

Term-Structure of Consumption Risk Premia in the Cross-Section of Currency Returns*

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Abstract

I quantify the risk-return relationship in the foreign-exchange (FX) market across different countries and investment horizons by focusing on the role of multiple sources of consumption risk. I estimate a flexible structural model of the joint dynamics of US aggregate consumption, inflation, nominal yield, and stochastic variance with cross-equation restrictions implied by recursive preferences. I identify four sources of consumption risk: short-run, long-run, inflation, and variance shocks. The long-run consumption risk plays a prominent role in the FX market: it contributes to the spread in returns between high and low interest rate currencies across multiple investment horizons from one to five quarters. The short-run consumption risk affects currencies only at the quarterly horizon, where it explains 40% of the spread. The difference in returns between high and low yield currencies disappears for horizons longer than four quarters.

Keywords: consumption risk, shock-elasticity, term-structure and cross-section of currency risk premium.

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1 Introduction

This paper quantifies the risk-return relationship in the foreign exchange (FX) market across different countries and investment horizons. I perform the analysis from the perspective of a US representative agent with recursive preferences over consumption. Multiple sources of consumption risk, such as shocks to expected consumption growth, stochastic variance of consumption growth, and consumption growth itself, reflect different aspects of macroeconomic risk. My focus is on the identification of such shocks, measuring their impact on exchange rates in the cross-section and at multi-period investment horizons, and understanding their relative importance for multi-period currency risk premia.

To describe the empirical properties of consumption risk in currency markets, I model consumption growth jointly with the cross-section of exchange rate growths for low, intermediate, and high interest rate currencies at a quarterly frequency. I estimate a flexible vector autoregression (VAR) that reflects two features of consumption growth as highlighted in the literature on long-run risk: time-variation in the expected consumption growth and in the variance of consumption growth (Bansal and Yaron, 2004; Bansal, Kiku, Shaliastovich, and Yaron, 2013; Campbell, Giglio, Polk, and Turley, 2014).

To capture the time-variation in the expected consumption growth, I jointly model consumption growth with inflation and nominal yield on a quarterly risk-free bond. The use of information contained in asset prices about the expected consumption growth is an important and novel feature of my approach. In equilibrium, an asset price is a function of conditional moments of consumption growth that are difficult to estimate on the basis of consumption data alone. Therefore, asset prices are an appealing source of information about consumption dynamics, providing the ability to estimate the expected consumption growth more precisely.

I choose nominal bond specifically because the theoretical literature (e.g., Bansal and Shaliastovich, 2013) stresses that nominal yields reflect the particular risks that relate to exchange rates. In addition, the use of the yield, as opposed to another asset price, does not require the modeling of extra cash flow dynamics. The model incorporates inflation in view of its ability to forecast future consumption growth

(Piazzesi and Schneider, 2006). Finally, I capture time-variation in the variance of consumption growth by allowing consumption growth, inflation, and nominal yield to share a common stochastic variance.

For the analysis, I use quarterly observations of US consumption growth, inflation, and three-month nominal yield from 1947 to 2011, together with twelve currency pairs against the US dollar from 1986 to 2011. I rely on Bayesian Markov Chain Monte Carlo (MCMC) methods to estimate the model and perform estimation in two stages. I first estimate the dynamics of consumption growth and identify structural shocks. Shocks are interpreted based on their empirical properties as short-run consumption risk, inflation risk, long-run consumption risk, and variance risk.¹ Next, I measure how the log exchange rate growth of each currency basket loads on the states and shocks of the consumption growth.

I explore the impact of consumption risk on the FX market by measuring currency shock-exposure elasticities (Borovička, Hansen, Hendricks, and Scheinkman, 2011, and Borovička and Hansen, 2013) that are multi-period risk sensitivities of exchange rates. The analysis leads to new evidence that high interest rate currencies exhibit higher exposures to long-run consumption and inflation shocks than low yield currencies across *multiple* investment horizons. The differences are economically sizeable and statistically significant. This result is robust to the shock-identifying assumptions. To measure how different multi-period currency risk exposures are priced, I adapt a structural approach. At this stage I introduce the US representative agent with recursive utility and an aversion to consumption risk.

Recursive preferences applied to the consumption growth process imply the dynamics of the pricing kernel. On the one hand, the pricing kernel depends on the nominal yield, because it is one of the states in the model. On the other hand, the pricing kernel values all assets in the economy, including the nominal risk-free bond. Hence, the nominal yield is an affine function of the states of the pricing kernel. The dual role of the nominal yield in the model implies a set of pricing restrictions on the structural parameters. As a result, the valuation exercise requires a constrained VAR of consumption growth with cross-equation restrictions derived under recursive preferences.

¹Henceforth, I use the terms “risk” and “shock” interchangeably.

The estimation results show that the model with structural restrictions fits the macroeconomic data and the data on asset prices quite well. First, the model captures important economic episodes such as the Great Moderation, recessions, and the recent financial crisis. Second, diagnostics of the fitting errors do not exhibit any noticeable misspecifications.

I examine asset pricing implications of the model across forty investment horizons, from one quarter up to ten years. Specifically, in addition to characterizing shock-exposure elasticities, I measure shock-price elasticities (Borovička, Hansen, Hendricks, and Scheinkman, 2011, and Borovička and Hansen, 2013) that assign prices to marginal multi-period currency risk exposures.

My main finding is that the risk of low frequency movements in the expected consumption growth plays the most prominent role in the FX market. The long-run consumption risk is the only source of consumption risk that affects differently the low- and high-yield currencies across *multiple* investment horizons. At the horizons from one to five quarters, the low-yield currencies exhibit significantly lower exposure to the risk than the high-yield currencies do. Moreover, this risk is associated with the highest compensation of 0.28% per each extra percent of risk exposure on a quarterly basis. Taken together, at least 39% of the quarterly cross-sectional spread in currency returns between the high and low interest rate currencies can be related to the long-run consumption risk.

The other shocks contribute to risk premia less prominently. Short-run consumption risk is priced in the cross section of currency baskets only at the horizon of one quarter. At least 40% of the spread in quarterly currency returns is related to the difference in the exposures of the high and low interest rate currencies to the risk. The role of the inflation risk is sensitive to the choice of shock identification. If the inflation shock affects the US consumption growth contemporaneously, it is priced in the cross-section of currency baskets at the horizon of one quarter and contributes 21% to the corresponding spread. The variance shock does not produce the cross-section of currency risk premia, yet it plays an important role in currency markets. This risk is the only source of time variation in currency risk premia.

At horizons longer than one year, the cross-section of currency risk premia due to currency sensitivity to the macroeconomic risk disappears. Hence, the profitability

of investments in bonds of high interest rate currencies when the money is borrowed in low interest rate environments is a short-horizon phenomenon.

Related literature

This paper is related to two strands of the international macro-finance literature that examines time-series and cross-sectional properties of currency risk premia. I limit my discussion to papers that interpret currency risk premia as compensation for consumption risk. On the empirical front, Cumby (1988), Sarkissian (2003) and Lustig and Verdelhan (2007) study the ability of the consumption growth factor to explain the cross-section of currency returns. Cumby (1988) shows that the ex-ante currency returns and the conditional covariances between currency excess returns and US consumption growth are linked in a way consistent with consumption-based models. Formal testing, however, reveals that basic models, e.g., models with a time-separable utility function, cannot quantitatively account for the behavior of currency returns.

Sarkissian (2003) adapts the framework of Constantinides and Duffie (1996) to a multi-country setting and shows that the cross-country variance of consumption growth has explanatory power for cross-sectional differences in returns on individual currencies, whereas the consumption growth itself does not. Lustig and Verdelhan (2007) use the framework of the durable CCAPM of Yogo (2006) to establish that the consumption growth is a priced factor in the cross-section of returns on currency baskets formed by sorting currencies by respective interest rates. There are two common threads in these papers. First, both studies recognize the presence of multiple sources of consumption risk but do not describe them explicitly. Second, both papers do not extend the analysis beyond a single horizon given by the decision interval of the representative agent (one quarter in the case of Sarkissian, 2003, and one year in the case of Lustig and Verdelhan, 2007).

The theoretical literature features different consumption-based models dedicated to rationalizing the time-series behavior of currency risk premia, e.g., the violation of the uncovered interest rate parity (UIP). The models include settings with habits (Heyerdahl-Larsen, 2014; Verdelhan, 2010), long-run risks (Bansal and Shaliastovich,

2013; Colacito, 2009; Colacito and Croce, 2013), and disasters (Gabaix and Farhi, 2014). My paper is closely related to the international literature on the long-run risk but my focus is different. Theoretical international long-run risk studies model a joint distribution of domestic and foreign macroeconomic quantities to pin down a theoretical equilibrium exchange rate consistent with the forward premium anomaly. As an alternative, I model multiple sources of consumption risk of the US representative agent, estimate them, and establish their relative importance for currency risk premia not only in the cross-section but also across alternative investment horizons.

This study also contributes to the debate about violations of the uncovered interest rate parity at long horizons, or alternatively, the long-run profitability of currency carry trades. Put simply, the two phenomena are different sides of the same coin. Boudoukh, Richardson, and Whitelaw (2013) reformulate the uncovered interest parity hypothesis by relating future exchange rate changes to the lagged forward interest rate differentials. They show that implied forecasts of the exchange rate growth work in line with the theoretical implications of the UIP at horizons longer than one year. Bekaert, Wei, and Xing (2007), Chinn and Meredith (2005), and Chinn and Quayyum (2012) find mixed evidence on the unbiasedness of the UIP hypothesis at long investment horizons for different currency pairs and different samples.

To date there has been no explicit study of the effect of the investment horizon on the profitability of carry trades. Recently Lustig, Stathopoulos, and Verdelhan (2013) analyzed the profitability of investing in bonds of different maturities across the globe, but their focus lay on one-period returns. My study characterizes the term-structure of carry profitability in relation to multi-period currency risk exposure to consumption risk, thereby providing direct evidence regarding those horizons at which the UIP holds and those at which it is violated.

Finally, this paper is related to Hansen, Heaton, and Li (2008), who provide evidence on the importance of the permanent shock to consumption in accounting for the value premium. The similarity is in terms of approach that is establishing the importance of consumption risks to explain the cross-section of asset prices by jointly modeling the stochastic discount factor (under the assumption of recursive preferences) and cash flow processes. My study differs from Hansen, Heaton, and Li (2008) in three principal dimensions. First, I study the foreign exchange market, which has been

examined to a lesser degree than the US equity market. Second, my model has stochastic variance: I therefore account for the variation in volatility of consumption growth, and therefore, in risk premia. Third, I quantify the relative importance of consumption risks for short and medium horizons, rather than at an infinite horizon.

2 Consumption risk in the cross-section of exchange rates

In this section, I establish novel empirical evidence on the relationship between consumption risk and the cross-section of exchange rates. I specify and estimate a joint reduced-form model of macroeconomic fundamentals and exchange rate growths that is informative about how multiple sources of consumption risk affect exchange rates. Next, I characterize the entire term-structure of currency exposures to the identified sources of consumption risk. My underlying goal is to relate the cross-sectional spread in returns between high and low interest rate currencies to the difference in currency exposures to consumption shocks at alternative investment horizons. I outline the necessary modeling ingredients to address this issue.

A. Property of currency returns

I start my analysis by forming test portfolios and reproducing the well-known result of the high average profitability of currency carry trades at short horizons (e.g., Lustig and Verdelhan (2007)). At the end of each quarter, the currencies of twelve large economies are sorted based on their quarterly interest rates and allocated to three equally weighted baskets: Basket “Low” (low interest rate currencies), Basket “Intermediate” (intermediate interest rate currencies), and Basket “High” (high interest rate currencies). Panel (a) of Table 1 shows the corresponding dynamic basket composition. Appendix A.1 describes the sample and data sources.

An investor follows a simple investment strategy. She borrows US dollars at the quarterly risk-free rate, converts the money into the foreign currencies of one of the baskets, invests them for one quarter at the corresponding foreign risk-free rates, and at the end of the quarter converts the proceeds back in USD to pay off the debt. The

Table 1
Composition of currency baskets

Currency	(a) Short-rate sorting Number of periods in			(b) Cross-term rate sorting Number of periods in		
	“Low”	“Intermed”	“High”	“Low”	“Intermed”	“High”
Australia	6	29	64	0	23	76
Canada	26	71	6	20	75	8
Denmark	8	70	15	11	70	12
Germany	34	8	10	34	16	2
Euro area	18	11	0	17	12	0
Japan	98	5	0	103	0	0
New Zealand	4	15	68	4	10	73
Norway	7	18	30	1	24	30
South Africa	0	0	58	0	0	58
Sweden	28	34	14	32	29	15
Switzerland	87	8	0	95	0	0
UK	6	41	56	5	50	48

Notes. Table entry shows the number of periods for which an individual currency belongs to a basket. (a) The three baskets are formed by sorting currencies by quarterly interest rates $\tilde{i}_{t,t+1}$, (b) The three baskets are formed by sorting currencies by cross-term interest rates, $\tilde{i}_t = \frac{1}{T} \sum_{\tau=1}^T \tilde{i}_{t,t+\tau}$. Quarterly rebalancing. Sample period: 1986 – 2011.

log excess returns for this strategy is

$$\log rx_{t,t+1} = \log s_{t+1} - \log s_t + \tilde{i}_{t,t+1} - i_{t,t+1},$$

where the exchange rate s_t is the nominal price of one unit of the currency basket in US dollars; $i_{t,t+1}$ ($\tilde{i}_{t,t+1}$) is a quarterly US (foreign) nominal yield.² The foreign yield $\tilde{i}_{t,t+1}$ is the average quarterly yield of the currencies in a specific basket. Double subscripts applied to a variable emphasize its horizon. For example, return $rx_{t,t+1}$ is a one-period return for an investment initiated at time t and realized at time $t + 1$; $i_{t,t+1}$ is a yield known at time t on a one-period nominal bond that matures at time $t + 1$. One period corresponds to one quarter. Panel (a) of Table 2 reports descriptive statistics of log excess returns for the currency baskets. The sorting produces a sizeable average spread in one-period returns between the high and low interest rate

²In what follows, the tilde denotes foreign variables.

currency baskets (a.k.a carry return) of $(0.0117 - 0.0015) \cdot 400 = 4.08\%$ per year.

Table 2
Properties of log excess returns

	Mean	Std Dev	Skewness	Kurtosis	Autocorrelation
(a) Short-rate sorting					
Basket “Low”	0.0015	0.0499	0.46	3.38	0.06
Basket “Intermediate”	0.0046	0.0431	-0.33	3.33	0.13
Basket “High”	0.0117	0.0532	-0.03	3.44	0.14
(b) Cross-term rate sorting					
Basket “Low”	0.0008	0.0514	0.41	3.07	0.02
Basket “Intermediate”	0.0054	0.0434	0.13	3.68	0.17
Basket “High”	0.0121	0.0507	-0.03	3.62	0.15

Notes. Descriptive statistics for log excess returns on currency baskets. Quarterly. (a) The three currency baskets are formed by sorting currencies by quarterly interest rates, $\tilde{i}_{t,t+1}$. (b) The three currency baskets are formed by sorting currencies by cross-term interest rates, $\tilde{i}_t = \frac{1}{T} \sum_{\tau=1}^T \tilde{i}_{t,t+\tau}$. Quarterly rebalancing. Sample period: 1986 – 2011.

B. Joint dynamics of consumption growth and exchange rates

I describe the joint dynamics of consumption growth and the cross-section of exchange rate growths in order to establish if currency baskets exhibit different sensitivities to consumption risk. I split this task in two steps. First, I specify and estimate a model of consumption growth and identify multiple sources of consumption risk. Further, I measure how log exchange rate growth of each currency basket loads on the states of consumption growth and shocks affecting it. The detailed discussion of the implementation follows below.

It is a well-known problem in asset pricing that high-quality consumption data are available at low frequency, and therefore the identification of multiple sources of consumption risk is a challenging task. To overcome this empirical difficulty, I model consumption growth jointly with inflation and quarterly nominal yield. These variables are known to have forecasting power for future consumption growth. Hall (1978) and Hansen and Singleton (1983) show that lagged consumption growth is useful in predicting future US consumption growth. Piazzesi and Schneider (2006) argue that inflation is a leading recession indicator. Bansal, Kiku, and Yaron (2012b),

Constantinides and Ghosh (2011), and Colacito and Croce (2011) argue that the real risk-free rate serves as a direct measure of the predictable component in future consumption growth. Instead of considering the real risk-free rate, I use a short-term nominal rate and inflation.³

In my modeling framework, the use of asset price to establish the process of consumption growth is especially important. Macro-finance models imply that equilibrium asset prices are functions of observable consumption growth and unobservable states of consumption growth. Consequently, incorporating an observable asset must provide additional information about multiple shocks affecting realized and expected consumption growth, as well as the conditional variance of consumption growth.

I choose the nominal bond as an asset specifically for two reasons. First, the extant theoretical finance literature relates interest rate risks to those in exchange rates (e.g., Bansal and Shaliastovich, 2013; Heyerdahl-Larsen, 2014; Verdelhan, 2010). Second, empirical macroeconomic studies emphasize that exchange rates do respond to shocks affecting interest rates. For example, Eichenbaum and Evans (1995) and Faust and Rogers (2003) quantify the impact of monetary policy shocks, identified through the dynamics of interest rates, on exchange rates.

I specify a parsimonious yet flexible model of consumption growth by positing a vector autoregressive process for $Y_{t+1} = (\log g_{t+1}, \log \pi_{t+1}, i_{t+1,t+2}, \sigma_{t+1}^2)'$, where $\log g_{t+1}$ is consumption growth, $\log \pi_{t+1}$ is inflation, $i_{t+1,t+2}$ is nominal yield, and σ_{t+1}^2 is stochastic variance

$$Y_{t+1} = \underset{4 \times 1}{F} + \underset{4 \times 4}{G} Y_t + \underset{4 \times 4}{H} \sigma_t \varepsilon_{t+1}. \quad (2.1)$$

Vector $\varepsilon_{t+1} = (\varepsilon_{g,t+1}, \varepsilon_{\pi,t+1}, \varepsilon_{i,t+1}, \varepsilon_{\sigma,t+1})'$ includes four structural shocks. Six parameter restrictions $G_{41} = G_{42} = G_{43} = H_{41} = H_{42} = H_{43} = 0$ are imposed in order to guarantee that the stochastic variance follows the discretized version of the continuous-time square-root process.⁴ Appendix A.1 describes the macro sample and lists the relevant data sources. Table 3 provides basic descriptive statistics.

³Data on real rates, such as inflation-indexed bonds, are not suitable as these are available only from 1997.

⁴I have to ensure that $G_{41} = G_{42} = G_{43} = 0$ because otherwise I would not be able to guarantee that the stochastic variance is well defined. If $G_{41} = G_{42} = G_{43} = 0$ then it is natural to think about stochastic variance as an exogenous variable, and therefore, $H_{41} = H_{42} = H_{43} = 0$. Thus, the model of stochastic variance looks like a discretized version of the square-root process. In continuous

I introduce stochastic variance to the vector autoregression for the following two reasons. First, the time variation in the volatility of consumption growth is a salient characteristic of consumption data (e.g., Kandel and Stambaugh, 1990, Whitelaw, 2000). Second, it serves as a direct source of time variation in asset risk premia and has implications for the cross-sectional properties of asset returns (Bansal, Kiku, Shaliastovich, and Yaron, 2013; Bansal and Shaliastovich, 2013; Campbell, Giglio, Polk, and Turley, 2014; Drechsler and Yaron, 2011).

The vector autoregression that features only one stochastic variance factor is a compromise between good fit, as suggested in previous literature and parsimony. Carriero, Clark, and Marcellino (2012) show that a vector autoregression with a common stochastic volatility factor efficiently summarizes the informative content of several macroeconomic variables, including consumption growth, GDP inflation, the 10-year Treasury bond yield, and the federal funds rate.

Table 3
Properties of macroeconomic variables

	Mean	Std Dev	Skewness	Kurtosis	N observations
Consumption growth	0.0048	0.0052	-0.45	4.04	259
Inflation	0.0083	0.0076	0.81	5.30	259
Nominal yield	0.0113	0.0076	0.93	4.13	259

Notes. Descriptive statistics for US consumption growth, inflation, and quarterly nominal yield. Sample period: second quarter of 1947 – fourth quarter of 2011. Quarterly.

I use Bayesian MCMC methods to estimate the vector autoregression

$$Y_{t+1} = F + GY_t + \Sigma^{1/2}\sigma_t w_{t+1}, \quad (2.2)$$

where the disturbances w_{t+1} are unknown linear functions of structural shocks ε_{t+1} : $H\varepsilon_{t+1} = \Sigma^{1/2}w_{t+1}$; and the vector of shocks w_{t+1} and ε_{t+1} follow the multivariate normal distribution: $w_{t+1} \sim \mathcal{MN}(0, I)$ and $\varepsilon_{t+1} \sim \mathcal{MN}(0, I)$. I augment the vector autoregression (2.2) with a number of identifying restrictions and use methods of

time, the Feller condition $2F_4 > H_{44}^2$ guarantees that the variance stays strictly positive. The formal modeling of this process in discrete time is achieved via a Poisson mixture of Gamma distributions (Gourieroux and Jasiak, 2006; Le, Singleton, and Dai, 2010). I use a direct discretization of the continuous-time square-root process to streamline the estimation of the model: I draw all parameters of the model together because the vector ε_{t+1} follows the multivariate normal distribution. I guarantee that the variance remains positive by drawing it in logs.

structural macroeconometrics to recover the structural shocks. I use contemporaneous zero restrictions and consider one of the globally just identified systems (Rubio-Ramirez, Waggoner, and Zha, 2010). Specifically, among twelve possible identifications, I choose an upper triangular identification scheme with the following order of the variables: $\log g_t$, $\log \pi_t$, $i_{t,t+1}$, and σ_t^2 . The results highlighted later in the section remain robust to different shock identifying assumptions.

The structural shocks ε_{t+1} are identified from the US data but are not necessarily specific to the US economy. Aggregate consumption of any country is an equilibrium outcome of risk sharing through trade in goods and asset markets, and hence reflects both local and global shocks. The relative importance of global shocks varies across different countries. Lustig, Roussanov, and Verdelhan (2011) and Verdelhan (2013) suggest that the global sources of risk must lie at the core of currency carry trade profitability. Because of the special role of the US economy in the world financial system and trade (e.g., Maggiori, 2013), it is natural to expect that the US data are the most informative about global consumption disturbances.

Based on the properties of the identified shocks, I interpret the shock $\varepsilon_{g,t+1}$ as the short-run consumption risk and the shock $\varepsilon_{i,t+1}$ as the long-run consumption risk. Four quarters after the shocks hit the economy the response of consumption growth to the shock $\varepsilon_{i,t+1}$ always dominates that to the shock $\varepsilon_{g,t+1}$. Therefore, the cumulative impact of the shock $\varepsilon_{g,t+1}$ on consumption growth is concentrated in the short-run, whereas the cumulative impact of the shock $\varepsilon_{i,t+1}$ on consumption growth dominates at long horizons.⁵ The online Appendix illustrates the corresponding impulse response functions. I label the shock $\varepsilon_{\sigma,t+1}$ the variance risk, because it is the only source of time variation in the variance of consumption growth. Finally, I refer to the shock $\varepsilon_{\pi,t+1}$ as the inflation risk because it affects primarily inflation and does not exhibit notable impact on consumption growth at any horizon.

Next, I rely on Bayesian methods to estimate how the log exchange rate growth $\log \delta_{t,t+1} = \log s_{t+1} - \log s_t - \log \pi_{t+1}$ of each currency basket loads on the states and

⁵The shock $\varepsilon_{i,t+1}$ exerts a long-lasting impact not only on consumption growth but also on inflation. I focus on the role of $\varepsilon_{i,t+1}$ on consumption growth rather than that on inflation because it is potentially more important for asset pricing. This is because the preferences of the representative agent are defined over the consumption stream.

shocks of US consumption growth:

$$\log \delta_{t,t+1} = \log \delta + \mu' Y_t + \xi' \sigma_t \varepsilon_{t+1} + \tilde{\xi}_v' \sigma_t \tilde{v}_{t+1}. \quad (2.3)$$

The elements of the vectors μ and ξ reflect the impact of both US and foreign variables on currency dynamics, as long as foreign variables are not orthogonal to the states Y_t and shocks ε_{t+1} . The process (2.3) has a component $\tilde{\xi}_v' \sigma_t \tilde{v}_{t+1}$, aggregating the impact of idiosyncratic (foreign specific) disturbances on the exchange rate growth, but omits a vector of foreign-specific states (states orthogonal to Y_t). The absence of foreign-specific states is not an impediment to the correct measurement of loadings μ and ξ that are of central importance for the analysis. This works in a similar way to the case of a linear regression with omitted regressors that are orthogonal to the rest of explanatory variables. The online Appendix provides further details.

I choose a two-stage estimation of the joint model of consumption growth and the cross-section of exchange rate growths for the following two reasons. First, because exchange rates do not Granger cause the economic states, the vector autoregressive process (2.2) can be viewed as an autonomous stochastic system.⁶ Further, the estimation procedure does not suffer from the problem of generated regressors because Bayesian methods account for the statistical uncertainty related to the components of the vector of shocks $\sigma_t \varepsilon_{t+1}$.

C. Cross-section of multi-period currency risk exposures

I study the importance of the four sources of consumption risk in the foreign exchange market by measuring the sensitivity of the cross-section of currency baskets to the shocks. Table 4 shows that at the quarterly horizon currencies belonging to different interest rate environments load differently on the inflation and the long-run consumption shocks. Moreover, the long-run shock stands out because the high interest rate

⁶The absence of Granger causality is an implication of a broad class of theoretical models (e.g., Bansal and Shaliastovich, 2013, Colacito and Croce, 2013, Verdelhan, 2010). It does not necessarily contradict literature documenting some forecasting ability of exchange rates for future fundamentals (e.g., Engel and West, 2005). Exchange rate growth is a function of states and shocks of the model. If one of the states of the model is an omitted regressor in the regression of a future fundamental on exchange rate growth and other variables of interest, then naturally exchange rate growth exhibits predictive power for the future fundamental.

currencies have a significantly positive exposure, whereas the low interest rate currencies have a significantly negative exposure to the source of risk. This evidence complements the finding of Lustig and Verdelhan (2007) who show that at the one-period investment horizon there is a cross-sectional variation in currency exposures to the realized consumption growth.

Table 4
Exposure of currency baskets to sources of consumption risk

Exposure to	Basket “Low”	Basket “Intermediate”	Basket “High”
Short-run risk, ξ_g	0.0069 (0.0011, 0.0130)	0.0023 (-0.0021, 0.0069)	0.0060 (0.0006, 0.0115)
Inflation risk, ξ_π	-0.0007 (-0.0055, 0.0040)	0.0081 (0.0040, 0.0121)	0.0121 (0.0077, 0.0163)
Long-run risk, ξ_i	-0.0116 (-0.0164, -0.0070)	-0.0021 (-0.0054, 0.0012)	0.0078 (0.0025, 0.0130)
Variance risk, ξ_σ	-0.0094 (-0.0220, 0.0028)	-0.0004 (-0.0095, 0.0097)	0.0041 (-0.0085, 0.0165)

Notes. I estimate the joint process for the state-vector Y_{t+1} and log exchange rate growth $\log \delta_{t,t+1}$ of the “Low”, “Intermediate”, and “High” currency baskets:

$$\begin{aligned} Y_{t+1} &= F + GY_t + H\sigma_t\varepsilon_{t+1}, \\ \log \delta_{t,t+1} &= \log \delta + \mu'Y_t + \sigma_t\xi'\varepsilon_{t+1} + \tilde{\xi}_v\sigma_t\tilde{v}_{t+1}, \end{aligned}$$

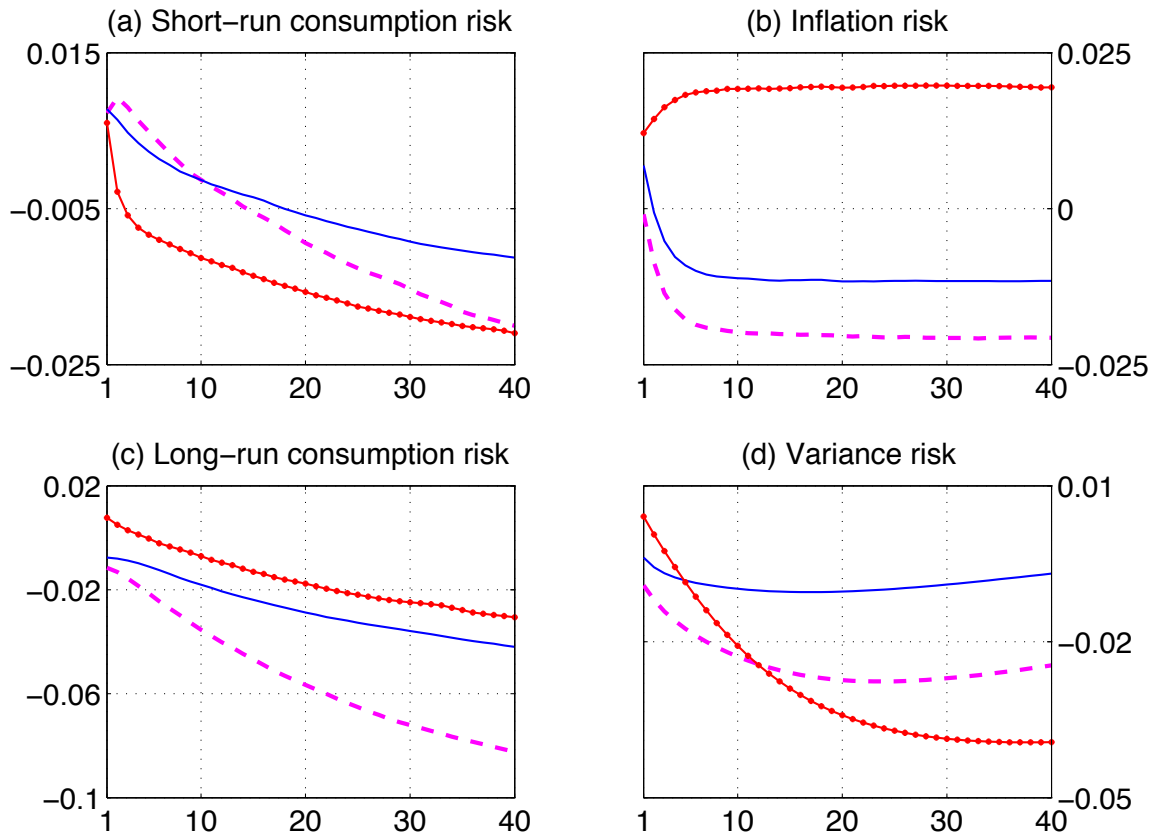
where $\xi = (\xi_g, \xi_\pi, \xi_i, \xi_\sigma)'$. To identify shocks $\varepsilon_{t+1} = (\varepsilon_{g,t+1}, \varepsilon_{\pi,t+1}, \varepsilon_{i,t+1}, \varepsilon_{\sigma,t+1})'$, I impose the recursive identification scheme: $H_{21} = H_{31} = H_{32} = H_{41} = H_{42} = H_{43} = 0$. Quarterly. The round brackets contain the 95% confidence intervals.

To explore multi-period currency risk exposures, I measure the shock-exposure elasticities of Borovička and Hansen (2013) for each currency basket. Shock-exposure elasticities are term-structures of marginal risk exposures. An appealing feature of these objects is their ability to measure the effect of each specific shock on the multi-period exchange rate growth when the variance is stochastic. The variance shock $\varepsilon_{\sigma,t+1}$ affects the expected multi-period exchange rate growth directly via term $\sigma_t\varepsilon_{\sigma,t+1}$ and indirectly together with other sources of risk via terms similar to $\sigma_{t+1}\varepsilon_{g,t+2}$. The shock-exposure elasticities take both effects into account. The concept of shock-exposure elasticities and their properties are discussed in the online Appendix.

Figure 1 shows that across multiple investment horizons the low and high interest rate currencies exhibit significantly different and economically sizeable spreads in

exposures to the long-run consumption and inflation risks. The spreads in currency exposures across horizons from one to sixteen quarters to the inflation shock and one to three quarters to the long-run risk shock are statistically and economically significant.⁷ The robust *multi-period* relationship among the long-run consumption risk, the inflation risk and the cross-section of currencies is a novel finding.

Figure 1
Shock-exposure elasticities. Recursive identification



Shock-exposure elasticities for the short-run consumption risk (panel (a)), inflation risk (panel (b)), long-run consumption risk (panel (c)), and variance risk (panel (d)). The magenta dashed line is for the basket “Low”, the blue solid line is for the basket “Intermediate”, the red marked line is for the basket “High”. Horizontal axes: from 1 quarter to 10 years. Quarterly.

⁷I do not plot the confidence bounds of the estimated shock-exposure elasticities to avoid clutter. Results are available upon request.

The differences in the multi-period currency exposures to the shocks have implications for the risk premia only if the risks are priced at the corresponding investment horizons. In the remainder of this section, I will illustrate that the standard approach of cross-sectional asset pricing is not a suitable framework for exploring the pricing implications of the identified shocks in my case. Instead, a structural model that describes the investor’s risk attitude, is a necessary ingredient of the analysis.

D. Asset pricing implications of the spreads in currency risk exposures

Prices of risk is a modeling ingredient that translates differences in risk exposures into differences in risk premia. The two-stage Fama and MacBeth (1973) procedure is a model-free methodology for estimating one-period prices of the orthogonal sources of risk ε . The online Appendix describes the mechanics of the procedure. In fact, the first stage involving the estimation of currency exposures to the multiple sources of consumption risk has already been undertaken. The parameters of the exposures $\xi = (\xi_g, \xi_\pi, \xi_i, \xi_\sigma)'$ are the “betas” (in the standard language of cross-sectional asset pricing) associated with the sources of risk.

The second stage is the estimation of the cross-sectional regression of the average returns of currency baskets $\log \bar{r}$ on the estimates for risky currency exposures $\hat{\xi}_g, \hat{\xi}_\pi, \hat{\xi}_i,$ and $\hat{\xi}_\sigma$. There are at least two requirements for this regression to yield consistent and unbiased results: (1) a large number of currency baskets (Lewellen, Nagel, and Shanken, 2010), (2) the absence of highly correlated cross-sectional exposures associated with different sources of risk (Ahn, Perez, and Gadarowski, 2013). It is difficult to satisfy these requirements in the current setting. There are only three currency baskets in the cross-section that are used to estimate prices of the four sources of risk. In addition, the risk exposures of currency baskets to the inflation, long-run, and variance shocks appear multicollinear. The loadings increase monotonically from the Basket “Low” to the Basket “High” (Table 4).

Similarly to measuring multi-period risk exposures to individual shocks, quantifying multi-period risk compensations is a challenging task in the presence of stochastic variance. The core of the problem lies in the convoluted role of the variance shock $\varepsilon_{\sigma,t+1}$, which affects the multi-period prices of risk directly (via term $\sigma_t \varepsilon_{\sigma,t+1}$) and indirectly (via terms similar to $\sigma_{t+1} \varepsilon_{g,t+2}$). The shock-price elasticities of Borovička and Hansen (2013) circumvent the problem but the necessary input for computation

of the elasticities is the pricing kernel. See the online Appendix for further details of the shock-price elasticity.

As a result, a structural model must be an essential element of the study. The use of the model allows the exploration of the pricing implications of individual shocks in isolation, facilitates the analysis of multi-period objects without requiring availability of longer samples of data, ensures consistent pricing of assets across multi-period horizons, and further narrows down the choice of shock identification strategies. As a result, the next step in the analysis is to augment the joint model of consumption growth and exchange rates, represented by expressions (2.1) and (2.3), with the investor's preferences.

3 The structural model

A. The pricing kernel

The dynamics of consumption growth (2.1) reflects multiple sources of consumption risk that, according to Figure 1, matter for currency markets in a non-trivial manner. In addition, depending on the shock identification scheme encoded in the matrix H , some shocks have a contemporaneous effect on consumption growth, e.g., $\varepsilon_{g,t+1}$, whereas others may affect consumption growth with a lag of one quarter and exhibit a different life time, e.g., $\varepsilon_{\pi,t+1}$, $\varepsilon_{i,t+1}$, and $\varepsilon_{\sigma,t+1}$. To understand importance of the shocks for valuation of currency investments, I model the attitude of the US representative investor towards macroeconomic risk.

I use the framework of the representative agent with recursive preferences in order to account for important aspects of the temporal distribution of risk on asset prices. Earlier notable applications of this framework for understanding the joint dynamics of exchange rates, macro quantities, and asset prices include Backus, Gavazzoni, Telmer, and Zin (2010), Bansal and Shaliastovich (2013), Colacito and Croce (2011), Colacito and Croce (2013), Colacito (2009), Tretvoll (2011a), and Tretvoll (2011b). The recursive utility is a constant elasticity of substitution recursion,

$$U_t = [(1 - \beta)c_t^\rho + \beta\mu_t(U_{t+1})^\rho]^{1/\rho}, \quad (3.4)$$

with the certainty equivalent function,

$$\mu_t(U_{t+1}) = [E_t(U_{t+1}^\alpha)]^{1/\alpha}, \quad (3.5)$$

where c_t is consumption at time t , U_t is utility from time t onwards, $(1 - \alpha)$ is the coefficient of relative risk aversion, $1/(1 - \rho)$ is the elasticity of intertemporal substitution (EIS), and β is the subjective discount factor. The pricing kernel derived by applying preferences (3.4)-(3.5) to the dynamics of the US consumption growth (2.1) is

$$\log m_{t,t+1} = \log m + \eta' Y_t + q' \sigma_t \varepsilon_{t+1}, \quad (3.6)$$

where $\eta = (\eta_g, \eta_\pi, \eta_i, \eta_\sigma)'$ and $q = (q_g, q_\pi, q_i, q_\sigma)'$. The parameters of the vectors η and q are functions of the structural parameters of the model (Appendix A.2).

Note that the pricing kernel depends on all the states Y_t , but one of the states $i_{t,t+1}$ is a transformed asset price. Because the pricing kernel must value all assets in the economy, including the nominal bond, the nominal yield plays a dual role in the model. A number of cross-equation restrictions on the parameters of matrices F , G , and $\Sigma = HH'$ guarantee that the one-period nominal yield, as an implication of the model, coincides with the state of the model $i_{t,t+1}$. As a result, the requirement of internal consistency implies that the model of consumption growth is a constrained vector autoregression.

The necessary parameter restrictions can be inferred from the equilibrium relationship between the nominal yield and the states of the model:

$$i_{t,t+1} \equiv A \log g_t + B \log \pi_t + C i_{t,t+1} + D \sigma_t^2 + E, \quad (3.7)$$

where A , B , C , D , and E are the functions of the preference parameters and structural parameters of the dynamics of consumption growth. Appendix A.2 derives the equilibrium nominal yield. The identity (3.7) holds if

$$A = B = D = E = 0, \quad (3.8)$$

$$C = 1. \quad (3.9)$$

The restrictions $A = B = E = 0$ and $C = 1$ are linear,

$$\frac{G_{21}}{G_{11}} = \frac{G_{22}}{G_{12}} = \frac{G_{23} - 1}{G_{13}} = \frac{F_2 - \log \beta}{F_1} = \rho - 1, \quad (3.10)$$

whereas the restriction $D = 0$ is nonlinear,

$$\begin{aligned} & \alpha(\alpha - \rho)(P + e_1)'HH'(P + e_1)/2 + e_2'Ge_4 - e_2'HH'e_2/2 \\ & - [(\alpha - \rho)P + e_1(\alpha - 1)]'HH'[(\alpha - \rho)P + e_1(\alpha - 1)]/2 \\ & + e_2'HH'[(\alpha - \rho)P + e_1(\alpha - 1)] - (\rho - 1)e_1'Ge_4 = 0. \end{aligned} \quad (3.11)$$

The vectors e_i are the corresponding coordinate vectors in a four-dimensional space. The elements of the vector $P = (p_g, p_\pi, p_i, p_\sigma)'$ are functions of the preference parameters and the parameters governing the dynamics of consumption growth (Appendix A.2).

The nonlinear nature of the restriction (3.11), combined with the presence of the endogenous parameters P , represents a challenge for the model estimation. It is critical to account for this nonlinear restriction, however, because otherwise the implied prices of risk in (3.6), $-q$, are biased in a non-trivial manner. To quantify this bias, I apply a recursive utility to the estimated dynamics of both unconstrained and constrained models for consumption growth, compute the corresponding prices of risk, and show that the differences between the prices of risk across different specifications are large. The online Appendix contains the corresponding results.

B. Currency sorting and exchange rate growth process

A τ -period currency carry return is the return on buying τ -maturity bonds of high interest rate countries by selling short τ -maturity bonds of low interest rate countries. Because the bonds are held until maturity, there is no interest rate risk involved. In order to relate the term-structure of carry returns to consumption risk, one has to measure τ -period risk exposures and corresponding risk premia of the low and high-yield currencies for alternative horizons τ .

When focusing on currency risk premia corresponding to a specific horizon, researchers sort currencies on the basis of yields of the corresponding maturity. It is impractical

for me to conduct multiple sorts one for each horizon in order to study the entire term-structure of risk premia. Therefore I define the cross-term foreign yield as

$$\tilde{i}_t = \frac{1}{T} \sum_{\tau=1}^T \tilde{i}_{t,t+\tau}, \quad (3.12)$$

and sort currencies by this rate. Symbol T denotes number of bonds in the foreign term structure and $\tilde{i}_{t,t+\tau}$ stands for a foreign yield of maturity τ . Because the average yield is an aggregate of information from the entire yield curve, this sorting separates low and high interest rate currencies across different investment horizons.

As in Section 2, at the end of each quarter I form three currency baskets – Basket “Low”, Basket “Intermediate”, and Basket “High” (panel (b) of Table 1). The spread between returns of the corner baskets is comparable to that when currencies are sorted by their short rate (Table 2). To ensure that the main findings of the study are not sensitive to the specific sorting procedure, I run the entire analysis twice, i.e., using the cross-term and short-rate sortings. I relegate material dedicated to the case of the short-rate sorting to the online Appendix.

The dynamics of the log exchange rate growth of the currency baskets is modeled in reduced form, as given in the expression (2.3) in Section 2. In the presence of the explicitly specified pricing kernel, one may argue that the process for exchange rate growth has direct implications for foreign interest rates. This is not the case in my setting, however, because the process (2.3) omits foreign-specific states. To extend the modeling approach and derive foreign risk-free rates, it is necessary to make additional strong assumptions and deal with a larger number of cross-equation restrictions at the estimation stage. In the present study, this approach is ignored because it is a computationally heavy modeling framework. The online Appendix carefully explains the costs and benefits of the extended modeling approach and stresses that the model of exchange rate growth in the form of the process (2.3) is a sufficient modeling ingredient for measuring how consumption risks are reflected in exchange rates.

4 Methodological approach

As in Section 2, I use the Bayesian MCMC methods to estimate a joint model of US consumption growth and exchange rate growth of each currency basket with cross-equation restrictions (3.10)-(3.11). In addition, I account for regularity conditions, ensuring that the nonlinear forward-looking difference equation (3.4), defining the recursive utility, has a unique solution, and shock-exposure and shock-price elasticities are defined at an infinite investment horizon. To derive the corresponding parameter restrictions, I follow the many results of Hansen and Scheinkman (2009) and Hansen and Scheinkman (2012). The online Appendix contains the relevant details. The key advantage of the Bayesian estimation approach is that it allows to impose directly the parameter restrictions, delivers the estimated time-series of stochastic variance σ_t^2 as a byproduct of the estimation routine, and accounts for parameter and state (σ_t^2) uncertainty when estimating the dynamics of exchange rate growths.

The pricing consistency restrictions (3.10) and (3.11) are functions of the twenty two structural parameters governing the dynamics of the vector autoregression (2.2) and the preference parameters α , β , and ρ . I estimate the elements of the matrices F , G , and Σ and calibrate the preference parameters: $\alpha = -9$, $\beta = 0.9924$, and $\rho = 1/3$. The values of the preference parameters α and ρ imply the preference for the early resolution of uncertainty and have been extensively used in the literature to address a number of asset pricing puzzles.⁸ The value of the subjective discount factor β is from Hansen, Heaton, and Li (2008).⁹

I identify structural shocks ε_{t+1} from the reduced-form innovations w_{t+1} , as is tra-

⁸For example, by utilizing these preference parameters, Bansal and Yaron (2004) explain salient features of the equity market in an equilibrium framework of endowment economy; Hansen, Heaton, and Li (2008) empirically explain the value premium puzzle; whereas Bansal and Shaliastovich (2013) rationalize properties of the term-structure of nominal interest rates and the violation of the uncovered interest rate parity. In addition, in the international setting Colacito and Croce (2013) advocate $EIS=3/2$ ($\rho = 1/3$) as a value supported by empirical evidence gained through the lens of their structural model.

⁹Long-run risk literature traditionally uses monthly calibrations with a risk aversion of 10 and an IES of 1.5, whereas I use quarterly data for estimation (Hansen, Heaton, and Li (2008) also use quarterly consumption data to identify the short-run and permanent consumption shocks, and assume similar preference parameters to price assets). Importantly, however, these preference parameters, if anything, are conservative at lower frequencies. See Bansal, Kiku, and Yaron (2012b) for a related discussion of how a model specified at a lower frequency pushes an estimate for the parameter of risk aversion upwards.

ditional in structural vector autoregressions in applied macroeconomics. Because I augment the model of consumption growth with preferences, there are important economic considerations that lead to a limited choice of identification schemes compared to the case of the reduced-form analysis of Section 2. Specifically, economic intuition suggests that only two out of twelve globally just identified systems with contemporaneous zero restrictions make sense. The identifications are labeled “Fast Consumption” and “Fast Inflation” and differ in terms of the identifying assumptions about shocks $\varepsilon_{g,t+1}$ and $\varepsilon_{\pi,t+1}$. I borrow the terminology of “fast variables” from structural VARs in applied macroeconomics. The empirical properties of the identified shocks are similar to those of the shocks from VAR in Section 2 (online Appendix). Therefore, I label the shocks in the same manner as before.

Under “Fast Consumption” (“Inflation”) consumption growth (inflation) reacts contemporaneously to an inflation (short-run consumption) shock, whereas inflation (consumption growth) reacts to a short-run consumption (inflation) shock with a one-quarter delay. Table 5 displays the corresponding location of the zero restrictions, and the online Appendix explains the motivation for the choice of identifying restrictions. I analyze the relative role of multiple sources of consumption risk in currency markets under both identification schemes. Henceforth, I focus my discussion on the case of “Fast Consumption” identification, stress whether the empirical results are sensitive to the choice of identification scheme, and relegate material dedicated to “Fast Inflation” identification to the Appendix A.3.

Table 5
Shock identification

	(a) “Fast Consumption”				(b) “Fast Inflation”			
	$\varepsilon_{g,t+1}$	$\varepsilon_{\pi,t+1}$	$\varepsilon_{i,t+1}$	$\varepsilon_{\sigma,t+1}$	$\varepsilon_{g,t+1}$	$\varepsilon_{\pi,t+1}$	$\varepsilon_{i,t+1}$	$\varepsilon_{\sigma,t+1}$
Consumption eq	H_{11}	H_{12}	0	H_{14}	H_{11}	0	0	H_{14}
Inflation eq	0	H_{22}	0	H_{24}	H_{21}	H_{22}	0	H_{24}
Interest rate eq	H_{31}	H_{32}	H_{33}	H_{34}	H_{31}	H_{32}	H_{33}	H_{34}
Variance eq	0	0	0	H_{44}	0	0	0	H_{44}

Notes. Globally just identified systems. Panel (a) illustrates zero restrictions for the “Fast Consumption” identification. Panel (b) illustrates zero restrictions for the “Fast Inflation” identification.

The shock $\varepsilon_{i,t+1}$ is identified in the spirit of Bansal and Yaron (2004). Specifically, the long-run risk shock contemporaneously affects the expected consumption growth

but not the consumption growth itself. In addition, it does not feed into the variance process. The identification of the variance shock $\varepsilon_{\sigma,t+1}$ is similar to the structural assumptions of Colacito (2009). The author allows for non-zero conditional correlations between consumption growth and stochastic variance and expected consumption growth and stochastic variance.

The detailed discussion of the remaining technical issues of the empirical implementation is relegated to the different sections of the online Appendix. Specifically, the online Appendix contains the estimation algorithm and reports the choice of priors. In addition, the online Appendix also explains how pricing restrictions are imposed, how shocks ε_{t+1} are identified, and why preference parameters are calibrated rather than estimated.

5 Results

This section presents the main findings in the following order. It starts with a discussion of the estimated dynamics of the structural VAR. Next, it analyzes the sensitivities of the low and high yield currencies to the four identified sources of consumption risk and the associated risk compensations across alternative investment horizons. Finally, it describes the term-structure of cross-sectional risk premia in the FX market.

A. Macro dynamics

I use the data displayed in panels (a)-(c) of Figure 2 to estimate the model (2.2) with the parameter restrictions (3.10) and (3.11). The diagnostics of the fitting errors suggest that the model does a good job of fitting the data. The details are given in the online Appendix. To emphasize the important role of the stochastic variance, I also estimate the homoscedastic version of the vector-autoregression (2.2) with parameter restrictions (3.10). I find that the original model with stochastic variance has a much better fit (the log Bayes-Odds ratio in favor of the model with stochastic variance is 333).¹⁰

¹⁰According to Kass and Raftery (1995), a log Bayes-Odds ratio greater than three provides strong evidence against the null model, i.e., the homoscedastic VAR in this case. The online Appendix describes how to compute the log Bayes-Odds ratios.

In addition to the parameter estimates, the estimation procedure delivers other useful outputs such as the estimated expected consumption growth $E_t \log g_{t,t+1}$ displayed in panel (a) of Figure 2 and the estimated path of the unobservable stochastic variance σ_t^2 . I take the square root of σ_t^2 and scale it appropriately, so that the series represents the stochastic volatility of consumption growth. I display this series in panel (d) of Figure 2. The annualized mean path of the estimated volatility of consumption growth varies from 0.65% to 2.26%, and it captures the important economic periods. For example, the volatility is high after the Second World War, during the oil crises, the monetary experiment, and the recent financial crisis, and the volatility is relatively low during the Great Moderation.

Table 6 reports the parameter estimates for the elements of the matrices F , G , and Σ . The element G_{44} is of special interest because it characterizes the persistence of the stochastic variance. The estimated half-life of the variance component is $\log 2 / (1 - G_{44}) = 13$ quarters. It is particularly interesting to compare the estimate of G_{44} with the corresponding values used in calibrations of the Bansal and Yaron (2004) model elsewhere in the literature, e.g., in Bansal and Yaron (2004), Bansal, Kiku, and Yaron (2012a) and Bansal, Kiku, and Yaron (2012b)¹¹. These values are 0.9615, 0.9949, and 0.997 on a quarterly basis, respectively; they are higher than my point estimate of $G_{44} = 0.9451$. Nonetheless, the persistence parameter used in Bansal and Yaron (2004) is within the confidence interval of the estimated parameter G_{44} .

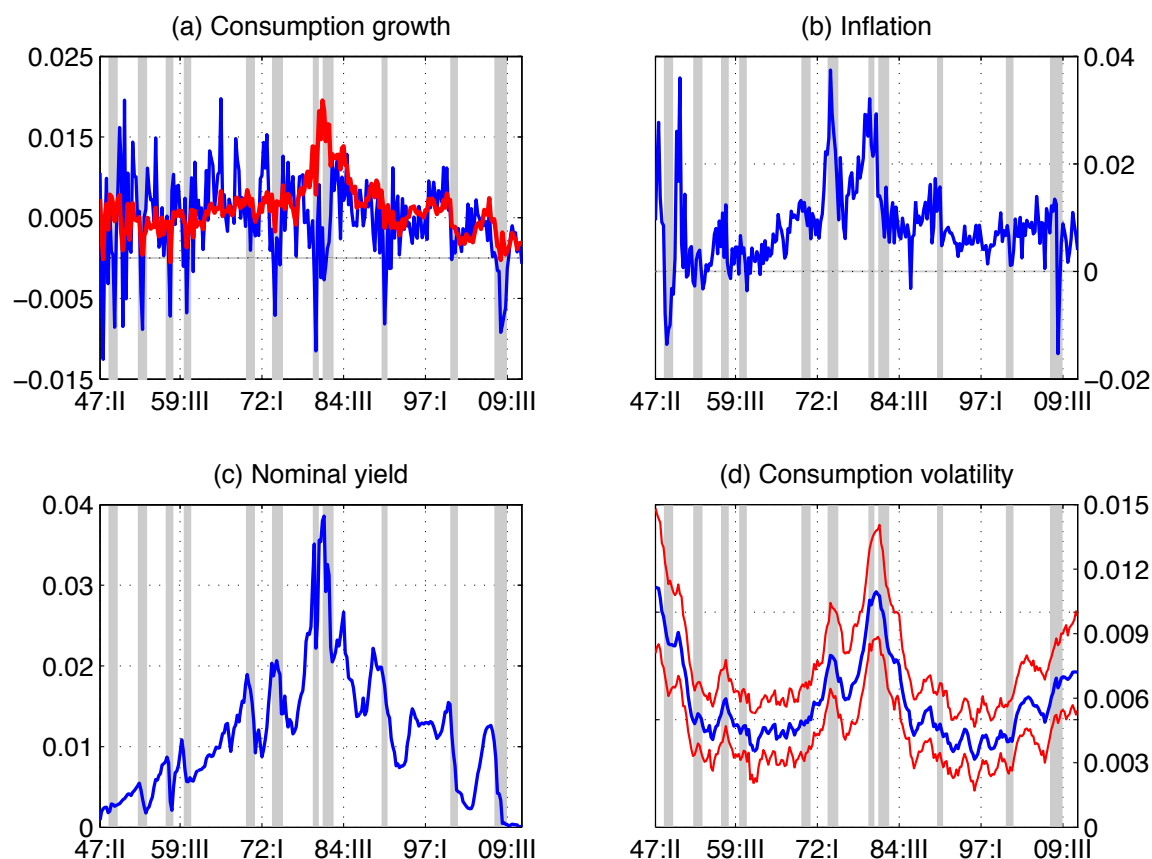
I compute the persistence of the expected consumption growth as an autocorrelation parameter of the expected consumption growth, $\text{corr}(E_t \log g_{t,t+1}, E_{t-1} \log g_{t-1,t})$. Its value is 0.85, with a 95% confidence interval from 0.76 to 0.92. These magnitudes are somewhat smaller than the values used in standard calibrations in the long-run risk literature. For example, Bansal and Yaron (2004) use the autoregressive parameter of 0.94, whereas Bansal, Kiku, and Yaron (2012a) use the value of 0.93.¹² Nevertheless, the model does not necessarily lose the ability of the long-run risk models to account for the equity risk premium. Specifically, the estimated confidence interval for the average annualized entropy of the pricing kernel is between 0.08 and 0.15; this is higher than the entropy values used in standard calibrations of the long-run risk

¹¹My model is similar to the specification of consumption growth in Bansal and Yaron (2004) in that it has only one variance factor. I use a discretized version of the square-root process, whereas Bansal and Yaron (2004) use an autoregressive model for the stochastic variance process.

¹²I refer to the parameter values converted to the consumption dynamics at a quarterly frequency.

models with stochastic variance only (no jumps).¹³

Figure 2
Dynamics of the model's states



Panel (a) displays log consumption growth (blue line) and estimated expected consumption growth (thin red line). Panel (b) displays inflation. Panel (c) displays quarterly nominal yield. Panel (d) displays consumption volatility $\sqrt{\Sigma_{11}}\sigma_t$. The blue line is the mean path of volatility, the red lines correspond to the 95% confidence bounds. Sample period: second quarter of 1947 – fourth quarter of 2011. Quarterly. Grey bars are the NBER recessions.

¹³Table 3 in Backus, Chernov, and Zin (2014) summarizes entropy values inherent to representative-agent pricing kernels with stochastic variance. They argue that a realistic macro-finance model should have the entropy of the pricing kernel of at least 0.06 per year. This is a necessary, but not a sufficient, condition to account for equity premium.

Table 6
The model of consumption growth. Parameter estimates

Parameter	Estimate	Confidence interval, 95%
F_1	-0.0006	(-0.0020, 0.0010)
F_2	-0.0072	(-0.0083, -0.0063)
F_3	-0.0003	(-0.0007, 0.0001)
F_4	0.0549	(0.0232, 0.0683)
G_{11}	0.2110	(0.0878, 0.3133)
G_{12}	-0.1687	(-0.2522, -0.0843)
G_{13}	0.3967	(0.3134, 0.4862)
G_{14}	0.0017	(0.0009, 0.0025)
G_{21}	-0.1407	(-0.2089, -0.0586)
G_{22}	0.1124	(0.0562, 0.1681)
G_{23}	0.7355	(0.6758, 0.7910)
G_{24}	0.0045	(0.0035, 0.0055)
G_{31}	0.0677	(0.0385, 0.1051)
G_{32}	0.0206	(-0.0076, 0.0451)
G_{33}	0.9536	(0.9317, 0.9768)
G_{34}	0.0005	(0.0003, 0.0007)
G_{44}	0.9451	(0.9020, 0.9764)
Σ_{11}	3.35e-5	(2.30e-5, 4.99e-5)
Σ_{12}	1.10e-5	(5.04e-6, 2.20e-5)
Σ_{13}	3.01e-6	(1.24e-6, 6.33e-6)
Σ_{14}	-0.0001	(-0.0004, 7.42e-5)
Σ_{22}	4.05e-5	(2.96e-5, 5.66e-5)
Σ_{23}	3.15e-6	(1.21e-6, 5.58e-6)
Σ_{24}	0.0003	(0.0001, 0.0006)
Σ_{33}	2.71e-6	(1.92e-6, 3.79e-6)
Σ_{34}	6.28e-5	(-2.16e-5, 0.0001)
Σ_{44}	0.0339	(0.0196, 0.0518)

Notes. I estimate a vector autoregression with stochastic variance

$$Y_{t+1} = F + GY_t + \sigma_t \Sigma^{1/2} w_{t+1}$$

and restrictions: (1) $G_{21}/G_{11} = G_{22}/G_{12} = (G_{23} - 1)/G_{13} = (F_2 - \log \beta)/F_1 = \rho - 1$ and (2) $\alpha(\alpha - \rho)(P + e_1)' \Sigma (P + e_1)/2 + e_2' G e_4 - e_2' \Sigma e_2/2 - [(\alpha - \rho)P + e_1(\alpha - 1)]' \Sigma [(\alpha - \rho)P + e_1(\alpha - 1)]/2 + e_2' \Sigma [(\alpha - \rho)P + e_1(\alpha - 1)] - (\rho - 1)e_1' G e_4 = 0$. Note that $\Sigma = H H'$, where H is from (2.1). Additional parameter restrictions, reflecting necessary regularity conditions are imposed. Vector $Y_t = (\log g_t, \log \pi_t, i_{t,t+1}, \sigma_t^2)'$ includes US consumption growth, inflation, quarterly nominal yield, and stochastic variance. To save space, I do not duplicate the symmetric entries of the matrix Σ . Sample period: second quarter of 1947 – fourth quarter of 2011. Quarterly.

The expected consumption growth loads significantly on all the observables used in the estimation with the largest loading in absolute terms on the nominal yield ($G_{13} = 0.40$). Because of the dominant role of the nominal yield, the cyclical properties of the expected growth and the nominal yield are similar. Occasionally, however, the expected consumption growth mirrors the dynamics of other variables. For example, during the recent financial crisis the dynamics of the expected consumption growth is mostly related to the dynamics of inflation with a negative sign, whereas during the economic downturn of 1958 the expected consumption growth closely tracks the evolution of the realized consumption growth.

Table 7 contains the estimates for the parameters of the matrix H . A positive variance shock leads to a positive contemporaneous move in inflation, a positive short-run consumption shock leads to a positive contemporaneous move in the nominal yield, while a positive inflation shock increases consumption growth and nominal yield. The impact of the structural shocks on the states of the model has direct implications for the magnitudes and signs of the prices of risk.

Table 7
Identification "Fast Consumption"

Parameter	Estimate	Confidence interval, 95%
H_{11}	0.0053	(0.0045, 0.0063)
H_{12}	0.0020	(0.0009, 0.0033)
H_{14}	-0.0008	(-0.0026, 0.0004)
H_{22}	0.0061	(0.0051, 0.0073)
H_{24}	0.0018	(0.0007, 0.0028)
H_{31}	0.0004	(0.0002, 0.0008)
H_{32}	0.0004	(0.0002, 0.0008)
H_{33}	0.0015	(0.0012, 0.0017)
H_{34}	0.0003	(-1.1e-5, 0.0006)
H_{44}	0.1828	(0.1399, 0.2280)

Notes. I identify structural shocks ε_{t+1} from the reduced form innovations w_{t+1} : $\Sigma^{1/2}w_{t+1} = H\varepsilon_{t+1}$. I consider "Fast Consumption" identification determined by the following zero restrictions: $H_{13} = H_{21} = H_{23} = H_{41} = H_{42} = H_{43} = 0$. Quarterly.

B. Term-structure of currency exposures to the multiple sources of consumption risk

Table 8 provides parameter estimates of the exchange rate growth process for all currency baskets. For the one-period exposures, the parameters ξ_g , ξ_π , ξ_i , and ξ_σ are of central interest. These parameters are quarterly quantities of the multiple sources of risk inherent in different currency baskets. Currencies exhibit statistically significant, economically meaningful, and comparable cross-sectional differences in risk sensitivities to the short-run, inflation, and long-run consumption shocks. The high interest rate currencies have significantly positive exposures to the risks. The low yield currencies have significantly negative exposure to the long-run risk and insignificant exposures to the other two shocks.

Next, I perform similar analysis across multiple investment horizons. Figure 3 displays the shock-exposure elasticities. Because the shock-exposure elasticities scale up and down depending on the magnitude of the stochastic variance, I set the stochastic variance σ_t^2 to be equal to 1. Shock-exposure elasticities for the short-run consumption shock, inflation shock, and long-run risk shock can be interpreted as multi-period quantities of risk in a standard sense. These shocks do not feed into the stochastic variance process; therefore, the average metrics of the price and quantity of risk coincide with their marginal counterparts. In contrast, shock-exposure elasticity for the variance shock has an interpretation in terms of the multi-period marginal quantity of risk – the marginal change in the expected cash flow due to a marginal change in the volatility of the underlying shock.

The high yield currencies load on the long-run consumption risk significantly larger than the low yield currencies do across horizons from one to five quarters.¹⁴ The cross-sectional difference in currency risk exposures to the variance risk grows with the horizon considered. After a positive variance shock, the marginal decline of the exchange rate growth of the basket “High” is more pronounced than that of the basket “Low”. Statistically, however, both cross-sectional differences in shock elasticities and individual baskets’ shock-elasticities for the variance risk are insignificant for all investment horizons. Similarly, neither the short-run nor the inflation shock are

¹⁴I do not plot confidence bounds of the estimated shock elasticities to avoid clutter. Results are available upon request.

associated with the cross-section of multi-period currency exposures. The impact of these shocks is limited to the horizon of one quarter only.

Table 8
Exchange rate growth process. Parameter estimates

Parameter	Basket “Low”	Basket “Intermediate”	Basket “High”
$\log \delta$	-0.0032 (-0.0377, 0.0279)	-0.0402 (-0.0646, 0.0001)	-0.0374 (-0.0641, 0.0059)
μ_g	1.4530 (0.6477, 2.2533)	-0.4648 (-1.1641, 0.3822)	-1.8467 (-2.7288, -0.9052)
μ_π	-0.8352 (-1.5798, -0.1445)	-3.1394 (-3.7379, -2.4473)	-2.1599 (-2.8617, -1.4217)
μ_i	-0.6547 (-1.8080, 0.3965)	1.6119 (0.7922, 2.7353)	1.0933 (0.1687, 2.3526)
μ_σ	0.0062 (-0.0307, 0.0418)	-0.0064 (-0.0338, 0.0372)	-0.0179 (-0.0467, 0.0276)
ξ_g	-0.0040 (-0.0078, 0.0001)	0.0084 (0.0045, 0.0123)	0.0188 (0.0121, 0.0261)
ξ_π	-0.0037 (-0.0099, 0.0031)	0.0083 (0.0025, 0.0136)	0.0173 (0.0105, 0.0238)
ξ_i	-0.0124 (-0.0157, -0.0090)	-0.0103 (-0.0131, -0.0076)	0.0063 (0.0030, 0.0095)
ξ_σ	-0.0013 (-0.0115, 0.0085)	0.0008 (-0.0087, 0.0101)	0.0081 (-0.0023, 0.0183)

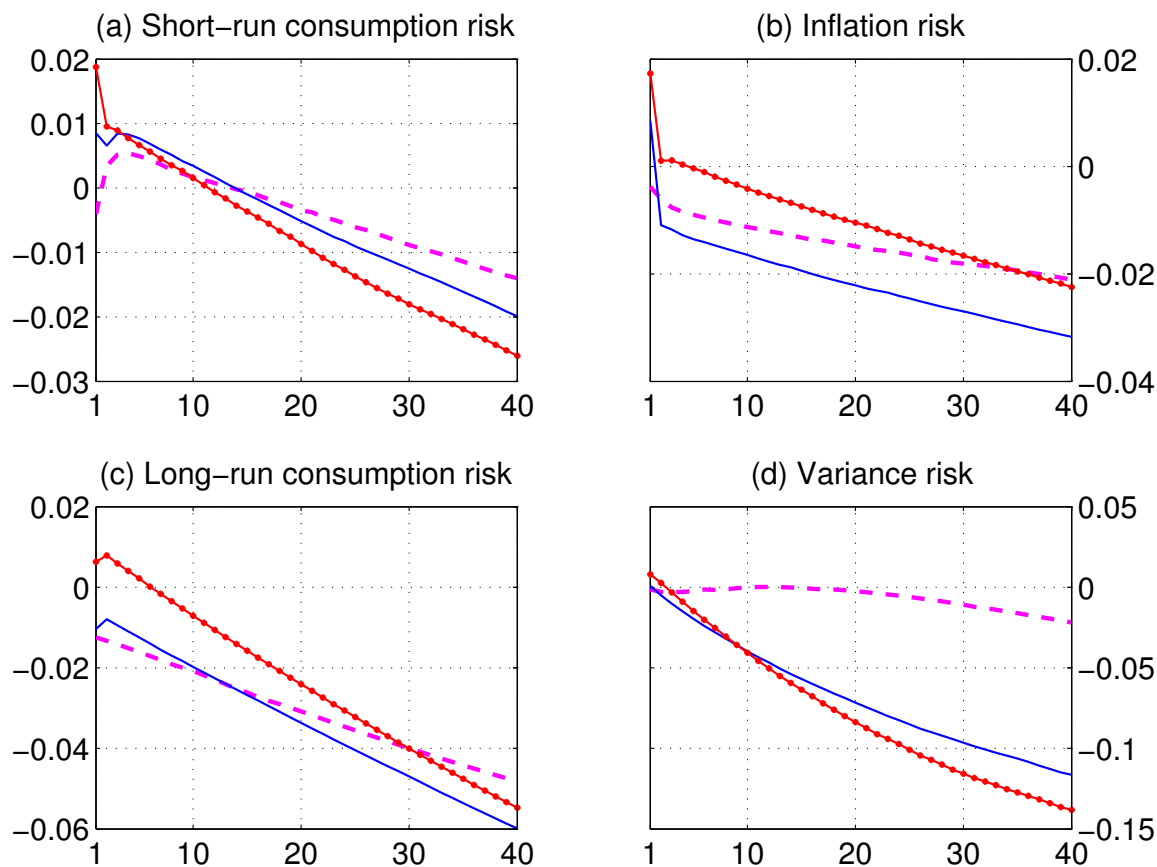
Notes. For each currency basket, I estimate the exchange rate growth process

$$\log \delta_{t,t+1} = \log \delta + \mu' Y_t + \xi' \sigma_t \varepsilon_{t+1} + \tilde{\xi}'_v \sigma_t \tilde{v}_{t+1},$$

where $\mu = (\mu_g, \mu_\pi, \mu_i, \mu_\sigma)'$ and $\xi = (\xi_g, \xi_\pi, \xi_i, \xi_\sigma)'$. Quarterly. The round brackets contain the 95% confidence intervals.

I come to similar conclusions with one important difference in the case of the identification “Fast Inflation”. The difference concerns the multi-period cross-section of currency risk exposures to the inflation shock. Specifically, if inflation responds to the short-run consumption shock contemporaneously, the exposures of the high interest rate currencies to the inflation risk are significantly higher than those of the low interest rate currencies for horizons from one to seven quarters. See Appendix A.3.

Figure 3
Shock-exposure elasticity. Identification “Fast consumption”



Shock-exposure elasticity for the short-run consumption risk (panel (a)), inflation risk (panel (b)), long-run consumption risk (panel (c)), variance risk (panel (d)). The magenta dashed line is for the basket “Low”, the blue solid line is for the basket “Intermediate”, the red marked line is for the basket “High”. Horizontal axes: from 1 quarter to 10 years. Quarterly.

Evidence from the vector autoregression with cross-equation restrictions is very similar to the empirical findings obtained from the reduced form model in the Section 2, which indicates the robustness of the earlier findings. To summarize: currencies load significantly differently on the long-run consumption risk and inflation shock (for some identification schemes) across *multiple* investment horizons. Further, the variance risk is not associated with the cross-section of currency risk exposures at any horizon. The only new result in this section is the significant role of the short-run

consumption risk in the cross-section of quarterly currency exposures. This evidence is consistent with Lustig and Verdelhan (2007).

C. Term-structure of prices of multiple sources of consumption risk

Previous section has documented that there are economically and statistically significant differences in currency exposures to (1) the long-run risk at multiple horizons from one quarter to five quarters, (2) inflation risk at multiple horizons from one quarter to seven quarters (at the horizon of one quarter only) under the “Fast Inflation” (“Fast Consumption”) identification, and (3) short-run consumption risk at the horizon matching the decision interval of the representative agent. These cross-sectional differences matter in currency markets, only if the sources of consumption risk are associated with significant and economically meaningful prices of risk at the corresponding investment horizons.

I start to characterize the prices of risks from a one-period perspective. Because the short-run consumption risk and inflation risk are identified differently across identification schemes, they have identification-dependent prices of risk. Table 9 describes the distribution of q_g , q_π , q_i , and q_σ (elements of the vector q) under the two identification schemes. The one-period prices of risk are the elements of the vector q but with negative sign. The only principal difference across different identifications is in the role of the inflation shock in the economy. Specifically, the inflation shock carries a statistically significant price of risk (equal to 0.13% for each extra percent of risk exposure on a quarterly basis) only under the “Fast Consumption” identification.¹⁵

The long-run risk shock is associated with the highest risk compensation of 0.28% for each extra percent of risk exposure on a quarterly basis. The estimated price of the variance risk is positive, similar to those of the other sources of risk, but has a much wider confidence interval. This is likely because the stochastic variance is estimated, and it is hence associated with statistical uncertainty, in contrast to other states of the model, which are observable.

The price of the variance risk is positive because the stochastic variance plays a dual role in this model. On the one hand, the representative agent, exhibiting preference

¹⁵Prices of risk are quoted per quarter. To obtain annualized values, they should be multiplied by 2.

Table 9
Parameters q

	Identification “Fast Consumption”		Identification “Fast Inflation”	
	Estimate	Confidence interval, 95%	Estimate	Confidence interval, 95%
q_g	-0.2356	(-0.3114, -0.1651)	-0.2667	(-0.3382, -0.1806)
q_π	-0.1293	(-0.2125, -0.0591)	-0.0363	(-0.1123, 0.0242)
q_i	-0.2764	(-0.3514, -0.2097)	-0.2764	(-0.3514, -0.2097)
q_σ	-0.1850	(-0.3095, -0.0075)	-0.1850	(-0.3095, -0.0075)

Notes. Vector q is the vector of loadings of the pricing kernel $\log m_{t,t+1}$ on the structural shocks $\sigma_t \varepsilon_{t+1}$:

$$\log m_{t,t+1} = \log m + \eta' Y_t + q' \sigma_t \varepsilon_{t+1}, \quad (2.6)$$

where $q = H'((\alpha - \rho)P + e_1(\alpha - 1))$, $q = (q_g, q_\pi, q_i, q_\sigma)'$. Preference parameters: $\alpha = -9$, $\rho = 1/3$, $\beta = 0.9924$. Quarterly.

for the early resolution of uncertainty, does not like a positive uncertainty shock. On the other hand, the representative agent does like a positive uncertainty shock, because stochastic variance positively predicts the future consumption growth. In this model (using assumed preferences parameters and estimated dynamics of consumption growth), the second effect dominates and this is why the variance shock is associated with a positive risk compensation.¹⁶

To put the magnitudes of Table 9 into perspective, I refer to a number of studies that report Sharpe ratios for different currency strategies. Table 3 in Ang and Chen (2010) reports a quarterly Sharpe ratio of 0.32 for a currency portfolio based on the level of the yield curve and 0.40 for a currency portfolio based on the slope of the yield curve; Table 1 in Burnside (2012) reports a quarterly Sharpe ratio of 0.45 for the equally weighted carry trade and 0.31 for the HML carry trade; Table 1 in Lustig, Roussanov, and Verdelhan (2013) documents a quarterly Sharpe ratio of 0.33 for the dollar carry trade.¹⁷

¹⁶The dual role of stochastic variance in representative agent models with recursive preferences is not new (Backus, Routledge, and Zin, 2010). While the negative price of risk is more standard in the literature, a recent study by Gill, Shaliastovich, and Yaron (2014) emphasizes the importance of the consumption volatility factor driven by the shock exhibiting a positive price of risk.

¹⁷Ang and Chen (2010) describe a currency strategy based on the level (slope) of the yield curve as one that entails going long in a currency with a high level factor (low term spread) and short in a currency with a low level factor (high term spread); Burnside (2012) defines the equally weighted

These numbers are not exact counterparts to the prices of risk documented in the paper. To facilitate the comparison, I compute the model-implied average Sharpe ratios associated with risk exposures of the dollar-neutral currency basket “High-Minus-Low” to individual sources of consumption risk.¹⁸ I find that quarterly Sharpe ratios of 0.28 and 0.23 for the long-run and short-run consumption risks, respectively, are comparable with the values quoted for currency strategies elsewhere in the literature.

I analyze the multi-period prices of risks by examining the shock-price elasticities shown in Figure 4. As in the case of exposure elasticity, I plot price elasticity by setting $\sigma_t^2 = 1$. The price elasticity for the short-run consumption shock, the inflation shock and the long-run risk shock corresponds to the negative of the impulse response function of the multi-period log pricing kernel. This works similarly to a linear model without stochastic variance because these shocks do not feed into the process for the stochastic variance. As a result, the marginal price of risk associated with any of these shocks is also the average price of risk. Such an interpretation is not appropriate for the price elasticity for the variance shock. The variance shock feeds into the variance process, and is therefore associated with important nonlinearities in the model. The shock-price elasticity is basket-dependent and interpreted as compensation for the marginal currency risk exposure.

The price for the long-run risk at all investment horizons is large. This result, coupled with the substantial cross-sectional spread in *multi-period* currency risk exposures to the shock is the main empirical finding of the paper. The long-run risk is the only source of consumption risk that matters in the cross-section of currency returns across *multiple* investment horizons.

The prices of the short-run consumption risk and inflation risk are also sizeable at all investment horizons. However, the pricing impact of the shocks on the cross-section

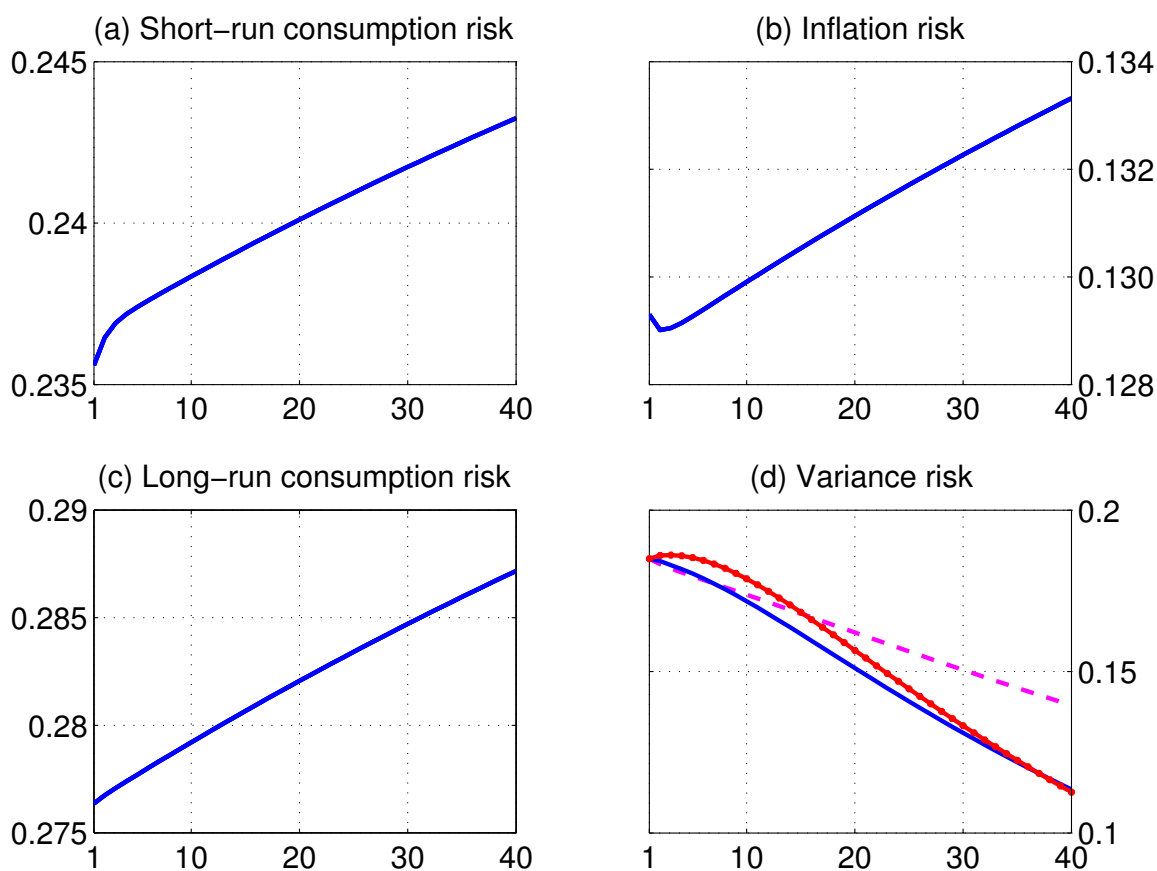
carry trade as the average of up to twenty individual currency carry trades against the US dollar; Lustig, Roussanov, and Verdelhan (2013) determine dollar carry trade as a strategy of going long in all available one-month currency forward contracts when the average forward discount of developed countries is positive and short otherwise.

¹⁸For example, the model-implied Sharpe ratio due to exposure of the currency basket to the short-run consumption shock is

$$SR_{gt} = \frac{e^{-\xi_g q_g \sigma_t^2} - 1}{e^{-\xi_g q_g \sigma_t^2} (e^{(\xi_g \sigma_t)^2} - 1)}.$$

of currency returns is limited to the horizon of one quarter. The underlying reason for this is the lack of the cross-sectional spread in multi-period currency risk exposures to the inflation and short-run consumption shocks.

Figure 4
Shock-price elasticity. Identification “Fast Consumption”



Shock-price elasticity for the short-run consumption risk (panel (a)), inflation risk (panel (b)), long-run consumption risk (panel (c)), variance risk (panel (d)). In panels (a), (b), and (c) shock-price elasticity is the same for the currency baskets. In panel (d) shock-price elasticity is basket-specific. The magenta dashed line is for the basket “Low”, the blue solid line is for the basket “Intermediate”, the red marked line is for the basket “High”. Horizontal axes: from 1 quarter to 10 years. Quarterly.

The marginal price of the variance risk is positive and significant for all currency baskets at all investment horizons. The price difference for the corner currency baskets

increases with the investment horizon. The effect is economically meaningful but not statistically significant. Because there is no spread, either in exposures or in prices of the variance risk, the variance shock does not contribute to the cross-sectional spread in currency returns. This fact does not suggest that stochastic variance is irrelevant, however. First, as emphasized earlier the presence of stochastic variance is key for modeling the dynamics of consumption growth and properly identifying multiple sources of consumption risk. Further, stochastic variance introduces an important channel of time-variation in currency risk premia. The cross-sectional spreads in currency risk premia and realized currency returns grow when the volatility of macro shocks is high, and decline when the volatility is low. This is observed in the data and implied by the model.

These findings are complementary to those of earlier studies examining the relationship between volatility dynamics and the cross-section of currency returns. Menkhoff, Sarno, Schmeling, and Schrimpf (2012) argue that carry returns are related to different exposures of high and low interest rate currencies to the innovations in global FX volatility. Lustig, Roussanov, and Verdelhan (2011) have a similar argument but focus on the innovations to equity market volatility. Due to the parsimony of my model, I do not distinguish time variation in consumption volatility from that in currency volatility and do not specify the equity volatility. I show that the macroeconomic volatility shock does not account for the cross-section of currency returns in the environment with multiple sources of consumption risk and leave it for future research to investigate differences in the information content of global currency, equity, and consumption volatility.

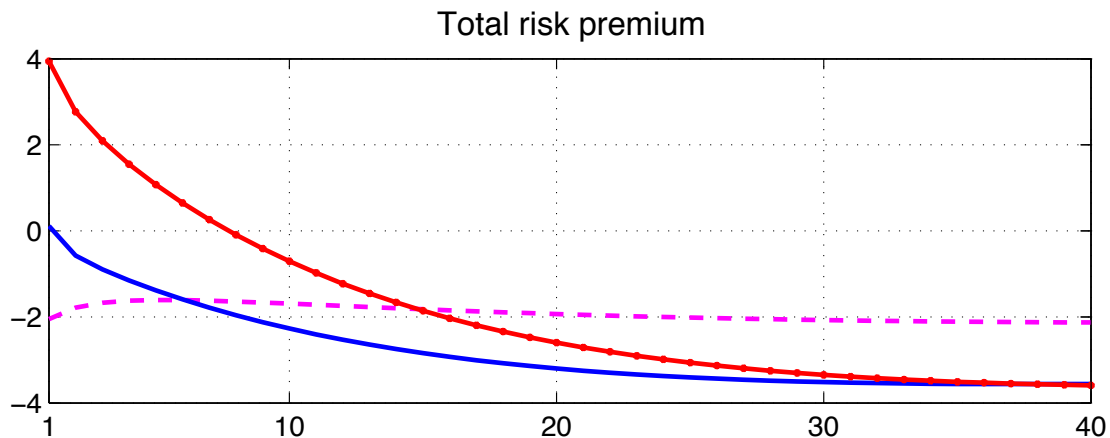
D. Consumption risk premia in the FX market: term-structure and decomposition

I have established how currencies from different interest rate environments are exposed to multiple sources of consumption risk across alternative investment horizons and how these risk exposures are priced. The next natural step is to aggregate this information and deduce implications for currency risk premium. Specifically, it would be instructive to characterize the term-structure of total risk premium and decompose it into risk premia due to currency exposures to individual sources of consumption risk.

Because stochastic variance introduces nonlinearities to the model, the decomposition result at horizons longer than one period poses a methodological challenge. In nonlinear models, multi-period risk premia are not a product of the shock-price elasticity and shock-exposure elasticity. Shock elasticities characterize marginal, but not average, risk exposures and prices of risk. The difference between marginal and average characteristics of risk is important at horizons at which the impact of the variance shock interacts with that of other sources of risk (i.e., horizons longer than one period). See Borovička, Hansen, Hendricks, and Scheinkman (2011) for further details.

Facing these technical challenges, I take a first step towards drawing out the implications of the characteristics of consumption risk in the FX market for the currency risk premium. Specifically, I characterize the entire term-structure of consumption risk premium in currency baskets and provide the decomposition result for the one-period risk premium.

Figure 5
Term-structure of risk premia. Identification “Fast Consumption”



Term-structure of risk premia associated with the currency baskets. The magenta dashed line is for the basket “Low”, the blue solid line is for the basket “Intermediate”, the red marked line is for the basket “High”. Horizontal axes: from 1 quarter to 10 years. Annualized, in percent.

Figure 5 shows the term-structure of total consumption risk premia associated with the three currency baskets. The spread in risk premia between corner baskets is significant across horizons from one to four quarters. At longer investment horizons, the economic difference in risk premia remains but loses its statistical significance.¹⁹ At an infinite investment horizon, the risk premia for all currency baskets converge to zero. This finding is consistent with Lustig, Stathopoulos, and Verdelhan (2013) who find that the term-structure of currency carry returns has a downward slope.

Table 10
One-period risk premium decomposition. Identification “Fast Consumption”

	Basket “Low”	Basket “Intermediate”	Basket “High”
Short-run risk	-0.3764 (-0.7727, 0.0087)	0.7911 (0.3964, 1.3123)	1.7732 (1.0010, 2.7495)
Inflation risk	-0.1935 (-0.6119, 0.1470)	0.4299 (0.1008, 0.9128)	0.8990 (0.3342, 1.6588)
Long-run risk	-1.3737 (-1.9254, -0.8906)	-1.1407 (-1.5847, -0.7166)	0.6938 (0.3201, 1.1317)
Variance risk	-0.1065 (-1.0421, 0.7338)	0.0349 (-0.7799, 0.7151)	0.5781 (-0.1632, 1.5966)
Total	-2.0501 (-3.1593, -0.9436)	0.1152 (-0.8973, 1.0450)	3.9441 (2.6174, 5.5569)

Decomposition of the quarterly currency risk premia into contributions of multiple sources of consumption risk. Stochastic variance σ_t^2 is set to 1. The round brackets contain the 95% confidence intervals. Annualized, in percent.

Table 10 shows the decomposition of the total one-period risk premium for each currency basket. For ease of interpretation, I report results in annualized terms in percent. The total one-period risk premium for Basket “Low” is -2% annualized, whereas that for Basket “High” is 4% annualized. I compute the spread in risk premia summing up contributions of the short-run, inflation, and long-run risks to which the low and high interest rate currencies have significantly different exposures:

¹⁹I do not plot confidence bounds for total currency risk premia associated with different baskets to avoid clutter. Results are available upon request.

$2.15\%+1.09\%+2.07\%=5.31\%$. The relative contributions of these risks to the spread are 40%, 21%, and 39%, respectively. As a result, in the cross-section of *quarterly* currency returns the short-run consumption risk is as important as the long-run consumption risk. The long-run consumption risk is special because of its *multi-period* importance in the cross-section of currency returns.

6 Conclusion

This paper provides novel evidence on the role of multiple sources of consumption risk in the foreign exchange market. The novelty is in terms of the economic questions and the methodological approach. On the methodological front, I identify multiple sources of consumption risk from the US macro and asset pricing data. To this end, I show how to use information content of asset prices to estimate the dynamics of consumption growth and identify systematic sources of risk. This methodology is generic and could be used in different macro-finance applications.

From an economic perspective, I carefully analyze the relative importance of various sources of consumption risk on the cross-section of currency returns across alternative investment horizons. Thus, the economic focus of the study is twofold, relating to the cross-section and term-structure of currency risk premia.

I reach the following three conclusions. First, the long-run consumption risk plays the most prominent role in currency markets. This is the only priced source of risk to which the low and high yield currencies exhibit significantly different exposures across multiple investment horizons from one to five quarters. Second, the short-run consumption risk matters for the cross-section of currency returns at the horizon of one quarter, where it explains 40% of the corresponding spread in excess returns. Finally, currency carry trade profitability as a compensation for currency exposures to consumption risk is a short-horizon phenomenon; it disappears at horizons longer than one year.

I leave at least two interesting avenues for the future research. The first broad and important question is the estimation of the preference parameters. The second direction relates to the further exploration of the role of the variance risk in macroeconomy

and asset markets by incorporating at the estimation stage the prices of assets that are informative about this risk.

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A Appendix

A.1 Data

Currency and interest rate data

I collect daily data on twelve spot exchange rates from Thomson Reuters provided by Datastream. The sample contains the price of the Australian dollar, the British pound, the Canadian dollar, the Danish krone, the Euro, the Deutsche mark, the Japanese yen, the New Zealand dollar, the Norwegian krone, the South African rand, the Swedish krona, and the Swiss frank in terms of USD. The sample runs from the beginning of 1986 until the end of 2011. According to the latest report of the Bank of International Settlements, these currencies are among the twenty one currencies with the highest daily turnover, as of April 2013.

I use fixed income data from Datastream, Bloomberg and the dataset of Wright (2011) providing detailed term-structure data for Australia, Canada, Germany, Japan, New Zealand, Norway, Sweden, Switzerland, and the UK until the first quarter of 2009. From the first quarter of 2009 until the last quarter of 2011, I compute the swap-implied interest rates for all these countries except for Germany. For Denmark, the Euro area, and South Africa, I compute the swap implied interest rates for the entire time interval. The term-structure data contain yields of forty maturities, from one quarter to ten years. Table A.1 describes data availability and sources of data for every country.

I choose currencies of big economies that are used elsewhere in the literature.²⁰ I use term-structures of foreign interest rates for sorting purposes. Because of the limited availability of term-structure data, my sample contains a smaller number of currencies. I work with quarterly currency quotes sampled at the end of the corresponding quarter to match the frequency of the consumption data.

²⁰See Burnside, Eichenbaum, Kleshchelski, and Rebelo (2011), Lustig, Roussanov, and Verdelhan (2011), Lustig and Verdelhan (2007), Menkhoff, Sarno, Schmeling, and Schrimpf (2012), Rafferty (2011) among others.

Table A.1
Data description

Country	Data availability	FX data/Datastream Mnemonics	Source of term-structure data/Mnemonics
Australia	1987:I – 2011:IV	AUSTDOL and USDOLLR	Wright [1987:I – 2009:I] B-g [2009:II – 2011:IV]: ADSW1 – ADSW10
Canada	1986:I – 2011:IV	CNDOLLR and USDOLLR	Wright [1986:I – 2009:I] B-g [2009:II – 2011:IV]: CDSW1 – CDSW10
Denmark	1988:III – 2011:IV	DANISHK and USDOLLR	B-g [1988:III – 2011:IV]: DKSW1 – DKSW10
Euro Area	2004:III – 2011:IV	EURSTER and USDOLLR	B-g [2004:III – 2011:IV]: EUSW1V3 – EUSW10V3
Germany	1986:I – 1998:IV	DMARKER and USDOLLR	B-g [1986:I – 1998:IV] Wright [1986:I – 2009:I]
Japan	1986:I – 2011:IV	JAPAYEN and USDOLLR	Wright [1986:I – 2009:I] B-g [2009:II – 2011:IV]: JYSW1 – JYSW10
New Zealand	1990:I – 2011:IV	NZDOLLR and USDOLLR	Wright [1990:I – 2009:I] B-g [2009:II – 2011:IV]: NDSW1 – NDSW10
Norway	1998:I – 2011:IV	NORKRON and USDOLLR	Wright [1998:I – 2009:I] B-g [2009:II – 2011:IV]: NKSW1 – NKSW10
South Africa	1997:II – 2011:IV	COMRAND and USDOLLR	B-g [1997:II – 2011:IV]: SASW1 – SASW10
Sweden	1992:IV – 2011:IV	SWEKRON and USDOLLR	Wright [1992:IV – 2009:I] B-g [2009:II – 2011:IV]: SKSW1 – SKSW10
Switzerland	1988:I – 2011:IV	SWISSFR and USDOLLR	Wright [1986:I – 2009:I] B-g [2009:II – 2011:IV]: SFSW1 – SFSW10
UK	1986:I – 2011:IV	USDOLLR	Wright [1986:I – 2009:I] B-g [2009:II – 2011:IV]: BPSW1 – BPSW10

Notes: FX and interest rate data availability and data sources. Abbreviation B-g stands for Bloomberg.

Macro data

I use quarterly data on US consumption growth, inflation, and three-month nominal yield from the second quarter of 1947 to the fourth quarter of 2011. In total, there are 259 observations. I collect consumption and price data from the NIPA tables of the Bureau of Economic Analysis. I use Table 2.1 (Personal income and its disposition), Table 2.3.4 (Personal indexes for personal consumption expenditures by major type of product) and Table 2.3.5 (Personal consumption expenditures by major type of product). I measure real consumption as per capita expenditure on non-durable goods and services. Non-durables and services is the sum of entries of row 8 from Table 2.3.5 divided by entries of row 8 from Table 2.3.4 and components of row 13 from Table 2.3.5 divided by components of row 13 from Table 2.3.4. I construct price index associated with personal consumption expenditures. Row 40 of the Table 2.1 provides population data. The nominal yield comes from CRSP. Panels (a)-(c) of Figure 2 display the dynamics of these variables.

A.2 Model's solution and pricing restrictions

In this Appendix, I solve the model. I briefly repeat the main building blocks for ease of explanation.

The representative agent has recursive preferences

$$U_t = [(1 - \beta)c_t^\rho + \beta\mu_t(U_{t+1})^\rho]^{1/\rho} \quad (\text{A.13})$$

with the certainty equivalent function

$$\mu_t(U_{t+1}) = [E_t(U_{t+1}^\alpha)]^{1/\alpha}, \quad (\text{A.14})$$

and preference parameters α (risk aversion is $1 - \alpha$), β (subjective discount factor), and ρ ($1/(1 - \rho)$ is the elasticity of intertemporal substitution).

The consumption growth process is described by a vector autoregressive system

$$Y_{t+1} = F + GY_t + H\sigma_t\varepsilon_{t+1}, \quad (\text{A.15})$$

where $Y_{t+1} = (\log g_{t+1}, \log \pi_{t+1}, i_{t+1,t+2}, \sigma_{t+1}^2)'$.

To solve the model, I follow closely the solution method of Backus, Chernov, and Zin (2014). Because the utility U_t is determined by a constant elasticity of substitution recursion (A.13), and the certainty equivalent function is also homogenous of degree one, scale (A.13) by consumption c_t :

$$u_t = [(1 - \beta) + \beta \mu_t (u_{t+1} g_{t+1})^\rho]^{1/\rho}, \quad (\text{A.16})$$

where $u_t = U_t/c_t$, and $g_{t+1} = c_{t+1}/c_t$.

The log pricing kernel under the recursive utility is

$$\log m_{t,t+1} = \log \beta + (\rho - 1) \log g_{t+1} + (\alpha - \rho) (\log (u_{t+1} g_{t+1}) - \log \mu_t (u_{t+1} g_{t+1})) \quad (\text{A.17})$$

Appendix A.5 of the NBER version of Backus, Chernov, and Zin (2014) provides the corresponding derivation.

Use a log-linear approximation of equation (A.16),

$$\log u_t \approx b_0 + b_1 \log \mu_t (g_{t+1} u_{t+1}), \quad (\text{A.18})$$

to obtain a closed-form solution to the value function $\log u_t$ and to compute the pricing kernel $\log m_{t,t+1}$. Parameters b_0 and b_1 in equation (A.18) are such that

$$b_1 = \beta e^{\rho \log \mu} / [(1 - \beta) + \beta e^{\rho \log \mu}], \quad (\text{A.19})$$

$$b_0 = \rho^{-1} \log [(1 - \beta) + \beta e^{\rho \log \mu}] - b_1 \log \mu. \quad (\text{A.20})$$

The equation (A.18) is exact if the elasticity of intertemporal substitution is equal to one. In such a case $b_0 = 0$ and $b_1 = \beta$. See Section III in Hansen, Heaton, and Li (2008) and Appendix A.7 in Backus, Chernov, and Zin (2014) for details about the log-linear approximation and its accuracy.

Guess that the solution to the equation (A.18) is an affine function of the four model's

states

$$\log u_t = \log u + P'Y_t, \quad (\text{A.21})$$

where P is a vector of loadings $P = (p_g, p_\pi, p_i, p_\sigma)'$.

Verify this guess. Compute the log of the certainty equivalent function

$$\log \mu_t(u_{t+1}g_{t+1}) = \underbrace{[\log u + e_1'F + P'F]}_{\log \mu} + [P'G + e_1'G]Y_t + \alpha[P + e_1]'\Sigma[P + e_1]\sigma_t^2/2, \quad (\text{A.22})$$

where $\Sigma = HH'$ and e_1 is a coordinate vector with the first element equal to 1. Substitute (A.21) and (A.22) to the equation (A.18) and collect and match the corresponding terms. The equation (A.18) has a constant term and four variables, hence obtain the system of five equations

$$\log u = b_0 + b_1 \log u + b_1 e_1'F + b_1 P'F, \quad (\text{A.23})$$

$$p_g = b_1(P + e_1)'Ge_1 \quad (\text{A.24})$$

$$p_\pi = b_1(P + e_1)'Ge_2, \quad (\text{A.25})$$

$$p_i = b_1(P + e_1)'Ge_3, \quad (\text{A.26})$$

$$p_\sigma = b_1(P + e_1)'Ge_4 + \alpha b_1(P + e_1)'\Sigma(P + e_1)/2, \quad (\text{A.27})$$

where e_i are the corresponding coordinate vectors.

Equations for p_g , p_π , and p_i are linear and therefore they result in unique solutions

$$p_g = A_g/B_g,$$

$$p_\pi = A_\pi/B_\pi,$$

$$p_i = A_i/B_i,$$

where

$$\begin{aligned} A_g &= -(G_{11}b_1 - G_{11}G_{22}b_1^2 + G_{12}G_{21}b_1^2 - G_{11}G_{33}b_1^2 + G_{13}G_{31}b_1^2 + G_{11}G_{22}G_{33}b_1^3 \\ &\quad - G_{11}G_{23}G_{32}b_1^3 - G_{12}G_{21}G_{33}b_1^3 + G_{12}G_{23}G_{31}b_1^3 + G_{13}G_{21}G_{32}b_1^3 - G_{13}G_{22}G_{31}b_1^3), \end{aligned}$$

$$\begin{aligned}
A_\pi &= -(G_{12}b_1 + G_{13}G_{32}b_1^2 - G_{12}G_{33}b_1^2), \\
A_i &= -(G_{13}b_1 + G_{12}G_{23}b_1^2 - G_{13}G_{22}b_1^2), \\
B_g &= B_\pi = B_i \\
&= G_{11}b_1 + G_{22}b_1 + G_{33}b_1 - G_{11}G_{22}b_1^2 + G_{12}G_{21}b_1^2 - G_{11}G_{33}b_1^2 + G_{13}G_{31}b_1^2 \\
&\quad - G_{22}G_{33}b_1^2 + G_{23}G_{32}b_1^2 + G_{11}G_{22}G_{33}b_1^3 - G_{11}G_{23}G_{32}b_1^3 - G_{12}G_{21}G_{33}b_1^3 \\
&\quad + G_{12}G_{23}G_{31}b_1^3 + G_{13}G_{21}G_{32}b_1^3 - G_{13}G_{22}G_{31}b_1^3 - 1.
\end{aligned}$$

The equation for p_σ is quadratic

$$A_\sigma p_\sigma^2 + B_\sigma p_\sigma + C_\sigma = 0,$$

where

$$\begin{aligned}
A_\sigma &= \alpha b_1 \Sigma_{44}/2, \\
B_\sigma &= \alpha b_1 (\Sigma_{34}p_i + \Sigma_{24}p_\pi + \Sigma_{14}(p_g + 1)) + b_1 G_{44} - 1, \\
C_\sigma &= \alpha b_1 ((p_g + 1)(\Sigma_{13}p_i + \Sigma_{12}p_\pi + \Sigma_{11}(p_g + 1)) + p_i(\Sigma_{33}p_i + \Sigma_{23}p_\pi + \Sigma_{13}(p_g + 1)) \\
&\quad + p_\pi(\Sigma_{23}p_i + \Sigma_{22}p_\pi + \Sigma_{12}(p_g + 1)))/2 + (b_1 p_g G_{14} + b_1 p_\pi G_{24} + b_1 p_i G_{34} + b_1 G_{14}).
\end{aligned}$$

This equation has two real roots if its discriminant $\text{Discr} = (B_\sigma^2 - 4A_\sigma C_\sigma)$ is positive. Choose the root that satisfies the requirement of stochastic stability (Hansen, 2012)

$$p_\sigma = \frac{-B_\sigma + \text{sign}(B_\sigma)\text{Discr}^{1/2}}{2A_\sigma}.$$

Finally, $\log u$ follows as

$$\log u = [b_0 + b_1 e_1' F + b_1 P' F]/[1 - b_1].$$

Plug the solution $\log u_t$ into (A.17) and obtain the final expression for the pricing kernel

$$\begin{aligned}
\log m_{t,t+1} &= [\log \beta + (\rho - 1)e_1' F] + (\rho - 1)e_1' G Y_t - \alpha(\alpha - \rho)(P + e_1)' \Sigma (P + e_1) \sigma_t^2 / 2 \\
&\quad + [(\alpha - \rho)P + e_1(\alpha - 1)]' H \sigma_t \varepsilon_{t+1} \tag{A.28}
\end{aligned}$$

or

$$\log m_{t,t+1} = \log m + \eta'Y_t + q'\sigma_t\varepsilon_{t+1}, \quad (\text{A.29})$$

where

$$\begin{aligned} \log m &= \log \beta + (\rho - 1)e_1'F, \\ \eta &= (\rho - 1)G'e_1 - \alpha(\alpha - \rho)e_4(P + e_1)'\Sigma(P + e_1)/2, \\ q &= H'[(\alpha - \rho)P + e_1(\alpha - 1)]. \end{aligned} \quad (\text{A.30})$$

Derive a one-period real risk-free rate

$$\begin{aligned} \log r_{t,t+1}^f &= -E_t(\log m_{t,t+1}) - \text{Var}_t(\log m_{t,t+1})/2 \\ &= -\log \beta - (\rho - 1)e_1'F - (\rho - 1)e_1'GY_t + \alpha(\alpha - \rho)(P + e_1)'\Sigma(P + e_1)\sigma_t^2/2 \\ &\quad - [(\alpha - \rho)P + e_1(\alpha - 1)]'\Sigma[(\alpha - \rho)P + e_1(\alpha - 1)]\sigma_t^2/2. \end{aligned} \quad (\text{A.31})$$

Finally, the nominal one-period yield is

$$\begin{aligned} i_{t,t+1} &= \log r_{t,t+1}^f + E_t(\log \pi_{t+1}) - \text{Var}_t(\log \pi_{t+1})/2 + \text{cov}_t(\log m_{t,t+1}, \log \pi_{t+1}) \\ &= -\log \beta - (\rho - 1)e_1'F + e_2'F - (\rho - 1)e_1'GY_t + e_2'GY_t \\ &\quad - [(\alpha - \rho)P + e_1(\alpha - 1)]'\Sigma[(\alpha - \rho)P + e_1(\alpha - 1)]\sigma_t^2/2 - e_2'\Sigma e_2\sigma_t^2/2 \\ &\quad + e_2'\Sigma[(\alpha - \rho)P + e_1(\alpha - 1)]\sigma_t^2 + \alpha(\alpha - \rho)(P + e_1)'\Sigma(P + e_1)\sigma_t^2/2. \end{aligned} \quad (\text{A.32})$$

Re-write the expression (A.32) in a compact form and note that it is an identity:

$$i_{t,t+1} \equiv A \log g_t + B \log \pi_t + C i_{t,t+1} + D \sigma_t^2 + E,$$

where

$$\begin{aligned} A &= -\log \beta - (\rho - 1)e_1'F + e_2'F, \\ B &= -(\rho - 1)e_1'Ge_1 + e_2'Ge_1, \\ C &= -(\rho - 1)e_1'Ge_2 + e_2'Ge_2, \\ D &= -(\rho - 1)e_1'Ge_3 + e_2'Ge_3, \end{aligned}$$

$$\begin{aligned}
E &= -[(\alpha - \rho)P + e_1(\alpha - 1)]'\Sigma[(\alpha - \rho)P + e_1(\alpha - 1)]/2 - e_2'\Sigma e_2/2 + e_2'Ge_4 \\
&+ e_2'\Sigma[(\alpha - \rho)P + e_1(\alpha - 1)] + \alpha(\alpha - \rho)(P + e_1)'\Sigma(P + e_1)/2 \\
&- (\rho - 1)e_1'Ge_4.
\end{aligned}$$

To guarantee consistent pricing of the nominal yield, the identity (A.32) must hold, i.e., the following five restrictions must be satisfied:

$$A = 0, \quad B = 0, \quad C = 1, \quad D = 0, \quad E = 0.$$

Four restrictions $A = B = E = 0, C = 1$ are linear and can be written as

$$\frac{G_{21}}{G_{11}} = \frac{G_{22}}{G_{12}} = \frac{G_{23} - 1}{G_{13}} = \frac{F_2 - \log \beta}{F_1} = \rho - 1.$$

The other restriction is nonlinear; it involves the endogenous parameters $p_g, p_\pi, p_i,$ and p_σ

$$\begin{aligned}
&-[(\alpha - \rho)P + e_1(\alpha - 1)]'\Sigma[(\alpha - \rho)P + e_1(\alpha - 1)]/2 - e_2'\Sigma e_2/2 + e_2'Ge_4 \\
&+ e_2'\Sigma[(\alpha - \rho)P + e_1(\alpha - 1)] + \alpha(\alpha - \rho)(P + e_1)'\Sigma(P + e_1)/2 \\
&- (\rho - 1)e_1'Ge_4 = 0.
\end{aligned} \tag{A.33}$$

A.3 Identification “Fast Inflation”

For completeness, this Appendix describes empirical results obtained in case of “Fast Inflation” identification.

Table A.2 contains the estimates for the parameters of the matrix H . Table A.3 provides parameter estimates of the exchange rate growth process for all currency baskets. Similarly to the case of “Fast Consumption”, currencies exhibit statistically significant, economically sizeable, and comparable cross-sectional differences in risk sensitivities to the short-run, inflation, and long-run risks. The low-yield currencies exhibit significantly lower exposures to the risks than the high interest rate currencies do.

Table A.2
Identification “Fast Inflation”

Parameter	Estimate	Confidence interval, 95%
H_{11}	0.0057	(0.0048, 0.0069)
H_{14}	-0.0008	(-0.0026, 0.0004)
H_{21}	0.0021	(0.0010, 0.0036)
H_{22}	0.0056	(0.0047, 0.0069)
H_{24}	0.0018	(0.0007, 0.0028)
H_{31}	0.0006	(0.0003, 0.0009)
H_{32}	0.0002	(-3.4e-5, 0.0006)
H_{33}	0.0015	(0.0012, 0.0017)
H_{34}	0.0003	(-1.1e-5, 0.0006)
H_{44}	0.1828	(0.1399, 0.2276)

Notes. I identify structural shocks ε_{t+1} from the reduced form innovations w_{t+1} : $\Sigma^{1/2}w_{t+1} = H\varepsilon_{t+1}$. I consider “Fast Inflation” identification determined by the following zero restrictions: $H_{12} = H_{13} = H_{23} = H_{41} = H_{42} = H_{43} = 0$. Quarterly.

The multi-period analysis shows that the exposures of the high interest rate currencies to the long-run risk (inflation risk) are significantly higher than those of the low interest rate currencies for horizons from one quarter to five (seven) quarters (Figure A.1). However, the price of the inflation risk is null for all investment horizons (Figure A.2), whereas that of the long-run consumption risk is positive and sizeable. As a result, the long-run consumption risk is the only source of consumption risk that contributes to the cross-section of currency returns across *multiple* investment horizons (from one to five quarters).

The short-run consumption risk contributes to the spread in currency returns at the horizon of one quarter only. Table A.4 reports the result of the one-period risk premia decomposition for all currency baskets. The total risk premium for basket “Low” is -2% annualized, whereas that for the basket “High” is 3.5% annualized. The short-run consumption risk contributes 54%, whereas the long-run consumption risk contributes 46% to the quarterly spread in returns between the corner baskets. Figure A.3 depicts the term-structure of risk premia associated with different currency baskets. Similar to the case of “Fast Consumption”, the cross-sectional spread in risk premia is significant across horizons from one quarter to one year.

Table A.3
Exchange rate growth process. Identification “Fast Inflation”

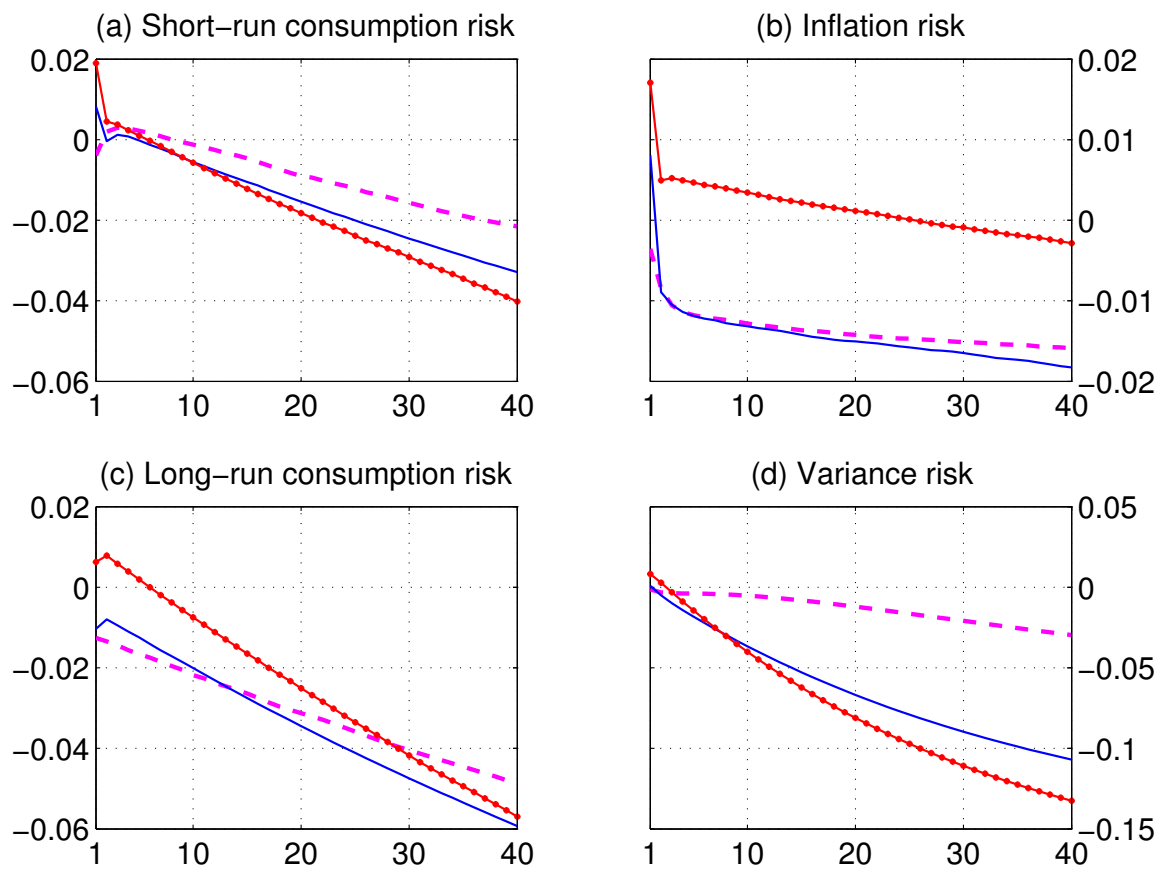
Parameter	Basket “Low”	Basket “Intermediate”	Basket “High”
$\log \delta$	-0.0037 (-0.0361, 0.0271)	-0.0392 (-0.0634, 0.0001)	-0.0365 (-0.0665, 0.0075)
μ_g	1.4338 (0.6096, 2.2304)	-0.4678 (-1.2026, 0.4320)	-1.8757 (-2.7720, -0.9857)
μ_π	-0.8610 (-1.5648, -0.1731)	-3.1396 (-3.7909, -2.4469)	-2.1909 (-2.8337, -1.4496)
μ_i	-0.6382 (-1.8235, 0.4043)	1.6173 (0.7963, 2.7514)	1.0810 (0.0914, 2.3769)
μ_σ	0.0061 (-0.0309, 0.0391)	-0.0065 (-0.0321, 0.0360)	-0.0191 (-0.0496, 0.0267)
ξ_g	-0.0039 (-0.0076, 0.0001)	0.0082 (0.0043, 0.0123)	0.0189 (0.0119, 0.0255)
ξ_π	-0.0035 (-0.0105, 0.0026)	0.0082 (0.0026, 0.0145)	0.0172 (0.0109, 0.0234)
ξ_i	-0.0126 (-0.0158, -0.0091)	-0.0103 (-0.0132, -0.0073)	0.0063 (0.0029, 0.0093)
ξ_σ	-0.0014 (-0.0116, 0.0092)	0.0009 (-0.0084, 0.0095)	0.0082 (-0.0023, 0.0188)

Notes. For each currency basket, I estimate the exchange rate growth process

$$\log \delta_{t,t+1} = \log \delta + \mu' Y_t + \xi' \sigma_t \varepsilon_{t+1} + \tilde{\xi}_v \sigma_t \tilde{v}_{t+1},$$

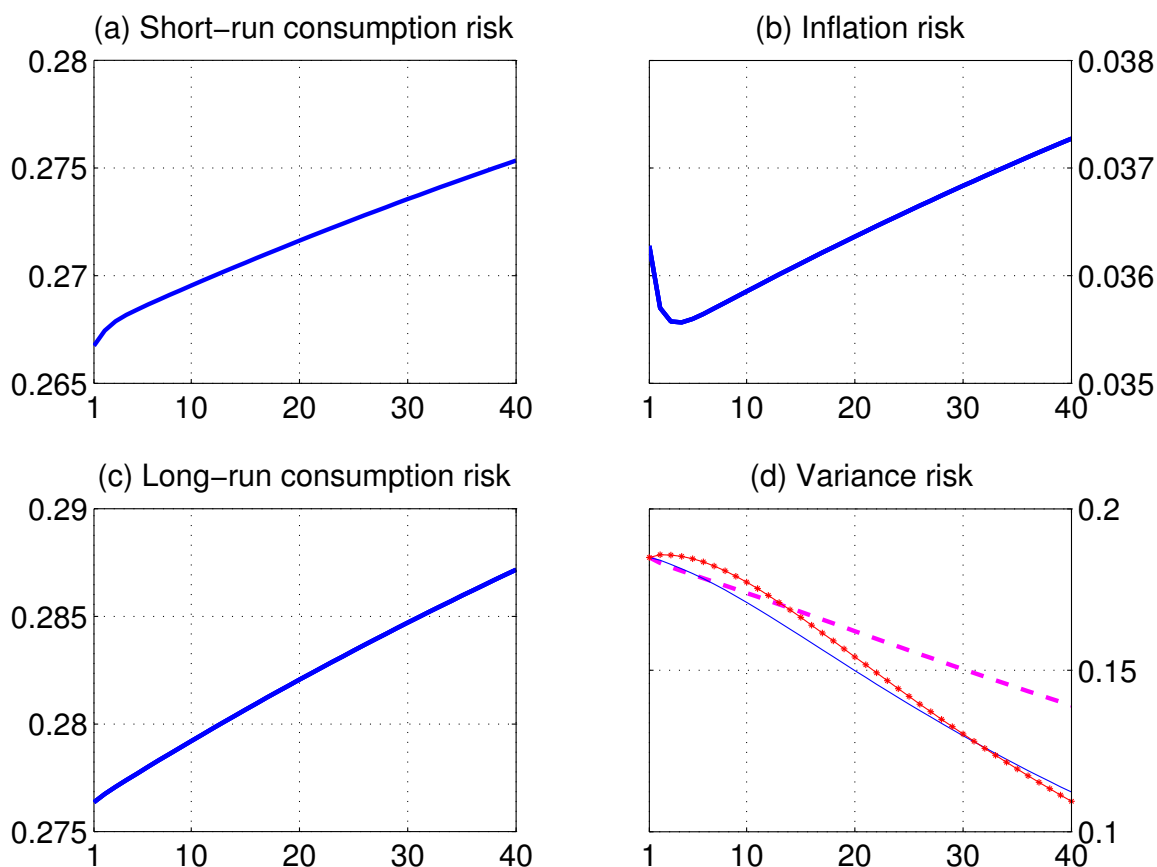
where $\mu = (\mu_g, \mu_\pi, \mu_i, \mu_\sigma)'$ and $\xi = (\xi_g, \xi_\pi, \xi_i, \xi_\sigma)'$. Quarterly. The round brackets contain the 95% confidence intervals.

Figure A.1
Shock-exposure elasticity. Identification “Fast Inflation”



Shock-exposure elasticity for the short-run consumption risk (panel (a)), inflation risk (panel (b)), long-run consumption risk (panel (c)), variance risk (panel (d)). The magenta dashed line is for the basket “Low”, the blue solid line is for the basket “Intermediate”, the red marked line is for the basket “High”. Horizontal axes: from 1 quarter to 10 years. Quarterly.

Figure A.2
Shock-price elasticity. Identification “Fast Inflation”



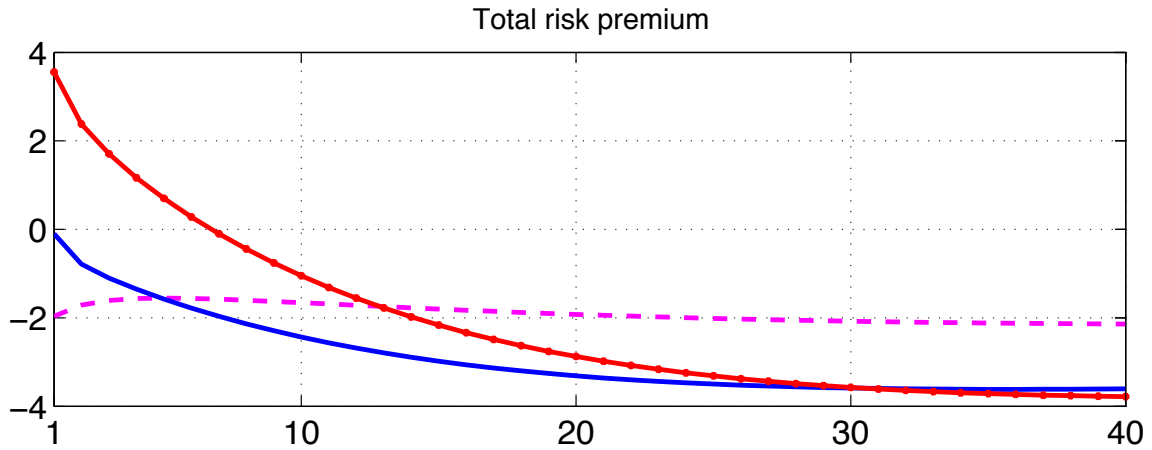
Shock-price elasticity for the short-run consumption risk (panel (a)), inflation risk (panel (b)), long-run consumption risk (panel (c)), variance risk (panel (d)). In panels (a), (b), and (c) shock-price elasticity is the same for the currency baskets. In panel (d) shock-price elasticity is basket-specific. The magenta dashed line is for the basket “Low”, the blue solid line is for the basket “Intermediate”, the red marked line is for the basket “High”. Horizontal axes: from 1 quarter to 10 years. Quarterly.

Table A.4
One-period risk premium decomposition. Identification “Fast Inflation”

	Basket “Low”	Basket “Intermediate”	Basket “High”
Short-run risk	-0.4215 (-0.8764, 0.0116)	0.8708 (0.4442, 1.4163)	2.0219 (1.1442, 3.0119)
Inflation risk	-0.0547 (-0.2635, 0.0550)	0.1189 (-0.0810, 0.4052)	0.2507 (-0.1588, 0.8059)
Long-run risk	-1.3895 (-1.9423, -0.9108)	-0.1387 (-1.5955, -0.7293)	0.6917 (0.3131, 1.1001)
Variance risk	-0.1008 (-1.0197, 0.7537)	0.0509 (-0.6750, 0.7440)	0.5912 (-0.1752, 1.5974)
Total	-1.9665 (-3.1151, -0.7964)	-0.0981 (-1.0858, 0.8092)	3.5555 (2.1682, 5.0084)

Notes: Decomposition of the quarterly currency risk premia into contributions of multiple sources of consumption risk. Stochastic variance σ_t^2 is set to 1. The round brackets contain the 95% confidence intervals. Annualized, in percent.

Figure A.3
Term-structure of risk premia. Identification “Fast Inflation”



Term-structure of risk premia associated with the currency baskets. The magenta dashed line is for the basket “Low”, the blue solid line is for the basket “Intermediate”, the red marked line is for the basket “High”. Horizontal axes: from 1 quarter to 10 years. Annualized, in percent.