(T - MU^2)\frac{\ell^2}{EI}\right] , \tag{16}$$

or

$$f_1^2(U) = f_o^2\left[1 - \frac{\beta_1 MU^2\ell^2}{(EI + \beta_1 T\ell^2)}\right] , \tag{17}$$

where f_o is the fundamental vibration frequency in static fluid, given by

$$f_o^2 = \frac{\beta_o^4 EI}{4\pi^2(M+m)\ell^4}\left(1 + \beta_1\frac{T\ell^2}{EI}\right) . \tag{18}$$

For simply-supported end conditions: $\beta_0 = \pi$ and $\beta_1 = 0.101$; and for fixed-fixed end conditions: $\beta_0 = 4.73$ and $\beta_1 = 0.0246$.

D. Flow-Velocity-Dependent Damping

The flow-velocity-dependence of system damping associated with the vibration of a flexible cylinder subjected to nominally axial flow has been identified [1] and studied preliminarily [12]. Results show a significant increase in damping factor with increasing mean flow velocity; therefore, this effect should be accounted for in predicting rms displacement response.

Flow-velocity-dependent damping is being studied in more detail [13]. These studies involve the use of an electromagnetic exciter assembly, with coils mounted in the wall of the test section, to apply a harmonic force to the cylinder. For selected mean axial flow velocities, forcing frequency is varied and an effective damping factor is computed from measured frequency-response curves. Two different measurement methods are used: (1) a modal magnification factor method, with constant force and with constant displacement; and (2) a frequency-bandwidth method.

Figure 6 is a typical plot of measurements made on a 0.5-in.-dia, 47-in.-long brass rod with fixed-fixed ends. The data show: (1) the flow-velocity-dependence of damping; (2) good agreement between the different measurement methods; and (3) good repeatability of measurement (Runs 1 and 2 were performed on different days).

The effects of end restraint, vibration frequency, and Reynolds number are also being investigated. In general, the data obtained thus far can be fitted with a quadratic equation of the form

$$\zeta_1(U) = \zeta_o + a_1 U + a_2 U^2 \quad , \qquad (19)$$

where ζ_o is the effective viscous damping factor in stagnant fluid and includes internal damping and external damping associated with friction at the supports. Values of the coefficients a_1 and a_2 given on Fig. 6 are typical of those being measured.

E. Flow-Noise Excitation

The near-field component of flow noise, in particular the random pressure fluctuations generated by turbulent eddies, has been identified as the primary excitation source in the parallel-flow-induced vibration of flexible rods [e.g., Ref. 10]. The resultant convecting random pressure field is characterized via a phenomenological model representing the cross-spectral density of the pressure field [8]. This model was used in deriving Eq. (14) for the rms displacement.

A number of independent studies of flow noise in air and water have been performed in which the mean-square spectral density of the pressure field was measured. In all instances, the data taken at high frequencies are in good agreement, but the low-frequency spectra vary widely. In general, it can be

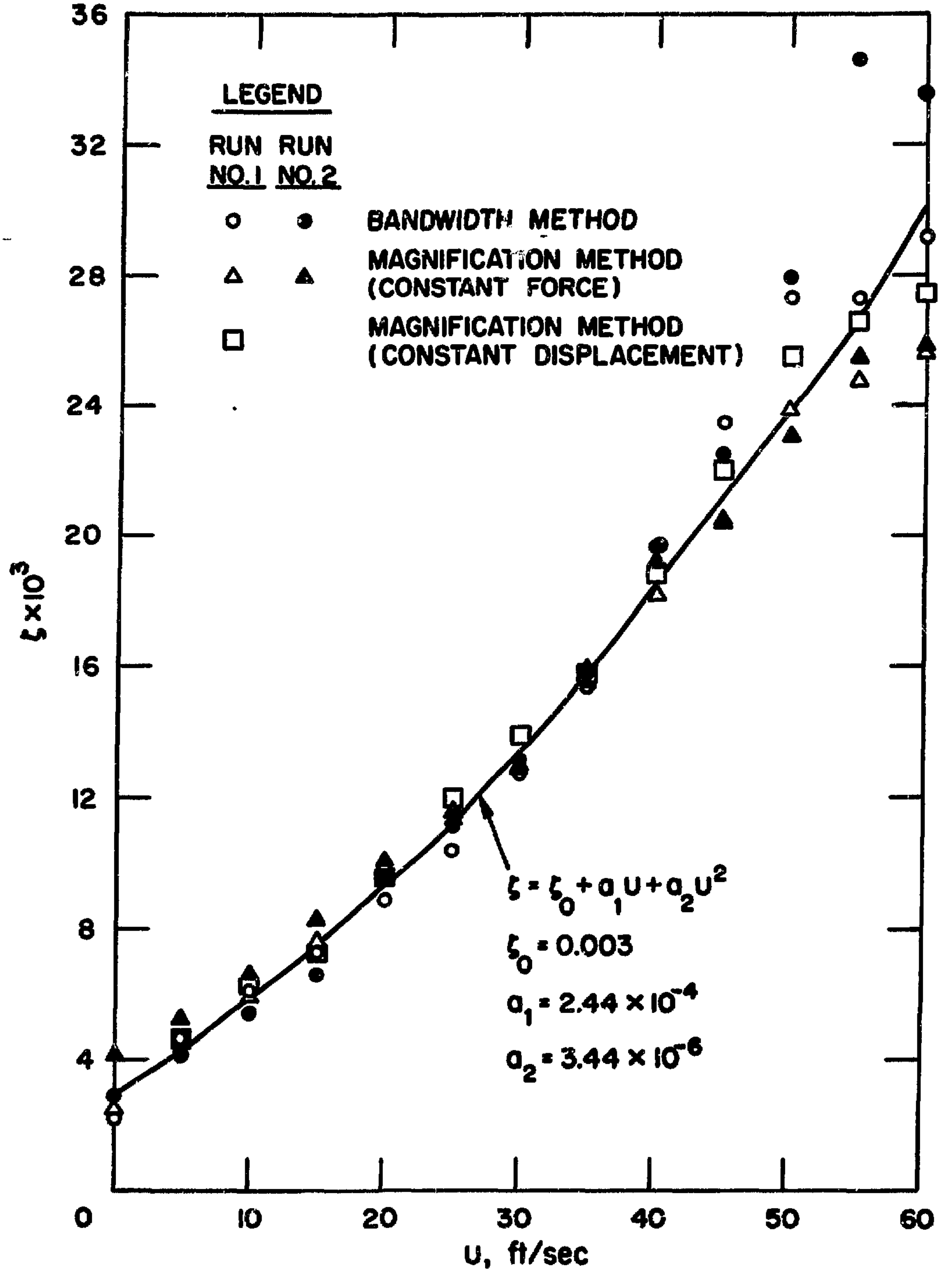

Fig. 6. Equivalent viscous damping factor (ζ_1) as a function of mean axial flow velocity (U) for a 0.5-in.-dia, 46$\frac{7}{8}$-in.-long brass rod with fixed-fixed ends

concluded that reliable results are not available in this frequency range ($f < 300$ Hz). However, this is precisely the range of interest for flow-induced motion studies of many reactor components (or flexible rods simulating reactor components), since it encompasses the range of first-mode natural frequencies at which the components predominantly respond.

The difficulty with measuring the low-frequency behavior of the near-field flow noise is caused primarily by the masking effect of extraneous, low-frequency acoustic noise. To circumvent this difficulty, a measurement method utilizing pairs of diametrically opposite, miniature, pressure transducers, flush-mounted on the surface of a rod, was developed [10]. In operation, the near-field mean-square spectra and rms pressure are computed from the pressure-difference signal, using the subtraction process to null out the far-field (acoustic) contribution to the flow noise. Typical results are given in Fig. 5, where the frequency has been scaled with a Strouhal number.

Equation (14), which describes rms displacement, is based on the assumption that for a given flow velocity the intensity or magnitude of the near-field spectrum Φ_o is constant out to a Strouhal number S of about 2.5. From measurements made to date, this appears to be a reasonable assumption, although there are some variations as can be seen from Fig. 5.

It would be desirable to characterize the flow-velocity-dependence of the intensity of the near-field spectra. However, attempts at scaling the intensity with a function of mean axial flow velocity have not been entirely successful [10]; additional study is required.

The experimental studies of damping at Argonne include measurements of the rms rod displacement induced by the flow excitation, and the resonant frequency. Therefore, having experimental values for $f_1(U)$, $\zeta_1(U)$, and $y_{rms}(\ell/2,U)$, and knowing the dimensions and material properties of the rod, Eq. (14) can be solved for $\Phi_o(U)$. Preliminary computations show that the intensity is approximately proportional to the mean axial flow velocity raised to the fourth power, and decreases with decreasing hydraulic diameter. Considering the small magnitude of the quantities involved, and the difficulties in making the associated measurements, the agreement between computed intensities and those measured with an instrumented test element [10] is good. This good agreement serves to validate the model and the derived relationship for rms displacement given by Eq. (14).

Based on the preliminary results from the above-mentioned analysis, the intensity of the near-field mean-square spectra in the low-frequency range and on the surface of a "smooth" rod in annular water flow can be characterized by the empirical relationship

$$\Phi_o(U) = K^2 d_h^3 U^4 \quad , \tag{20}$$

where

$$K^2 \approx 3.16 \times 10^{-10} (\mathrm{lb}^2)(\mathrm{sec}^5)/(\mathrm{in.}^4)(\mathrm{ft}^7) \quad .$$

Equation (20) is tentative, since it is based on limited values of d_h ($0.5 < d_h < 1.5$) and only one rod diameter; additional investigation is planned.

F. Summary

Prediction of the rms response of a flexible rod in nominally axial flow requires a random-vibration analysis. A one-mode approximation to a series solution was shown to be sufficient, and a representation of the mean-square spectral density of the displacement was obtained utilizing a phenomenological model to characterize the convecting random pressure field. A parameter study gives rise to approximations which permit evaluation of the integral in the equation for mean-square displacement. A relationship for rms displacement (Eq. (14)) is obtained as a function of natural frequency, damping factor, and the intensity of the near-field mean-square spectrum in the low frequency range; these three variables are functions of mean axial flow velocity.

The flow-velocity-dependence of natural frequency is obtained from analytical considerations, and is given by Eq. (17). The natural frequency is also a function of axial load, which may vary under operating conditions.

Damping increases with mean axial flow velocity. Although the energy dissipation mechanism is not fully understood, a quadratic equation as given by Eq. (19) seems to satisfactorily represent flow-velocity dependence as measured from experiments.

The flow-velocity-dependence of the intensity of the near-field spectra is the least understood of the three quantities. However, for a smooth rod in annular water flow, Eq. (20) is a reasonable approximation.

Substituting Eqs. (17), (19), and (20) into Eq. (14) yields the following expression for rms displacement of a flexible rod in annular water flow, with minimal entrance turbulence:

$$y_{rms}(x,U) = \frac{(0.0180)Kd^{1.5}d_h^{1.5}U^2\phi_1(x)}{\ell^{0.5}f_o^{1.5}(M+m)(\zeta_o + a_1U + a_2U^2)^{0.5}\left[1 - \dfrac{\beta_1 MU^2\ell^2}{(EI + \beta_1 T\ell^2)}\right]^{0.75}} \quad (21)$$

where

$$K = 2.56 \times 10^{-3}(\text{lb})(\text{sec}^{2.5})/\text{ft}^{5.5} \quad .$$

For a simply-supported rod

$$\beta_1 = 0.101, \ \phi_1(x) = \sqrt{2}\,\sin(\pi x/\ell) \ , \ 0 < x < \ell \quad ;$$

and for a fixed-fixed rod

$$\beta_1 = 0.0246$$

$$\phi_1(x) = \cosh(4.73\ x/\ell) - \cos(4.73\ x/\ell) - 0.983[\sinh(4.73\ x/\ell)$$

$$- \sin(4.73\ x/\ell)] \quad , \quad 0 < x < \ell \quad .$$

In application, it is necessary:

(1) To assume or measure the damping factor (ζ_o) in stagnant fluid.

(2) To assume values for a_1 and a_2 based on independent studies of damping ($a_1 = 2.4 \times 10^{-4}$ sec/ft, and $a_2 = 3.4 \times 10^{-6}$ sec^2/ft^2 are typical values). If they were to be measured, one might just as easily measure y_{rms}.

(3) To assume a value for the axial load T, or to calculate it from Eq. (18), using a measured value of natural frequency in static fluid.

The remaining quantities are known from the system geometry and the material properties of the rod.

Amplitudes are of primary importance, since the mode of failure most likely will be fretting or wear. However, for those cases in which stresses are of concern, they are easily obtained from displacements as given by

$$[\sigma_y(x,U)]_{rms} = \frac{Ed}{2} \frac{\partial^2 y_{rms}(x,U)}{\partial x^2} \tag{22}$$

or, in terms of midpoint displacement as given by

$$[\sigma_y(x,U)]_{rms} = \left[\frac{Ed\phi_1''(x)}{2\phi_1(\ell/2)}\right] y_{rms}(\ell/2,U) \quad , \tag{23}$$

where a prime (') denotes differentiation with respect to x. Values for the mode shape ϕ_1, and its second derivative ϕ_1'', are tabulated in Ref. 14.

III. AMPLITUDE DISTRIBUTION

Absolute values of amplitude are meaningless because we are dealing with a random vibration phenomenon. Therefore, we have developed a relationship to predict the rms displacement response. Although this information is useful, a complete description of the random signal requires knowledge of the probability law describing the amplitude distribution. For example, two random motions can have the same rms displacement, but one could experience peak amplitudes significantly greater than the other and/or more frequently per given sample length.

Displacement-time histories from a number of different flow tests have been processed on an amplitude-distribution analyzer. The shapes of the curves obtained suggest a normal or Gaussian distribution. As shown in Fig. 7, which is a typical probability density representation of vibration amplitude, the normal probability law approximates the data quite well.

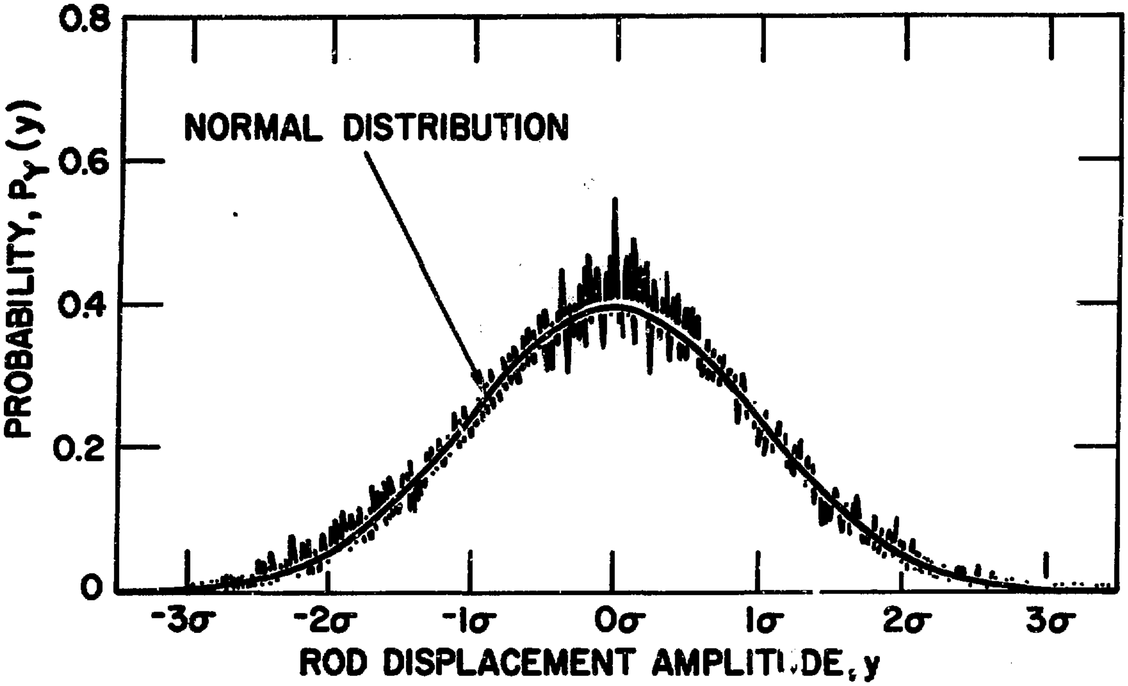

Fig. 7. Typical probability-density representation of displacement of flexible cylinder vibrating in parallel-flowing fluid (σ = rms value = 2.34 mils; U = 16 ft/sec)

Based on this agreement, we can assume a normal distribution for the distribution of vibration amplitude in a given direction, and write the probability law as

$$p_Y(y) = \frac{1}{\sqrt{2\pi}\,\sigma} \exp\left[-\frac{1}{2}\left(\frac{y}{\sigma}\right)^2\right] , \tag{24}$$

which implies a mean value of zero (σ is the standard deviation, and for a zero mean value is equal to the rms value). The probability that an observed value of displacement will range between $-y_o$ and y_o is determined by the integration

$$P[-y_o \le y \le y_o] = \int_{-y_o}^{y_o} p_Y(y)dy = \frac{1}{\sqrt{2\pi}\,\sigma} \int_{-y_o}^{y_o} \exp\left[-\frac{1}{2}\left(\frac{y}{\sigma}\right)^2\right] dy . \tag{25}$$

The normal probability integral has been evaluated and is tabulated in various sources [e.g., Ref. 15]. Several values are listed below:

n	$P[-n \le y \le n]$
0.5	0.383
1.0	0.683
1.5	0.866
2.0	0.954
2.5	0.988
3.0	0.997

Observe the probability of 0.997 that the absolute vibration amplitude will be less than 3σ; that is, 99.7% of the time the amplitude can be expected to be less than 3σ, or $3y_{rms}$. Such information is useful in fatigue and wear studies and in determining if impacting with adjacent components or support members may occur.

IV. EXAMPLE OF APPLICATION

Consider the case of a long, slender tube concentrically located within a containment tube and subjected to coolant flow; system dimensions and material properties are given in Table I. The end conditions can be approximated as fixed, and axial loading will be assumed negligible. The problem: Find the flow-induced, rms displacement of the midpoint of the tube.

TABLE I

Data for Sample Problem	
Tube	
Outside Diameter (d)	0.4 in.
Inside Diameter (d_i)	0.3 in.
Length (ℓ)	96 in.
Material: stainless steel	
Density (ρ)	15.6 $(lb)(sec^2)/ft^4$
Young's Modulus (E)	28×10^6 $lb/in.^2$
Containment Tube	
Inside Diameter (D)	1.5 in.
Coolant	
Water	
Density (ρ_f)	1.94 $(lb)(sec^2)/ft^4$
Velocity (U)	40 ft/sec

The following calculational steps are required:

(1) Hydraulic diameter (d_h)

$$d_h = \frac{4\ (\text{Flow Area})}{(\text{Wetted Perimeter})}$$

$$d_h = \frac{4\pi(D^2 - d^2)}{4\pi(D + d)} = D - d$$

$$d_h = 0.0917 \text{ ft}$$

(2) Mass per unit length of tube (m)

$$m = \rho[\pi(d^2 - d_i^2)/4]$$

$$m = 5.96 \times 10^{-3} (lb)(sec^2)/ft^2$$

(3) Added mass per unit length of fluid (M)

From Eq. (5)

$$M = C_m \rho_f (\pi d^2/4)$$

For $D/d = 3.75$, we read $C_m = 1.15$ from Fig. 2 and obtain

$$M = 1.95 \times 10^{-3}\ (lb)(sec^2)/ft^2$$

(4) Moment of inertia (I)

$$I = \pi(d^4 - d_i^4)/64$$

$$I = 4.14 \times 10^{-8}\ ft^4$$

(5) Natural vibration frequency in stagnant fluid (f_o)

From Eq. (18), with no axial preload (T = 0),

$$f_o = \frac{\beta_o^2}{2\pi\ell^2}\left[\frac{EI}{(M + m)}\right]^{1/2} \quad .$$

For fixed-fixed end conditions $\beta_o = 4.73$ and

$$f_o = 8.1\ Hz$$

The next step is to compute the Strouhal numbers $f_o d_h/U$, $\omega_o d/U_c$, and $\omega_o \ell/U_c$, and verify that the values fall within the ranges specified in Eqs. (11), for which the analysis is valid. From the above computations,

$$\omega_o = 2\pi f_o = 50.9\ rad/sec;$$

U_c is the convection velocity of the pressure field and is approximately 0.8 U; therefore,

$$U_c = 0.8\ U = 32\ ft/sec \quad .$$

The Strouhal numbers are

$$f_o d_h/U = 0.019 \quad ,$$

$$\omega_o d/U_c = 0.053 \quad ,$$

and

$$\omega_o \ell/U_c = 12.7 \quad .$$

From Eqs. (11) we see that the values lie within the range over which the approximation is valid.

For displacement at the midpoint, $x = \ell/2$, and $\phi_1(\ell/2) = 1.588$, as obtained from Eq. (4a) or Ref. 14. Let us assume: (1) the effective viscous damping factor in stagnant fluid ζ_o is 0.008; and (2) the flow-velocity-dependence of damping is that given in Fig. 6; therefore $a_1 = 2.44 \times 10^{-4}$ sec/ft, and $a_2 = 3.44 \times 10^{-6}\ sec^2/ft^2$. Substituting these values into Eq. (21), we compute

$$y_{rms} = 3.1\ mils$$

as the expected flow-induced, rms displacement of the tube for a coolant flow velocity of 40 ft/sec.

V. *DISCUSSION*

Equation (14), which describes the rms displacement response, is derived from a theoretical analysis using the results of a parameter study to simplify the solution. It is applicable in the design evaluation of many reactor system components which are subjected to nominally axial coolant flow, specifically those with parameters satisfying the Strouhal number ranges specified in Eq. (11). It is also applicable to pipes conveying fluid.

The primary difficulty associated with direct usage of Eq. (14) is that the system and flow-velocity dependence of the intensity of the near-field mean-square spectra in the low-frequency range of interest is not known. Therefore, since near-field flow noise is strongly system dependent, accurate prediction of displacement requires measurement of the pressure field in a prototypic assembly closely simulating the geometry and flow paths of the real system. However, the pressure measurements are difficult to perform; moreover, if a prototypic assembly was available, one might as well measure displacement response directly.

Equation (21) is an empirical relationship for rms displacement derived from Eq. (14) and based on experiments involving a smooth, flexible cylinder in annular water flow. It is in terms of known quantities and is, therefore, easily used by designers. However, since it is based on results from a smooth cylinder in flow with minimal entrance effects, the computed rms response represents a lower bound on the actual displacement. It is the response the designer can expect in a system designed for minimum turbulence by streamlining flow paths and providing isolation from external, structural-borne vibrations. The upper bound will depend on the system.

In addition to providing insight into the response problem and a "feel" for the magnitude of the displacement induced by near-field flow noise, the results can be useful in design evaluation. For example, if the response computed from Eq. (21) exceeds the allowable response determined from considerations of damage and failure mechanisms, the designer will know immediately that his design must be revised.

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