



## Test for Aggregational Gaussianity (AG) in Petroleum Prices Returns

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### Authors' contributions

This work was carried out in collaboration between both authors. Both authors read and approved the final manuscript.

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### Abstract

The work aims at investigating and establishing if Aggregational Gaussianity, (AG) is in the dynamics of petroleum prices. This AG aspect is the phenomenon in which the empirical distribution of log-returns tends to normality (or as the time scale over which the returns are calculated increases). In order to achieve this, the petroleum price series was tested for arch effects. In addition, tests for Aggregational Gaussianity, (AG) were carried out using qualitative (graphical) approach and inferential approach, (involving statistical inference). The study shows that the presence of arch effects does not guarantee existence of AG. It is also observed that qualitative (graphical) approach may suggest normality and hence, presence of AG, on the other hand, inferential approach (involving statistical tests) gives a better picture of the actual conclusion, of the presence (or otherwise) of AG in the data set, with a 99.97% rejection from normality by the three tests-Kolmogorov-simonov, Shapiro-Wilks, and Anderson-darling. In the circumstance, there is no evidence to confirm a discernible presence of AG in the dynamics of petroleum prices. The non-existence of AG in the study shows the instability in the dynamics of petroleum prices, since one cannot invoke normality as an invariant property this, among other factors, make the economy unstable as it is oil- driven. However, since the highest percentage of the budget for the country is based on the petroleum sales, which as this study reveals is unstable, hence, diversification of the economy is proposed. The softwares used in the work are Eviews 10, Minitab 18, Spss 17, Easy-fit 5.6 professional, and R 3.2.2.

*Keywords:* Stylized facts; Aggregational Gaussianity (AG); arch effects; asset returns; economy.

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## 1 INTRODUCTION

In the context of financial time-series, log-returns computed over shorter time periods are known to be leptokurtotic (heavytailed) and often skewed. As the time-interval over which the returns are calculated is increased, the distribution of returns is propagated normally. According to Daniel, David & Obeid [1], Aggregational Gaussianity (AG) is the phenomenon in which the empirical distribution of log-returns tends to normality (or as the time scale over which the returns are calculated increases). This implies that the shape of the distribution varies at different time scales, or terms. Embrechts, Kluppelberg, & Mikosch [2] observed that Extreme-value theory is mostly useful in modelling heavy tails associated with returns in finance. Kulikova and Taylor [3] opined that Intermediate returns such as daily returns frequently exhibit log-linear properties which could be modeled using normal Inverse Gaussian or hyperbolic distributions.

On the other hand, Eberlein & Keller [4] noted that the absence of AG suggests that stable distributions are unsuitable models for log-returns which imply that the underlying distribution of price changes is a normal-mixture. Moreover, many experts in financial time series have made similar observations. It is in this light that Bingham, Kiesel, & Schmidt [5] remarked that the general rule of thumb is that terms in excess of 16 days typically conform to normality. This is evident in the work of Herlemont [6] where AG is documented from three months on the cac-40, and Boavida [7] where AG is documented for six months, but not for twelve months in the US markets. The specification of appropriate volatility model for capturing variations in stock returns cannot be overemphasized, as it helps investors in their risk management decision and portfolio adjustment [8]. Also, Engle [9] proposed the autoregressive conditional heteroscedastic (ARCH) model to capture volatility of stock returns. Bollerslev [10] and Taylor [11] proposed the generalized autoregressive conditional heteroscedastic (GARCH) model. In this study, we test for the existence of AG in petroleum prices using graphical and inferential approaches.

## 2 METHODOLOGY

### 2.1 Data

The data for this work are monthly Petroleum Prices (sales) in US dollar per barrel from January, 2000 to July, 2017 from the Central Bank of Nigeria database website [www.cbn.gov.ng](http://www.cbn.gov.ng) under the Data & Statistics heading and the Petroleum Crude Oil Price subheading

### 2.2 Testing for ARCH Effects

The Oil Price was plotted against time to discover the volatile nature of the variable after which it proceeded to test for arch effects. The steps for arch tests using LM test of Engle (9) are as follows:

- (a) Run a postulated linear regression of the form

$$y_t = \beta_1 + \beta_2 X_{2t} + \beta_3 X_{3t} + \beta_4 X_{4t} + u_t \quad (1)$$

- (b) Square the residuals and regress on  $m$  own lags to test for ARCH of order  $m$ , i.e., run the regression

$$\hat{U}_t^2 = \gamma_0 + \gamma_1 \hat{U}_{t-1}^2 + \dots + \gamma_m \hat{U}_{t-m}^2 + V_t \quad (2)$$

Where  $V_t$  is the error term? Obtain  $R^2$  from this equation.

- (c) The test statistic is defined as  $TR^2$  (the number of observations multiplied by the coefficient of determination [multiple correlation]) from the last regression and is distributed as  $\chi_m^2$  i.e.,  $\chi_m^2 \sim TR^2$ .

- (d) The null and alternative hypotheses are:

$$H_0 : \gamma_1 = 0 \text{ and } \gamma_2 = 0 \text{ and } \gamma_3 = 0 \text{ and } \dots \gamma_m = 0 \rightarrow \text{no arch effect}$$

$$H_1 : \gamma_1 \neq 0 \text{ or } \gamma_2 \neq 0 \text{ or } \gamma_3 \neq 0 \text{ or } \dots \text{ or } \gamma_m \neq 0 \rightarrow \text{there is arch effect}$$

(e) We use the model (2) in the form

$$U_t^2 = \alpha_0 + \alpha_1 U_{t-1}^2 + \dots + \alpha_m U_{t-m}^2 + \varepsilon_t \tag{3}$$

Where  $t = m+1, \dots, T$ ,  $m$  is a pre-specified integer, and  $T$  is the sample size.

Let

$$\begin{aligned} SSR_0 &= \sum_{t=m+1}^T (U_t^2 - \varpi)^2 \\ SSR_1 &= \sum_{t=m+1}^T \varepsilon_t^2 \\ \varpi &= \frac{1}{T} \sum_{t=1}^T U_t^2 \end{aligned} \tag{4}$$

Which is asymptotically distributed as a Chi – Squared distribution with  $m$  degrees of freedom under the null hypothesis:

$$H_0 : \alpha_1 = \dots = \alpha_m = 0.$$

(f) The decision rule is to reject  $H_0$  if  $F > \chi_m^2(\alpha)$ , Where  $\chi_m^2(\alpha)$  is the upper  $100(1 - \alpha)^{th}$  percentile of  $\chi_m^2$  or when  $p$  value of  $F$  is less than  $\alpha$ , the level of significance.

The study used LM test of Engle (9) with arch test results given in the section ahead.

The last test for conditional heteroscedasticity (also known as arch effects) is that which uses Ljung – Box statistic  $Q(m)$ , and can be seen in McLeod and Li [12]. The null hypothesis is that the first  $m$  lags of autocorrelation function (ACF) of the  $\{U_t^2\}$  series are zero. On the whole, we run tests of Aggregational Gaussianity (AG), for normality, by plotting each one of the samples and also all samples of the data to test for normality in which the Aggregational Gaussianity (AG), principles follows. We made probability-probability (p-p), Quantile-Quantile(Q-Q) plots. Also carried out are: Anderson-Darling (AD), Shapiro-Wilks (SW), and Kolmogorov-Smirnov (KS) tests, with their test statistic results along sides their records of associated tail- probabilities or  $p$ -values.

### 3 RESULTS

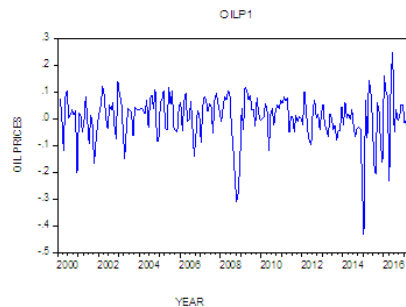


Fig. 1. The volatility nature of oil price

This Fig. 1 shows the volatility clustering nature of our data, the Petroleum Prices, which also shows that the data has what it takes to contain all the styles facts, including Aggregational Gaussianity (AG).

This Table 1 shows the Arch Effect results on the Semi-annual data. The result has that F-statistic with probability, Prob.F(5,189) 0.0000 which shows that there is Arch Effect. The sample size is 189 after adjustments, and the number of variables used is five (5). By this result of the presence of Arch Effect, the use of Garch for the analysis was the best option.

Also, this Table 2 shows the Arch Effect results on the Annual data. The result has that F-statistic with probability, Prob. F (5,183) 0.0000 which shows that there is Arch Effect. The sample size is 183 after adjustments, and the number of variables used is five (5). By this result of the presence of Arch Effect, the use of Garch for the analysis was the best option. However, it is believe that whenever Arch Effect is encountered in a data set, it will contain stylize facts, including Aggregational Gaussianity (AG), being one of them under which we can use GARCH to model it.

The Results of the Data from Monthly to Annual Returns are shown:

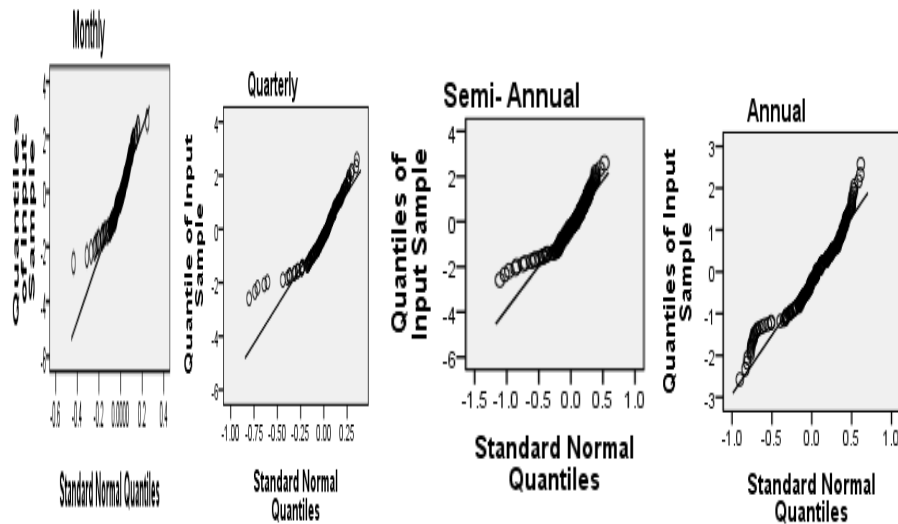


Fig. 2. Is the returns for monthly data, quarterly data, semi-annual data and annual data

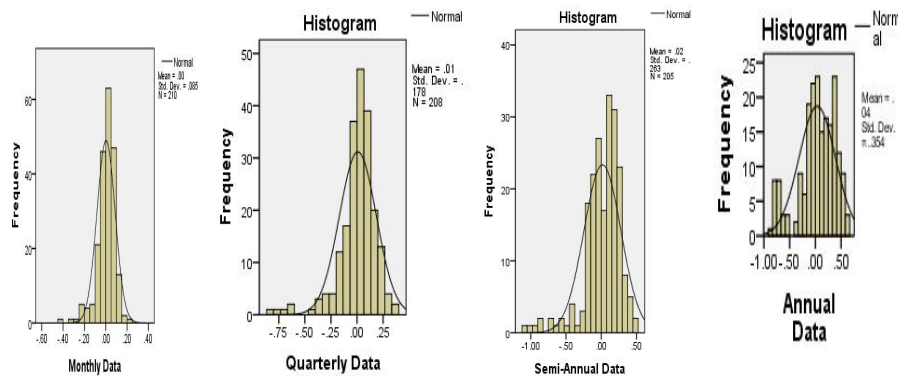


Fig. 3. The corresponding histogram for the data from monthly through annual data

**Table 1. The arch effect on semi -annual data**

F-statistic	11.42521	Prob. F(5,189)	0.0000	
Obs*R-squared	45.25965	Prob. Chi-Square(5)	0.0000	
Test Equation:				
Dependent Variable: RESID^2				
Method: Least Squares				
Date: 07/16/20 Time: 07:18				
Sample (adjusted): 11 205				
Included observations: 195 after adjustments				
Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.024942	0.006188	4.030836	0.0001
RESID^2(-1)	0.523858	0.072724	7.203363	0.0000
RESID^2(-2)	-0.137568	0.082121	-1.675189	0.0956
RESID^2(-3)	0.067588	0.082595	0.818315	0.4142
RESID^2(-4)	0.010693	0.082115	0.130222	0.8965
RESID^2(-5)	-0.042033	0.072776	-0.577572	0.5642
R-squared	0.232101	Mean dependent var		0.043323
Adjusted R-squared	0.211786	S.D. dependent var		0.066803
S.E. of regression	0.059309	Akaike info criterion		-2.781836
Sum squared resid	0.664811	Schwarz criterion		-2.681128
Log likelihood	277.2290	Hannan-Quinn criter.		-2.741061
F-statistic	11.42521	Durbin-Watson stat		2.002090
Prob(F-statistic)	0.000000			

**Table 2. The arch effect on annual data**

<b>Heteroskedasticity Test: ARCH</b>				
F-statistic	30.90741	Prob. F(5,183)	0.0000	
Obs*R-squared	86.53126	Prob. Chi-Square(5)	0.0000	
Test Equation:				
Dependent Variable: RESID^2				
Method: Least Squares				
Date: 07/16/20 Time: 07:26				
Sample (adjusted): 11 199				
Included observations: 189 after adjustments				
Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.028774	0.008844	3.253510	0.0014
RESID^2(-1)	0.738361	0.073216	10.08474	0.0000
RESID^2(-2)	-0.146127	0.091461	-1.597702	0.1118
RESID^2(-3)	0.051622	0.092128	0.560332	0.5759
RESID^2(-4)	0.003545	0.091519	0.038732	0.9691
RESID^2(-5)	0.020980	0.073444	0.285654	0.7755
R-squared	0.457837	Mean dependent var		0.088930
Adjusted R-squared	0.443024	S.D. dependent var		0.106304
S.E. of regression	0.079336	Akaike info criterion		-2.199021
Sum squared resid	1.151835	Schwarz criterion		-2.096109
Log likelihood	213.8075	Hannan-Quinn criter.		-2.157329
F-statistic	30.90741	Durbin-Watson stat		2.008475
Prob(F-statistic)	0.000000			

### 3.1 Analytical Tests Procedures

Histograms with inferential remarks before sampling.  
 These histograms are presented below in Figs. 4 and 5.  
 Sample Size: (N = 204).

### 3.2 Histograms with Inferential Remarks after Sampling

Inferential remarks on the histograms after Sampling are presented in Tables 4 and 5.

### 3.3 Normality Tests with Parents Data

Parent's data normality Tests results are given in Table 6.

### 3.4 Normality Tests with Samples

The normality tests results are given in Tables 7 and 8.

### 3.5 Failure of Goodness- of –Fit Test for Normality

The percentages of Trials that fail the goodness –of –fit tests for normality are presented in Table 9 with an equivalent illustration in Fig. 6.

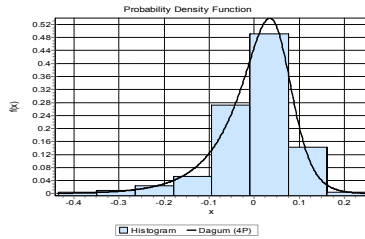


Fig. 4. Parent data (Semi-Annually)

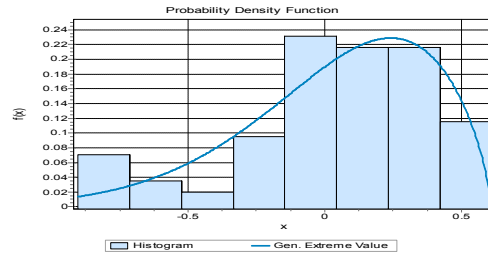


Fig. 5. Parent data (Annually)

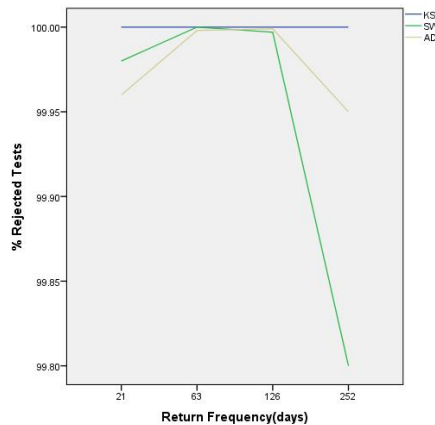


Fig. 6. Graphical illustration of the proportion of trials failing normality tests

**Table 3. Inferential remarks on histogram before sampling**

<b>Data</b>	<b>Histogram Name</b>	<b>Remarks</b>
Monthly	Dagum (4P)	None normal
Quarterly	Dagum (4P)	None normal
Semi – Annual	Dagum (4P)	None normal
Annual	Gen Extreme value	None normal

**Table 4. Semi-annually sample no, size, histogram name**

<b>Data</b>	<b>Sample no</b>	<b>Sample size</b>	<b>Histogram Name</b>	<b>Remarks</b>	<b>Pdf</b>	
Semi -Annual	1	50	Log-Logistic(3P)	None normal		
		100	Weibull (3P)	None normal		
		150	Weibull (3P)	None normal		
	2	50	Burr (4P)	None normal		
		100	Burr (4P)	None normal		
		150	Triangular	None normal		
	3	50	Cauchy	None normal		
		100	Dagum (4P)	None normal		
		150	Dagum (4P)	None normal		
	4	50	Triangular	None normal		
		100	Gumbel Min	None normal		
		150	Burr (4P)	None normal		
	5	50	Dagum (4P)	None normal		
		100	Dagum (4P)	None normal		
		150	Dagum (4P)	None normal		
	6	50	Burr (4P)	None normal		
		100	Gen Gamma(4P)	None normal		
		150	Burr (4P)	None normal		
	7	50	Kumaraswamy	None normal		
		100	Burr (4P)	None normal		
		150	Dagum (4P)	None normal		
	8	50	Log-Logistic(3P)	None normal		
		100	Log-Logistic(3P)	None normal		
		150	Burr (4P)	None normal		
	9	50	Dagum (4P)	None normal		
		100	Burr (4P)	None normal		
		150	Dagum (4P)	None normal		
	*			*	*	
	*			*	*	
	*			*	*	
20	50	50	Dagum (4P)	None normal		
		100	Kumaraswamy	None normal		
		150	Log-Logistic(3P)	None normal		

**Table 5. The annual sample no, size, histogram name**

Data	Sample no	Sample size	Histogram name	Remarks	Pdf
Annual	1	50	Gen Extreme value	None normal	
		100	Gumbel Min	None normal	
		150	Gen Extrem Value	None normal	
	2	50	Gen Extreme value	None normal	
		100	Weibull (3P)	None normal	
		150	Log – Logistic (3P)	None normal	
	3	50	Cauchy	None normal	
		100	Dagum (4P)	None normal	
		150	Dagum (4P)	None normal	
	4	50	Triangular	None normal	
		100	Gumbel Min	None normal	
		150	Burr (4P)	None normal	
	5	50	Dagum (4P)	None normal	
		100	Dagum (4P)	None normal	
		150	Dagum (4P)	None normal	
	6	50	Burr (4P)	None normal	
		100	Gen Gamma(4P)	None normal	
		150	Burr (4P)	None normal	
	7	50	Kumaraswamy	None normal	
		100	Burr (4P)	None normal	
150		Dagum (4P)	None normal		
8	50	Log-Logistic(3P)	None normal		
	100	Log-Logistic(3P)	None normal		
	150	Burr (4P)	None normal		
9	50	Dagum (4P)	None normal		
	100	Burr (4P)	None normal		
	150	Dagum (4P)	None normal		
*			*	*	
*			*	*	
*			*	*	
20	50	50	Log – Logistic (3P)	None normal	
		100	Gen Extreme value	None normal	
		150	Gen Extreme value	None normal	

**Table 6. Parent data normality tests results**

Data	Author	Value (Test Stat)	P-value	Decision
Monthly}	KS	D = 0.431	2.2e -16	None normal
	SW	W = 0.9148	1.239 e -09	None normal
	AD	A = 3.9195	8.583 e -10	None normal
Quarterly}	KS	D = 0.3736	2.2e -16	None normal
	SW	W = 0.8824	1.17 e -11	None normal
	AD	A = 4.9417	2.907 e -12	None normal
Semi- Annual}	KS	D = 0.34	2.2e -16	None normal
	SW	W = 0.8826	1.516 e -11	None normal
	AD	A = 5.1159	1.105 e -11	None normal
Annual}	KS	D = 0.2814	4.108 e-14	None normal
	SW	W = 0.9325	5.713 e – 08	None normal
	AD	A = 3.6471	3.918 e - 09	None normal

Note: KS means Kolmogorov-Smirnov; SW means Shapiro-Wilks  
AD means Anderson-Darling



Table 7. Semi-annual data samples normality tests results

Data	Sample No	Author	Value	P-Value	Decision
Semi -Annual	1	KS	0.3501	2.22 e -16	None normal
		SW	0.8693	3.401 e -10	None normal
	2	AD	4.1062	2.929 e -10	None normal
		KS	0.3431	8.882 e -16	None normal
		SW	0.9108	5.645 e-08	None normal
	3	AD	2.4792	2.714 e -06	None normal
		KS	0.3231	5.018 e -14	None normal
		SW	0.9077	3.706 e -08	None normal
	4	AD	2.8627	3.124 e -07	None normal
		KS	0.3433	8.882 e -16	None normal
		SW	0.92	2.128 e -09	None normal
	5	AD	2.6014	1.362 e -06	None normal
		KS	0.3445	6.661 e -16	None normal
		SW	0.8634	1.803 e -10	None normal
	6	AD	5.2346	5.503 e -13	None normal
		KS	0.3578	2.2 e -16	None normal
		SW	0.8544	7.034 e -11	None normal
	7	AD	4.3879	6.083 e -11	None normal
		KS	0.3431	8.884 e -16	None normal
		SW	0.9173	1.431 e -07	None normal
	8	AD	2.3758	4.868 e -07	None normal
		KS	0.3457	5.551 e -16	None normal
		SW	0.8729	5.079 e -10	None normal
	9	AD	3.9363	7.568 e -10	None normal
		KS	0.3364	3.553 e -15	None normal
		SW	0.87	3.66 e -10	None normal
	10	AD	4.0907	3.193 e -10	None normal
		KS	0.3431	8.882 e -16	None normal
		SW	0.8786	9.661 e -10	None normal
	11	AD	3.9653	9.04 e -05	None normal
		KS	0.3617	2.2 e -16	None normal
		SW	0.9441	1.096 e -05	None normal
	12	AD	1.8595	9.04 e -05	None normal
		KS	0.3299	1.321 e -14	None normal
		SW	0.8814	1.337 e -09	None normal
	13	AD	3.8726	1.081 e -09	None normal
		KS	0.3433	8.882 e -16	None normal
		SW	0.8616	1.491 e -10	None normal
	14	AD	5.4507	1.664 e -13	None normal
		KS	0.3364	3.553 e -15	None normal
		SW	0.8386	1.472 e -11	None normal
	15	AD	7.3582	2.2 e -16	None normal
		KS	0.3252	3.308 e -14	None normal
		SW	0.8821	1.448 e -09	None normal
	16	AD	4.5262	2.815 e -11	None normal
		KS	0.3298	1.354 e -14	None normal
		SW	0.9186	1.726 e -07	None normal
	17	AD	2.0453	3.155 e -05	None normal
		KS	0.3457	5.551 e -16	None normal
		SW	0.8323	8.144 e -12	None normal
	18	AD	5.7042	4.096 e -14	None normal

Data	Sample No	Author	Value	P-Value	Decision
19		KS	0.3659	2.2 e -16	None normal
		SW	0.9437	1.023 e -05	None normal
		AD	1.4768	0.0007931	None normal
20		KS	0.3431	8.882 e -16	None normal
		SW	0.8664	2.476 e -10	None normal
		AD	3.8693	1.1 e -09	None normal
		KS	0.3366	3.442 e -15	None normal
		SW	0.8786	9.668 e -10	None normal
		AD	4.5301	2.755 e -11	None normal

Table 8. Annual data samples normality tests results

Data	Sample No	Author	Value	P-value	Decision
Annual	1	KS	0.2898	2.282 e -11	None normal
		SW	0.9308	1.125 e -06	None normal
		AD	2.8369	3.613 e -07	None normal
2		KS	0.2831	7.178 e -11	None normal
		SW	0.9364	2.843 e -06	None normal
		AD	2.6655	9.49 e -07	None normal
3		KS	0.2831	7.178 e -11	None normal
		SW	0.9428	8.656 e -06	None normal
		AD	2.1156	2.12 e -05	None normal
4		KS	0.2784	1.605 e -10	None normal
		SW	0.9362	2.748 e -06	None normal
		AD	2.3528	5.543 e -06	None normal
5		KS	0.2831	7.178 e -11	None normal
		SW	0.9309	1.155 e -06	None normal
		AD	2.7293	6.624 e -07	None normal
6		KS	0.2831	7.178 e -11	None normal
		SW	0.9356	2.508 e -06	None normal
		AD	2.4902	2.551 e -06	None normal
7		KS	0.2873	3.497 e -11	None normal
		SW	0.9232	3.424 e -07	None normal
		AD	3.1052	7.984 e -08	None normal
8		KS	0.2765	2.198 e -10	None normal
		SW	0.9368	2.037 e -06	None normal
		AD	0.6224	1.26 e -06	None normal
9		KS	0.2831	7.178 e -11	None normal
		SW	0.93	9.877 e -07	None normal
		AD	2.8531	3.297 e -07	None normal
10		KS	0.2765	2.198 e -10	None normal
		SW	0.9379	3.678 e -06	None normal
		AD	2.2721	8.749 e -06	None normal
11		KS	0.285	5.202 e -11	None normal
		SW	0.9329	1.585 e -05	None normal
		AD	2.8434	3.483 e -07	None normal
12		KS	0.2831	7.178 e -11	None normal
		SW	0.933	1.616 e -06	None normal
		AD	2.6367	1.116 e -06	None normal
13		KS	0.2831	7.178 e -11	None normal
		SW	0.9211	2.483 e -07	None normal
		AD	3.4173	1.384 e -08	None normal

Data	Sample No	Author	Value	P-value	Decision
	14	KS	0.2831	7.178 e -11	None normal
		SW	0.9335	1.743 e -06	None normal
		AD	2.7469	5.999 e -07	None normal
	15	KS	0.285	5.202 e -11	None normal
		SW	0.9389	1.489 e -06	None normal
		AD	2.1902	1.39 e -05	None normal
	16	KS	0.2765	2.198 e -10	None normal
		SW	0.9325	1.489 e -06	None normal
		AD	2.8784	2.86 e -07	None normal
	17	KS	0.2765	2.198 e -10	None normal
		SW	0.9396	4.882 e -06	None normal
		AD	2.5696	1.63 e -06	None normal
	18	KS	0.2831	7.178 e -11	None normal
		SW	0.9337	1.82 e -06	None normal
		AD	2.6488	1.043 e -06	None normal
	19	KS	0.2831	7.178 e -11	None normal
		SW	0.9246	4.222 e -07	None normal
		AD	3.3853	1.656 e -08	None normal
	20	KS	0.2831	7.178 e -11	None normal
		SW	0.9216	2.694 e -07	None normal
		AD	3.35048	2.014 e -08	None normal

**Table 9. Percentage of trial that fail the goodness- of- fit tests for normality**

Term	KS	SW	AD
Monthly	99.9997	99.980	99.96
Quarterly	99.9997	99.9998	99.998
Semi-Annual	99.9997	99.997	99.999
Annual	99.9996	99.800	99.95

(via: KS, SW & AD Tests)

%Rejected KS, SW & AD Tests per Return Frequency

## 4 DISCUSSION

The volatility nature of the data set (oil prices from 2000 to 2017) used in this study is shown in Fig. 1. In Fig. 1, there is a clear evidence of volatility clustering; thus, Garch model was considered appropriate for modeling the data set. On this note, we proceeded to test for Arch Effect on the oil prices data and the results are shown in Tables 2 and 3.

The results of the Q-Q plots and the normality tests that were conducted as presented in Fig. 2 and their corresponding histogram plots are shown in Fig. 3. The data in this figure shows a seemingly tending to Normality though at a very slow space. Here, it seems that at a mere looking at the Q – Q plots, one may conclude that there is AG in the Petroleum price Returns (PPR). However, a closer look revealed that there is appreciable errant tail behavior in Semi-annual and Annual returns to be concerned about. So, since two of these distributions do not aggregate to the normal law and are not closed under convolution, this is problematic.

More so, evidences from Tables 4 and 5, and those of Tables 6, 7, 8 and 9, talks about the statistical tests we have conducted, our findings do not support any theory of the presence of AG in the dynamics of Petroleum Price Returns (PPR). All showing none normal plots of the three (3) tests (Kolmogorov – Smirnov, Shapiro-Wilks, Anderson-Darling).Normality Tests, even as evident by their P-values.

The question now arises regarding the plausibility of these findings within the Q – Q plot framework. Unfortunately, this qualitative analysis does not provide the necessary statistical rigor required to support or refute the existence of AG. For this objective, one needs to move beyond Q – Q plots and into an inferential framework (involving Tests) as we have done as shown by those Tables 4, 5, 6, 7, 8 and 9. Thus, the overall results of our findings as presented in Table 9 and depicted in Fig. 6 that shows the percentages (99.97%) of failure of normality of our tests.

## **5 CONCLUSION**

We have already shown that the Statistical properties of the dynamics of Petroleum Price Returns (PPR) of which Aggregational Gaussianity (AG) had been assumed as one of the stylized facts is not a stable property. We wish to state as a consequence that since most of the previous studies had reported that Aggregational Gaussianity (AG) is among the stylized facts in the context of the Nigerian Asset Returns, it may be stated here that those documents that assumed AG as one of the stylized facts had been statistically inadequate, either by virtue of their non-inferential (i.e., Testing) framework backings or by virtue of their lack of regard for the impact of auto-correlated returns data. We wish to further state that the data used in this study exhibits auto-correlation, which has a consequential impact on the non-inferential evidence. We hope that this study will motivate the world body to revisit the understanding of AG in the world capital markets.

With these facts in mind, we hope that the methods used here, has made some advancement even as the managers in the Nigerian Assets control Markets will have a rethink concerning the said AG. Consequently:

- i. AG is not seen as a clear feature of the Nigerian Assets control Market.
- ii. AG is not a feature even out to terms of 12-months

## **COMPETING INTERESTS**

Authors have declared that no competing interests exist.

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