# Test mass materials for a new generation of gravitational wave detectors

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## ABSTRACT

To obtain improved sensitivities in future generations of interferometric gravitational wave detectors, beyond those proposed as upgrades of current detectors, will require different approaches in different portions of the gravitational wave frequency band. However the use of silicon as an interferometer test mass substrate, along with all-reflective interferometer topologies, could prove to be a design enabling sensitivity improvements at both high and low frequencies. In this paper the thermo-mechanical properties of silicon are discussed and the potential benefits from using silicon as a mirror substrate material in future gravitational wave detectors are outlined.

**Keywords:** Gravitational waves, thermal noise, silicon test masses, cooling

## 1. INTRODUCTION

The current generation of long-baseline gravitational wave interferometers[1,2,3,4] is in the initial stages of operation with periods of coincident running between detectors on different continents already having taken place.[1] These detectors, when operating at their design sensitivities, should form an array having an estimated probability of detecting one event (Black Hole/Black hole coalescence) over the next three years, of approximately 50%.[5] Plans are already in place to upgrade some of the currently operating detectors – those of the LIGO project - over the next 4 to 6 years to give approximately a ten-fold increase in instrumental strain sensitivity across a wide range of frequencies.[6] This anticipated increase in sensitivity should allow the probability of detecting gravitational waves to increase to around 99%[5], moving from a phase of simply detecting gravitational waves to carrying out gravitational wave astronomy.

These improvements will be enabled through developments in various areas of detector instrumentation. At low frequencies, the adoption of active seismic isolation systems and multiple pendulum suspension should reduce the effects of seismic motion on the interferometer test masses.[6] At high frequencies the use of increased laser powers, along with the technique of signal recycling[6], should allow improvements in the shot-noise-limited sensitivity of interferometers. In the mid and low frequency ranges, thermal noise will be reduced by the use of quasi-monolithic test mass suspensions incorporating fused silica fibers or ribbons jointed to test masses of sapphire or fused silica.[6] The proposed upgrades to current detectors are discussed in more detail elsewhere in these proceedings in a paper by Peter Fritschel et al.[7]

The plans outlined for these advanced detectors are such that designs for upgraded interferometers have sensitivities limited by three main areas: effects related to the use of high power lasers; thermal/thermo-elastic noise in the test mass substrates and their suspensions; and at very low frequencies, gravity gradient noise. These upgraded detector designs are not yet influenced by the limits of the detector facilities (e.g. by noise resulting from fluctuations of refractive index in the light path along the interferometer arms from residual gas in the vacuum pipes or the effects of stray light). Further improvements in interferometer sensitivity in various frequency ranges should thus be possible; some approaches to developing relevant technologies, based on the use of silicon as a test mass substrate material, are discussed here.

# 2. DESIGNS FOR NEW GENERATIONS OF DETECTORS

The challenges faced in attaining sensitivity goals for future interferometers, as outlined by Shoemaker[8] are different in the high and low frequency regimes and result in conflicting requirements for interferometer designs. For example the increased laser power needed for improved shot-noise-limited interferometer sensitivity at high frequencies (above a few hundred Hertz) can result in decreased interferometer sensitivity at low frequencies (below approximately one hundred Hertz) due to the effects of radiation pressure on the test masses. High frequency detectors require topologies and materials that can handle high laser powers without performance degradation by thermal loading effects. Low frequency detectors require massive mirrors to minimize radiation pressure effects. Meeting all the requirements with a single interferometer design seems difficult. However the use of silicon as the mirror substrate material for the test mass mirrors – with non-transmissive topology where necessary - has the potential to give improved sensitivities at either high or low frequency.[9]

## 2.1 Improvements at performance at high frequencies (greater than a few 100Hz)

At frequencies where interferometer performance is limited by shot noise, improvements in sensitivity may be achieved through using higher circulating powers in an interferometer. However, depending on the levels of power absorbed in the interferometer test masses and mirror coatings, high levels of circulating power can stress the interferometer stability limits through thermally induced deformations of the optics.[10, 11] When power is absorbed locally in a transmissive optic, the temperature gradient inside the optic causes spatially varying changes in the temperature-dependent refractive index $n$ of the material resulting in a thermally induced lens. The magnitude of this effect is also dependent on the thermal conductivity, $K$, of the substrate. The quantity $[K^{-1}dn/dT]$, along with the bulk substrate absorption, can be thus used as a lensing figure of merit, with which to compare material performance.[10] In addition, local heating causes deformation of the surfaces of transmissive optics, or non-transmissive mirrors through the coefficient of thermal expansion, $\alpha$, of the substrate material. In this case the quantity $\alpha/K$ can be used as a comparative figure of merit.[10]

A survey of materials suitable for use in the upgraded detector designs described in section 1, including evaluation of expected thermal noise performance, availability in suitable sizes etc, has left two candidates for consideration as transmissive substrates: - sapphire and fused silica.

| Material At 300K | Lensing Figure of merit $(K^{-1}dn/dt)$ (nm/W) | Expansion Figure of merit $(\alpha/K)$ (nm/W) | Substrate Absorption (ppm/cm) | Power limit inside cavity (kW) |
|---|---|---|---|---|
| **Transmissive** | | | | |
| Sapphire | 250 | 125 | 20 | 630 |
| Fused silica | 7250 | 362 | 1 | 196 |

Table 1: Following the analysis described in Winkler[10] and adopted by Beyersdorf[11] this table shows the comparative figures of merit for fused silica and sapphire substrates when evaluating levels of thermally induced lensing and surface deformation. Also shown is the power inside a transmissive Fabry-Perot cavity of finesse ~100 which produces thermally induced distortions equal to the sagitta of confocally spaced mirrors of radius of curvature equal to 4km. A coating absorption of 1ppm is assumed in each case.

From Table 1 it can be seen that the use of sapphire substrates rather than silica allows a higher power level to be supported, even including the effects of expected higher substrate absorption for sapphire.[12] This is due to the higher thermal conductivity of sapphire. Reduction in the absorption of sapphire through current efforts at improving growth and processing would further increase its relative power handling advantage.

Changing from a transmissive to a reflective topology for the arms of the interferometer allows non-transmissive materials such as silicon to be considered for use as a mirror substrate material and eliminates thermal loading due to substrate absorption. Table 2 compares the performance of sapphire, fused silica and silicon under thermally loaded conditions when used in such a topology. It can be seen that the power handling of both sapphire and silica improves

when used in an all-reflective topology. However by using silicon as the mirror substrate material, powers approximately seven times higher than those supported by sapphire may be tolerated for the same induced surface deformation. This is due to the higher thermal conductivity and lower coefficient of thermal expansion of silicon compared to sapphire.

Silicon thus has thermal properties that make it of considerable interest as a test mass substrate material capable of supporting high laser powers at room temperature. Preliminary studies of its suitability as a mirror substrate have been initiated with pieces having been polished, figured and coated to the performance level required for gravitational wave detectors.[13, 11] However more work is required to demonstrate that large mirrors of the specifications necessary for long-baseline detectors can be fabricated.

| Material At 300K | Lensing Figure of merit $(K^{-1}\mathrm{d}n/\mathrm{d}t)$ (nm/W) | Expansion Figure of merit $(\alpha/K)$ (nm/W) | Substrate Absorption (ppm/cm) | Power limit inside cavity (kW) |
|---|---|---|---|---|
| **Reflective** | | | | |
| Sapphire | - | 125 | - | $1.7 \times 10^4$ |
| Fused silica | - | 362 | - | $5.87 \times 10^3$ |
| Silicon | - | 17 | - | $1.27 \times 10^5$ |

Table 2: This table shows the comparative figures of merit for thermally induced surface deformation of fused silica, sapphire and silicon mirror substrates. Also shown is the power inside a transmissive Fabry-Perot cavity of finesse ~100 which produces thermally induced distortions equal to the sagitta of confocally spaced mirrors of radius of curvature equal to 4km. A coating absorption of 1ppm is assumed in each case.

## 2.2 Improvements in low and mid-frequency performance (few Hz to few 100Hz)

At low frequencies, up to a few hundred Hertz, laser powers higher than those proposed for upgraded detectors are unnecessary, since interferometer sensitivity in this frequency range is not shot-noise limited. Higher laser powers could even prove undesirable due to the effects of radiation pressure on the test masses. At low frequencies, improved performance will require reduction of the level of thermal noise expected from the test masses and their suspensions. Achieving this goal may require a careful choice of test mass substrate material and possibly involve cooling the test masses and their suspensions. Cooling has been used for many years in resonant mass 'bar' gravitational wave detectors and is a possible route for decreasing interferometer thermal noise, provided that suitable materials are chosen. Fused silica is not a promising candidate for use in cooled detectors since it has a broad mechanical dissipation peak centered on ~40K.[14] Sapphire and silicon are however good candidates for cooling; work is currently underway in Japan on developing cooled sapphire test masses and suspension fibers for use in a transmissive Fabry-Perot based interferometer system.[15] However the temperature dependence of the thermal properties of silicon make it a promising alternative to sapphire.

At room temperature, both the intrinsic mechanical dissipation[16,17] and the expected thermo-elastic loss of silicon are comparable to sapphire. Braginsky et al[18] have pointed out that at room temperature, in proposed interferometer designs, thermal noise resulting from thermo-elastic effects can be a significant noise source in the frequency band of interest for gravitational wave detection. Thus it is important to evaluate the temperature dependence of both intrinsic mechanical dissipation and thermo-elastic dissipation in silicon.

Measurements suggest the intrinsic dissipation of silicon in general decreases on cooling, but can exhibit dissipation peaks at particular temperatures.[19] In Figure 1 the expected level of intrinsic thermal noise in a single test mass of single crystal silicon at 10Hz is calculated as a function of temperature, using measured values for intrinsic dissipation from McGuigan et al.[19] Also shown is the calculated level of thermo-elastic noise as a function of temperature, using material parameters from Touloukian[20] and a version of the formula for the level of thermo-elastic noise from Braginsky et al[18] modified by us to allow for short thermal diffusion timescales.[21, 22]

This figure illustrates some interesting features of the expected behavior of silicon when cooled. At two temperatures, close to 120K and 18K, the coefficient of thermal expansion, $\alpha$, of silicon goes to zero.[20] Since the linear spectral density of thermo-elastic noise is directly proportional to $\alpha^{18}$ the calculated thermo-elastic noise at these temperatures also goes to zero. However, very close to these temperatures peaks in the intrinsic dissipation have been measured,[19] giving corresponding peaks in the intrinsic thermal noise. Measurements by other workers have also detected dissipation peaks, but at a variety of temperatures, see for example Yasamura.[24] There are a number of tentative explanations postulated for the existence of these peaks, but their origin is not well understood, and in no case is identified as being connected with the zeros in the coefficient of thermal expansion.

**Displacement**
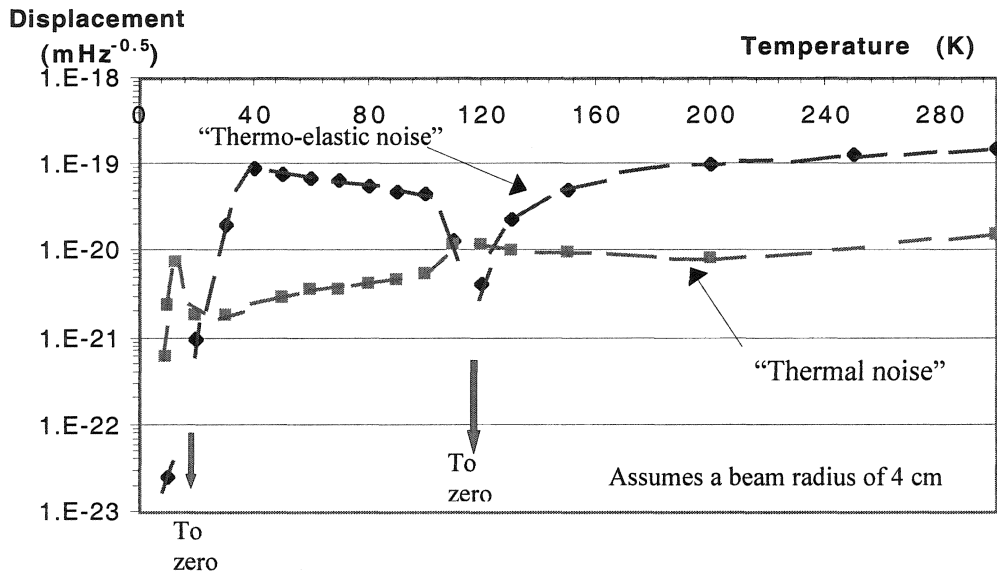**(mHz$^{-0.5}$)**

**Temperature (K)**



Fig.1. Calculated temperature dependence of thermo-elastic noise and thermal noise at 10Hz in a single silicon mirror substrate, assuming illumination by a laser beam of radius 4cm.

To take advantage of the minima at 120K or 18K in the thermo-elastic noise it is thus important to understand the sources of, and to reduce, the intrinsic dissipation at these temperatures. It is also important to understand the requirements on temperature stability of the test masses[23].

Only moderate cooling is required to reach the first thermo-elastic noise minimum at 120K. Significantly more effort is required to reach the second minimum at 18K. In between these temperatures the thermo-elastic noise is proportional to the square root of the thermal conductivity of the substrate. The thermal conductivity of silicon in this temperature can vary widely, by up to two orders of magnitude depending on the type and doping of the silicon used.[20] Thus should operation near a minimum of thermo-elastic noise prove undesirable due to the existence of peaks in intrinsic dissipation there exists the potential for significantly decreasing the thermo-elastic noise at other temperatures.

It should be noted that at low temperatures the mean free path for phonons in a sample can become comparable to the sample dimensions.[20] The dimensions of a given sample can thus have a significant effect on the measured thermal conductivity. The values for the thermal conductivity used in estimating the thermo-elastic noise at low temperatures in the figure above were taken from Touloukian[20], using the largest values of thermal conductivity reported. Further work is underway on this.

### 2.3 Use of cooled silicon substrates under thermally loaded conditions

In section 2.1 the potential benefits of using silicon substrates illuminated by high optical powers rather than fused silica or sapphire at room temperature was discussed. From Winkler et al[10] the thermally induced surface deformations of a substrate scale as $\alpha/K$. By using silicon mirrors cooled to one of the temperatures at which the coefficient of thermal expansion goes to zero the thermally induced distortions of the mirrors essentially become negligible. This effect has

motivated the development of cryogenically cooled silicon optics for use as monochromator optics in high flux x-ray synchrotron sources[25] and has the potential to be useful in the analogous problem of controlling the deformations in gravitational wave detector optics under high optical powers.

# 3. CONCLUSIONS

Non-transmissive silicon has thermo-mechanical properties that make it a potentially useful test mass substrate for future generations of interferometric gravitational wave detectors. In particular silicon has thermal properties that make it attractive for detectors operating at high powers/high frequencies and elastic properties that make it attractive for use as a low-thermal-noise/low frequency test mass material. However, further investigations of the temperature dependence of the material properties of silicon are required to inform design choices for future detectors.

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