Term-Structure of Consumption Risk Premia in the Cross-Section of Currency Returns[*]

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Abstract

I quantify the risk-return relationship in the foreign exchange market in the cross-section and across investment horizons by focusing on the role of multiple sources of US consumption risk. To this end, I estimate a flexible structural model of the joint dynamics of aggregate consumption, inflation, nominal interest rate, and stochastic variance with cross-equation restrictions implied by recursive preferences. I identify the following four structural shocks: inflation, short-run, long-run and variance consumption risks. To measure their relative importance, I compute marginal quantities and prices of risk (marginal Sharpe ratios) in the cross-section of currency baskets for alternative investment horizons. I find that the long-run consumption risk plays a prominent role: it carries an average quarterly Sharpe ratio of 0.28 and contributes to the spread in excess returns between baskets of high and low interest rate currencies across investment horizons from one to five quarters. The short-run consumption risk has an effect on currencies at the horizon of one quarter only, where it explains at least 40% of the corresponding spread in excess returns. The carry trade profitability disappears at horizons longer than four quarters.

Keywords: consumption risks, term-structure of risk, term-structure of risk compensation, cross-section of currencies, shock elasticity.

1 Introduction

This paper quantifies the role of macro-economic risk in the foreign exchange market. Multiple sources of consumption risk, such as shocks to expected consumption growth, stochastic variance of consumption growth, and consumption growth itself, reflect different aspects of macro-economic risk. The central question is how much these shocks contribute to generating observed cross-section of short-horizon currency returns and what happens with the profitability of currency carry trades at longer investment horizons. To answer this question, this study identifies multiple sources of consumption risk and measures their relative importance for currency dynamics and currency risk premia both in the cross-section and at different investment horizons.

I document that the risk of low frequency movements in the expected consumption growth plays the most prominent role in the FX market. This is the only source of consumption risk that affects differently low and high yield currencies across multiple investment horizons. Moreover, this risk is associated with the highest risk compensation: its quarterly average log Sharpe ratio is about 0.28. Taken together, at least 39% of the quarterly cross-sectional spread in currency excess returns can be related to the risk of low frequency movements in the expected consumption growth. Next, I find that the profitability of currency carry trades is a short-horizon phenomenon. At horizons longer than one year, cross-section of currency risk premia due to currency exposures to the macro-economic risk degenerates.

Similarly to influential study by Lustig and Verdelhan (2007), I perform my analysis from the perspective of a US representative agent with recursive preferences over consumption. I identify sources of consumption risk from the US macro-economic and asset pricing data and subsequently model the dynamics of the pricing kernel of the US representative agent. The key issue here is that the identified sources of consumption risk are not necessarily US specific but reflect both local and global components, whereas the pricing kernel denominated in the US dollars is a sufficient modelling ingredient to quantify how systematic sources of macro-economic risk are priced in the cross-section of currency returns at any investment horizon.

The main interest of the study lies in describing the empirical properties of consumption risk. Therefore, instead of taking a stand on a specific process for consumption growth, I estimate a flexible vector autoregression (VAR) with stochastic variance that captures the spirit of the long-run risk models. Specifically, two insights from the long-run risk

literature are important – time-variation in expected consumption growth and time-variation in the variance of consumption growth. As it has been widely shown in the literature, shocks in expected consumption growth (e.g., Bansal and Yaron, 2004) and stochastic variance of consumption growth (e.g., Bansal, Kiku, Shaliastovich, and Yaron, 2013; Campbell, Giglio, Polk, and Turley, 2013) have first order implications for the macro dynamics and properties of asset prices.

To capture time-variation in expected consumption growth, I model consumption growth jointly with inflation and nominal yield on a quarterly risk-free bond. The use of additional information about first and second moments of consumption growth contained in asset prices is an important novel feature of my approach. An asset price is an appealing source of information about the dynamics of consumption growth. In equilibrium, it reflects the unobservable components of the consumption growth process that are difficult to estimate on the basis of consumption data alone. Thus, the use of asset pricing data adds power to identify the process of consumption growth better and estimate the expected consumption growth more precisely.

I choose nominal bond as an asset because the theoretical literature (e.g., Bansal and Shaliastovich, 2013) has emphasized that nominal yields reflect risks relevant for exchange rates. In addition, the use of the yield, as opposed to another asset price, is especially convenient because it does not require modeling of any cash flow dynamics. I incorporate inflation for the following two reasons. Firstly, inflation has forecasting ability for the future consumption growth (Piazzesi and Schneider, 2006). Secondly, I need the inflation dynamics to convert the model implied real risk-free rate to the observed nominal interest rate.

The pricing kernel derived by applying recursive preferences to the consumption growth process naturally depends on the nominal yield because it is one of the states of the model. On the other hand, the pricing kernel values all assets in the economy, including the risk-free bond. Therefore, the nominal yield is an equilibrium outcome of the model. The twofold role of the nominal yield implies a set of pricing restrictions on the parameters of the vector autoregression and preference parameters. Thus, the model of consumption growth has the form of a vector autoregression with stochastic variance and structural restrictions derived under recursive preferences.

I estimate the model by using quarterly data for US consumption growth, inflation, and a three-month nominal yield from the second quarter of 1947 until the fourth quarter of 2011. For estimation, I employ the Bayesian Markov Chain Monte Carlo (MCMC) methods. The key advantage of this methodology is that it allows imposing the required

pricing restrictions directly; in addition, this approach delivers the estimated time series of stochastic variance.

I identify structural shocks from the estimated reduced-form innovations choosing to work with globally identified systems. There is a choice of only two systems because of various restrictions based on economic intuition and regularity conditions (Rubio-Ramirez, Waggoner, and Zha, 2010). The model features four structural shocks to consumption growth that I interpret based on their empirical properties: (1) the short-run consumption risk, (2) the inflation risk, (3) the long-run consumption risk, and (4) the variance consumption risk. The only difference between the identification schemes is in the underlying identifying assumptions about the short-run consumption and inflation risks.

To measure the relative importance of the sources of consumption risk in the foreign exchange market, I form the cross-section of testing assets. I use data on twelve currencies of developed economies over the period from 1986 to 2011 at a quarterly frequency. The choice of currencies is limited by the availability of the term-structure data required for the cross-sectional sorting. At the end of each quarter, I sort currencies into three currency baskets based on the level of average foreign yield: low, intermediate and high interest rate currency baskets. I project the exchange rate growth of each currency basket in real US dollars on the states of the model and sources of consumption risk. Thereby, I augment the model of consumption growth with the process for currency dynamics.

I find that the model fits the macroeconomic data and data on asset prices well. First, the model captures important economic episodes such as the Great Moderation, recessions, and the recent financial crisis. Second, diagnostics of fitting errors do not exhibit noticeable misspecification. Hence, there is a realistic setup for examining the model's asset pricing implications. I perform such an analysis across forty investment horizons, from one quarter to ten years. I use the shock-exposure and shock-price elasticities of Borovička, Hansen, Hendricks, and Scheinkman (2011) and Borovička and Hansen (2013) to characterize the term-structure of consumption risks and their prices in the cross-section of currency baskets at alternative horizons. Shock elasticities measure the sensitivity of expected cash flows and returns with respect to the change in the amount of the underlying risk and account for the presence of stochastic variance. The elasticities represent marginal quantities and marginal prices of risk (marginal Sharpe ratios).

Related literature

This paper is related to two strands of international macro-finance literature that examines time-series and cross-sectional properties of currency risk premia. I limit my discussion to papers that interpret currency risk premia as compensation for consumption risk. On the empirical front, Sarkissian (2003) and Lustig and Verdelhan (2007) study the ability of the consumption growth factor to explain the cross-section of currency returns. Sarkissian (2003) adapts the framework of Constantinides and Duffie (1996) to a multi-country setting and documents that the cross-country variance of consumption growth exhibits explanatory power for cross-sectional differences in returns on individual currencies, whereas the consumption growth itself does not. Lustig and Verdelhan (2007) establish in the framework of the durable CCAPM of Yogo (2006) that the consumption growth is a priced factor in the cross-section of returns on currency baskets formed by sorting currencies by respective interest rates. There are two common features in these papers. First, both studies recognize the presence of multiple sources of consumption risk but do not describe them explicitly. Second, both papers do not extend the analysis beyond a fixed horizon given by a decision interval of the representative agent (one quarter in the case of Sarkissian, 2003, and one year in the case of Lustig and Verdelhan, 2007).

Part of the theoretical literature features different consumption-based models dedicated to rationalizing the time-series behavior of currency risk premia, e.g., the violation of the uncovered interest rate parity (UIP). Models include, but not limited to, settings with habits (Heyerdahl-Larsen, 2012; Verdelhan, 2010), long-run risks (Bansal and Shaliastovich, 2013; Colacito, 2009; Colacito and Croce, 2013), and disasters (Gabaix and Farhi, 2013). My paper is closely related to the international long-run risk literature, but my focus is different. Theoretical international long-run risk studies model a joint distribution of domestic and foreign macroeconomic quantities to pin down a theoretical equilibrium exchange rate consistent with the forward premium anomaly. Instead, I model multiple sources of consumption risk of the US representative agent, estimate them, and establish their relative importance for currency risk premia in the cross-section of currencies and across alternative investment horizons.

This work also contributes to the debate about violations of the uncovered interest rate parity at long horizons, alternatively, long-run profitability of currency carry trades. Boudoukh, Richardson, and Whitelaw (2013) reformulate the uncovered interest parity hypothesis by relating future exchange rate changes to the lagged forward interest rate differential and show that implied forecasts of the exchange rate growth works in line

with the theoretical implications of the UIP at horizons longer than one year. Bekaert, Wei, and Xing (2007), Chinn and Meredith (2005), and Chinn and Quayyum (2012) find mixed evidence on the unbiasedness of the UIP hypothesis at long investment horizons for different currency pairs and different samples. Lustig, Stathopoulos and Verdelhan (2013) characterize carry trade profitability at the ten-year investment horizon (proxy for the infinite investment horizon) and argue that the term structure of currency carry trade risk premia is downward slopping. I complement this discussion by providing direct evidence regarding violation/plausibility of the uncovered interest rate parity hypothesis at various investment horizons in relation to currency exposures to multiple sources of consumption risk.

Finally, this paper is related to Hansen, Heaton, and Li (2008), who provide evidence on the importance of the permanent shock to consumption growth in account for the value premium. The similarity is in terms of approach, that is establishing the importance of consumption risks for explaining the cross-section of asset prices by joint modeling the stochastic discount factor (under the assumption of recursive preferences) and cash flow processes. My study differs from Hansen, Heaton, and Li (2008) in three principal dimensions. First, I study the foreign exchange market, which has been less examined than the US equity market. Second, my model has stochastic variance, so I account for variation in volatility of consumption growth, and therefore, in risk premia. Third, I quantify the relative importance of consumption risks at short and medium horizons, rather than at infinite horizons.

2   Returns on currency speculation

To illustrate the basic risk-return relationship in the foreign exchange market, consider the following investment strategy. At time t, the US representative investor buys a zero-coupon foreign bond of maturity τ and pays $\exp\left(-\tilde{i}_t^\tau\right)s_t/p_t$ US dollars in real terms. At a future date $t + \tau$, the foreign bond pays back one unit of the foreign currency, i.e., $s_{t+\tau}/p_{t+\tau}$ USD in real terms. The excess τ- period log real return on this strategy,

$$\log rx_{t,t+\tau} = [\log s_{t+\tau} - \log s_t + \tilde{i}_t^\tau - i_t^\tau - \log \pi_{t,t+\tau}]/\tau,$$

is called a currency return because the currency price is the only risky part of the return if the investor holds the foreign bond until maturity.[1]   I use the following notation:

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[1]Note that there is also risk associated with realization of the future US inflation. However, it affects all currency returns in a similar way if the cross-section of currency returns has the same base currency.

exchange rate s is the nominal price of one unit of foreign currency in terms of US dollars; i^τ ($\tilde{i}^\tau$) is the US (foreign) nominal yield of maturity τ; p is the level of the US price index.

Violation of the uncovered interest rate parity at short horizons implies an interesting cross-section of currency returns: higher short yield currencies ($\tau = 1$ month, 1 quarter, or 1 year) are on average associated with higher excess returns at the corresponding horizon τ. There has been lots of research dedicated to rationalize this salient feature of currency data in equilibrium contexts with a purpose of providing a risk-based explanation. Specifically, since the study by Lustig and Verdelhan (2007), it has become a standard practice in the literature to sort currencies into baskets depending on the level of the respective short interest rates (equivalently, on interest rate differentials) and examine the covariance of the baskets' returns with some macroeconomic variables or return-based factors.[2]

No-arbitrage principle across international bond markets implies that the price of a τ-maturity foreign bond captures covariation between a τ- period domestic pricing kernel and a τ- horizon exchange rate growth. Hence, sorting currencies into baskets based on the level of foreign yields at a fixed maturity τ isolates currencies with similar risk profiles at the corresponding investment horizon. Equivalently, the existing literature has focused on understanding the nature of the risk-return relationship in the cross-section of currency returns at a fixed investment horizon.

In contrast, this study aims to quantify how the exposures of currencies to the multiple sources of risk are priced at alternative horizons. Thus, there is a need to form a new cross-section of currency testing assets that isolates currencies with similar risk profiles across many investment horizons. At the end of each quarter I sort currencies into three currency baskets (low, intermediate and high interest rate currency baskets) based on the average yield in the corresponding foreign term-structures

$$\tilde{y}_t = \sum_{\tau=1}^{T} \tilde{i}_t^\tau.$$

The average foreign yield $\tilde{y}_t$ proxies for the price of the multi-period currency exposure to the sources of risk. This sorting produces a spread in real quarterly currency returns

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[2]Examples include, but not limited to, Burnside (2012), Burnside, Eichenbaum, Kleshchelski, and Rebelo (2011), Della Corte, Riddiough, and Sarno (2014), Lustig, Roussanov, and Verdelhan (2011), Lettau, Maggiori, and Weber (2013), Menkhoff, Sarno, Schmeling, and Schrimpf (2011), Mueller, Stathopoulos, and Vedolin (2013).

between Basket "High" and Basket "Low" (basket of high and low interest rate currencies, respectively) of 4.52% per year (Table 1). Other sorting strategies are possible. I perform robustness check and sort currencies into baskets based on the respective short interest rates as in the rest of the literature. I repeat the entire analysis with such a cross-section of testing assets. My results remain broadly similar.[3]

3 The model

This paper studies the importance of multiple sources of consumption risk for a cross-section of currency baskets across multiple investment horizons from the viewpoint of the US representative agent. In particular, I examine how currencies covary with the sources of consumption risk identified from the US macro-economic data and quantify how this covariation is priced across alternative investment horizons. It is important to emphasise that ex-ante there is no need to take a stand on whether the identified sources of consumption risk are US-specific or have global nature. The empirical results will speak about the nature of the shocks, and therefore, I postpone this discussion until later.

The key modeling ingredients of the study are: (1) the pricing kernel implied by the preferences of the representative agent and the dynamics of the consumption growth process and (2) the evolution of exchange rate growth of the currency baskets. The formal model of consumption dynamics is a necessary element of the analysis in order to identify the sources of consumption risk. The process for exchange rate growth is key to measure currencies' exposure to the sources of risk. Finally, the pricing kernel encodes information about how risks are priced from the representative agent point of view.

In this section, I proceed by describing each modeling component in turn. Specifically, I will outline (1) how to specify the pricing kernel, (2) how to develop a realistic model of consumption growth, and (3) how to measure currency sensitivity to the identified sources of risk. In the subsequent sections, I will discuss (1) how to identify sources of consumption risk, and (2) how to quantify the risk-return relationship in the foreign exchange market across alternative investment horizons.

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[3]The Online Appendix provides these results.

3.1 Recursive preferences

I use a standard framework of the representative agent model with recursive preferences. The recursive utility of Epstein and Zin (1989) and Weil (1989) is designed to account for the temporal distribution of risks; therefore, it is a natural setting for studying the role of multiple sources of risk. Notable applications of this framework for understanding the joint dynamics of exchange rates, macro quantities, and asset prices include, but not limited to, Backus, Gavazzoni, Telmer, and Zin (2010), Bansal and Shaliastovich (2013), Colacito and Croce (2011), Colacito and Croce (2013), Colacito (2009), Tretvoll (2011a), and Tretvoll (2011b).

The recursive utility is a constant elasticity of substitution recursion,

$$U_t \;=\; [(1-\beta)c_t^\rho + \beta\mu_t(U_{t+1})^\rho]^{1/\rho}, \tag{3.1}$$

with the certainty equivalent function,

$$\mu_t(U_{t+1}) = [E_t(U_{t+1}^\alpha)]^{1/\alpha}, \tag{3.2}$$

where c_t is consumption at time t, U_t is utility from time t onwards, $(1-\alpha)$ is the coefficient of relative risk aversion, $1/(1-\rho)$ is the elasticity of intertemporal substitution (EIS), and β is the subjective discount factor.

Under recursive preferences, the stochastic discount factor $m_{t,t+1}$ has two components, consumption growth and a forward looking component:

$$m_{t,t+1} \;=\; \beta(c_{t+1}/c_t)^{\rho-1}(U_{t+1}/\mu_t(U_{t+1}))^{\alpha-\rho}. \tag{3.3}$$

Appendix A.5 of the NBER version of Backus, Chernov, and Zin (2014) provides the derivation. There are two alternative ways to consider the component $U_{t+1}/\mu_t(U_{t+1})$ and to further derive the pricing kernel. One possibility is to use the connection between U_t and the equilibrium value of the aggregate consumption stream. This link would imply that the log stochastic discount factor is a function of consumption growth, $\log g_{t,t+1} = \log(c_{t+1}/c_t)$, and the return to a claim on future wealth, $r_{t,t+1}^w$, (Epstein and Zin, 1991):

$$\log m_{t,t+1} \;=\; \alpha/\rho \log \beta - \alpha(1-\rho)/\rho \log g_{t,t+1} - (1-\alpha/\rho) \log r_{t,t+1}^w. \tag{3.4}$$

The other possibility is to specify the process for consumption growth explicitly and derive this component of the pricing kernel as a function of the model's states and fundamental shocks.

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The latter approach serves my purpose of describing the relative importance of multiple sources of consumption risk for currency pricing across multiple horizons. First, under the null of a structural model, the multi-period objects (e.g., consumption growth, pricing kernel) directly follow from the dynamics of the corresponding one-period objects. Therefore, a multi-period characterization of currency risk exposures and corresponding prices of risk does not require more data than its one-period counterpart. Second, this setting allows the decomposition of the total risk premium into the contributions of different sources of risk across multiple horizons (Borovička and Hansen, 2013).

3.2   Consumption growth process

It is a well known problem in asset pricing that high-quality consumption data are available at low frequency, and consequently, the identification of multiple sources of consumption risk is a challenging task. As a result, most studies of the joint behaviour of macro economic quantities and asset prices are theoretical. Authors calibrate various empirically plausible processes for consumption growth and study the implications of these models for asset prices.

The common critique of this approach is that different consumption growth processes are observationally equivalent given small sample sizes. Nonetheless, they have very different implications for asset prices. This observation has two implications. On the one hand, an econometrician working with consumption based models faces a serious challenge. On the other hand, this observation suggests that theoretical asset prices are informative about the consumption growth process. Indeed, in equilibrium asset prices are functions of observable consumption growth and unobservable states of consumption growth process. As a result, one can learn about the data-generating process for consumption growth by observing asset prices. I exploit this implication to identify consumption growth empirically.

I specify a parsimonious yet flexible model of consumption growth. I posit a vector autoregressive process for $Y_{t+1} = (\log g_{t,t+1},\ \log \pi_{t,t+1},\ i^1_{t+1},\ \sigma^2_{t+1})'$ that includes US consumption growth $\log g_{t,t+1}$, inflation $\log \pi_{t,t+1}$, short-term nominal yield i^1_{t+1}, and the stochastic variance σ^2_{t+1}

$$Y_{t+1} \;\;=\;\; F + GY_t + H\sigma_t\varepsilon_{t+1}, \tag{3.5}$$

where F is a four-by-one column-vector, and G and H are four-by-four matrices.[4] The

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[4] I use double subscripts for log consumption growth and inflation to indicate the time period of the

vector ε_{t+1} contains four structural shocks, $\varepsilon_{t+1} = (\varepsilon_{g,t+1}, \ \varepsilon_{\pi,t+1}, \ \varepsilon_{i,t+1}, \ \varepsilon_{\sigma,t+1})'$. Shocks $\varepsilon_{g,t+1}$, $\varepsilon_{\pi,t+1}$, and $\varepsilon_{\sigma,t+1}$ are the direct consumption risk, the inflation risk, and the variance risk, respectively. I interpret the fourth shock $\varepsilon_{i,t+1}$ later, when I obtain the estimation results. I impose six parameter restrictions $G_{41} = G_{42} = G_{43} = H_{41} = H_{42} = H_{43} = 0$ to ensure that the stochastic variance follows the discretized version of the continuous-time square-root process.[5]

The structural shocks ε could be both US local or global risks. The intuition behind this observation is straightforward. In a world with multiple economies and trade, country specific endowment shocks enter consumption processes of all countries that trade and share risk. This fact does not necessarily imply that consumption growth across countries has to be highly correlated: if there is consumption home bias, the relative importance of various shocks differ across countries. In this paper, I do not write down a fully fledged structural model starting from micro-foundations, yet it is possible to tell if any of the identified sources of risk ε are purely local or global. To this end, I consider the projection of the foreign pricing kernel on the US states and shocks and describe the implied distribution of the prices of risk. A source of risk is global, if it is associated with the significant price of risk under the viewpoint of the representative agent.

I select a variable to be included in Y_t if the variable has forecasting power for the future consumption growth. Hall (1983) and Hansen and Singleton (1983) show that lagged consumption growth is useful in predicting future US consumption growth. Piazzesi and Schneider (2006) argue that inflation is a leading recession indicator. Bansal, Kiku, and Yaron (2012b), Constantinides and Ghosh (2011), and Colacito and Croce (2011) argue that the real risk-free rate serves as a direct measure of the predictable component in future consumption growth. Instead of including the real risk-free rate in Y_t, I use a short-term nominal yield and inflation.[6]

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corresponding change in consumption or price level. For example, $\log \pi_{t,t+\tau}$ is a $\tau-$period inflation from t to $t + \tau$. I use superscripts for interest rates to indicate the type of the corresponding yields. For example, i_t^τ corresponds to the yield of the τ-period nominal bond at time t. σ_t^2 is a one-period stochastic variance, σ_t is a one-period stochastic volatility.

[5]I have to ensure that $G_{14} = G_{24} = G_{34} = 0$ because otherwise I would not be able to guarantee that the stochastic variance is well defined. If $G_{14} = G_{24} = G_{34} = 0$ then it is natural to think about stochastic variance as an exogenous variable, and therefore, $H_{14} = H_{24} = H_{34} = 0$. Thus, the model of stochastic variance looks like a discretised version of the square-root process. In continuous time, the Feller condition $2F_{41} > H_{44}^2$ guarantees that the variance stays strictly positive. A formal modeling of this process in discrete time is achieved via a Poisson mixture of Gamma distributions (Gourieroux and Jasiak, 2006; Le, Singleton, and Dai, 2010). I use a direct discretization of the continuous-time square-root process to streamline the estimation of the model: I draw all parameters of the model together because the vector ε_{t+1} follows the multivariate normal distribution. I ensure that the variance remains positive by drawing it in logs.

[6]Price-dividend ratio and default premia are other variables used in consumption growth predictive

Among the possible asset prices, I use the nominal yield for a number of reasons. First, the extant empirical and theoretical literature on the violation of the uncovered interest rate parity has documented that risks in exchange rates and interest rates are related (e.g., Bansal and Shaliastovich, 2013; Heyerdahl-Larsen, 2012; Verdelhan, 2010). Second, the use of the yield does not require modeling of an extra cash flow process, e.g., the dividend process, or taking a stand on whether the stock market return is a good proxy for the return on the aggregate consumption claim.

I introduce stochastic variance to the model because the time-variation in the volatility of consumption growth is a salient feature of consumption data, which in its turn serves as a direct source of time variation in risk premia (Bansal and Shaliastovich, 2013; Drechsler and Yaron, 2011). Recently, Bansal, Kiku, Shaliastovich, and Yaron (2013) and Campbell, Giglio, Polk, and Turley (2013) have revisited the importance of the stochastic variance of consumption growth and emphasized its first-order implications for understanding the macro dynamics, as well as the time-series and cross-sectional properties of asset prices. In general, it is a challenging task to identify the stochastic variance in consumption data. My strategy of using a multi-variate system of consumption growth, inflation, and nominal yield to do so has a higher power because several variables have a stronger informative content regarding common unobserved variance.[7]

The pricing kernel derived by applying preferences (3.1)-(3.2) to the consumption growth process (3.5) is

$$\log m_{t,t+1} \quad = \quad \log m + \eta' Y_t + q' \sigma_t \varepsilon_{t+1}, \tag{3.6}$$

where $\eta = (\eta_g, \ \eta_\pi, \ \eta_i, \ \eta_\sigma)'$ and $q = (q_g, \ q_\pi, \ q_i, \ q_\sigma)'$. The parameters of the vectors η and q are functions of the structural parameters of the model (Appendix A.1). Note that the pricing kernel naturally depends on all the states Y_t, but one of the states i_t^1 is a transformed asset price. Because the pricing kernel must value all assets in the economy, including the nominal bond, I impose a number of cross-equation restrictions on the VAR (3.5). As a result, the requirement of internal consistency leads to a constrained vector autoregression.

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regressions. See Colacito and Croce (2011) for details. I do not use these variables because connecting them to the pricing kernel requires an additional modeling effort. For example, the use of the price-dividend ratio must be accompanied by the model of the dividend growth process.

[7]Carriero, Clark, and Marcellino (2012) document that a vector autoregression with common stochastic volatility factor efficiently summarizes the informative content of several macroeconomic variables, such as GDP growth, consumption growth, growth of payroll employment, the unemployment rate, GDP inflation, the 10-year Treasury bond yield, the federal funds rate, and growth of business fixed investment. The authors justify this modeling approach using the observation that the pattern of estimated volatilities is often similar across variables.

The equilibrium nominal yield is an affine function of all the model's states,

$$i_t^1 = A \log g_{t-1,t} + B \log \pi_{t-1,t} + C i_t^1 + D \sigma_t^2 + E,$$

where A, B, C, D, and E are the functions of the structural parameters describing the dynamics of the consumption growth and the preference parameters. I provide the full derivation of the equilibrium nominal yield in the Appendix A.1. To ensure that the price of the nominal bond is internally consistent, I require

$$A = B = D = E = 0, \tag{3.7}$$

$$C = 1. \tag{3.8}$$

The restrictions $A = B = E = 0$ and $C = 1$ are linear

$$\frac{G_{21}}{G_{11}} = \frac{G_{22}}{G_{12}} = \frac{G_{23} - 1}{G_{13}} = \frac{F_2 - \log \beta}{F_1} = \rho - 1, \tag{3.9}$$

whereas the restriction $D = 0$ is nonlinear

$$\alpha(\alpha - \rho)(P + e_1)'HH'(P + e_1)/2 + e_2'Ge_4 - e_2'HH'e_2/2$$
$$-[(\alpha - \rho)P + e_1(\alpha - 1)]'HH'[(\alpha - \rho)P + e_1(\alpha - 1)]/2$$
$$+e_2'HH'[(\alpha - \rho)P + e_1(\alpha - 1)] - (\rho - 1)e_1'Ge_4 = 0, \tag{3.10}$$

and depends on the endogenous parameters $P = (p_g, \ p_\pi, \ p_i, \ p_\sigma)'$ that show up in the solution of the value function

$$\log u_t = \log (U_t/c_t) = \log u + p_g \log g_{t-1,t} + p_\pi \log \pi_{t-1,t} + p_i i_t^1 + p_\sigma \sigma_t^2. \tag{3.11}$$

The parameters of the vector P are functions of the preference parameters and the parameters governing the dynamics of consumption growth. Vectors e_i that enter the non-linear restriction (3.10) are the corresponding coordinate vectors in a four-dimensional space. The nonlinear nature of the restriction (3.10), combined with the presence of the endogenous parameters, represents a serious challenge for the estimation. See Appendix A.1 for the model's solution and further details.

In summary, I model consumption growth via its joint dynamics with inflation and nominal interest rate. I allow for common stochastic variance and impose restrictions required for internal pricing consistency. This process for consumption growth, combined with recursive preferences, leads me to a fully articulated model of the pricing

kernel.

3.3 Foreign cash flow process

Next, I introduce the notion of the FX cash flow: $\delta_{t,t+\tau} = s_{t+\tau}/[s_t \pi_{t,t+\tau}]$. Strictly speaking, it is the real normalized cash flow growth of a foreign bond (alternatively, exchange rate growth in real terms). I prefer to work with $\delta_{t,t+\tau}$ rather than with the real currency price $s_{t+\tau}/p_{t+\tau}$ because the primer is stationary.

I augment the dynamics of the pricing kernel described in the previous section with the law of motion of the FX cash flow for each currency basket. To do so, I project the baskets' FX cash flows on the information set of the representative agent and the structural shocks and omit the orthogonal component:

$$\log \delta_{t,t+1} \quad = \quad \log \delta + \mu' Y_t + \xi' \sigma_t \varepsilon_{t+1} + \xi_v \sigma_t v_{t+1}, \qquad (3.12)$$

where $\mu = (\mu_g, \ \mu_\pi, \ \mu_i, \ \mu_\sigma)'$, $\xi = (\xi_g, \ \xi_\pi, \ \xi_i, \ \xi_\sigma)'$ and v_{t+1} is an idiosyncratic shock. The omitted orthogonal component is irrelevant for the US pricing and does not affect statistical inference. The latter acts similarly to a linear regression, with an omitted variable that is orthogonal to regressors. By using the process (3.12), I make an additional assumption that all economies share the same volatility factor. Having a separate volatility factor for a foreign economy would be appealing; however, at the estimation stage the FX data at a quarterly frequency are not informative enough about it.

The process (3.12) summarizes the impact of the sources of consumption risk ε on each currency basket at the horizon matching the decision interval of the representative agent – parameters $\xi = (\xi_g, \ \xi_\pi, \ \xi_i, \ \xi_\sigma)'$. This process is an essential part of the model, because the covariation of the US pricing kernel with the FX cash flow growth reveals how multiple sources of consumption risks are valued in different currency baskets.

Note, in the process (3.12) there is no implicit assumption that foreign states are spanned by the US states. To illustrate this, consider the following thought experiment. Assume that markets are complete and the foreign pricing kernel is [8]

$$\log \tilde{m}_{t,t+1} \quad = \quad \log \tilde{m} + \tilde{\eta}' \tilde{Y}_t + \sigma_t \tilde{q}' \tilde{\varepsilon}_{t+1},$$

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[8] I assume completeness only for the ease of exposition. If markets are complete, exchange rate growth is pinned down in a unique way.

where

$$\begin{aligned}
\tilde{Y}_t &= aY_t + bY_t^{\perp}, \\
\tilde{\varepsilon}_{t+1} &= c\varepsilon_{t+1} + d\varepsilon_{t+1}^{\perp}.
\end{aligned}$$

Size of matrices a and c is the number of foreign states and shocks, respectively, by four; size of matrices b and d is the number of foreign states and shocks, respectively, by the number of foreign specific states and shocks (states and shocks orthogonal to the US states Y_t and shocks ε_{t+1}), respectively. Consider that b and d are non-zero matrices, i.e., there are foreign specific shocks and foreign states are not spanned by the US states.

The "true" exchange rate growth (in real terms) process implied by the two pricing kernels is [9]

$$\begin{aligned}
\log \delta_{t,t+1} &= \log \tilde{m}_{t,t+1} - \log m_{t,t+1} - \log \tilde{\pi}_{t,t+1} \\
&= (\log \tilde{m} - \log m - \tilde{e}_2' aF) + (\tilde{\eta}' a - \eta' - \tilde{e}_2' aG)Y_t + \sigma_t(\tilde{q}'c - q' - \tilde{e}_2' aH)\varepsilon_{t+1} \\
&+ \tilde{\eta}' bY_t^{\perp} + \sigma_t \tilde{q}' d\varepsilon_{t+1}^{\perp} + \tilde{e}_2' b\tilde{Y}_{t,t+1}.
\end{aligned}$$

However, I recover true loadings μ and ξ as the result of the estimation of the process (3.12)

$$\begin{aligned}
\mu &= a'\tilde{\eta} - \eta - G'a'\tilde{e}_2, \\
\xi &= c'\tilde{q} - q - H'a'\tilde{e}_2.
\end{aligned}$$

The corresponding estimates are not biased and inconsistent because omitted states $Y_t^{\perp}$, $Y_{t+1}^{\perp}$ and shocks $\varepsilon_{t+1}^{\perp}$ are orthogonal to the regressors – US states Y_t and shocks $\sigma_t\varepsilon_{t+1}$.

Furthermore, because $cov_t(\tilde{\eta}' bY_t^{\perp} + \sigma_t \tilde{q}' d\varepsilon_{t+1}^{\perp} + \tilde{e}_2' b\tilde{Y}_{t,t+1}^{\perp}, m_{t,t+1}) = 0$, I do not need to worry about the true values of the loadings $\tilde{\eta}' b$, $\tilde{q}' d$, and $\tilde{e}_2' b$ while computing risk premia. Hence, by projecting cash flows of currency baskets on states Y_t and shocks ε_{t+1}, I do not lose any information important to the exercise: other US variables are spanned by the states of the model, whereas foreign variables may contain extra information useful to predicting exchange rates but such an information is irrelevant to the US representative agent.

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[9]Note that I write down the process for the exchange rate growth in real US money, $\log s_{t+1} - \log s_t - \log \pi_{t,t+1}$, rather than a process for the real exchange rate growth, $\log s_{t+1} - \log s_t - \log \pi_{t,t+1} + \log \tilde{\pi}_{t,t+1}$. Hence, I subtract foreign inflation $\log \tilde{\pi}_{t,t+1}$ from the difference in pricing kernels.

4   Data

4.1   Macro data

I use quarterly data on consumption growth, inflation, and three-month nominal yield from the second quarter of 1947 to the fourth quarter of 2011. In total, there are 259 observations. I collect consumption and price data from the NIPA tables of the Bureau of Economic Analysis. The nominal yield comes from CRSP. Appendix A.2 contains detailed data description. Panels (a)-(c) of Figure 1 display the dynamics of these variables. Table 2 provides basic descriptive statistics. The unconditional standard deviation of consumption growth is slightly higher than 1% annualized. This value is at least twice as low as the value over a longer time interval, including the Great Depression.

4.2   Currency and interest rate data

I collect daily data on twelve spot exchange rates from Thomson Reuters provided by Datastream. The sample contains the price of the Australian dollar, the British pound, the Canadian dollar, the Danish krone, the Euro, the Deutsche mark, the Japanese yen, the New Zealand dollar, the Norwegian krone, the South African rand, the Swedish krone, and the Swiss frank in terms of USD. The sample runs from the beginning of 1986 until the end of 2011. According to the latest report of the Bank of International Settlements, these currencies are among the twenty two currencies with the highest daily turnover, as of April 2010.

I use fixed income data from Datastream, Bloomberg and the dataset of Wright (2011). Wright (2011) provides detailed term-structure data for Australia, Canada, Germany, Japan, New Zealand, Norway, Sweden, Switzerland, and the UK until the first quarter of 2009. From the first quarter of 2009 until the last quarter of 2011, I compute swap implied interest rates for all of these countries but Germany. For Denmark, the Euro area, and South Africa, I compute the swap implied interest rates for the entire time interval. The term-structure data contain yields of forty maturities, from one quarter to ten years. Appendix A.2 (Table 14) describes data availability and sources of data for every country.

I choose currencies of developed countries that are used elsewhere in the literature.[10] Because of limited data availability on the term-structure of interest rates, my sample contains a smaller number of currencies. I work with quarterly currency quotes sampled at the end of the corresponding quarter. The choice of the data frequency corresponds to the frequency of consumption growth data.[11]

At the end of each quarter, I sort currencies into three baskets by the average yield in the foreign term-structure $\tilde{y}_t$. Because the number of currencies is small, I use only three portfolios. Basket "Low" contains the low interest rates currencies, basket "Intermediate" contains the intermediate interest rate currencies, and basket "High" contains the high interest rate currencies.

Table 1 provides descriptive statistics of currency portfolio returns. The average return is monotonically increasing from basket "Low" to basket "High". Similar to Lustig and Verdelhan (2007), I find a spread in excess returns between basket "High" and basket "Low" of approximately 4.52% per year. Table 3 displays the dynamic composition of the baskets. Evidently, some currencies, e.g., the Japanese yen or the Swiss franc, remain in the same basket during the entire time period, so the basket re-balancing does not affect them. Other currencies, for example, the Canadian dollar or the Swedish krone, belong to each basket for several quarters.

5   Methodology

In this section, I describe my estimation approach. My ultimate goal is to estimate the joint dynamics of the pricing kernel and the FX cash flow. It suffices, however, to estimate the joint process of consumption growth and the FX cash flow and to identify the sources of consumption risk. Recursive preferences applied to the dynamics of consumption growth automatically pin down the dynamics of the pricing kernel.

I assume that the idiosyncratic foreign exchange shock does not affect the dynamics of consumption growth; therefore, the estimation of the joint process is equivalent to a three-stage procedure: (1) estimation of the consumption growth process (3.5) with

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[10]See Burnside, Eichenbaum, Kleshchelski, and Rebelo (2011), Lustig, Roussanov, and Verdelhan (2011), Lustig and Verdelhan (2007), Menkhoff, Sarno, Schmeling, and Schrimpf (2011), Rafferty (2011) among others.

[11]US consumption data are available at monthly, quarterly, and annual frequencies. It is well known that annual data are preferable but there are few observations to carry empirical work. I choose consumption data at a quarterly frequency as a compromise of quality and the number of available observations.

pricing restrictions (3.9) and (3.10), (2) identification of the structural shocks ε_{t+1} from the estimated reduced-form innovations in the vector autoregression, and (3) estimation of the foreign exchange cash flow process, taking into account identified structural shocks ε_{t+1}.[12] Because I use the Bayesian methods, my approach is free of the generated regressors' problem and provides the full distribution for all parameter estimates, stochastic variance, and structural shocks. Below, I explain every stage in detail, and in addition, discuss a number of regularity conditions that must be satisfied to ensure that all modelling objects are well defined.

5.1 Estimation of the consumption growth process

I employ the Bayesian MCMC methods to estimate a vector autoregressive model of consumption growth,

$$Y_{t+1} = F + GY_t + \Sigma^{1/2}\sigma_t w_{t+1}, \qquad (5.13)$$

with restrictions (3.9) and (3.10) and stochastic variance, where w_{t+1} are reduced-form innovations that are unknown linear functions of structural shocks: $H\varepsilon_{t+1} = \Sigma^{1/2}w_{t+1}$. Matrix $\Sigma^{1/2}$ is the Cholesky lower triangular matrix and vectors of shocks $w_{t+1} \sim \mathcal{N}(0, I)$ and $\varepsilon_{t+1} \sim \mathcal{N}(0, I)$ follow the multivariate normal distribution.

The key advantage of this estimation approach is that it allows to impose the required pricing restrictions (3.9) and (3.10) directly and delivers the estimated time-series of stochastic variance as a byproduct of the estimation routine. I carefully design the simulation method for the stochastic variance. In particular, I draw the log of variance; therefore, the variance itself never becomes negative or zero. My approach to estimating a vector autoregression with stochastic variance is different from standard methods used in applied macroeconomics.[13] In particular, I draw all the parameters of the vector autoregression jointly because the stochastic variance is a part of the vector of state variables.

The pricing consistency restrictions (3.9) and (3.10) are functions of the structural parameters governing the dynamics of consumption growth (3.5) and the preference

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[12]Assumption that idiosyncratic foreign shock does not affect US consumption growth does not mean that only local US shocks impact US consumption growth. As it has been already discussed above, shocks ε contain both US local and global components. Because I do not write down my model starting from micro foundations, I cannot say how much global versus US local shocks matter for the US consumption growth.

[13]See, for example, Cogley and Sargent (2005), Justiniano and Primiceri (2008), Primiceri (2005), Sargent and Surico (2010).

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parameters α, β, and ρ. Therefore, in addition to the twenty two parameters of the consumption growth process, there are three more preference parameters to estimate. This is a very challenging task, considering that the restriction (3.10) is nonlinear and requires a solution to the fixed point problem. Appendix A.3 discusses the nature of the fixed point problem.

This study follows an easier yet still challenging route. I assume all the preference parameters and estimate the remaining twenty two parameters of the dynamics of consumption growth. Parameter α enters the model only through the nonlinear restriction (3.10). For this reason, it is not clear how to set up a prior for α and how to characterize its posterior distribution. Therefore, I have decided to fix α at some assumed value. If the preference parameter α is fixed, it does not pay off to estimate the other preference parameters ρ and β – estimates for ρ and β will be necessarily distorted. Estimation of the preference parameters is a big important topic that I leave for future research.

I assume the following values for the preference parameters: $\alpha = -9$, $\beta = 0.9924$, and $\rho = 1/3$. The parameters α and ρ imply the preference for the early resolution of uncertainty and have been extensively used in the literature to address a number of asset pricing puzzles. For example, by utilizing these preference parameters, Bansal and Yaron (2004) explain salient features of the equity market in an equilibrium framework of endowment economy; Hansen, Heaton, and Li (2008) empirically explain the value premium puzzle; whereas Bansal and Shaliastovich (2013) rationalize properties of the term-structure of nominal interest rates and the violation of the uncovered interest rate parity. In addition, in the international setting Colacito and Croce (2013) advocate EIS=3/2 ($\rho = 1/3$) as a value supported by empirical evidence gained through the lens of their structural model. Finally, I borrow the value of the subjective discount factor β from Hansen, Heaton, and Li (2008).[14]

I account for the linear restrictions (3.9) by incorporating them directly in the parameter posterior distribution. I reject all the MCMC draws that violate the nonlinear restriction (3.10). To evaluate the nonlinear restriction, I solve the fixed-point problem at each draw; this process makes the estimation problem very time-consuming. It is worthwhile

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[14]Long-run risk literature traditionally uses monthly calibrations, whereas I use quarterly data to estimate my model (Hansen, Heaton, and Li (2008) also use quarterly consumption data to identify the short-run and permanent consumption shocks, and assume similar preference parameters to price assets). In this study, I choose quarterly decision interval of the representative agent because long time-series of high quality consumption data are not available at a higher frequency. Hence, in the model set up at a quarterly frequency I use preference parameters that have been widely employed for monthly calibrations. Importantly, however, these preference parameters if anything are conservative in such a case. See Bansal, Kiku, and Yaron (2012b) for a discussion on how a model specified at a lower frequency pushes an estimate for the risk aversion parameter up.

mentioning that if I had to estimate the preference parameters as well, the problem would be even more complicated.

As an additional exercise, I check if the nonlinear structural restriction is a crucial component of the estimation technology. To this end, I estimate vector-autoregression (5.13) without taking restriction (3.10) into account. Next, I identify multiple sources of consumption risk and describe distribution of associated with them prices. I find that restriction (3.10) is essential in order to analyze pricing implications of the model. Prices of risk are non-linear functions of the models' parameters. If the structural restriction is not imposed, the prices of risk are distorted in a non-trivial manner. Online Appendix provides further details.

5.2 Identification

To recover structural shocks ε_{t+1} from the reduced-form innovations w_{t+1}, I augment the model with a number of economically motivated identifying restrictions as is usually done in structural vector autoregressions in applied macroeconomics.[15] The natural question is the number and the type of restrictions that should be imposed.

Rothenberg (1971) provides a necessary condition, also known as the order condition, which says that to identify a system of n equations there must be $n(n-1)/2$ zero restrictions imposed. I have a system of four equations. Therefore, to identify the structural shocks ε_{t+1}, it is necessary to impose six restrictions. Theorem 1 in Rubio-Ramirez, Waggoner, and Zha (2010) provides a sufficient condition, also known as the rank condition, stating that the location of restrictions in the matrix H matters.

I choose to work with zero restrictions. The necessary and sufficient conditions together with several additional considerations guide me towards particular identification schemes. In principle, there is an ample choice of identification schemes to consider. I prefer to work with those that identify shocks uniquely and are similar to ones used in the long-run risk literature.[16]

Firstly, the stochastic variance σ_t^2 follows the square root process, meaning that three restrictions on matrix H have been imposed from the beginning: $H_{41} = H_{42} = H_{43} = 0$.

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[15]See Blanchard and Quah (1989), Cochrane (1994), Christiano, Eichenbaum, and Evans (1999), Eichenbaum and Evans (1995), Leeper, Sims, Zha, Hall, and Bernanke (1996), Stock and Watson (2012), Uhlig (2005) among others.

[16]Note that every structural model dogmatically implies only one identification scheme. In this sense, I allow for more flexibility by considering several identification schemes and analyzing subsequently their implications.

Next, economically, there must be no zero restrictions on the elements of the third row of the matrix H. The third variable in the system is the nominal rate. It is an equilibrium outcome, and hence, an affine function of the model's states. In principle, the nominal rate might not load materially on one state or another, but a priori, it would be unreasonable to restrict the model in any possible way. Finally, two equations and three more restrictions remain. Here, I follow Theorem 1 from Rubio-Ramirez, Waggoner, and Zha (2010) and find that the only two combinations of three zero restrictions (1) $H_{12} = H_{13} = H_{23} = 0$ and (2) $H_{13} = H_{21} = H_{23} = 0$ guarantee that the model is globally identified.

Identification $H_{12} = H_{13} = H_{23} = 0$ is labeled "Fast Inflation" because inflation reacts contemporaneously to a direct consumption shock, whereas consumption growth reacts to a current inflation shock with a one-quarter delay. Table 4 displays the corresponding location of zero restrictions. Identification $H_{13} = H_{21} = H_{23} = 0$ is labeled "Fast Consumption" because consumption reacts contemporaneously to an inflation shock, whereas inflation reacts to a direct consumption growth shock with a one-quarter delay. Table 5 displays the corresponding location of zero restrictions. I borrow the terminology of "fast variables" from structural VARs in applied macroeconomics.

Based on the properties of the identified shocks, I name the direct consumption shock $\varepsilon_{g,t+1}$ the short-run consumption shock and the shock $\varepsilon_{i,t+1}$ the long-run risk shock. Specifically, four quarters after the shocks hit the economy the response of consumption growth to the shock $\varepsilon_{i,t+1}$ always dominates that to the shock $\varepsilon_{g,t+1}$. Therefore, the cumulative impact of the shock $\varepsilon_{g,t+1}$ on consumption growth is concentrated in the short-run, whereas the cumulative impact of the shock $\varepsilon_{i,t+1}$ on consumption growth dominates at long horizons.

The shock $\varepsilon_{i,t+1}$ exerts a long-lasting impact not only on consumption growth but also on inflation. However, from the pricing perspective, the impact of the shock on real consumption growth is much more important than that on inflation. To make such a conclusion, I perform simple back-of-envelope calculations: I consider two scenarios – (1) zero out element G_{13} so that $\varepsilon_{i,t+1}$ has only immediate affect on consumption growth and (2) zero out element G_{23} so that $\varepsilon_{i,t+1}$ has only immediate affect on inflation. Under both scenarios, I compute the respective prices of risk for both identification schemes. If $G_{13} = 0$ the prices of risk change materially and have even opposite signs; however, if $G_{23} = 0$ – prices of risk change only slightly. Based on this evidence, I refer to $\varepsilon_{i,t+1}$ as the long-run *consumption* risk. The Online Appendix describes empirical properties of the shocks ε_{t+1}.

The identification of the long-run risk shock $\varepsilon_{i,t+1}$ and the variance shock $\varepsilon_{\sigma,t+1}$ is exactly the same in both identification schemes. The shock $\varepsilon_{i,t+1}$ is identified in the spirit of Bansal and Yaron (2004), i.e., the long-run risk shock affects expected consumption growth but not consumption growth itself, and does not feed into the variance process. The identification of the variance shock $\varepsilon_{\sigma,t+1}$ has a flavor of the structural assumptions of Colacito (2009) who allows for non-zero conditional correlations between consumption growth and stochastic variance and expected consumption growth and stochastic variance. Identification of the short-run consumption shock $\varepsilon_{g,t+1}$ and the inflation shock $\varepsilon_{\pi,t+1}$ is different across the identification schemes, as discussed above.

5.3   Estimation of the FX cash flow process

The estimation of the FX cash flow process (3.12) becomes straightforward once the structural shocks ε_{t+1} from the VAR (3.5) are identified. Intuitively, the cash flow process is a part of the vector autoregression that also includes the dynamics of consumption growth, inflation, nominal yield, and stochastic variance. The FX cash flow does not Granger-cause the economic states, whereas the economic states do cause the FX cash flow. In other words, there is nothing new to learn from the dynamics of foreign exchange cash flow that is not already contained in the dynamics of economic states. Given this property, the estimation of the joint distribution of the economic states and foreign exchange cash flow can be performed in two steps, as follows: (1) estimate the model of consumption growth (3.5) and (2) use the results from (1) to estimate the foreign exchange cash flow, i.e., measure the loadings of the corresponding cash flow on economic states and structural shocks. Because a two-stage estimation is equivalent to the estimation of the joint process, the problem of generated regressors does not arise.

Effectively, estimating the FX cash flow process is almost identical to running a linear regression because the full distribution of the stochastic variance σ_t^2 and structural shocks ε_{t+1} are already known, as a byproduct of the Bayesian MCMC approach. Components such as $\sigma_t \varepsilon_{g,t+1}$ in the process (3.12) act as additional regressors to the economic states. I use the Bayesian MCMC methods to estimate the FX cash flow process. I provide the details of the estimation algorithm and discuss the corresponding choice of priors in the Online Appendix.

5.4 Shock elasticity

In this section, I describe how I quantify prices and quantities of consumption risks in the cross-section of currency baskets at alternative horizons. I follow the idea of dynamic value decomposition of Hansen (2012) and, in particular, I use shock-exposure and shock-price elasticities of Borovička and Hansen (2013) and Borovička, Hansen, Hendricks, and Scheinkman (2011). Shock-exposure elasticity and shock-price elasticity are marginal metrics of quantity and price of risk, respectively.

The importance of a distinct source of risk for a cash flow is measured by the magnitude of the risk premium earned because of the cash flow's exposure to the risk. Two metrics matter: quantity of risk (exposure) and price of risk (compensation per unit of exposure). In a dynamic world with multiple sources of risk, the total risk premium is a compensation for cash flow's exposure to all the sources of risk at many horizons. Thus, to shed light on the relative importance of one source of risk, it is necessary to isolate one shock of that type and study its pricing implications for cash flow $\delta_{t,t+\tau}$ across different horizons τ. In this case, the quantity and price of risk depend on the time gap $\tau - 1$ between the moment when the shock is realized and the moment when the shock impacts the cash flow. This dependence on time creates a term-structure of risks and their prices.

Borovička and Hansen (2013) describe in detail how to characterize the term-structure of risks and their prices in a structural model with stochastic variance in discrete time. I illustrate their approach in a simple example by examining the role of the variance risk. Appendix A.4 provides the formal derivation of shock elasticities in the context of my model.

To characterize the role of the variance risk ε_σ, Borovička and Hansen (2013) propose to undertake the following steps. First, they change the exposure of the cash flow $\log \delta_{t,t+\tau}$ to the risk $\sigma_t \varepsilon_{\sigma,t+1}$.[17] To do so, they introduce a perturbation

$$\log h(\mathrm{v}) = \gamma(\mathrm{v}, \sigma_t) + \mathrm{v}\sigma_t \varepsilon_{\sigma,t+1},$$

where the functional form of $\gamma(\mathrm{v}, \sigma_t)$ is not important and v is a scalar, and add this perturbation to the original multi-period cash flow $\delta_{t,t+\tau}$:

$$\log \bar{\delta}_{t,t+\tau} \quad = \quad \log \delta_{t,t+\tau} + \log h(\mathrm{v}).$$

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[17]Without loss of generality, assume that $\sigma_t \varepsilon_{\sigma,t+1}$ has a unit standard deviation.

As a result, they change the amount of the variance risk in the cash flow by the value v at time $t + 1$. Next, the authors study how the log of the expected cash flow changes in response to a change in the amount of risk, when the change is marginal, i.e., they compute the following derivative

$$\ell_\delta(Y_t, \tau) = \left.\frac{\mathrm{d}\log E_t[\delta_{t,t+\tau}]}{\mathrm{d}\log h(\mathrm{v})}\right|_{\mathrm{v}=0} = \left.\frac{\mathrm{d}\log E_t[\bar{\delta}_{t,t+\tau}]}{\mathrm{d}\mathrm{v}}\right|_{\mathrm{v}=0} \tag{5.14}$$

and call the result the shock-exposure elasticity. Similarly, they study how the log risk premium changes in response to a change in the amount of risk, when the change is marginal, i.e., they compute the following derivative

$$\ell_p(Y_t, \tau) = \left.\frac{\mathrm{d}\log E_t[rx_{t,t+\tau}]}{\mathrm{d}\log h(\mathrm{v})}\right|_{\mathrm{v}=0} = \left.\frac{\mathrm{d}\log E_t[\bar{\delta}_{t,t+\tau}]}{\mathrm{d}\mathrm{v}}\right|_{\mathrm{v}=0} - \left.\frac{\mathrm{d}\log E_t[\bar{\delta}_{t,t+\tau} m_{t,t+\tau}]}{\mathrm{d}\mathrm{v}}\right|_{\mathrm{v}=0} \tag{5.15}$$

and call this object the shock-price elasticity. The derivative with respect to $\log h(\mathrm{v})$ is effectively a derivative with respect to the random variable $\mathrm{v}\sigma_t\varepsilon_{\sigma,t+1}$. In continuous time, such a derivative is known as the Malliavin derivative. Borovička, Hansen, Hendricks, and Scheinkman (2011) show that it is equal to the directional derivative in the right-hand side of (5.14) or (5.15) in continuous time. Borovička and Hansen (2013) simply adopt the directional derivative as a definition of the shock elasticity in discrete time.

The shock-exposure elasticity $\ell_\delta(Y_t, \tau)$ is marginal quantity of risk, whereas the shock-prices elasticity $\ell_p(Y_t, \tau)$ is marginal price of risk, or the marginal Sharpe ratio. The elasticities depend on the time elapsed since the shock has been realized until it impacts the cash flow and on the information set Y_t. These marginal metrics can be viewed as asset pricing counterparts to cumulative impulse response functions. Shock elasticities are specifically designed to study asset pricing implications of structural models with stochastic variance (or other types of nonlinearity).

In a linear model, marginal metrics of quantity and price of risk correspond to their average counterparts. Therefore, the shock-exposure elasticity is the cumulative impulse response function of the multi-period log cash flow (multi-period quantity of risk), whereas the shock-price elasticity is the cumulative impulse response function of the negative of the multi-period log stochastic discount factor (average multi-period Sharpe ratio). Section 2.2 of Borovička and Hansen (2013) illustrates this equivalence.[18] How-

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[18]Intuitively, the equivalence holds because the nonlinearities, for which shock elasticities additionally account, are absent. Roughly speaking, the cumulative impulse response function requires computing the expectation of the log, whereas the shock elasticity requires computing the opposite, i.e., the log of the expectation. In a linear model the order does not matter.

ever, in a model with stochastic variance (or other types of nonlinearities), shock elasticities do not coincide with cumulative impulse response functions, and have a different interpretation. This difference is critical because only shock elasticities can describe risks in isolations in the presence of nonlinearities.

My model contains three types of risk, namely, the short-run consumption risk, the inflation risk, and the long-run consumption risk, which enter the model linearly. Therefore, the shock-exposure and shock-price elasticity for these risks have a standard interpretation of average quantity and price of risk. Shock elasticities for the variance risk has interpretation of marginal quantity and price of risk. In this case, prices of risk associated with different currency baskets are different, i.e., they are basket specific. This is a direct manifestation of nonlinearity.

5.5   Regularity conditions

In addition to cross-equation and shock identifying restrictions, a number of regularity conditions has to be imposed on the parameters of the model so that all modeling objects are well defined. First, the state vector process

$$Y_{t+1} \;\; = \;\; F + GY_t + H\sigma_t\varepsilon_{t+1}$$

has to be stationary and stochastic variance σ_t^2 has to be positive at each point of time. The property of covariance-stationarity requires all eigenvalues of G to lie inside the unit cycle. In addition, the Feller condition that guarantees that the stochastic variance is positive for any t is $2(1 - G_{44}) > \Sigma_{44}$.

Secondly, there must exist a unique solution to the nonlinear forward-looking difference equation (3.1), i.e., the utility function is defined. Hansen and Scheinkman (2012) establish existence and uniqueness results in the infinite-horizon specifications of the difference equation in a Markov environment. I follow their lead and discuss corresponding parameter restrictions in Appendix A.5.

Finally, shock elasticities $\ell_\delta(Y_t, \tau)$, $\ell_p(Y_t, \tau)$ have to be defined in the limit $(\tau \to \infty)$. Appendix A.4 illustrates that shock elasticities are exponentially affine functions of the model states. This result arises naturally from multiplicative factorisations of the cash-flow functional $\delta_{t,t+\tau}$ and value functional $\delta_{t,t+\tau}m_{t,t+\tau}$ that are associated with alternative probability measures. The changes of measure are related to the problem of finding the principal eigenvalues and eigenfunctions. Hansen and Scheinkman (2009)

discuss conditions that lead to a stochastically stable change of measure in the continuous time environment. I use a discrete time counterpart of their result. Appendix A.5 provides further technical details.

6 Results

I present my findings in the following order. I start with a discussion of the estimated dynamics of the structural VAR. Next, I analyze how foreign exchange cash flows are sensitive to the four identified sources of consumption risk and how these sensitivities are priced at alternative investment horizons. Finally, I examine term-structure of cross-sectional risk premia in the FX market.

6.1 Macro dynamics

I use the data displayed in panels (a)-(c) of Figure 1 to estimate the model (5.13) with the consistency restrictions (3.9) and (3.10). The Online Appendix summarizes the diagnostics of fitting errors based on which I conclude that the model has a good fit. To emphasize the important role of the stochastic variance in the model, I also estimate the homoscedastic version of the vector-autoregression (5.13). As a result, I find that the original model with stochastic variance has superior fit of data (log Bayes-Odds ratio in favour of the model with stochastic variance is 565).[19]

In addition to the parameter estimates, the estimation procedure delivers such useful outputs as the estimated expected consumption growth $E_t \log g_{t,t+1}$ displayed in panel (a) of Figure 1 and estimated path of the unobservable stochastic variance σ_t^2. I take the square root of σ_t^2 and scale it appropriately, so that the series represents the stochastic volatility of consumption growth. I display this series in panel (d) of Figure 1. The annualized mean path of estimated volatility of consumption growth varies from 0.65% to 2.26%. It captures the important economic periods: the volatility is high after the Second World War, during the oil crises, the monetary experiment, and the recent financial crisis, and the volatility is relatively low during the Great Moderation.

Table 6 reports the parameter estimates for the elements of the matrices F, G, and Σ. The element G_{44} is of special interest because it characterizes the persistence of

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[19]According to Kass and Raftery (1995), a log Bayes-Odds ratio greater than three is a strong evidence against the null model. The homoscedastic VAR is the null model here. Online Appendix contains estimation results and describes the methodology of computing the log Bayes-Odds ratios.

the stochastic variance. The estimated half-life of the variance component is $\log 2/(1 - G_{44}) = 13$ quarters. It is particularly interesting to compare the estimate of G_{44} with the corresponding values used in calibrations elsewhere in the literature. Similar to the specification of the consumption growth process in Bansal and Yaron (2004), my model has only one stochastic variance factor.[20] I proceed by comparing the estimate of G_{44} with the corresponding parameter values used in different calibrations of the Bansal and Yaron (2004) model, e.g., in Bansal and Yaron (2004), Bansal, Kiku, and Yaron (2012a) and Bansal, Kiku, and Yaron (2012b). These values are 0.9615, 0.9949, and 0.997 on a quarterly basis, respectively; they are higher than my point estimate of G_{44} which is 0.9451. However, the persistence parameter used by Bansal and Yaron (2004) is within the confidence interval of the estimated parameter G_{44}.

I compute the persistence of the expected consumption growth as an autocorrelation parameter of the expected consumption growth: $corr(E_t \log g_{t,t+1}, E_{t-1} \log g_{t-1,t})$. Its value is 0.85 with the 95% confidence interval from 0.76 to 0.92. These magnitudes are somewhat smaller than the values used in standard calibrations of the long-run risk models. For example, Bansal and Yaron (2004) use the autoregressive parameter of 0.94, whereas Bansal, Kiku, and Yaron (2012a) use the value of 0.93 (I refer to the parameter values corresponding to the consumption dynamics at a quarterly frequency). Nevertheless, the model does not necessarily lose ability of long-run risk models to account for the equity risk premium. Specifically the estimated confidence interval for the annualized entropy of the pricing kernel is between 0.08 and 0.15; that is everywhere higher than the entropy values used in standard calibrations of long-run risk models with stochastic variance only (no jumps).[21]

The expected consumption growth loads significantly on all the observables used in the estimation with the largest in absolute terms loading on the nominal yield ($G_{13} = 0.38$). Because of the dominant role of the nominal yield, the cyclical properties of the expected growth and the nominal yield are similar. Occasionally, however, the expected consumption growth mirrors the dynamics of other variables. For example, during the recent financial crisis the dynamics of the expected consumption growth is mostly related to the dynamics of inflation with a negative sign, whereas during the economic downturn of 1958 the expected consumption growth closely tracks the evolution of the realized consumption growth.

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[20]In contrast to the theoretical long-run risk literature, I specify the stochastic variance not as an autoregressive process but as a discretized version of the square-root process.

[21]Table 3 in Backus, Chernov, and Zin (2011) summarises entropy values inherent to representative-agent pricing kernels with stochastic variance. They argue that a realistic macro-finance model should have the entropy of the pricing kernel of at least 0.12 per year.

Table 7 contains the estimates for the parameters of the matrix H. Under both identification schemes, I find that a positive variance shock leads to a positive contemporaneous move in inflation, whereas a positive short-run consumption shock leads to a positive contemporaneous move in the nominal yield. Additionally, under "Fast Inflation" a positive short-run consumption shock leads to an increase in inflation, whereas under "Fast Consumption", a positive inflation shock increases consumption growth. This impact of the structural shocks on the states of the model has direct implications for the prices of risk, and hence, magnitude of the entropy of the pricing kernel.

6.2 Term-structure of exposures of FX cash flows to the multiple sources of consumption risk

Table 8 and Table 9 describe the distribution of the parameters of the cash flow process estimated for all currency baskets under both identification schemes. For the one-period exposures, the parameters ξ_g, ξ_π, ξ_i, and ξ_σ are of central interest. These parameters are the loadings of the foreign exchange cash flows on the vector of structural shocks $\sigma_t \varepsilon_{t+1}$, and, therefore, can be interpreted as the quantity of the short-run risk, inflation risk, long-run risk, and variance risk, respectively.

Figure 2 and Figure 3 summarize information contained in Table 8 and Table 9. Specifically, they illustrate one-period cross-sectional differences in currencies' exposures to the risks under both identification schemes. There are separate panels dedicated to each currency basket that show how baskets' cash flow load on the consumption risks. To emphasize cross-sectional differences, I form the basket of currencies that takes short position in the low yield currencies and long position in the high yield currencies, and document how its cash flow loads on the risks. Blue bars correspond to significant exposures, whereas grey bars correspond to insignificant sensitivities to the risks.

The cross-sections of the one-period currency risk exposures to the consumption risks look merely identical under both identification schemes. In particular, because of positive significant exposures of the high interest rate currencies to the short-run, long-run, and inflation risks coupled with the significant negative exposure of the low interest rate currencies to the long-run risk and negative but insignificant exposures to the short-run and inflation risks, currencies exhibit economically meaningful and statistically significant cross-sectional differences in risk sensitivities. At the horizon of one period, these cross-sectional differences have similar magnitudes for the short-run, inflation, and long-run risk shocks.

Next natural question is to analyze the cross-section of currency risk sensitivities across multiple investment horizons. For multi-period currency risk sensitivities, the parameters μ_g, μ_π, μ_i, and μ_σ matter. In conjunction with the parameters of the matrices G and H, they determine how shocks propagate across time in the cross-section of FX cash flows. Shock-exposure elasticities summarize information encoded in cash flow dynamics and the state evolution and characterize cumulative effects of the shocks on multi-period foreign exchange cash flows.

Figure 4 and Figure 5 display the shock-exposure elasticity under the "Fast Inflation" identification and the "Fast Consumption" identification, respectively. To plot the graphs, I set the stochastic variance σ_t^2 to be equal to 1, i.e., to its long-run mean.[22] Shock-exposure elasticities for short-run consumption shock, inflation shock, and long-run risk shock can be interpreted as quantities of risk in a standard sense (for example, $\xi_g \sigma_t$ is a one-period quantity of the short-run risk associated with the FX cash flow of one of the currency baskets). These shocks do not feed into the stochastic variance process; therefore, the average metrics of price and quantity of risk coincide with their marginal counterparts. In contrast, shock exposure elasticity for the variance shock has an interpretation of the marginal quantity of risk: marginal change in the expected cash flow due to a marginal change in the volatility of the underlying shock.

Under both identification schemes, the high yield currencies load on the long-run consumption risk significantly higher than the low yield currencies do across horizons from one to five quarters.[23] In addition, under the "Fast Inflation" identification, currencies of the basket "High" have significantly higher exposure to the inflation shock than currencies of the basket "Low" do across horizons from one to seven quarters. Neither short-run consumption risk, nor variance risk produce significant cross-sections of currency risk exposures at horizons beyond one quarter under any of the identification schemes. It is important to mention, however, that economically the cross-sectional difference in currency risk exposures to the variance risk is high, especially at longer horizons: FX cash flow of of the basket "High" declines stronger after a positive variance shock. Statistically, however, both cross-sectional differences in shock elasticities and individual baskets' shock-elasticities for the variance risk are insignificant at any investment horizon.

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[22]The shock-exposure elasticities scale up and down depending on the magnitude of the stochastic variance.

[23]I do not plot confidence bounds of the estimated shock elasticities not to overcrowd the figures. Results are available upon request.

6.3 Term-structure of prices of multiple sources of consumption risk

In the previous section, I have documented that there are economically and statistically significant differences in currency exposures to (1) the long-run risk at multiple horizons from one quarter to five quarters, (2) inflation risk at multiple horizons from one quarter to seven quarters under the "Fast Inflation" identification, and (3) short-run consumption risk at the horizon matching the decision interval of the representative agent. These cross-sectional differences matter in the FX market, only if the sources of consumption risk are associated with significant and economically meaningful term-structures of risk prices.

I start characterizing the prices of risks from a one-period perspective. Table 11 describes the distribution of q_g, q_π, q_i, and q_σ (elements of the vector q) under both identification schemes. The one-period prices of risk (log Sharpe ratios) are the negative of the elements of the vector q. Because the short-run consumption risk and inflation risk are identified differently across identification schemes, they are associated with identification-dependent prices of risk. In particular, the inflation shock carries a statistically significant price of risk (log Sharpe ratio of 0.13) only under the "Fast Consumption" identification. The impact of different identifying assumptions on the short-run consumption risk is less meaignful. One-period log Sharpe-ratio associated with the short consumption risk is 0.27 under the "Fast Inflation" identification versus 0.24 under the the "Fast Consumption" identification.

The distribution of q_i and q_σ is identical across the identification schemes because the long-run and variance risks are identified in exactly the same manner. The long-run risk shock is associated with the highest risk compensation among all the sources of risk under both identification schemes. It exhibits the log one-period Sharpe ratio of 0.28.[24] The estimated price of the variance risk is positive similar to other sources of risk but it is associated with high uncertainty. This is likely because the stochastic variance is estimated itself, and hence it is associated with uncertainty, in contrast to other states of the model that are observable.

It is worth mentioning that the price of the variance risk is positive because stochastic variance plays a twofold role in this model. On the one hand, the representative agent, exhibiting preference for the early resolution of uncertainty, does not like a positive uncertainty shock. On the other hand, the representative agent does like a positive

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[24]Sharpe ratios are quoted per quarter. For annualized numbers, Sharpe ratio should be multiplied by 2.

uncertainty shock, if stochastic variance positively predicts future consumption growth. In this model (under assumed preferences parameters), the second effect dominates and this is why the variance shock is associated with a positive risk compensation.[25]

I analyze the multi-period prices of risks by examining the shock price elasticities displayed in Figure 6 and Figure 7.[26] The price elasticity of the short-run consumption shock, the inflation shock and the long-run risk shock corresponds to the negative of the cumulative impulse response function of the multi-period log pricing kernel. This works similarly to a linear model without stochastic variance because these shocks do not feed into the process for the stochastic variance. Therefore, the marginal price of risk associated with these shocks is also the average price of risk, or average Sharpe ratio for log returns.

Such an interpretation is not appropriate for the price elasticity for the variance shock. The variance shock feeds into the variance process, and therefore, it is associated with important nonlinearities in the model. The price elasticity of the variance shock is a marginal change in the risk premium caused by the marginal change in the exposure of cash flow to the source of risk, i.e., a marginal Sharpe ratio for log returns associated with a specific shock. The price elasticity for the variance shock is cash flow dependent because its marginal price of risk is not equal to its average price of risk. At longer investment horizons, currencies of the basket "Low" exhibit higher sensitivity of the risk premia associated with the compensation to the variance risk than currencies of the basket "High" do. However, these differences are not statistically significant.

The risk premia of all currency baskets at all investment horizons are especially sensitive to the long-run risk under both identification schemes. This finding, in conjunction with the substantial spread in quantity of the long-run risk between high and low interest rate currency baskets across multiple horizons, is the main result of the paper. The term-structure of the shock price-elasticities for the long-run risk is slightly upward slopping. Thus, similar cross-sectional differences in the amount of currency exposure to the long-run risk will be associated with a more pronounced cross-section of the long-run risk premia in currency baskets at longer investment horizons.

In addition, currency risk premia are sensitive to the short-run consumption risk, inflation risk (only under "Fast Consumption" identification), and variance risk across

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[25]The twofold role of stochastic variance in representative agent models with recursive preferences is not new (Backus, Routledge, and Zin, 2010). While the negative price of risk is more standard in the literature, recent study by Gill, Shaliastovich, and Yaron (2013) emphasizes the importance of the second consumption volatility factor with the shock associated with a positive price of risk.

[26]As in the case of exposure elasticity, I plot price elasticity by setting $\sigma_t^2 = 1$.

all investment horizons from one quarter to ten years. However, the role of these risks for the cross-section of currency returns is limited. The reason is intuitive. Currencies of baskets "High" and "Low" have significantly different shock exposure elasticities for the short-run consumption risk and inflation risk (under "Fast Consumption" identification) at the one-period horizon only. Therefore, the pricing impact of these risks on the cross-section of currency returns is restricted to the horizon of one quarter.

Because there is no cross-section of currency risk exposures and prices of risk to the variance shock, there is no cross-section of currency variance risk premia. This fact, however, does not mean that stochastic variance is irrelevant for the macro-economy and currency risk premia. First, the presence of stochastic variance is key to fit data well, and hence, to properly identify multiple sources of macro-economic risk. Second, stochastic variance introduces an important channel of time-variation in currency risk premia. The cross-sectional spreads in currency risk premia and realized currency returns grow, when the volatility of macro shocks is high, and decline, when the volatility is low. This is observed in the data and implied by the model.

To put the magnitudes displayed in Figure 6 and Figure 7 into perspective, I refer to a number of studies that report Sharpe ratios for different currency strategies. Table 3 in Ang and Chen (2010) reports a quarterly Sharpe ratio of 0.32 for a currency portfolio based on the level of the yield curve and 0.40 for a currency portfolio based on the slope of the yield curve; Table 1 in Burnside (2012) reports an annualized Sharpe ratio of 0.45 for the equally-weighted carry trade and 0.31 for the HML carry trade; Table 1 in Lustig, Roussanov, and Verdelhan (2012) documents an annualized Sharpe ratio of 0.33 for the dollar carry trade.[27]

These numbers are not exact counterparts to the prices of risk that I document in the paper. In particular, I report Sharpe ratios for log returns, consider different strategies, and use different data. However, I believe these numbers are still informative and could be used as a rough benchmark. The one period log Sharpe ratios for the short-run consumption shock and long-run risk shock are smaller than their multi-period counterparts but already substantial enough against the numbers quoted for currency strategies elsewhere in the literature.

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[27]Ang and Chen (2010) describe a currency strategy based on the level (slope) of the yield curve as one that entails going long in a currency with a high level factor (low term spread) and short in a currency with a low level factor (high term spread); Burnside (2012) defines the equally weighted carry trade as the average of up to twenty individual currency carry trades against the US dollar; Lustig, Roussanov, and Verdelhan (2012) determine dollar carry trade as a strategy of going long in all available one-month currency forward contracts when the average forward discount of developed countries is positive and short otherwise.

6.4 Total currency risk premia: term-structure and decompositon

In this section, I characterize the model implied term-structure of consumption risk premia in the cross-section of currency baskets and decompose one-period risk premia into risk compensations due to currency exposures to multiple sources of consumption risk in isolation. This analysis complements the previous discussion of the role of multiple sources of consumption risk in the cross-section of currency returns.[28]

Figure 8 illustrates the term-structure of total risk premia associated with the three currency baskets under both identification schemes. The spread in risk premia between basket "Low" and basket "High" is significant across horizons from one quarter to four quarters irrespective of identifying assumptions employed. At longer investment horizons, the economic difference in risk premia between high and low yield currencies remains but losses its statistical significance. At infinite investment horizon, the risk premia for all currency baskets converge to zero. This evidence is consistent with the result from the contemporaneous study by Lustig, Stathopoulos, and Verdelhan (2013) obtained in a model-free setting – the term-structure of currency carry trade risk premia (risk premia on the basket of high minus low interest rate currencies) is downward slopping.

As it has been already emphasized, the long-run consumption risk plays the most prominent role in the cross-section of currency returns. At the horizon of one quarter, however, the short-run consumption risk and the inflation risk (under "Fast Consumption" identification) make contribution to the spread in excess currency returns as well. Therefore, it is instructive to study decomposition of the one-period total risk premia. Tables 12 and 13 show the overall level of baskets' risk premia and illustrate how total risk premia split up in accordance to the contributions of different sources of risk. For the ease of interpretation, I report results in annualized terms in percent.

The overall level of total risk premia is such that Basket "Low" is associated with a negative risk premium (-2% annualized), whereas Basket "High" is associated with a positive risk premium (3.5% annualized or 4% annualized under "Fast Inflation" identification and "Fast Consumption" identification, respectively). Currency baskets' historical average returns fall within the 95% confidence intervals associated with the estimated

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[28]It would be instructive to provide a risk premia decomposition result for alternative investment horizons, as well. However, because stochastic variance introduces nonlinearities to the model, the decomposition result is a serious methodological challenge. In nonlinear models, multi-period risk premia is not a product of shock price elasticity and shock exposure elasticity. See Borovička, Hansen, Hendricks, and Scheinkman (2011) for further details.

currency total risk premia. Therefore, the level and the spread of the excess returns in the cross-section of currencies is fully explained by the exposures of currencies to the priced sources of consumption risk under assumed preference parameters.

Under the "Fast Inflation" identification, different exposures of currency baskets to the long-run risk shock accounts for 46% of the one-period spread of excess returns, whereas the remaining 54% are due to the different exposures of the currency baskets to the short-run consumption shock. Under the "Fast Consumption" identification, different currency exposures to the long-run risk shock, the short-run consumption shock, and the inflation shock contribute 39%, 40%, and 21%, respectively, to the spread of the real currency excess returns.

6.5   A new look at existing puzzles

The carry trade profitability is a global phenomenon. It other words, it does not matter for the cross-section of currency returns which currency is the base currency – the higher interest rate currencies are on average more profitable. Lustig, Roussanov, and Verdelhan (2011) argue that different currency exposures to some global risk is at the core of currency carry trade profitability.

This study sheds light on what this global risk is and suggests that this is not a unique source of risk. The projection of foreign pricing kernels on the US states and shocks, i.e., $\log \tilde{m}_{t,t+1} = \log m_{t,t+1} + \log \delta_{t,t+1} + \log \tilde{\pi}_{t,t+1}$, loads negatively and significantly on the short-run, long-run and inflation risks identified from the US macro and asset pricing data. Hence, these shocks are priced under the viewpoint of any foreign representative agent. In addition, the low and high yield currencies, as it has been shown in section 6.2, have very different exposures to these sources of risk. Thus, the short-run consumption risk, the long-run consumption risk and the inflation risk are global risks to which currencies have heterogenous exposures.

Finally, empirical results of this paper suggest an answer to a longstanding question: why it is difficult to document a relationship between macro-fundamentals and exchange rates. The global macro-economy is subject to multiple sources of risk with stochastic variance. Measuring unconditional correlations between macro fundamentals (e.g., consumption growth differential) and exchange rate growth is misleading. Such an estimate measures the correlation between two linear combinations of shocks with stochastic variance. Therefore, the impact of individual shocks is muted. To document how macroeconomic risks affect currency dynamics, it is important (1) to identify multiple sources

of risk, (2) measure their individual impact on exchange rates, and (3) model investor's risk attitude in such a way that the temporal distribution of risk matters.

7   Conclusion

This paper provides novel evidence on the role of multiple sources of consumption risk in the foreign exchange market. The novelty is in terms of economic questions and methodological approach. On the methodological front, I innovatively identify multiple sources of consumption risk from the US macro and asset pricing data. To this end, I show how to use informative content of asset prices to estimate the dynamics of consumption growth and identify systematic sources of risk. From the economic perspective, I carefully analyze the relative importance of various sources of consumption risk on the cross-section of currency baskets across alternative investment horizons. Thus, the focus of the study is twofold – cross-section of currency risk premia and term-structure of currency risk premia.

The most interesting findings of the study are the following three. First, the long-run consumption risk plays the most prominent role in currency markets. On the one hand, it is associated with the largest price of risk across multiple horizons (e.g., average quarterly log Sharpe ratio is 0.28). On the other hand, there is a stable cross-section of currency exposures to the long-run risk across investment horizons from one to five quarters. Second, there are multiple sources of risk – short-run and long-run consumption risks and inflation risk – to which high and low yield currencies exhibit different risk exposures. Therefore, this study describes the economic nature of global risk that is at the core of carry trade profitability (Lustig, Roussanov, and Verdelhan, 2011). Finally, carry trade profitability is a short-horizon phenomenon; it disappears at horizons longer than one year.

I leave at least two interesting avenues for the future research. The first big and important question is the estimation of the preference parameters. The second direction of research is further exploration of the role of the variance risk in macroeconomy and asset markets by utilizing assets that are informative about this type of risk at the estimation stage.

34

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Table 1
Properties of real log excess returns

	Mean	Std Dev	Skewness	Kurtosis	Autocorrelation
Basket "Low"	-0.0063	0.0517	0.38	3.07	0.01
Basket "Intermediate"	-0.0018	0.0432	0.09	3.81	0.15
Basket "High"	0.0050	0.0502	0.03	3.62	0.12

Notes. The three currency baskets are formed by sorting currencies by their corresponding average yields at a quarterly basis. Average yields are computed for each currency's term-structure at each point of time. Sample period: 1986 – 2011. Quarterly.

Table 2
Properties of macro economic variables

	Mean	Std Dev	Skewness	Kurtosis	N observations
$\log g_{t,t+1}$	0.0048	0.0052	-0.45	4.04	259
$\log \pi_{t,t+1}$	0.0083	0.0076	0.81	5.30	259
i_t^1	0.0113	0.0076	0.93	4.13	259

Notes. Descriptive statistics for consumption growth, inflation, and nominal yield. Sample period: second quarter of 1947 – fourth quarter of 2011. Quarterly.

Table 3
Composition of currency baskets

Currency	Basket "Low"	Number of periods in Basket "Intermediate"	Basket "High"
Australia	0	23	76
Canada	20	75	8
Denmark	11	70	12
Germany	34	16	2
Euro area	17	12	0
Japan	103	0	0
Norway	1	24	30
New Zealand	4	10	73
Sweden	32	29	15
Switzerland	95	0	0
UK	5	50	48
South Africa	0	0	58

Notes. Table entry shows the number of periods each currency belongs to each basket. Sample period: 1986 – 2011, at a quarterly frequency.

Table 4
Identification "Fast Inflation"

	$\varepsilon_{g,t+1}$	$\varepsilon_{\pi,t+1}$	$\varepsilon_{i,t+1}$	$\varepsilon_{\sigma,t+1}$
Consumption eq	H_{11}	0	0	H_{14}
Inflation eq	H_{21}	H_{22}	0	H_{24}
Interest rate eq	H_{31}	H_{32}	H_{33}	H_{34}
Variance eq	0	0	0	H_{44}

Notes. A globally identified system. Inflation reacts to a consumption shock ε_g contemporaneously, whereas consumption growth reacts to an inflation shock ε_π with a delay of one period.

Table 5
Identification "Fast Consumption"

	$\varepsilon_{g,t+1}$	$\varepsilon_{\pi,t+1}$	$\varepsilon_{i,t+1}$	$\varepsilon_{\sigma,t+1}$
Consumption eq	H_{11}	H_{12}	0	H_{14}
Inflation eq	0	H_{22}	0	H_{24}
Interest rate eq	H_{31}	H_{32}	H_{33}	H_{34}
Variance eq	0	0	0	H_{44}

Notes. A globally identified system. Consumption growth reacts to an inflation shock ε_π contemporaneously, whereas inflation reacts to a consumption shock ε_g with a delay of one period.

Table 6
The model of consumption growth. Parameter estimates

Parameter	Estimate	Confidence interval, 95%
F_1	-0.0006	(-0.0020, 0.0010)
F_2	-0.0072	(-0.0083, -0.0063)
F_3	-0.0003	(-0.0007, 0.0001)
F_4	0.0549	(0.0232, 0.0683)
G_{11}	0.2110	(0.0878, 0.3133)
G_{12}	-0.1687	(-0.2522, -0.0843)
G_{13}	0.3967	(0.3134, 0.4862)
G_{14}	0.0017	(0.0009, 0.0025)
G_{21}	-0.1407	(-0.2089, -0.0586)
G_{22}	0.1124	(0.0562, 0.1681)
G_{23}	0.7355	(0.6758, 0.7910)
G_{24}	0.0045	(0.0035, 0.0055)
G_{31}	0.0677	(0.0385, 0.1051)
G_{32}	0.0206	(-0.0076, 0.0451)
G_{33}	0.9536	(0.9317, 0.9768)
G_{34}	0.0005	(0.0003, 0.0007)
G_{44}	0.9451	(0.9020, 0.9764)
Σ_{11}	3.35e-5	(2.30e-5, 4.99e-5)
Σ_{12}	1.10e-5	(5.04e-6, 2.20e-5)
Σ_{13}	3.01e-6	(1.24e-6, 6.33e-6)
Σ_{14}	-0.0001	(-0.0004, 7.42e-5)
Σ_{22}	4.05e-5	(2.96e-5, 5.66e-5)
Σ_{23}	3.15e-6	(1.21e-6, 5.58e-6)
Σ_{24}	0.0003	(0.0001, 0.0006)
Σ_{33}	2.71e-6	(1.92e-6, 3.79e-6)
Σ_{32}	6.28e-5	(-2.16e-5, 0.0001)
Σ_{44}	0.0339	(0.0196, 0.0518)

Notes. I estimate a vector autoregression with stochastic variance

$$Y_{t+1} = F + GY_t + \sigma_t \Sigma^{1/2} w_{t+1}$$

and restrictions: (1) $G_{21}/G_{11} = G_{22}/G_{12} = (G_{23}-1)/G_{13} = (F_2 - \log \beta)/F_1 = \rho - 1$ and (2) $\alpha(\alpha-\rho)(P+e_1)'\Sigma(P+e_1)/2 + e_2'Ge_4 - e_2'\Sigma e_2/2 - [(\alpha-\rho)P+e_1(\alpha-1)]'\Sigma[(\alpha-\rho)P+e_1(\alpha-1)]/2 + e_2'\Sigma[(\alpha-\rho)P+e_1(\alpha-1)] - (\rho-1)e_1'Ge_4 = 0$. Note that $\Sigma = HH'$, where H is from (2.5).

Vector $Y_t = (\log g_{t-1,t}, \ \log \pi_{t-1,t}, \ i_t^1, \ \sigma_t^2)'$ includes US consumption growth, inflation, one-period nominal yield, and stochastic variance.

To save space, I do not duplicate the symmetric entries of the matrix Σ. Sample period: second quarter of 1947 – fourth quarter of 2011. Quarterly.

Table 7
Global identification

	Identification "Fast Inflation"			Identification "Fast Consumption"	
Parameter	Estimate	Confidence interval, 95%	Parameter	Estimate	Confidence interval, 95%
H_{11}	0.0057	(0.0048, 0.0069)	H_{11}	0.0053	(0.0045, 0.0063)
H_{14}	-0.0008	(-0.0026, 0.0004)	H_{12}	0.0020	(0.0009, 0.0033)
H_{21}	0.0021	(0.0010, 0.0036)	H_{14}	-0.0008	(-0.0026, 0.0004)
H_{22}	0.0056	(0.0047, 0.0069)	H_{22}	0.0061	(0.0051, 0.0073)
H_{24}	0.0018	(0.0007, 0.0028)	H_{24}	0.0018	(0.0007, 0.0028)
H_{31}	0.0006	(0.0003, 0.0009)	H_{31}	0.0004	(0.0002, 0.0008)
H_{32}	0.0002	(-3.4e-5, 0.0006)	H_{32}	0.0004	(0.0002, 0.0008)
H_{33}	0.0015	(0.0012, 0.0017)	H_{33}	0.0015	(0.0012, 0.0017)
H_{34}	0.0003	(-1.1e-5, 0.0006)	H_{34}	0.0003	(-1.1e-5, 0.0006)
H_{44}	0.1828	(0.1399, 0.2276)	H_{44}	0.1828	(0.1399, 0.2280)

Notes. I identify structural shocks ε_{t+1} from the reduced form innovations w_{t+1}: $\Sigma^{1/2} w_{t+1} = H \varepsilon_{t+1}$. I consider two globally exactly identified models. Identification "Fast Inflation" is determined by the following zero restrictions: $H_{12} = H_{13} = H_{23} = H_{41} = H_{42} = H_{43} = 0$. Identification "Fast Consumption" is determined by the following zero restrictions: $H_{13} = H_{21} = H_{23} = H_{41} = H_{42} = H_{43} = 0$. Quarterly.

Table 8
Estimated FX cash flow process (identification "Fast Inflation")

Parameter	Basket "Low"	Basket "Intermediate"	Basket "High"
$\log \delta$	-0.0037	-0.0392	-0.0365
	(-0.0361, 0.0271)	(-0.0634, 0.0001)	(-0.0665, 0.0075)
μ_g	1.4338	-0.4678	-1.8757
	(0.6096, 2.2304)	(-1.2026, 0.4320)	(-2.7720, -0.9857)
μ_π	-0.8610	-3.1396	-2.1909
	(-1.5648, -0.1731)	(-3.7909, -2.4469)	(-2.8337, -1.4496)
μ_i	-0.6382	1.6173	1.0810
	(-1.8235, 0.4043)	(0.7963, 2.7514)	(0.0914, 2.3769)
μ_σ	0.0061	-0.0065	-0.0191
	(-0.0309, 0.0391)	(-0.0321, 0.0360)	(-0.0496, 0.0267)
ξ_g	-0.0039	0.0082	0.0189
	(-0.0076, 0.0001)	(0.0043, 0.0123)	(0.0119, 0.0255)
ξ_π	-0.0035	0.0082	0.0172
	(-0.0105, 0.0026)	(0.0026, 0.0145)	(0.0109, 0.0234)
ξ_i	-0.0126	-0.0103	0.0063
	(-0.0158, -0.0091)	(-0.0132, -0.0073)	(0.0029, 0.0093)
ξ_σ	-0.0014	0.0009	0.0082
	(-0.0116, 0.0092)	(-0.0084, 0.0095)	(-0.0023, 0.0188)

Notes. For each currency basket, I estimate the FX cash flow process:

$$\log \delta_{t,t+1} = \log \delta + \mu' Y_t + \sigma_t \xi' \varepsilon_{t+1} + \xi_v \sigma_t v_{t+1},$$

where $\mu = (\mu_g, \ \mu_\pi, \ \mu_i, \ \mu_\sigma)'$ and $\xi = (\xi_g, \ \xi_\pi, \ \xi_i \ \xi_\sigma)'$. Quarterly. There are 95% confidence intervals in the brackets.

Table 9
Estimated FX cash flow process (identification "Fast Consumption")

Parameter	Basket "Low"	Basket "Intermediate"	Basket "High"
$\log \delta$	-0.0032	-0.0402	-0.0374
	(-0.0377, 0.0279)	(-0.0646, 0.0001)	(-0.0641, 0.0059)
μ_g	1.4530	-0.4648	-1.8467
	(0.6477, 2.2533)	(-1.1641, 0.3822)	(-2.7288, -0.9052)
μ_π	-0.8352	-3.1394	-2.1599
	(-1.5798, -0.1445)	(-3.7379, -2.4473)	(-2.8617, -1.4217)
μ_i	-0.6547	1.6119	1.0933
	(-1.8080, 0.3965)	(0.7922, 2.7353)	(0.1687, 2.3526)
μ_σ	0.0062	-0.0064	-0.0179
	(-0.0307, 0.0418)	(-0.0338, 0.0372)	(-0.0467, 0.0276)
ξ_g	-0.0040	0.0084	0.0188
	(-0.0078, 0.0001)	(0.0045, 0.0123)	(0.0121, 0.0261)
ξ_π	-0.0037	0.0083	0.0173
	(-0.0099, 0.0031)	(0.0025, 0.0136)	(0.0105, 0.0238)
ξ_i	-0.0124	-0.0103	0.0063
	(-0.0157, -0.0090)	(-0.0131, -0.0076)	(0.0030, 0.0095)
ξ_σ	-0.0013	0.0008	0.0081
	(-0.0115, 0.0085)	(-0.0087, 0.0101)	(-0.0023, 0.0183)

Notes. For each currency basket, I estimate the FX cash flow process:

$$\log \delta_{t,t+1} = \log \delta + \mu' Y_t + \sigma_t \xi' \varepsilon_{t+1} + \xi_v \sigma_t v_{t+1},$$

where $\mu = (\mu_g, \ \mu_\pi, \ \mu_i, \ \mu_\sigma)'$ and $\xi = (\xi_g, \ \xi_\pi, \ \xi_i, \ \xi_\sigma)'$. Quarterly. There are 95% confidence intervals in the brackets.

Table 10
Parameters of the fixed point problem

Parameter	p_g	p_π	p_i	p_σ
Estimate	2.02	-0.10	20.37	0.08
Conf interval	(1.12, 3.19)	(-0.79, 0.54)	(15.49, 26.03)	(2.2e-3, 0.16)

Parameter	b_0	b_1
Estimate	-4e-4	0.9912
Conf interval	(-1e-3, 0)	(0.9901, 0.9932)

Notes. I solve the approximate equation:

$$\log u_t \approx b_0 + b_1 \log \mu_t(u_{t+1}g_{t+1})$$

The value function is $\log u_t = \log u + p_g \log g_{t-1,t} + p_\pi \log \pi_{t-1,t} + p_i i_t^1 + p_\sigma \sigma_{t,1}^2$. Quarterly.

Table 11
Parameters q. Negative of the prices of risk (one-period log Sharpe ratios)

	Identification "Fast Inflation"		Identification "Fast Consumption"	
Parameter	Estimate	Confidence interval, 95%	Estimate	Confidence interval, 95%
q_g	-0.27	(-0.34, -0.18)	-0.24	(-0.31, -0.17)
q_π	-0.04	(-0.11, 0.02)	-0.13	(-0.21, -0.06)
q_i	-0.28	(-0.35, -0.21)	-0.28	(-0.35, -0.21)
q_σ	-0.19	(-0.31, -0.01)	-0.19	(-0.31, -0.01)

Notes. Vector q is the vector of loadings on the structural shocks $\sigma_t \varepsilon_{t+1}$ in the pricing kernel $\log m_{t,t+1}$:

$$\log m_{t,t+1} \;=\; \log m + \eta' Y_t + q' \sigma_t \varepsilon_{t+1}, \qquad (2.6)$$

where $q = H'((\alpha - \rho)P + e_1(\alpha - 1))$, $q = (q_g, \; q_\pi, \; q_i, \; q_\sigma)'$. Preference parameters: $\alpha = -9$, $\rho = 1/3$, $\beta = 0.9924$. Quarterly.

Table 12
One-period risk premia (identification "Fast Inflation")

|	Basket "Low"	Basket "Intermediate"	Basket "High"
Short-run risk	-0.4215	0.8708	2.0219
	(-0.8764, 0.0116)	(0.4442, 1.4163)	(1.1442, 3.0119)
Inflation risk	-0.0547	0.1189	0.2507
	(-0.2635, 0.0550)	(-0.0810, 0.4052)	(-0.1588, 0.8059)
Long-run risk	-1.3895	-0.1387	0.6917
	(-1.9423, -0.9108)	(-1.5955, -0.7293)	(0.3131, 1.1001)
Variance risk	-0.1008	0.0509	0.5912
	(-1.0197, 0.7537)	(-0.6750, 0.7440)	(-0.1752, 1.5974)
Total	-1.9665	-0.0981	3.5555
	(-3.1151, -0.7964)	(-1.0858, 0.8092)	(2.1682, 5.0084)
Data	-2.52	-0.72	2

Notes: One period risk premia associated with multiple sources of risk. Stochastic variance σ_t^2 is set to be equal 1. Quarterly. In percent. I report 95% confidence intervals in the round brackets. The last row "Data" reports the level of the observed average excess returns.

Table 13
One-period risk premia (identification "Fast Consumption")

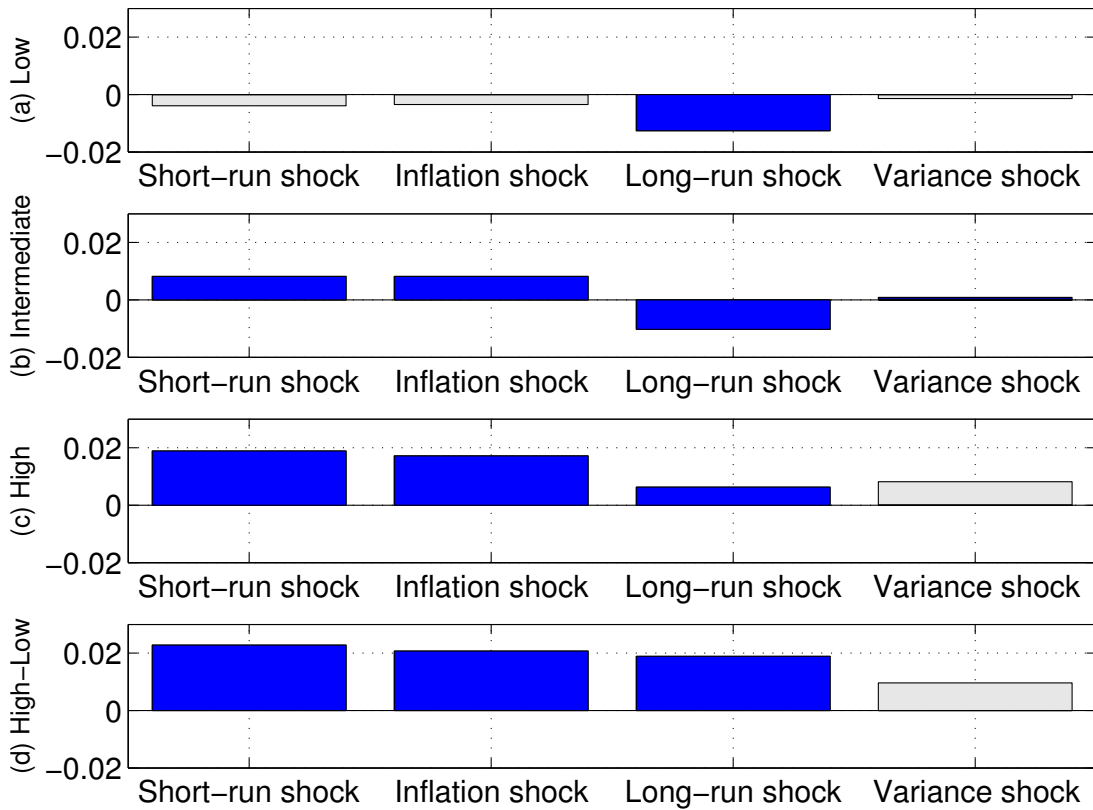|	Basket "Low"	Basket "Intermediate"	Basket "High"
Short-run risk	-0.3764	0.7911	1.7732
	(-0.7727, 0.0087)	(0.3964, 1.3123)	(1.0010, 2.7495)
Inflation risk	-0.1935	0.4299	0.8990
	(-0.6119, 0.1470)	(0.1008, 0.9128)	(0.3342, 1.6588)
Long-run risk	-1.3737	-1.1407	0.6938
	(-1.9254, -0.8906)	(-1.5847, -0.7166)	(0.3201, 1.1317)
Variance risk	-0.1065	0.0349	0.5781
	(-1.0421, 0.7338)	(-0.7799, 0.7151)	(-0.1632, 1.5966)
Total	-2.0501	0.1152	3.9441
	(-3.1593, -0.9436)	(-0.8973, 1.0450)	(2.6174, 5.5569)
Data	-2.52	-0.72	2

Notes: One period risk premia associated with multiple sources of risk. Stochastic variance σ_t^2 is set to be equal 1. Quarterly. In percent. I report 95% confidence intervals in the round brackets. The last row "Data" reports the level of the observed average excess returns.

Figure 1: **Dynamics of the model's states**
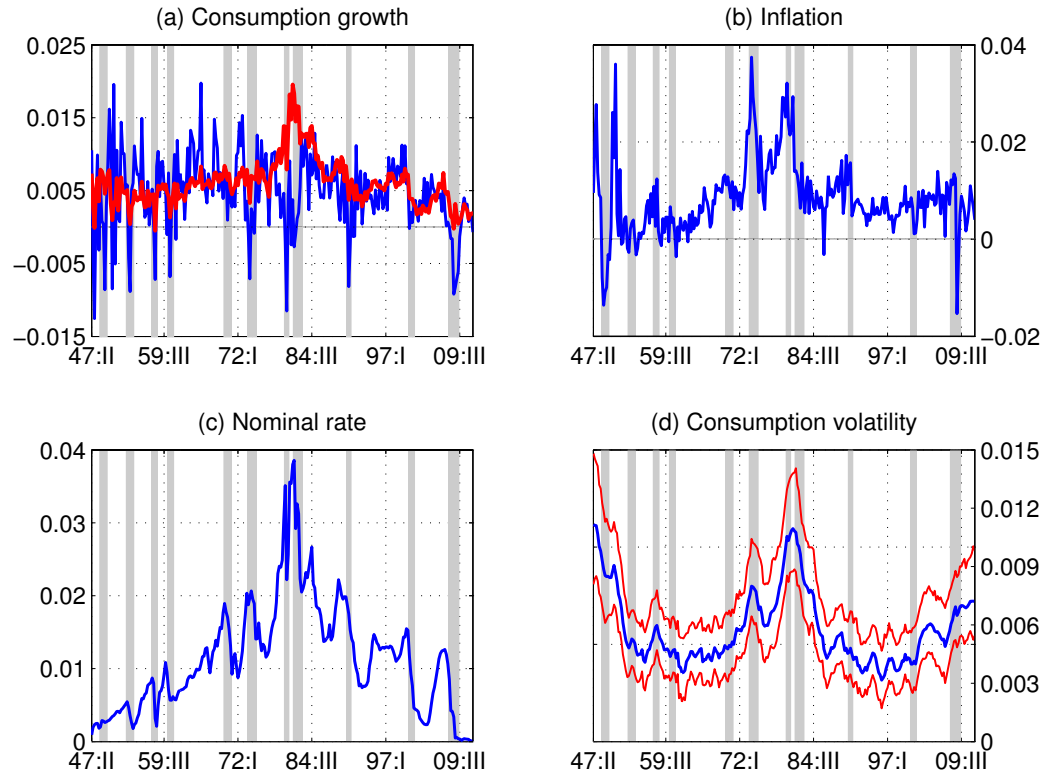
Panel (a) displays quarterly log consumption growth (blue line) and estimated expected consumption growth (thin red line). Panel (b) displays quarterly inflation. Panel (c) displays the 3-month nominal yield, quarterly. Panel (d) displays consumption volatility $\sqrt{\Sigma_{11}}\sigma_t$, quarterly. Blue line is the mean path of volatility, red lines correspond to the 95% confidence bounds. Grey bars are the NBER recessions.

Figure 2: **Cross-section of one-period shock exposures (identification "Fast Inflation")**

Panels (a), (b), and (c) displays loadings of the cash-flow of Basket "Low", Basket "Intermediate", and Basket "High" on the consumption risks. Panel (d) shows the loadings of the cash flow of Basket "High-Low" (long position in the high yield currencies and short position in the short yield currencies) on the consumption risks. Blue bars correspond to statistically significant exposures, whereas grey bars correspond to insignificant exposures. Shocks are identified according to the "Fast Inflation" identification scheme.

Figure 3: **Cross-section of one-period shock exposures (identification "Fast Consumption")**
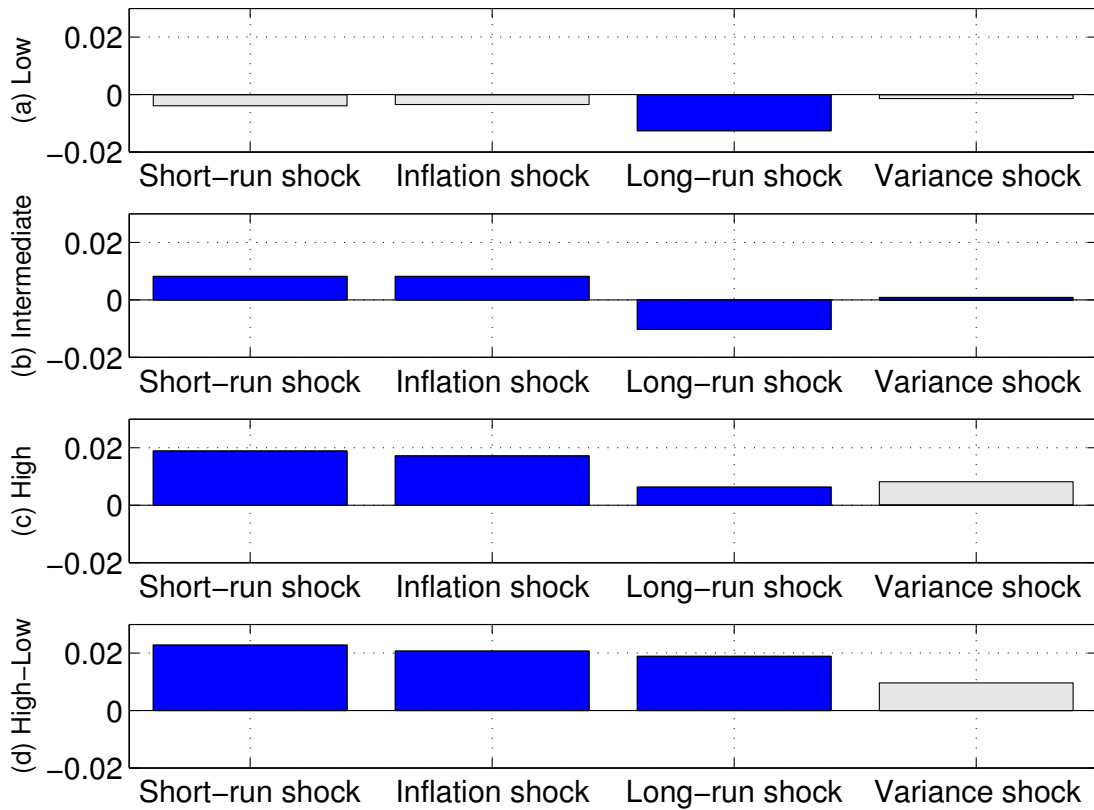

Panels (a), (b), and (c) displays loadings of the cash-flow of Basket "Low", Basket "Intermediate", and Basket "High" on the consumption risks. Panel (d) shows the loadings of the cash flow of Basket "High-Low" (long position in the high yield currencies and short position in the short yield currencies) on the consumption risks. Blue bars correspond to statistically significant exposures, whereas grey bars correspond to insignificant exposures. Shocks are identified according to the "Fast Consumption" identification scheme.

Figure 4: **Shock-exposure elasticity (identification "Fast Inflation")**
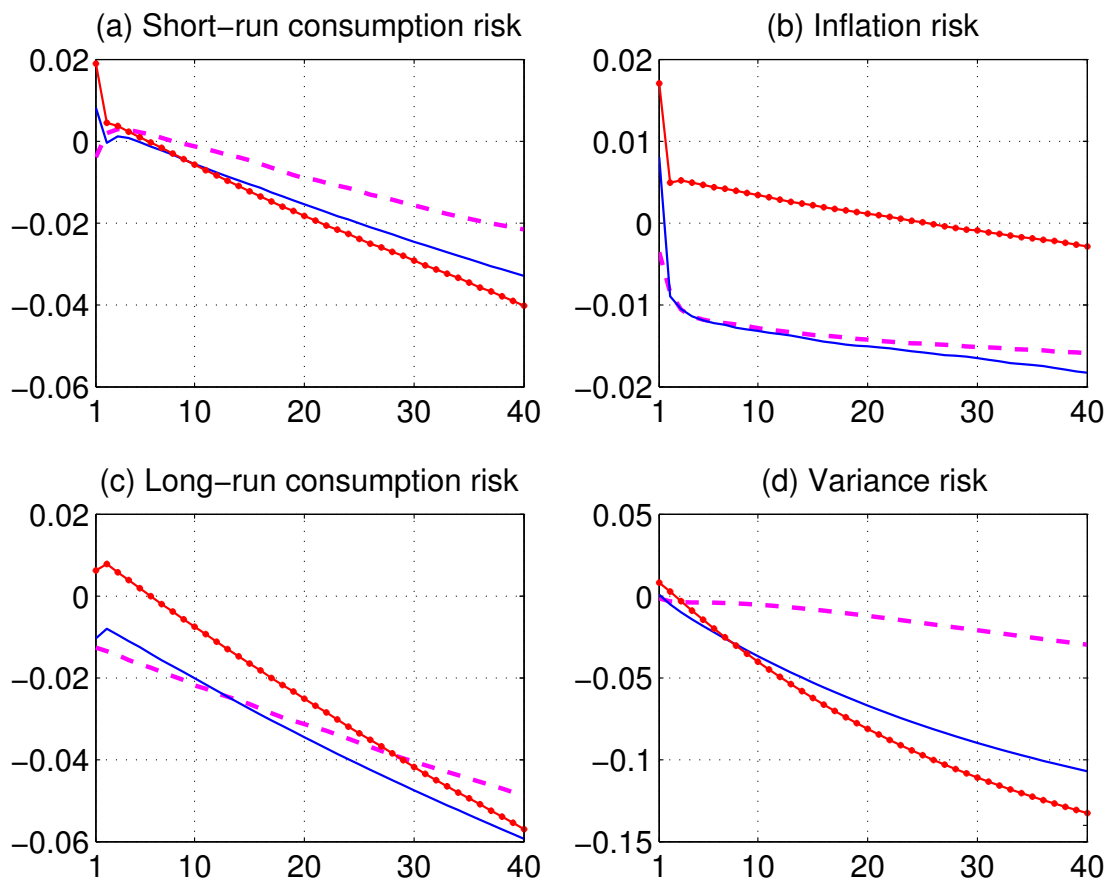
Panel (a) displays shock-exposure elasticity for the short-run consumption risk. Panel (b) displays shock-exposure elasticity for the inflation risk. Panel (c) displays shock-exposure elasticity for the long-run consumption risk. Panel (d) displays shock-exposure elasticity for the variance risk. Identification "Fast Inflation". Quarterly. The magenta dashed line is for the basket "Low", the blue solid line is for the basket "Intermediate", the red marked line is for the basket "High". The horizontal axes: from 1 quarter to 10 years.

Figure 5: **Shock-exposure elasticity (identification "Fast consumption")**
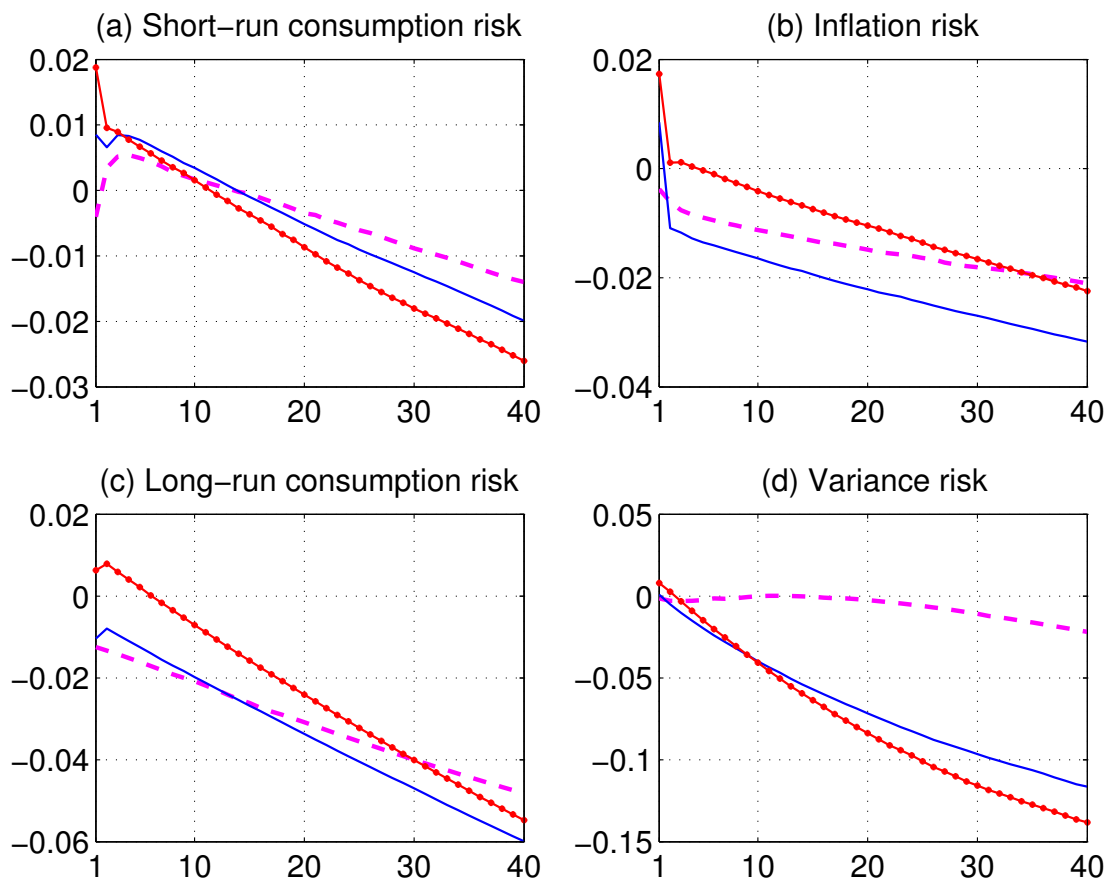
(a) Short–run consumption risk

(b) Inflation risk

(c) Long–run consumption risk

(d) Variance risk

Panel (a) displays shock-exposure elasticity for the short-run consumption risk. Panel (b) displays shock-exposure elasticity for the inflation risk. Panel (c) displays shock-exposure elasticity for the long-run consumption risk. Panel (d) displays shock-exposure elasticity for the variance risk. Identification "Fast Consumption". Quarterly. The magenta dashed line is for the basket "Low", the blue solid line is for the basket "Intermediate", the red marked line is for the basket "High". The horizontal axes: from 1 quarter to 10 years.
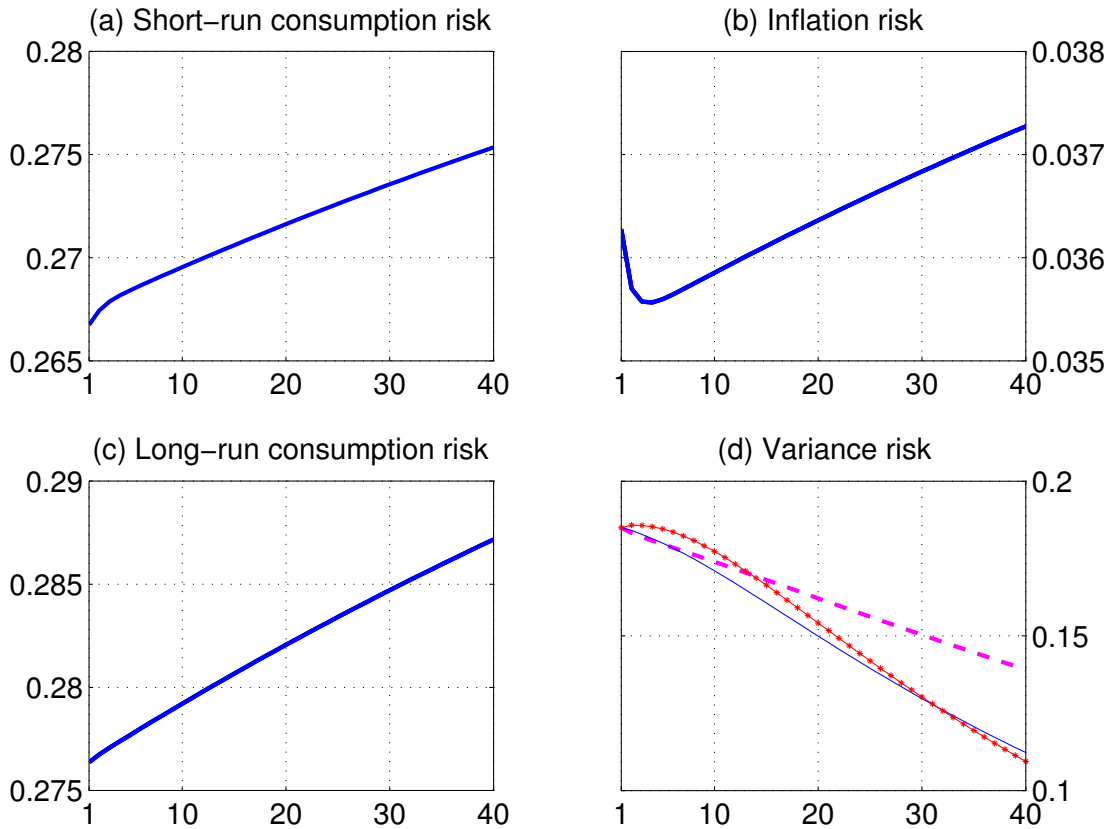
Figure 6: **Shock-price elasticity (identification "Fast Inflation")**

Panel (a) displays shock-price elasticity for the short-run consumption risk. Panel (b) displays shock-price elasticity for the inflation risk. Panel (c) displays shock-price elasticity for the long-run consumption risk. Panel (d) displays shock-price elasticity for the variance risk. The magenta dashed line is for the basket "Low", the blue solid line is for the basket "Intermediate", the red marked line is for the basket "High". The horizontal axes: from 1 quarter to 10 years. Identification "Fast Consumption". Quarterly.
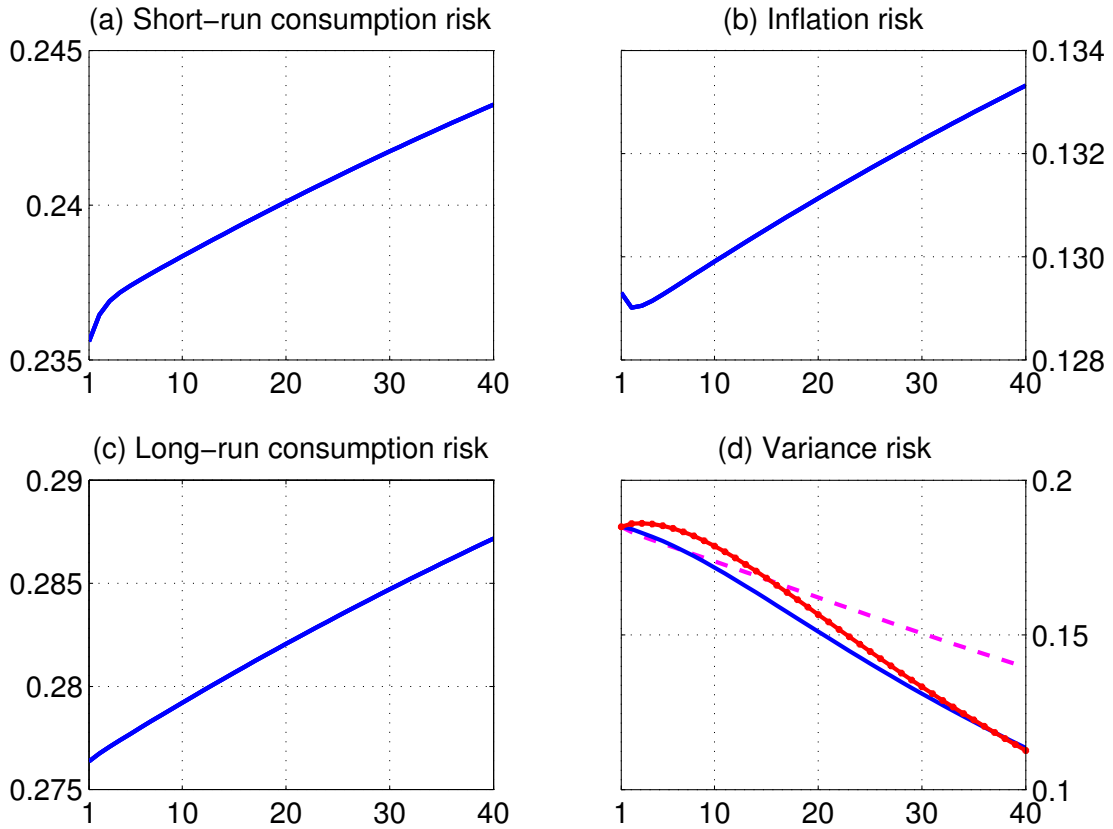
Figure 7: **Shock-price elasticity (identification "Fast Consumption")**

Panel (a) displays shock-price elasticity for the short-run consumption shock. Panel (b) displays shock-price elasticity for the inflation shock. Panel (c) displays shock-price elasticity for the long-run consumption risk shock. Panel (d) displays shock-price elasticity for the variance shock. The magenta dashed line is for the basket "Low", the blue solid line is for the basket "Intermediate", the red marked line is for the basket "High". Identification "Fast Inflation". Quarterly.

Figure 8: **Term-structure of currency risk premia**

(a) Fast Inflation
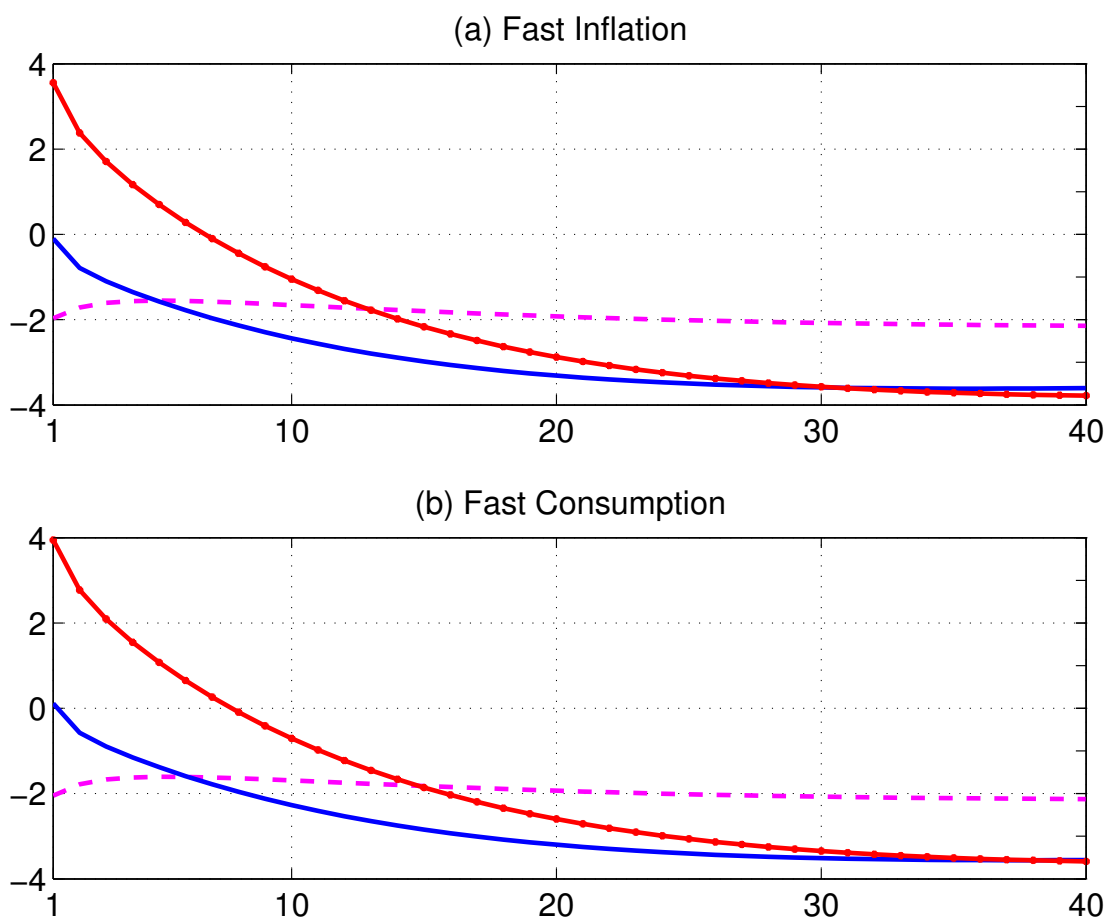
(b) Fast Consumption

Panel (a) displays the term-structure of currency risk premia under the identification "Fast Inflation", panel (b) displays the term-structure of currency risk premia under the identification "Fast Consumption". The magenta dashed line is for the basket "Low", the blue solid line is for the basket "Intermediate", the red marked line is for the basket "High". Quarterly. Annualized. In percent.

A Appendix

A.1 Model's solution and pricing restrictions

In this Appendix, I derive the model's solution. I briefly repeat the main building blocks for the ease of explicating.

The representative agent has recursive preferences

$$U_t = [(1 - \beta)c_t^\rho + \beta\mu_t(U_{t+1})^\rho]^{1/\rho} \qquad \text{(A.16)}$$

with the certainty equivalent function

$$\mu_t(U_{t+1}) = [E_t(U_{t+1}^\alpha)]^{1/\alpha}, \qquad \text{(A.17)}$$

and preference parameters α (risk aversion is $1 - \alpha$), β (subjective discount factor), and ρ ($1/(1 - \rho)$ is the elasticity of intertemporal substitution).

The consumption growth process is described by a vector autoregressive system

$$Y_{t+1} = F + GY_t + H\sigma_t\varepsilon_{t+1}, \qquad \text{(A.18)}$$

where $Y_{t+1} = (\log g_{t,t+1}, \ \log \pi_{t,t+1}, \ i_{t+1}^1, \ \sigma_{t+1}^2)'$.

To solve the model, I follow closely the solution method of Backus, Chernov, and Zin (2014). Since the utility U_t is determined by a constant elasticity of substitution recursion (A.16) and the certainty equivalent function is also homogenous of degree one, I scale (A.16) by consumption c_t:

$$u_t = [(1 - \beta) + \beta\mu_t(u_{t+1}g_{t,t+1})^\rho]^{1/\rho}, \qquad \text{(A.19)}$$

where $u_t = U_t/c_t$, and $g_{t,t+1} = c_{t+1}/c_t$.

The log pricing kernel under the recursive utility is

$$\log m_{t,t+1} = \log \beta + (\rho - 1)\log g_{t,t+1} + (\alpha - \rho)(\log(u_{t+1}g_{t,t+1}) - \log\mu_t(u_{t+1}g_{t,t+1}))$$

$$\text{(A.20)}$$

Appendix A.5 of Backus, Chernov, and Zin (2011) provides the corresponding derivation.

To derive the pricing kernel, I need to solve the equation (A.19). I use a log-linear approximation of (A.19) to obtain a closed-form solution to the value function $\log u_t$ and to the pricing kernel:

$$\log u_t \approx b_0 + b_1 \log \mu_t(g_{t,t+1} u_{t+1}), \tag{A.21}$$

where

$$
\begin{aligned}
b_1 &= \beta e^{\rho \log \mu} / [(1-\beta) + \beta e^{\rho \log \mu}], &\text{(A.22)}\\
b_0 &= \rho^{-1} \log\left[(1-\beta) + \beta e^{\rho \log \mu}\right] - b_1 \log \mu. &\text{(A.23)}
\end{aligned}
$$

The equation is exact if the elasticity of intertemporal substitution is equal to one. In such a case $b_0 = 0$ and $b_1 = \beta$. See Section III in Hansen, Heaton, and Li (2008) and Appendix A.7 in Backus, Chernov, and Zin (2014) for details about the log-linear approximation and its accuracy.

I guess that the solution to the equation (A.21) is an affine function of the four model's states:

$$\log u_t = \log u + P' Y_t, \tag{A.24}$$

where P is a vector of loadings $P = (p_g,\ p_\pi,\ p_i,\ p_\sigma)'$.

Next, I verify my guess. I compute the log of the certainty equivalent function

$$\log \mu_t(u_{t+1} g_{t,t+1}) = \underbrace{[\log u + e_1' F + P' F]}_{\log \mu} + [P'G + e_1' G] Y_t + \alpha [P + e_1]' \Sigma [P + e_1] \sigma_t^2 / 2,$$

$$\tag{A.25}$$

where $\Sigma = HH'$ and e_1 is a coordinate vector with the first element equal to 1. Then I substitute (A.24) and (A.25) to the equation (A.21) and collect and match the corresponding terms. The equation (A.21) has a constant term and four variables, hence I obtain the system of five equations:

$$
\begin{aligned}
\log u &= b_0 + b_1 \log u + b_1 e_1' F + b_1 P' F, &\text{(A.26)}\\
p_g &= b_1 (P + e_1)' G e_1 &\text{(A.27)}\\
p_\pi &= b_1 (P + e_1)' G e_2, &\text{(A.28)}\\
p_i &= b_1 (P + e_1)' G e_3, &\text{(A.29)}\\
p_\sigma &= b_1 (P + e_1)' G e_4 + \alpha b_1 (P + e_1)' \Sigma (P + e_1)/2, &\text{(A.30)}
\end{aligned}
$$

where e_i are the corresponding coordinate vectors.

Equations for p_g, p_π, and p_i are linear and therefore they result in unique solutions:

$$
\begin{aligned}
p_g &= A_g/B_g, \\
p_\pi &= A_\pi/B_\pi, \\
p_i &= A_i/B_i,
\end{aligned}
$$

where

$$
\begin{aligned}
A_g &= -(G_{11}b_1 - G_{11}G_{22}b1^2 + G_{12}G_{21}b_1^2 - G_{11}G_{33}b_1^2 + G_{13}G_{31}b_1^2 + G_{11}G_{22}G_{33}b_1^3 \\
&\quad - G_{11}G_{23}G_{32}b_1^3 - G_{12}G_{21}G_{33}b_1^3 + G_{12}G_{23}G_{31}b_1^3 + G_{13}G_{21}G_{32}b_1^3 - G_{13}G_{22}G_{31}b_1^3), \\
A_\pi &= -(G_{12}b_1 + G_{12}G_{23}b_1^2 - G_{13}G_{22}b_1^2), \\
A_i &= -(G_{13}b_1 + G_{13}G_{32}b_1^2 - G_{12}G_{33}b_1^2), \\
B_g &= B_\pi = B_i \\
&= G_{11}b_1 + G_{22}b_1 + G_{33}b_1 - G_{11}G_{22}b_1^2 + G_{12}G_{21}b_1^2 - G_{11}G_{33}b_1^2 + G_{13}G_{31}b_1^2 \\
&\quad - G_{22}G_{33}b_1^2 + G_{23}G_{32}b_1^2 + G_{11}G_{22}G_{33}b_1^3 - G_{11}G_{23}G_{32}b_1^3 - G_{12}G_{21}G_{33}b_1^3 \\
&\quad + G_{12}G_{23}G_{31}b_1^3 + G_{13}G_{21}G_{32}b_1^3 - G_{13}G_{22}G_{31}b_1^3 - 1
\end{aligned}
$$

The equation for p_σ is quadratic:

$$
A_\sigma p_\sigma^2 + B_\sigma p_\sigma + C_\sigma = 0,
$$

where

$$
\begin{aligned}
A_\sigma &= \alpha b_1 \Sigma_{44}/2, \\
B_\sigma &= \alpha b_1(\Sigma_{34}p_i + \Sigma_{24}p_\pi + \Sigma_{14}(p_g + 1)) + b_1 G_{44} - 1, \\
C_\sigma &= \alpha b_1((p_g + 1)(\Sigma_{13}p_i + \Sigma_{12}p_\pi + \Sigma_{11}(p_g + 1)) + p_i(\Sigma_{33}p_i + \Sigma_{23}p_\pi + \Sigma_{13}(p_g + 1)) \\
&\quad + p_\pi(\Sigma_{23}p_i + \Sigma_{22}p_\pi + \Sigma_{12}(p_g + 1)))/2 + (b_1 p_g G_{14} + b_1 p_\pi G_{24} + b_1 p_i G_{34} + b_1 G_{14}).
\end{aligned}
$$

This equation has two real roots if its discriminant Discr $= (B_\sigma^2 - 4A_\sigma C_\sigma)$ is positive. Only one real root is good, however. It has to be selected based on the property of stochastic stability (Hansen, 2012),

$$
p_\sigma = \frac{-B_\sigma + \text{sign}(B_\sigma)\text{Discr}^{1/2}}{2A_\sigma}.
$$

Finally, $\log u$ follows as

$$\log u = [b_0 + b_1 e_1' F + b_1 P' F]/[1 - b_1].$$

I plug the solution $\log u_t$ into (A.20) and obtain the final expression for the pricing kernel

$$
\begin{aligned}
\log m_{t,t+1} &= [\log \beta + (\rho - 1)e_1' F] + (\rho - 1)e_1' G Y_t - \alpha(\alpha - \rho)(P + e_1)' \Sigma (P + e_1)\sigma_t^2/2 \\
&+ [(\alpha - \rho)P + e_1(\alpha - 1)]' H \sigma_t \varepsilon_{t+1}
\end{aligned}
\tag{A.31}
$$

or

$$\log m_{t,t+1} = \log m + \eta' Y_t + q' \sigma_t \varepsilon_{t+1},$$

where

$$
\begin{aligned}
\eta &= (\rho - 1)G' e_1 - \alpha(\alpha - \rho)e_4(P + e_1)' \Sigma (P + e_1)/2, \\
q &= H'[(\alpha - \rho)P + e_1(\alpha - 1)].
\end{aligned}
$$

Next, I derive a one-period real risk-free rate

$$
\begin{aligned}
r_{f,t}^1 &= -E_t(\log m_{t,t+1}) - Var_t(\log m_{t,t+1})/2 \\
&= -\log \beta - (\rho - 1)e_1' F - (\rho - 1)e_1' G Y_t + \alpha(\alpha - \rho)(P + e_1)' \Sigma (P + e_1)\sigma_t^2/2 \\
&- [(\alpha - \rho)P + e_1(\alpha - 1)]' \Sigma [(\alpha - \rho)P + e_1(\alpha - 1)]\sigma_t^2/2.
\end{aligned}
\tag{A.32}
$$

Finally, the nominal one-period rate is

$$
\begin{aligned}
i_t^1 &= r_{f,t}^1 + E_t(\log \pi_{t,t+1}) - Var_t(\log \pi_{t,t+1})/2 + cov_t(\log m_{t,t+1}, \log \pi_{t,t+1}) \\
&= -\log \beta - (\rho - 1)e_1' F + e_2' F - (\rho - 1)e_1' G Y_t + e_2' G Y_t + \alpha(\alpha - \rho)(P + e_1)' \Sigma (P + e_1)\sigma_t^2/2 \\
&- [(\alpha - \rho)P + e_1(\alpha - 1)]' \Sigma [(\alpha - \rho)P + e_1(\alpha - 1)]\sigma_t^2/2 - e_2' \Sigma e_2 \sigma_t^2/2 \\
&+ e_2' \Sigma [(\alpha - \rho)P + e_1(\alpha - 1)]\sigma_t^2.
\end{aligned}
\tag{A.33}
$$

Note that i_t^1 enters both the left-hand side and the right-hand side of (A.33), because the nominal yield i_t^1 is a part of the state-vector Y_t.

$$i_t^1 = A \log g_{t-1,t} + B \log \pi_{t-1,t} + C i_t^1 + D \sigma_t^2 + E,$$

where

$$A = -\log \beta - (\rho - 1)e_1'F + e_2'F,$$
$$B = -(\rho - 1)e_1'Ge_1 + e_2'Ge_1$$
$$C = -(\rho - 1)e_1'Ge_2 + e_2'Ge_2,$$
$$D = -(\rho - 1)e_1'Ge_3 + e_2'Ge_3,$$
$$E = -[(\alpha - \rho)P + e_1(\alpha - 1)]'\Sigma[(\alpha - \rho)P + e_1(\alpha - 1)]/2 - e_2'\Sigma e_2/2 + e_2'Ge_4$$
$$+e_2'\Sigma[(\alpha - \rho)P + e_1(\alpha - 1)] + \alpha(\alpha - \rho)(P + e_1)'\Sigma(P + e_1)/2$$
$$-(\rho - 1)e_1'Ge_4.$$

The expression (A.33) is not an equation which nails down the nominal rate, it is an identity. Therefore, to guarantee consistent pricing of the nominal yield, the following five restrictions must be satisfied:

$$A = 0, \quad B = 0, \quad C = 1, \quad D = 0, \quad E = 0.$$

Four restrictions $A = B = E = 0$, $C = 1$ are linear and can be written as

$$\frac{G_{21}}{G_{11}} = \frac{G_{22}}{G_{12}} = \frac{G_{23} - 1}{G_{13}} = \frac{F_2 - \log \beta}{F_1} = \rho - 1.$$

The other restriction is nonlinear and it involves the endogenous parameters p_g, p_π, p_i, and p_σ:

$$-[(\alpha - \rho)P + e_1(\alpha - 1)]'\Sigma[(\alpha - \rho)P + e_1(\alpha - 1)]/2 - e_2'\Sigma e_2/2 + e_2'Ge_4$$
$$+e_2'\Sigma[(\alpha - \rho)P + e_1(\alpha - 1)] + \alpha(\alpha - \rho)(P + e_1)'\Sigma(P + e_1)/2$$
$$-(\rho - 1)e_1'Ge_4 = 0. \tag{A.34}$$

A.2 Data description

Macro data come from the NIPA tables of the Bureau of Economic Analysis and CRSP. I use Table 2.1 (Personal income and its disposition), Table 2.3.4 (Personal indexes for personal consumption expenditures by major type of product) and Table 2.3.5 (Personal consumption expenditures by major type of product). I measure real consumption as per capita expenditure on non-durable goods and services. Non-durables and services is the sum of entries of the row 8 from Table 2.3.5 divided by entries of the row 8 from Table 2.3.4 and components of row 13 from Table 2.3.5 divided by components of row 13 from Table 2.3.4. I construct price index associated with personal consumption expenditures. Row 40 of the Table 2.1 provides population data.

Table 14 describes sources and availability of currency and fixed income data.

A.3 Fixed point problem

In this Appendix, I sketch the fixed point problem embedded in the equation (A.21).

1. I guess b_0 and b_1 and solve equations (A.26)-(A.30).

2. I compute $\log \mu$ from (A.25). Next, I evaluate (A.22) and (A.23) to obtain b_0' and b_1':

$$
\begin{aligned}
b_1' &= \beta e^{\rho \log \mu}/[(1-\beta) + \beta e^{\rho \log \mu}], \\
b_0' &= \rho^{-1} \log \left[(1-\beta) + \beta e^{\rho \log \mu}\right] - b_1 \log \mu.
\end{aligned}
$$

3. If b_0' and b_1' are not close enough to the initial values of b_0 and b_1, I set $b_0 = b_0'$ and $b_1 = b_1'$ and return to Stage 2.

I iterate until I achieve convergence. I set the following convergence criterion: $(b_0 - b_0')^2 + (b_1 - b_1')^2 < 10^{-18}$.

A.4 Shock elasticity

In this section, I follow lead of Borovička and Hansen (2013) and derive the shock-exposure and the shock-price elasticity for the four sources of consumption risk ε_{t+1}.

Shock-exposure elasticity

The shock-exposure elasticity quantifies the term-structure of marginal quantities of risk. It depends on the functional form of the cash flow process and the evolution of the model's states.

The cash flow process is

$$
\log \delta_{t,t+1} = \log \delta + \mu' Y_t + \xi' \sigma_t \varepsilon_{t+1},
$$

where without loss of generality, I omit the idiosyncratic shock v_{t+1}.

The dynamics of the model's states is summarized in the vector autoregression:

$$
Y_{t+1} = F + G Y_t + H \sigma_t \varepsilon_{t+1}.
$$

The shock-exposure elasticity has the following mathematical representation

$$\ell_\delta(Y_t, \tau) = \left. \frac{\mathrm{d} \log E[\tilde{\delta}_{t,t+\tau}|Y_t]}{\mathrm{dv}} \right|_{v=0} = \alpha_h(Y_t) \cdot \tilde{E}_\delta(\varepsilon_{t+1}|Y_t),$$

where $\alpha_h(Y_t)$ is a vector which selects one source of risk ($\alpha_h(Y_t)\cdot\varepsilon_{t+1}$ has a unit standard deviation) and $\tilde{E}_\delta$ is an operator of the mathematical expectation under the change of measure represented by the random variable[29]

$$L_{t,\tau}^\delta = \frac{\delta_{t,t+1}E(\delta_{t,t+\tau}/\delta_{t,t+1}|Y_{t+1})}{E(\delta_{t,t+1}E(\delta_{t,t+\tau}/\delta_{t,t+1}|Y_{t+1})|Y_t)}.$$

I derive the shock exposure elasticity by using the multiplicative factorization of the multi-period cash flow and applying the law of iterated expectations a number of times.

First, I compute $L_{t,1}^\delta$:

$$L_{t,1}^\delta = \frac{\delta_{t,t+1}}{E(\delta_{t,t+1}|Y_t)} = \frac{\exp\left(\xi'\varepsilon_{t+1}\sigma_t\right)}{\exp\left(\xi'\xi\sigma_t^2/2\right)} = \frac{\exp\left(\tilde{e}_\delta'(0,Y_t)\varepsilon_{t+1}\right)}{\exp\left(\tilde{e}_\delta'(0,Y_t)\tilde{e}_\delta(0,Y_t)/2\right)},$$

where $\tilde{E}_\delta(\varepsilon_{t+1}|Y_t) = \tilde{e}_\delta(0,Y_t)$ and note that

$$\ell_\delta(Y_t, 1) = \alpha_h(Y_t) \cdot \xi\sigma_t.$$

Next, I use the law of iterated expectations

$$
\begin{aligned}
E(\delta_{t,t+\tau}|Y_t) &= E(\delta_{t,t+1}\delta_{t+1,t+2}\cdots\delta_{t+\tau-1,t+\tau}|Y_t)\\
&= E(\delta_{t,t+1}E(\delta_{t+1,t+2}\cdots E(\delta_{t+\tau-1,t+\tau}|Y_{t+\tau-1})|\cdots|Y_{t+1})|Y_t)
\end{aligned}
$$

and compute $E(\delta_{t,t+\tau}|Y_t)$ recursively.

I start with

$$
\begin{aligned}
E(\delta_{t+\tau-1,t+\tau}|Y_{t+\tau-1}) &= \exp\left(\log\delta + \mu Y_{t+\tau-1} + \xi'\xi\sigma_{t+\tau-1}^2/2\right)\\
&= \exp\left(\mathcal{A}_0(1) + \mathcal{A}_g(1)\log g_{t+\tau-2,t+\tau-1} + \mathcal{A}_\pi(1)\log \pi_{t+\tau-2,t+\tau-1} + \mathcal{A}_i(1)i_{t+\tau-1}^1 + \mathcal{A}_\sigma(1)\sigma_{t+\tau-1}^2\right),
\end{aligned}
$$

---

[29]For example, $\alpha_h(Y_t) = (1,\ 0,\ 0,\ 0)'\sigma_t$, where $E(\sigma_t^2) = 1$, or $\alpha_h(Y_t) = (1,\ 0,\ 0,\ 0)'$ selects the short-run consumption shock. Other specifications of $\alpha_h(Y_t)$ are possible.

66

where

$$\begin{aligned}
\mathcal{A}_0(1) &= \log \delta, \\
\mathcal{A}_g(1) &= \mu_g, \\
\mathcal{A}_\pi(1) &= \mu_\pi, \\
\mathcal{A}_i(1) &= \mu_i, \\
\mathcal{A}_\sigma(1) &= \mu_\sigma + \xi'\xi/2.
\end{aligned}$$

Next, I compute

$$E(\delta_{t+\tau-2,t+\tau-1}E(\delta_{t+\tau-1,t+\tau}|Y_{t+\tau-1})|Y_{t+\tau-2}) =$$
$$= \exp\left(\mathcal{A}_0(2) + \mathcal{A}_g(2)\log g_{t+\tau-3,t+\tau-2} + \mathcal{A}_\pi(2)\log \pi_{t+\tau-3,t+\tau-2} + \mathcal{A}_i(2)i^1_{t+\tau-2} + \mathcal{A}_\sigma(2)\sigma^2_{t+\tau-2}\right),$$

where

$$\begin{aligned}
\mathcal{A}_0(2) &= \log \delta + \mathcal{A}_0(1) + [\mathcal{A}_g(1),\ \mathcal{A}_\pi(1),\ \mathcal{A}_i(1),\ \mathcal{A}_\sigma(1)]F \\
&= \log \delta + \mathcal{A}_0(1) + \mathcal{A}_g(1)F_1 + \mathcal{A}_\pi(1)F_2 + \mathcal{A}_i(1)F_3 + \mathcal{A}_\sigma(1)F_4, \\
\mathcal{A}_g(2) &= \mu_g + \mathcal{A}_g(1)G_{11} + \mathcal{A}_\pi(1)G_{21} + \mathcal{A}_i(1)G_{31} + \mathcal{A}_\sigma(1)G_{41}, \\
\mathcal{A}_\pi(2) &= \mu_\pi + \mathcal{A}_g(1)G_{12} + \mathcal{A}_\pi(1)G_{22} + \mathcal{A}_i(1)G_{32} + \mathcal{A}_\sigma(1)G_{42}, \\
\mathcal{A}_i(2) &= \mu_i + \mathcal{A}_g(1)G_{13} + \mathcal{A}_\pi(1)G_{23} + \mathcal{A}_i(1)G_{33} + \mathcal{A}_\sigma(1)G_{43}, \\
\mathcal{A}_\sigma(2) &= \mu_\sigma + \mathcal{A}_g(1)G_{14} + \mathcal{A}_\pi(1)G_{24} + \mathcal{A}_i(1)G_{34} + \mathcal{A}_\sigma(1)G_{44} \\
&+ 0.5([\mathcal{A}_g(1),\ \mathcal{A}_\pi(1),\ \mathcal{A}_i(1),\ \mathcal{A}_\sigma(1)]H + \xi')([\mathcal{A}_g(1),\ \mathcal{A}_\pi(1),\ \mathcal{A}_i(1),\ \mathcal{A}_\sigma(1)]H + \xi')'.
\end{aligned}$$

Finally, for a generic τ,

$$E(\delta_{t,t+\tau}|Y_t) = \exp\left(\mathcal{A}_0(\tau) + \mathcal{A}_g(\tau)\log g_{t-1,t} + \mathcal{A}_\pi(\tau)\log \pi_{t-1,t} + \mathcal{A}_i(\tau)i^1_t + \mathcal{A}_\sigma(\tau)\sigma^2_t\right),$$

where the parameters of the conditional expectation are determined by the system of

difference equations:

$$
\begin{aligned}
\mathcal{A}_0(\tau) &= \log\delta + \mathcal{A}_0(\tau-1) + [\mathcal{A}_g(\tau-1),\ \mathcal{A}_\pi(\tau-1),\ \mathcal{A}_i(\tau-1),\ \mathcal{A}_\sigma(\tau-1)]F, \\
\mathcal{A}_g(\tau) &= \mu_g + \mathcal{A}_g(\tau-1)G_{11} + \mathcal{A}_\pi(\tau-1)G_{21} + \mathcal{A}_i(\tau-1)G_{31} + \mathcal{A}_\sigma(\tau-1)G_{41}, \\
\mathcal{A}_\pi(\tau) &= \mu_\pi + \mathcal{A}_g(\tau-1)G_{12} + \mathcal{A}_\pi(\tau-1)G_{22} + \mathcal{A}_i(\tau-1)G_{32} + \mathcal{A}_\sigma(\tau-1)G_{42}, \\
\mathcal{A}_i(\tau) &= \mu_i + \mathcal{A}_g(\tau-1)G_{13} + \mathcal{A}_\pi(\tau-1)G_{23} + \mathcal{A}_i(\tau-1)G_{33} + \mathcal{A}_\sigma(\tau-1)G_{43}, \\
\mathcal{A}_\sigma(\tau) &= \mu_\sigma + \mathcal{A}_g(\tau-1)G_{14} + \mathcal{A}_\pi(\tau-1)G_{24} + \mathcal{A}_i(\tau-1)G_{34} + \mathcal{A}_\sigma(\tau-1)G_{44} \\
&\quad + 0.5([\mathcal{A}_g(\tau-1),\ \mathcal{A}_\pi(\tau-1),\ \mathcal{A}_i(\tau-1),\ \mathcal{A}_\sigma(\tau-1)]H + \xi') \\
&\quad\ ([\mathcal{A}_g(\tau-1),\ \mathcal{A}_\pi(\tau-1),\ \mathcal{A}_i(\tau-1),\ \mathcal{A}_\sigma(\tau-1)]H + \xi')'.
\end{aligned}
$$

In this case, the random variable associated with the change of measure is

$$
L_{t,\tau}^\delta = \frac{\exp\left(\tilde{e}_\delta'(\tau-1,Y_t)\varepsilon_{t+1}\right)}{\exp\left(0.5(\tilde{e}_\delta'(\tau-1,Y_t)\tilde{e}_\delta(\tau-1,Y_t))'\right)},
$$

where

$$
\tilde{e}_\delta(\tau-1,Y_t) = ([\mathcal{A}_g(\tau-1),\ \mathcal{A}_\pi(\tau-1),\ \mathcal{A}_i(\tau-1),\ \mathcal{A}_\sigma(\tau-1)]H + \xi')'\sigma_t.
$$

The shock-exposure elasticity immediately follows

$$
\begin{aligned}
\ell_\delta(Y_t,\tau) &= \alpha_h(Y_t)\cdot\tilde{e}_\delta(\tau-1,Y_t) \\
&= \alpha_h(Y_t)\cdot([\mathcal{A}_g(\tau-1),\ \mathcal{A}_\pi(\tau-1),\ \mathcal{A}_i(\tau-1),\ \mathcal{A}_\sigma(\tau-1)]H + \xi')'\sigma_t.
\end{aligned}
$$

Shock-price elasticity

To compute the shock-price elasticity (5.15), I need to evaluate the following object

$$
\ell_v(Y_t,\tau) = \frac{\mathrm{d}\log E[\tilde{\delta}_{t,t+\tau}m_{t,t+\tau}|Y_t]}{\mathrm{d}v}\bigg|_{v=0}
$$

which has a similar mathematical structure to the shock-exposure elasticity. Borovička and Hansen (2013) call this object the shock-value elasticity. The shock-price elasticity, $\ell_p(Y_t,\tau)$, follows by means of subtracting the shock-value elasticity from the shock-exposure elasticity:

$$
\ell_p(Y_t,\tau) = \ell_\delta(Y_t,\tau) - \ell_v(Y_t,\tau).
$$

The derivation of the shock-value elasticity mirrors one of the shock-exposure elasticity.

Therefore, the solution has a similar mathematical representation:

$$\ell_v(Y_t, \tau) = \alpha_v(Y_t) \cdot ([\mathcal{B}_g(\tau-1),\ \mathcal{B}_\pi(\tau-1),\ \mathcal{B}_i(\tau-1),\ \mathcal{B}_\sigma(\tau-1)]H + \xi' + q')'\sigma_t,$$

where $\mathcal{B}_g$, $\mathcal{B}_\pi$, $\mathcal{B}_i$, and $\mathcal{B}_\sigma$ solve the system of difference equations:

$$
\begin{aligned}
\mathcal{B}_0(\tau) &= \log\delta + \log m + \mathcal{B}_0(\tau-1) + [\mathcal{B}_g(\tau-1),\ \mathcal{B}_\pi(\tau-1),\ \mathcal{B}_i(\tau-1),\ \mathcal{B}_\sigma(\tau-1)]F, \\
\mathcal{B}_g(\tau) &= \mu_g + \eta_g + \mathcal{B}_g(\tau-1)G_{11} + \mathcal{B}_\pi(\tau-1)G_{21} + \mathcal{B}_i(\tau-1)G_{31} + \mathcal{B}_\sigma(\tau-1)G_{41}, \\
\mathcal{B}_\pi(\tau) &= \mu_\pi + \eta_\pi + \mathcal{B}_g(\tau-1)G_{12} + \mathcal{B}_\pi(\tau-1)G_{22} + \mathcal{B}_i(\tau-1)G_{32} + \mathcal{B}_\sigma(\tau-1)G_{42}, \\
\mathcal{B}_i(\tau) &= \mu_i + \eta_i + \mathcal{B}_g(\tau-1)G_{13} + \mathcal{B}_\pi(\tau-1)G_{23} + \mathcal{B}_i(\tau-1)G_{33} + \mathcal{B}_\sigma(\tau-1)G_{43}, \\
\mathcal{B}_\sigma(\tau) &= \mu_\sigma + \eta_\sigma + \mathcal{B}_g(\tau-1)G_{14} + \mathcal{B}_\pi(\tau-1)G_{24} + \mathcal{B}_i(\tau-1)G_{34} + \mathcal{B}_\sigma(\tau-1)G_{44} \\
&+ 0.5(q' + \xi' + [\mathcal{B}_g(\tau-1),\ \mathcal{B}_\pi(\tau-1),\ \mathcal{B}_i(\tau-1),\ \mathcal{B}_\sigma(t-1)]H) \\
&\quad (q' + \xi' + [\mathcal{B}_g(\tau-1),\ \mathcal{B}_\pi(\tau-1),\ \mathcal{B}_i(\tau-1),\ \mathcal{B}_\sigma(\tau-1)]H)'.
\end{aligned}
$$

with the following initial conditions

$$
\begin{aligned}
\mathcal{B}_0(1) &= \log m + \log\delta, \\
\mathcal{B}_g(1) &= \mu_g + \eta_g, \\
\mathcal{B}_\pi(1) &= \mu_\pi + \eta_\pi, \\
\mathcal{B}_i(1) &= \mu_i + \eta_i, \\
\mathcal{B}_\sigma(1) &= \mu_\sigma + \eta_\sigma + (\xi + q)'(\xi + q)/2
\end{aligned}
$$

and

$$\ell_v(Y_t, 1) = \alpha_h(Y_t) \cdot (\xi + q)\sigma_t.$$

A.5   Regularity conditions

Existence and uniqueness of the fixed point problem

I work with a model that features recursive utility. Recursive utility is the solution to the non-linear forward looking difference equation with infinite horizon in the Markov environment. I use regularity conditions of Hansen and Scheinkman (2012) to check that estimated parameterizations satisfy the existence and uniqueness conditions of continuation value processes, i.e., there exists a unique solution to the fixed point problem (A.21).

Hansen and Scheinkman (2012) show that the solution to the fixed point problem (A.21) is closely related to a Perron-Frobenius eigenvalue equation of the following type:

$$\mathcal{P}v(Y_t) = e^\nu v(Y_t),$$

where

$$\mathcal{P}v(Y_t) = E_t(\exp{(\alpha \log g_{t,t+1})}v(Y_{t+1})), \tag{A.35}$$

ν is an eigenvalue, $v(Y)$ is a principal eigenvector, and α is the preference parameter as above.

Solve the problem (A.35) by a guess and verify method. Guess that the solution is exponentially affine in the model states:

$$v(Y_t) = \exp{(\log v + K'Y_t)}, \tag{A.36}$$

where $K = (K_g,\ K_\pi,\ K_i,\ K_\sigma)'$.

Use functional form (A.36) in equation (A.35):

$$
\begin{aligned}
\log E_t(\exp{(\alpha \log g_{t,t+1})}v(Y_{t+1})) &= \log E_t(\exp{(\alpha e_1' Y_{t+1})}\exp{(\log v + K'Y_{t+1})})\\
&= \log E_t(\exp{(\log v + (\alpha e_1 + K)'F + (\alpha e_1 + K)'GY_t + (\alpha e_1 + K)'H\sigma_t\varepsilon_{t+1})})\\
&= \log v + (\alpha e_1 + K)'F + (\alpha e_1 + K)'GY_t + (\alpha e_1 + K)'HH'(\alpha e_1 + K)\sigma_t^2/2.\\
&= \log v + (\alpha e_1 + K)'F + (\alpha e_1 + K)'GY_t + (\alpha e_1 + K)'\Sigma(\alpha e_1 + K)\sigma_t^2/2.
\end{aligned}
$$

Verify guess by collecting matching terms:

$$
\begin{aligned}
\nu &= (\alpha e_1 + K)'F,\\
K &= G'(\alpha e_1 + K) + e_4(\alpha e_1 + K)'\Sigma(\alpha e_1 + K)/2.
\end{aligned}
$$

Thus, I have four equations to solve for K_g, K_π, K_i, and K_σ:

$$
\begin{aligned}
K_g &= e_1'G'(\alpha e_1 + K), & \text{(A.37)}\\
K_\pi &= e_2'G'(\alpha e_1 + K), & \text{(A.38)}\\
K_i &= e_3'G'(\alpha e_1 + K), & \text{(A.39)}\\
K_\sigma &= e_4'G'(\alpha e_1 + K) + (\alpha e_1 + K)'\Sigma(\alpha e_1 + K)/2. & \text{(A.40)}
\end{aligned}
$$

Equations (A.37-A.39) are linear. Therefore, there are unique solutions $K_g = A_g/B_g$,

$K_\pi = A_\pi/B_\pi$, and $K_i = A_i/B_i$, where

$$A_g = -(G_{11}\alpha - G_{11}G_{22}\alpha + G_{12}G_{21}\alpha - G_{11}G_{33}\alpha + G_{13}G_{31}\alpha + G_{11}G_{22}G_{33}\alpha$$
$$-G_{11}G_{23}G_{32}\alpha - G_{12}G_{21}G_{33}\alpha + G_{12}G_{23}G_{31}\alpha + G_{13}G_{21}G_{32}\alpha - G_{13}G_{22}G_{31}\alpha),$$
$$A_\pi = -\alpha(G_{12} - G_{12}G_{33} + G_{13}G_{32}),$$
$$A_i = -\alpha(G_{13} + G_{12}G_{23} - G_{13}G_{22}),$$
$$B_g = B_\pi = B_i = (G_{11} + G_{22} + G_{33} - G11G_{22} + G_{12}G_{21} - G_{11}G_{33} + G_{13}G_{31} - G_{22}G_{33}$$
$$+G_{23}G_{32} + G_{11}G_{22}G_{33} - G_{11}G_{23}G_{32} - G_{12}G_{21}G_{33} + G_{12}G_{23}G_{31} + G_{13}G_{21}G_{32}$$
$$-G_{13}G_{22}G_{31} - 1),$$

The last equation (A.39) has two real roots if $Discr = B_\sigma^2 - 4A_\sigma C_\sigma > 0$, where

$$
\begin{aligned}
A_\sigma &= \Sigma_{44}/2, \\
B_\sigma &= (\Sigma_{34}K_i + \Sigma_{24}K_\pi + \Sigma_{14}(K_g + \alpha)) + G_{44} - 1, \\
C_\sigma &= (K_g + \alpha)(\Sigma_{11}(K_g + \alpha)/2 + K_i\Sigma_{13}/2 + K_\pi\Sigma_{12}/2) + K_i(\Sigma_{13}(K_g + \alpha) + K_i\Sigma_{33} + K_\pi\Sigma_{23})/2, \\
&+ K_\pi(\Sigma_{12}(K_g + \alpha) + K_i\Sigma_{23} + K_\pi\Sigma_{22})/2 + (\alpha + K_g)G_{14} + K_\pi G_{24} + K_i G_{34}.
\end{aligned}
$$

I choose such a root that satisfies the requirement of stochastic stability:

$$K_\sigma = \frac{-B_\sigma + sign(B_\sigma)\sqrt{Discr}}{2A_\sigma}.$$

Finally I compute $\nu = \alpha F_{11} + K'F$ and check that

$$-\log \beta > \frac{\rho\nu}{\alpha}. \tag{A.41}$$

Parameterizations that satisfy this inequality remain in the set. Parameterizations for which real solutions to the quadratic equation (A.40) do not exist or exist but inequality (A.41) is not satisfied are left out from consideration. See Hansen and Scheinkman (2012) for further details.

Limiting dynamics of shock elasticities

I consider economic environment with stochastic growth and study cash flow risk exposures and associated with them compensations (prices of risk) across alternative horizons. These objects require studying steady distributions under alternative probability

measures. One change of measure is associated with the cash-flow functional $\delta_{t,t+\tau}$, whereas the other one is associated with the valuation functional $\delta_{t,t+\tau}m_{t,t+\tau}$. The changes of measure arise naturally from a multiplicative factorization of the functionals and are related to the problem of finding the principal eigenvalues and eigenfunctions. Hansen and Scheinkman (2009) discuss the sufficient conditions for the existence and uniqueness of the relevant eigenvalue and eigenfunction that lead to a stochastically stable change of measure in the continuous-time environment. Their result has a direct counterpart for discrete-time processes that I use in this paper.

To streamline presentation, I illustrate the solution to the eigenvalue-eigenvector problem for the growth functional $\delta_{t,t+1}$. Similar logic and computational algorithm applies to the valuation functional $\delta_{t,t+1}m_{t,t+1}$:

$$\delta_{t,t+1}m_{t,t+1} = \exp\left(\log\delta + \log m + (\eta + \mu)'Y_t + (q + \xi)'\sigma_t\varepsilon_{t+1}\right). \tag{A.42}$$

The log cash-flow process is

$$\log\delta_{t,t+1} = \log\delta + \mu'Y_t + \xi'\sigma_t\varepsilon_{t+1}.$$

Without loss of generality, I ignore the idiosyncratic shock.

To study the limiting behaviour of this functional, solve the eigenvalue-eigenvector problem:

$$\mathcal{P}v(Y_t) = \exp(\nu)v(Y_t),$$

where

$$\mathcal{P}v(Y_t) = E_t[\delta_{t,t+1}v(Y_{t+1})].^{30}$$

Solve by a guess and verify method. Guess that the solution has the following functional form: $e(Y_t) = \exp(\log v + K'Y_t)$, where $K = (K_g, K_\pi, K_i, K_\sigma)'$. Verify the guess and solve for K_g, K_π, K_i, and K_σ correspondingly:

$$\mu_g + K_g G_{11} + K_\pi G_{21} + K_i G_{31} = K_g,$$
$$\mu_\pi + K_g G_{12} + K_\pi G_{22} + K_i G_{32} = K_\pi,$$
$$\mu_i + K_g G_{13} + K_\pi G_{23} + K_i G_{33} = K_i,$$
$$\mu_\sigma + K_g G_{14} + K_\pi G_{24} + K_i G_{34} + K_\sigma G_{44} + 0.5(\xi + KH)(\xi + KH)' = K_\sigma.$$

Finally, $\nu = \log v + K'F$.

Three first equations determining K_g, K_π, and K_i are linear and hence there is a unique solution to this system. The equation for K_σ is quadratic, so potentially there are two real solutions. I choose solution based on the criterion of stochastic stability. In particular, $K_g = A_g/B_g$, $K_\pi = A_\pi/B_\pi$, $K_i = A_i/B_i$, where

$$
\begin{aligned}
A_g &= -(\mu_g + G_{21}\mu_\pi - G_{22}\mu_g + G_{31}\mu_i - G_{33}\mu_g + G_{21}G_{32}\mu_i - G_{22}G_{31}\mu_i \\
&\quad - G_{21}G_{33}\mu_\pi + G_{23}G_{31}\mu_\pi + G_{22}G_{33}\mu_g - G_{23}G_{32}\mu_g), \\
A_\pi &= -(\mu_\pi - G_{11}\mu_\pi + G_{12}\mu_g + G_{32}\mu_i - G_{33}\mu_{pi} - G_{11}G_{32}\mu_i + G_{12}G_{31}\mu_i \\
&\quad + G_{11}G_{33}\mu_\pi - G_{13}G_{31}\mu_\pi - G_{12}G_{33}\mu_g + G_{13}G_{32}\mu_g) \\
A_i &= -(\mu_i - G_{11}\mu_i + G_{13}\mu_g - G_{22}\mu_i + G_{23}\mu_{pi} + G_{11}G_{22}\mu_i - G_{12}G_{21}\mu_i \\
&\quad - G_{11}G_{23}\mu_\pi + G_{13}G_{21}\mu_\pi + G_{12}G_{23}\mu_g - G_{13}G_{22}\mu_g) \\
B_g &= B_\pi = B_i = (G_{11} + G_{22} + G_{33} - G_{11}G_{22} + G_{12}G_{21} - G_{11}G_{33} + G_{13}G_{31} \\
&\quad - G_{22}G_{33} + G_{23}G_{32} + G_{11}G_{22}G_{33} - G_{11}G_{23}G_{32} - G_{12}G_{21}G_{33} + G_{12}G_{23}G_{31} \\
&\quad + G_{13}G_{21}G_{32} - G_{13}G_{22}G_{31} - 1).
\end{aligned}
$$

and if $Discr > 0$

$$
K_\sigma = \frac{-B_\sigma + sign(B_\sigma)\sqrt{Discr}}{2A_\sigma},
$$

where

$$
\begin{aligned}
Discr &= B_\sigma^2 - 4A_\sigma C_\sigma, \\
A_\sigma &= \Sigma_{44}/2, \\
B_\sigma &= H_{41}\xi_g + H_{43}\xi_i + H_{42}\xi_\pi + H_{44}\xi_\sigma + K_g\Sigma_{14} + K_i\Sigma_{34} + K_\pi\Sigma24 + G44 - 1, \\
C_\sigma &= K_g(K_g\Sigma_{11} + K_i\Sigma_{13} + K_\pi\Sigma12)/2 + K_i(K_g\Sigma_{13} + K_i\Sigma_{33} + K_\pi\Sigma_{23})/2 \\
&\quad + K_\pi(K_g\Sigma_{12} + K_i\Sigma_{23} + K_\pi\Sigma22)/2 + K_g(H_{11}\xi_g + H_{13}\xi_i + H_{12}\xi_\pi + H_{14}\xi_\sigma) \\
&\quad + K_i(H_{31}\xi_g + H_{33}\xi_i + H_{32}\xi_\pi + H_{34}\xi_\sigma) + K_\pi(H_{21}\xi_g + H_{23}\xi_i + H_{22}\xi_\pi + H_{24}\xi_\sigma) \\
&\quad + \mu_\sigma + K_gG_{14} + K_\pi G_{24} + KiG_{34} + \xi\xi'/2.
\end{aligned}
$$

If there is a solution to the eigenvalue-eigenvector problem, the limiting behaviour of shock elasticities is well-defined. However, not all model parameterizations, that deliver solutions to the Perron-Frobenius problem associated with the cash-flow and value functionals, are feasible. Specifically, I have to ensure that the recursions for the parameters

of shock elasticities converge to their corresponding limiting values. For example, for the shock-elasticity case

$$\mathcal{A}_g(\tau) \xrightarrow[\tau \to \infty]{} K_g,$$
$$\mathcal{A}_\pi(\tau) \xrightarrow[\tau \to \infty]{} K_\pi$$
$$\mathcal{A}_i(\tau) \xrightarrow[\tau \to \infty]{} K_i$$
$$\mathcal{A}_\sigma(\tau) \xrightarrow[\tau \to \infty]{} K_\sigma$$

Because there are two real solutions K_σ, feasibility of a particular parameterization depends on whether initial conditions $(\mathcal{A}_g(1), \mathcal{A}_\pi(1), \mathcal{A}_i(1), \mathcal{A}_\sigma(1))$ are proper, i.e., the recursive system converges to a stochastically stable solution. This is a well known problem in the theory of differential and difference equations.[31] Thus, I check it if for a given estimated parameterization of cash-flow process (i.e., parameters of vectors μ and ξ), $\mathcal{A}_\sigma(\tau)$ converges in the limit to a stable K_σ. I select only those parametrizations for which this is true.

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[31]Barnett and Cameron (1985) discuss stability of solutions to non-linear differential (difference) equations in section 5.5 (Liapunov's linearisation theorem).

Table 14
Data description

Country	Data availability	FX data/Datastream Mnemonics	Source of term-structure data/Mnemonics
Australia	1987:I – 2011:IV	AUSTDOL and USDOLLR	Wright [1987:I – 2009:I] Bloomberg [2009:II – 2011:IV]: ADSW1, ..., ADSW10
Canada	1986:I – 2011:IV	CNDOLLR and USDOLLR	Wright [1986:I – 2009:I] Bloomberg [2009:II – 2011:IV]: CDSW1, ..., CDSW10
Denmark	1988:III – 2011:IV	DANISHK and USDOLLR	Bloomberg [1988:III – 2011:IV]: DKSW1, ..., DKSW10
Euro Area	2004:III – 2011:IV	EURSTER and USDOLLR	Bloomberg [2004:III – 2011:IV]: EUSW1V3, ..., EUSW10V3
Germany	1986:I – 1998:IV	DMARKER and USDOLLR	Wright [1986:I – 1998:IV]
Japan	1986:I – 2011:IV	JAPAYEN and USDOLLR	Wright [1986:I – 2009:I] Bloomberg [2009:II – 2011:IV]: JYSW1, ..., JYSW10
New Zealand	1990:I – 2011:IV	NZDOLLR and USDOLLR	Wright [1990:I – 2009:I] Bloomberg [2009:II – 2011:IV]: NDSW1, ..., NDSW10
Norway	1998:I – 2011:IV	NORKRON and USDOLLR	Wright [1998:I – 2009:I] Bloomberg [2009:II – 2011:IV]: NKSW1, ..., NKSW10
South Africa	1997:II – 2011:IV	COMRAND and USDOLLR	Bloomberg [1997:II – 2011:IV]: SASW1, ..., SASW10
Sweden	1992:IV – 2011:IV	SWEKRON and USDOLLR	Wright [1992:IV – 2009:I] Bloomberg [2009:II – 2011:IV]: SKSW1, ..., SKSW10
Switzerland	1988:I – 2011:IV	SWISSFR and USDOLLR	Wright [1986:I – 2009:I] Bloomberg [2009:II – 2011:IV]: SFSW1, ..., SFSW10
UK	1986:I – 2011:IV	USDOLLR	Wright [1986:I – 2009:I] Bloomberg [2009:II – 2011:IV]: BPSW1, ..., BPSW10

Notes: Data availability and data sources.