Terahertz imaging with compressed sensing and phase retrieval

Wai Lam Chan, Matthew L. Moravec, Richard G. Baraniuk and Daniel M. Mittleman

Rice University, Department of Electrical and Computer Engineering, MS 366,
Houston, TX 77251-1892, USA
Phone: (713) 348-5452 Fax: (713) 348-5686
E-mail: daniel@rice.edu

Abstract: We describe a new terahertz imaging method for high-speed image acquisition using a compressed sensing phase retrieval algorithm. This technique permits image reconstruction using a limited and randomly chosen subset of a Fourier image.

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With applications to homeland security, medical imaging, and quality control of packaged goods, commercial time-domain THz imaging systems can achieve a spatial resolution of less than 1 mm. However, these systems are generally limited by slow image acquisition rate [1,2]. Meanwhile, developments of THz imaging techniques using more sophisticated image processing approaches, such as the Radon transform [3,4] and interferometric imaging [5], have shown preliminary successes but face similar limitations in speed, resolution and/or hardware requirements. Here we describe a new approach which addresses these problems by partial sampling of the amplitude image in the Fourier plane and reconstruction of the target based on its *spatial structure*. This work is motivated by the possibility of reconstructing an image using many fewer measurements than are traditionally required.

Integral to our approach is a new signal processing scheme that combines the recent theory of *compressed sensing* (CS) [7-9] with traditional *phase retrieval* (PR) algorithms [6]. Traditional PR algorithms recover the Fourier phase from modulus-only measurements of an image's entire Fourier transform. CS theory enables image recovery from a small, random subset of Fourier measurements (magnitude and phase). In general, an infinite number of signals can be found that match these few measurements; CS uses an optimization procedure to find the "best" solution. This notion of "best" is based on assumptions of the objects' spatial structure, e.g., the sparsest solution in terms of some basis. We combine CS and PR to reconstruct the object with a small subset of the Fourier transform modulus. Our Compressed Sensing Phase Retrieval algorithm (CSPR) iterates in a way similar to the classic Hybrid Input-Output (HIO) algorithm [6] in order to find the phase of the limited measurements, but in each step also performs a CS optimization. The CS-scheme we use is orthogonal matching pursuit (OMP) [10].

Our imaging system consists of a THz transmitter, a receiver, and two lenses, one of which collimates the THz beam while the other focuses the beam (Fig. 1). The object mask, placed in between the two lenses, diffracts the THz waves. The focusing lens forms the Fourier transform of the object at its focal plane. The receiver, mounted on a translation stage, performs a raster scan in the focal plane, over an area

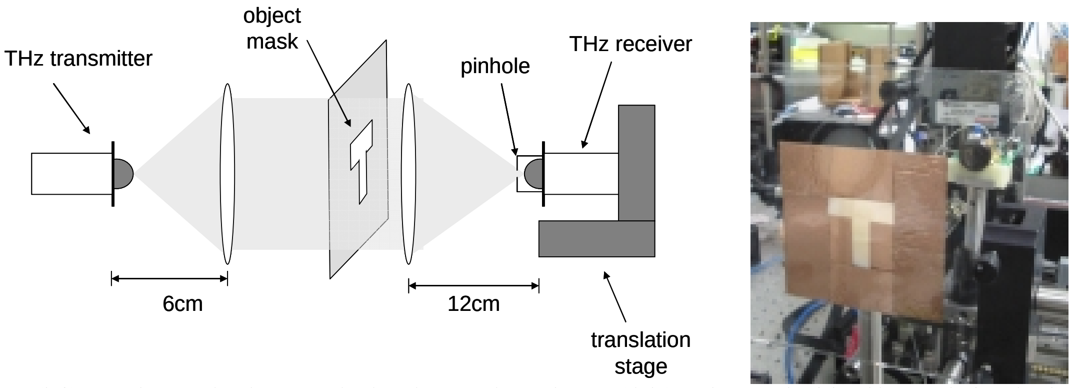

Fig.1. (left) THz phase-retrieval compressive imaging experimental setup. (right) A photograph of the object mask, focusing lens (partially occluded by the object), and the THz receiver.

of 64×64 mm, measuring a THz waveform at every 1 mm interval. We mount a pinhole over the receiver antenna so that it only samples a small area of the Fourier pattern (in our case, 1 mm in diameter), rather than relying on the ~6 mm receiver aperture. The object mask is made up of copper tape on a plastic plate. In our experiments, our object mask has a T-shaped hole, 35 mm by height and by width.

Figure 2 compares the traditional image formation via raster scanning and PR to a random scanning approach enabled by CS. The left panel shows the raw data at one particular wavelength ($\lambda = 2.1$ mm), which is the modulus of the measured pattern in the Fourier plane. A direct Fourier inversion of this pattern, using both amplitude and phase information, would produce a reconstruction of the object with a spatial resolution in each dimension given by $\Delta x = \lambda f/X$, where X is the length of the raster scan area (64 mm, in this case) and f is the focal length of the focusing lens (120 mm). Thus we obtain a resolution of $\Delta x = 3.93$ mm in both dimensions, at the chosen wavelength.

To compare traditional PR to CS, we first down-sample the original 4096 measurements to a 20×20 grid, which is the minimum size required for an accurate reconstruction using PR. We then reconstruct the object accurately with the HIO algorithm. Then, using only 150 measurements randomly selected from the 4096 measurements, we reconstruct the image with our CSPR algorithm. We observe from Figure 2 that CSPR can reconstruct an image with fewer measurements than PR, thus decreasing the required acquisition time. In addition, the CSPR image has a superior signal-to-noise (S/N), 26.5dB, with improved background suppression, compared to 19.6dB in PR. However, CSPR has longer post-processing time (30 seconds, vs. 1 second for PR) and does not always converge to the correct image. The origins of this convergence issue are under investigation.

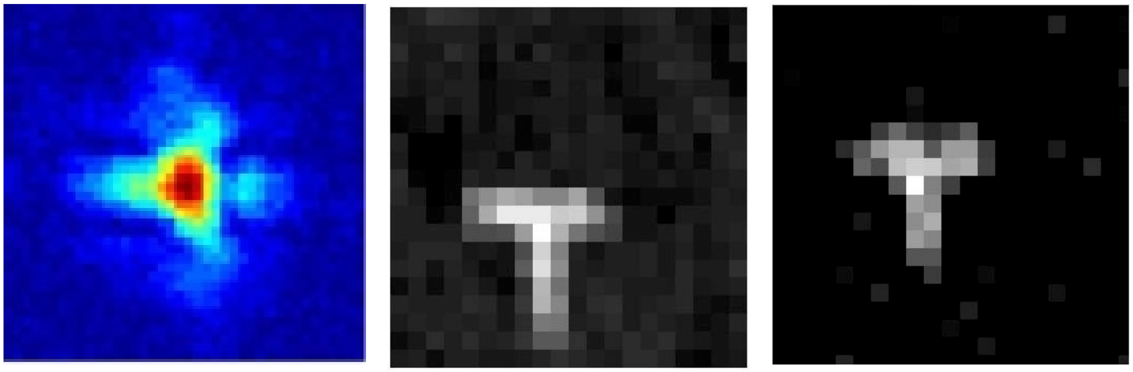

Fig. 2 (left) Acquired THz data in Fourier plane (modulus of Fourier transform) at wavelength $\lambda = 2.1$mm. Red and blue colors indicate large and small pixel values respectively. Object reconstructed via PR (middle) with 400 measurements and CSPR (right) with only 150 measurements. Dark areas have small pixel values. Object reconstructed from PR/CSPR can be shifted or inverted.

We believe that the advantages of CSPR over PR will become more significant when we modify the system to image a larger object at a higher resolution. In particular, the fact that a randomly selected subset of the image data is adequate for image reconstruction is a significant advantage, because it means that a fixed collection of randomly generated masks can be used to collect data for any arbitrary image.

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