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TESTING FOR COUNTRY HETEROGENEITY IN GROWTH MODELS USING A FINITE MIXTURE APPROACH

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SUMMARY

We define a bivariate mixture model to test whether economic growth can be considered exogenous in the Solovian sense. For this purpose, the multivariate mixture approach proposed by Alfò and Trovato is applied to the Bernanke and Gürkaynak extension of the Solow model. We find that the explanatory power of the Solow growth model is enhanced, since growth rates are not statistically significantly associated with investment rates, when cross-country heterogeneity is considered. Moreover, no sign of convergence to a single equilibrium is found. Copyright © 2008 John Wiley & Sons, Ltd.

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1. INTRODUCTION

Many recent papers have pointed out that cross-country models based on the Mankiw, Romer and Weil (MRW hereafter; Mankiw *et al*., 1992) specification of the Solow (1956) growth model do not allow for heterogeneity among countries. Bianchi (1997), Bloom *et al*. (2003), Brock and Durlauf (2000), Desdoigts (1999), Durlauf (2001), Durlauf *et al*. (2001), Durlauf and Johnson (1995), Kalaitzidakis *et al*. (2001), Liu and Stengos (1999), Masanjala and Papageorgiou (2004), Paap and Van Dijk (1998), Paap *et al*. (2005) and Quah (1996, 1997) all found strong parameter heterogeneity in cross-country or panel type growth regressions. This evidence is in contrast to the homogeneity assumptions of the standard Solow model. It is common practice to identify three possible sources of heterogeneity: varying parameters across countries, omitted variables and nonlinearities in the production function. Each of these sources has been studied applying specific theoretical or statistical modifications to the MRW model under both parametric and semi-parametric approaches. The present paper aims at testing the explanatory capability of the MRW model while defining a parsimonious statistical approach to model different sources of heterogeneity. To simplify model estimation, we assume that heterogeneity sources can be simply modeled by introducing a latent effect to each country growth experience allowing for a posterior classification of countries based on the latent variable values (see, for example, Paap *et al*., 2005; Paap and van Dijk, 1998). Similar statistical approaches have also been discussed by Canova (2004) and Bloom *et al*. (2003). Canova (2004) discusses a Bayesian approach to model regional data; while the approach is explicitly based on a finite mixture representation, model parameters are estimated using permutation-based tools for detecting structural break points in time series. Thus, it could be computationally cumbersome when the sample size or the number of components

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in the finite mixture is high. The approach followed by Bloom *et al.* (2003) does not directly test for the existence of more than two groups. Moreover, these proposals are based on extensions of the standard growth equation.

According to the Solow model, in the long-run steady state the level of real output per worker is a function of two variables: the saving rate and the labor force growth rate. Since saving and growth rates vary across countries, each country has a specific steady state; if country steady state correctly describes the distribution of output per worker, the Solow model assumes that the long-run growth rate is independent of saving rates and capital accumulation. According to MRW and Barro and Sala-i-Martin (1992), if economies are homogeneous in technologies and preferences and we assume diminishing returns of capital, countries' growth rates of per capita income should converge to a single equilibrium. The evidence of cross-country (conditional) convergence is considered as supporting the Solow model, while absence of convergence has been considered evidence in favor of endogenous growth theories. However, Azariadis and Drazen (1990), Durlauf and Johnson (1995), Bernard and Durlauf (1996) and Galor (1996) show that the Solow model may also be consistent with the existence of convergence clubs. Thus, failure to allow for unobservable heterogeneity in country steady states may lead to spurious rejection of the Solow model.

Our proposal concerns both modeling heterogeneity in country-specific steady states and convergence issues. According to the intuition of Bernanke and Gürkaynak (2001), levels of per capita income and growth rates are correlated and thus have to be studied in a bivariate framework. If convergence occurs, poor countries grow faster than rich ones; therefore we should find a negative (significant) correlation between residuals in the regression models for, respectively, levels and growth rates. By adopting the proposed model, we assume that, given steady-state characteristics, the latent effects impact the steady-state level of income in the univariate profile, and both steady levels and growth rates in the bivariate process, accounting for dependence between the two responses. This means that if poor countries catch up with the levels of per capita income of rich countries, the corresponding latent structures should be negatively correlated. The sensitivity of the Bernanke and Gürkaynak (2001) approach to parameter heterogeneity has not yet been addressed. We find strong evidence that allowing for parameter heterogeneity leads us to cast doubt on the conclusions of Bernanke and Gürkaynak (2001), since we find no evidence of *global* convergence; as shown in the following, the latent effects in the levels and growth rates models are positively correlated. Data on human capital are from the World Bank's World Development Report.

The paper is divided into four sections. Section 2 illustrates the univariate model for levels of per capita income, discusses the extension to joint modeling of levels and growth rates and reviews computational details of the corresponding EM algorithm. Section 3 introduces the empirical section, presents the results obtained and shows how unobserved heterogeneity can help explain differences among countries. Section 4 concludes.

2. THE ECONOMETRIC MODEL

Following MRW, we start by assuming that the output Y_{it} for country $i = 1, \ldots, n$ at time $t = 1, \ldots, T$ can be described by a Cobb–Douglas type function of the following inputs: raw labor L_{it}, physical capital K_{it}, human capital H_{it} and state of technology A_{it}. This relationship may be elicited as

$$f(Y_{it}|A_{it}, K_{it}, H_{it}, L_{it}) = K_{it}^{\alpha} H_{it}^{\beta} (A_{it} L_{it})^{(1-\alpha-\beta)} \tag{1}$$

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where α, $\beta \in (0, 1)$ represent the shares of physical and human capital, respectively. Technology (broadly defined) and labor are assumed to grow at exogenous rates, respectively g and n_i, i.e.

$$A_{it} = A_{i0}e^{gt} \tag{2}$$

$$L_{it} = L_{i0}e^{n_i t} \tag{3}$$

According to the deterministic version of the Solow model, country-specific laws of motion for capital inputs determine the corresponding accumulating process. Using lower-case letters to denote per-worker quantities, i.e. $y_{it} = Y_{it}/Lit$, $k_{it} = K_{it}/Lit$ and $h_{it} = H_{it}/Lit$, we can rewrite the production function and the capital accumulation equations in a standard way as follows:

$$f(y_{it}|A_{it}, k_{it}, h_{it}) = A_{it}^{(1-\alpha-\beta)}k_{it}^{\alpha}h_{it}^{\beta} \tag{4}$$

$$\dot{k}_{it} = s_{ki}Y_{it} - \delta K_{it} \tag{5}$$

$$\dot{h}_{it} = s_{hi}Y_{it} - \delta H_{it} \tag{6}$$

where δ is the depreciation rate, common to both inputs, while s_{ki} and s_{hi} represent the share of output invested in physical and human capital, respectively.

We can now solve explicitly for the balanced growth path of output per worker. The output per worker in the balanced growth path is therefore given by the following log-linear function:

$$\ln(y_{it}) = \gamma_0 + \frac{\alpha}{(1-\alpha-\beta)}\ln\left[\frac{sk_{it}}{(n_i+g+\delta)}\right] + \frac{\beta}{(1-\alpha-\beta)}\ln\left[\frac{sh_{it}}{(n_i+g+\delta)}\right] + \varepsilon_{it} \tag{7}$$

where ε_{it} represent independent mean-zero homoscedastic Gaussian errors, and parameter estimates are usually obtained through ordinary least squares, under standard i.i.d. hypotheses. Several authors (see, for example, Brock and Durlauf, 2000; Durlauf, 2001; Durlauf *et al.*, 2001; Durlauf and Johnson, 1995; Liu and Stengos, 1999; Kalaitzidakis *et al.*, 2001; Masanjala and Papageorgiou, 2004) have found empirical evidence for heterogeneity in regression parameters. This may be for various reasons. For example, the relationship between output and factors could be far from linear; a homogeneous Cobb–Douglas function may not correctly describe the factor allocation process, since countries may have different production functions. A further possibility is that residuals can be correlated among units over time. While heteroscedasticity may be corrected through traditional parametric applications, Zellner (1969) shows that standard OLS estimates are averages of group-specific parameters only if no correlation exists between observations in the groups. However, economic series are only rarely uncorrelated. Many other potential variables could be used to explain the growth rates. The literature on growth is full of tests on the effects of *not yet considered* new covariates; Levine and Renelt (1992) show that new variables are not robustly explicative in growth equations, while Sala-i-Martin (1997) finds that new covariates can be used to model economic growth. The problem is that potential regressors could number 100 or more. Inserting them we could gain in explanation power but lose simplicity and model interpretation would not be an easy matter. According to Aitkin (1999), we assume that some fundamental covariates were not considered in the model specification and that their joint effect can be accounted for by adding latent effects to the linear predictor, thus relaxing the assumption of i.i.d. residuals.

Let us start assuming that, conditional on a set of individual latent effects u_i which represent the effects of unobserved sources of heterogeneity, the observed log output per worker $\ln(y_{it})$,

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$i = 1, \ldots, n$, $t = 1, \ldots, T$, are realizations of independent variates, drawn from a normal density. For the sake of simplicity, we will denote by

$$\gamma_1 = \frac{\alpha}{(1-\alpha-\beta)},\ \gamma_2 = \frac{\beta}{(1-\alpha-\beta)},\ x_{1it} = \ln\left[\frac{sk_{it}}{(n_i+g+\delta)}\right],\ x_{2it} = \ln\left[\frac{sh_{it}}{(n_i+g+\delta)}\right]$$

the set of model parameters. The regression model for log-output per worker is defined as

$$\mathrm{E}[\ln(y_{it})|u_i] = \gamma_0 + x_{it1}\gamma_1 + x_{it2}\gamma_2 + u_i \tag{8}$$

Latent effects appear additively in the linear predictor, but this assumption can be relaxed by associating random parameters to some elements of the covariates set, generalizing to a *random coefficient* model. Given the assumption of conditionally independence, we have

$$\begin{aligned} f_i = f(\mathbf{y}_i|\mathbf{x}_i, u_i) &= \prod_{t=1}^{T}\{f(y_{it}|\mathbf{x}_{it}, u_i)\} = \prod_{t=1}^{T} f_{it} \\ &= \prod_{t=1}^{T}\left\{\frac{1}{\sqrt{2\pi}\sigma}\exp\left[-\frac{1}{2\sigma^2}(\ln(y_{it}) - \gamma_{0i} - \gamma_1 x_{it1} - \gamma_2 x_{it2})^2\right]\right\} \end{aligned} \tag{9}$$

Treating the latent effects as nuisance parameters, and integrating them out, we obtain for the likelihood function the following expression:

$$L(\cdot) = \prod_{i=1}^{n}\left\{\int_u f_i \mathrm{d}G(u)\right\} \tag{10}$$

where u represents the support for $G(u)$, the distribution function of ui. Model parameters in (8) can be estimated through the marginal likelihood in (10), where the intercept term $\gamma_{0i} = [\ln(A)_0 + gt + u_i]$ varies across countries in order to capture country-specific features. In this context, the random component in γ_{0i} represents mean zero deviations from the fixed part $\ln(A)_0 + gt$; strictly speaking, country-specific latent effects u_i capture country variability in the dynamic process of the 'technological' factor. In the standard formulation of equation (2), technology differs between countries only to the extent that each country starts from a different (nonrandom) initial condition, $\ln(A)_{i0}$. The focus of our paper is rather on the estimation of the latent variables affecting countries' growth experiences, namely u_i. Various alternative parametric specifications may be proposed for modeling random effect distribution. A standard approach is to turn to numerical quadrature techniques, such as Gaussian or Adaptive quadrature (see, for example, Liu and Pierce, 1994). Other alternatives are simulation methods such as Markov chain Monte Carlo methods (McCulloch, 1994), or simulated maximum likelihood methods (Geyer and Thompson, 1992; Munkin and Trivedi, 1999). Parametric specifications of the mixing distribution can, however, be restrictive and are generally unverifiable; for this reason, we propose to avoid specifying a parametric distribution for the random effects, and leave $G(\cdot)$ completely unspecified. As proved by Lindsay (1983a, 1983b), the MLe of $G(\cdot)$ is concentrated on a support of cardinality at most equal to the number of *distinct* points in the analyzed sample. For fixed $\hat{\gamma}$, the likelihood is maximized with respect to $G(\cdot)$ by at least one discrete distribution $\hat{G}_n(\cdot)$ with at most n

support points. Therefore, the integral in (10) may be approximated by a sum on a finite number of locations, say K:

$$L(\cdot) = \prod_{i=1}^{n} \left\{ \sum_{k=1}^{K} f(\mathbf{y}_i|\mathbf{x}_i, u_k)\pi_k \right\} = \prod_{i=1}^{n} \left\{ \sum_{k=1}^{K} [f_{ik}\pi_k] \right\} \tag{11}$$

where $f(\mathbf{y}_i|\mathbf{x}_i, u_k) = f_{ik}$ denotes the response distribution in the kth component of the finite mixture. Locations u_k and corresponding masses π_k (prior probabilities) represent unknown parameters, as does K, which is treated as fixed and estimated via penalized likelihood criteria. Denoting by δ the *complete* parameter vector, we obtain

$$\frac{\partial \log[L(\delta)]}{\partial \delta} = \frac{\partial \ell(\delta)}{\partial \delta} = \sum_{i=1}^{n} \sum_{k=1}^{K} \left(\frac{\pi_k f_{ik}}{\sum_{k=1}^{K} \pi_k f_{ik}} \right) \frac{\partial \log f_{ik}}{\partial \delta} = \sum_{i=1}^{n} \sum_{k=1}^{K} w_{ik} \frac{\partial \log f_{ik}}{\partial \delta} \tag{12}$$

where w_{ik} represents the posterior probability that the ith unit comes from the kth component of the mixture. The corresponding likelihood equations are weighted sums of those for an ordinary log-linear regression model with weights w_{ik}. Solving these equations for a given set of weights, and updating the weights from the current parameter estimates, defines an EM algorithm (see, for example, McLachlan and Peel, 2000).

2.1. The Multivariate Case

Up to now, we have discussed only the univariate case. However, Bernanke and Gürkaynak (2001) observe that testing the Solow model is equivalent to testing independence between the steady-state national growth rates and both saving and human capital formation rates. They note that levels and growth rates of output per worker are different dimensions of the same phenomena, and reject the Solow hypothesis using seemingly unrelated regression equations (SURE) for levels and growth rates of per capita output. As noted by Caselli (2001), one possible explanation of their results is that the analyzed economies are not on a balanced growth path; should this be the case, the Solow model would still be consistent with reality. A further reason is that statistical models used to test Solow conclusions may lead to wrong conclusions if unobserved country-specific heterogeneity is present. As outlined before, our aim is to test whether the assumption of technological exogeneity is a helpful approximation of the driving process of per capita income levels and growth rates when heterogeneity across countries is considered. We assume that the analyzed sample is composed of n countries, with y_{1it} and y_{2it} denoting, respectively, per capita national output and growth rate at time $t = 1, \ldots, T$. Vectors of, possibly outcome-specific, p_{jt} covariates have been recorded for each country and will be denoted by $\mathbf{x}_{1it}$ and $\mathbf{x}_{2it}$. To simplify the discussion, consider the case where covariates do not differ across outcome s and are, respectively, equal to $x_{1it} = \ln[sk_{it}/(n_i + g + \delta)]$ and $x_{2it} = \ln[sh_{it}/(n_i + g + \delta)]$. Following the usual notation for multivariate data, let $\mathbf{y}_i = (\mathbf{y}_{i1}, \mathbf{y}_{i2})$ denote respectively the vector of observed per capita and growth rate of output for the ith country, $i = 1, \ldots, n$ in the analyzed time-window. The

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univariate approach above should be extended in order to take into account potential dependence among outcomes (Davidson and Mackinnon, 1993).

For example, endogeneity of regressors in cross-country or longitudinal estimation is a well-known problem which has not been given a final solution, as pointed out, among others, by Mankiw (1995). Furthermore, omitted covariates may affect both per capita output levels and growth rates; therefore, modeling the association among the two outcomes can be a fundamental aspect of research. For complete generality, we assume that unobserved heterogeneity affects outcomes in different ways, i.e., that the latent effects in the two regression equations are *correlated*. In the MRW context, the *t–period* difference of equation (7) along the balance growth path leads to the conclusion that only time-related changes in technology affect country-specific growth rates; therefore, the growth rate is independent of capital or physical accumulation. Assuming that technology may differ across countries reflecting 'resource endowments, climate, institutions and so on'(MRW, p. 411), we can write it in classical statistical notation as

$$\mathrm{E}\ln(A_{it}) = \overline{a}_t + \varepsilon_{it} \tag{13}$$

while balanced growth per worker output, $\ln(y_{it}^*)$, matches the actual one, $\ln(y_{it})$, minus the stochastic and stationary term, ξ_{it}, which represents the cyclical deviation from that path:

$$\ln(y_{it}) = \ln(y_{it}^*) - \xi_{it} \tag{14}$$

Therefore, we have the following system:

$$\mathrm{E}(y_i) = \begin{cases} y_{1it} = \ln(y_{it}^*) = \overline{a}_t + \dfrac{\alpha}{1-\alpha-\beta}\ln\left[\dfrac{sk_{it}}{n_i+g+\delta}\right] \\ \quad + \dfrac{\beta}{1-\alpha-\beta}\ln\left[\dfrac{sh_{it}}{n_i+g+\delta}\right] + \varepsilon_{it} + \xi_{it} \\ y_{2it} = gy_{it} = \ln(y_{it})^* - \ln(y_{i0}^*) = \ln(A_{it}) - \ln(A_{i0}) = tg(\cdot) + \xi_{it} - \xi_{i0} \end{cases} \tag{15}$$

Indeed, differentiating the output per worker in equation (7) along the BGP we obtain the second expression of the system. Let us assume, according to Bernanke and Gürkaynak (2001), that *sh, sk* and the population rate of growth are included as predictors in the estimated regression equations; therefore, if none of these covariates is significant, the assumption of an exogenous 'technology' cannot be rejected. The authors reject the hypothesis, finding that some of the specified covariate effects are statistically significant. However, even if the assumption is not rejected, the technology cannot be considered really exogenous; rather, we can still use the Solow model to describe, in a parsimonious way, the role of human and physical capital in countries' growth rates. As Romer (2001) points out, Bernanke and Gürkaynak (2001) do not really test for exogeneity of technology; rather they evaluate whether asymmetries in growth among countries can be explained by differences in capital accumulation. Azariadis and Drazen (1990) show that multiple steady-state equilibria can be generated by capitals rates thresholds, producing a non-convex production function. Durlauf and Johnson (1995) and Bernard and Durlauf (1996) show that multiple steady-state regimes can be correctly estimated only if one uses subsets of countries; otherwise, formal convergence tests based on common linear models may be biased. Starting from the model of Bernanke and Gürkaynank (2001), we propose to admit multiple BGPs entailing heterogeneous groups of countries. Specifically, since the error terms in system (15) are correlated, we use a multivariate expansion of the univariate model (7), allowing for correlated latent effects.

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Let $\mathbf{u}i = (u_{i1}, u_{i2})$ denote the corresponding set of country- and outcome-specific random effects. The hypothesis is that (Y_{1it}, Y_{2it}) represents conditionally independent Gaussian variates given the latent effects, which vary over outcomes and account for both cross-country variation and dependence among outcomes. These models are sometimes referred to as multifactor models (see, for example, Winkelmann, 2000). Given the modeling assumptions, the bivariate regression model can be written as follows:

$$y_{ijt} = \gamma_0 + x_{1it}\gamma_{1j} + x_{2it}\gamma_{2j} + u_{ij}, \quad j = 1, 2 \tag{16}$$

As mentioned before, u_{ij} represent subject- and outcome-specific heterogeneity in the intercept parameters. The corresponding likelihood function can be rewritten as follows:

$$L(\cdot) = \prod_{i=1}^{n} \left\{ \int_u f(\mathbf{y}_i|\mathbf{x}_i, \mathbf{u}_i) \right\} = \prod_{i=1}^{n} \left\{ \int_u \left[\prod_j \prod_{t=1}^{T} f(y_{ijt}|\mathbf{x}_{it}, u_{ij}) \right] \mathrm{d}G(\mathbf{u}_i) \right\} \tag{17}$$

This multiple integral cannot be solved in closed form, even if some simplifications are possible. We provide a nonparametric ML estimation of the mixing distribution $G(\cdot)$ following the path discussed for the univariate case (for a detailed discussion, see Alfó and Trovato, 2004). In this case, the likelihood function becomes

$$L(\cdot) = \prod_{i=1}^{n} \left\{ \sum_{k=1}^{K} f_{ik}\pi_k \right\} = \prod_{i=1}^{n} \left\{ \sum_{k=1}^{K} \left[\prod_j \prod_{t=1}^{T} f(y_{ijt}|\mathbf{x}_{it}, u_{jk}) \right] \pi_k \right\} \tag{18}$$

where $\pi_k = \Pr(\mathbf{u}_k) = \Pr(u_{k1}, u_{k2})$, $k = 1, \ldots, K$ represents the joint probability of locations $\mathbf{u}_k$.

As before, locations $\mathbf{u}_k$ and corresponding masses π_k represent unknown parameters. The elements u_{kj} of $\mathbf{u}_k$ can be estimated by introducing, in the linear predictor, the interaction between a K-level factor and the indicator variables d_{jit}, where $d_{jit} = 1\ \forall i = 1, \ldots, n$, $j = 1, 2$ and $t = 1, \ldots, T$ iff the jth outcome is modeled, 0 otherwise. Deriving w.r.t. the vector of model parameters, γ, we have

$$\frac{\partial \log[L(\gamma)]}{\partial \gamma} = \frac{\partial \ell(\gamma)}{\partial \gamma} = \sum_{i=1}^{n} \sum_{k=1}^{K} \left(\frac{\pi_k f_{ik}}{\sum_{k=1}^{K} \pi_k f_{ik}} \right) \frac{\partial \log(f_{ik})}{\partial \gamma} = \sum_{i=1}^{n} \sum_{k=1}^{K} w_{ik} \frac{\partial \log(f_{ik})}{\partial \gamma} \tag{19}$$

where

$$f_{ik} = f(\mathbf{y}_i|\mathbf{x}_i, \mathbf{u}_k) = \prod_{j=1}^{2} \prod_{t=1}^{T} f(y_{ijt}|\mathbf{x}_{it}, u_{kj}) = \prod_{j=1}^{2} \prod_{t=1}^{T} f_{ijk} \tag{20}$$

and w_{ik} represents the posterior probability that the ith unit comes from the kth component of the mixture. The corresponding likelihood equations are weighted sums of those for an ordinary multivariate regression model with weights w_{ik}; the adopted EM algorithm can be sketched as follows.

2.2. Computational Details

As is well known (see, among others, Aitkin, 1999; Wang *et al.*, 1996), the EM algorithm is designed to maximize the complete data likelihood in expression (10). Let us start denoting by $\mathbf{z}_i = (z_{i1}, \ldots, z_{iK})$ the unobservable vector of component indicators, where $z_{ik} = 1$, if the country has been sampled from the component of the mixture, and 0 otherwise. Since the component labels in $\mathbf{z}$ are unobservable, they have to be treated as missing data. We therefore denote the complete data by $\mathbf{y}_c = \{\mathbf{y}, \mathbf{z}\}$. The likelihood for the *complete* data is defined by the following expression:

$$L(\cdot) = \prod_{i=1}^{n}\prod_{k=1}^{K}\{\pi_k f(\mathbf{y}_i|\mathbf{x}_i, \mathbf{u}_k)\}^{z_{ik}} \tag{21}$$

while the corresponding log-likelihood function is given by

$$\ell_c(\cdot) = \sum_{i=1}^{n}\sum_{k=1}^{K}\hat{z}_{ik}\left[\log(\pi_k) + \sum_{i}\log\{f(\mathbf{y}_i|\mathbf{u}_k)\}\right] \tag{22}$$

where

$$f_{ik} = f(\mathbf{y}_i|\mathbf{x}_i, \mathbf{u}_k) = \prod_{j=1}^{2}\prod_{t=1}^{T} f(y_{ijt}|\mathbf{x}_{it}, u_{kj}) = \prod_{j=1}^{2}\prod_{t=1}^{T} f_{jik}$$

Since the z_{ik} are treated as missing data, in the rth iteration of the E-step, we take the expectation of the log-likelihood for *complete* data over the unobservable component indicator vector $\mathbf{z}_i$ given the observed data $\mathbf{y}_i$ and the current parameter estimates, say $\boldsymbol{\delta}^{(r)} = \{\boldsymbol{\gamma}^{(r)}, \mathbf{u}^{(r)}, \boldsymbol{\pi}^{(r)}\}$. In other words, we replace z_{ik} with its conditional expectation:

$$\hat{z}_{ik}(\boldsymbol{\delta}^{(r)} = w_{ik}^{(r)} = \frac{\pi_k f_{ik}}{\sum_{k=1}^{K}\pi_k f_{ik}} \tag{23}$$

where $\hat{z}_{ik}$ $(\boldsymbol{\delta}^{(r)} = w_{ik}^{(r)}$ is the posterior probability that the ith unit belongs to the kth component of the mixture. The conditional expectation of the log likelihood for complete data is expressed by the function

$$Q(\cdot) = E_{\boldsymbol{\delta}^{(r)}}\{\ell_c(\cdot)|\mathbf{y}_i\} = \sum_{i=1}^{n}\sum_{k=1}^{K} w_{ik}^{(r)}\{\log(\pi_k) + \log(f_{ik})\} \tag{24}$$

The M-step aims at maximizing the expected value of the complete data likelihood given the observed data and the current parameter estimates. The estimated parameters are the solution of the following M-step equations:

$$\frac{\partial Q}{\partial \pi_k} = \sum_{i=1}^{n}\left\{\frac{w_{ik}^{(r)}}{\hat{\pi}_k} - \frac{w_{iK}^{(r)}}{\hat{\pi}_K}\right\} = 0 \tag{25}$$

$$\frac{\partial Q}{\partial \gamma} = \sum_{i=1}^{n}\sum_{j=1}^{J} w_{ik}^{(r)}\frac{\partial}{\partial \gamma}\log(f_{ik}) \tag{26}$$

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To obtain updated estimates of the unconditional probability π_k we replace each z_{ik} by $\hat{z}_{ik}(\boldsymbol{\delta}^{(r)})$, and, solving equation (25), we obtain

$$\hat{\pi}_k^{(r)} \sum_{i=1}^{n} \frac{w_{ik}^{(r)}}{n} \tag{27}$$

which represents a well-known result from ML in finite mixtures. Solutions of equation (26) can be obtained through an iteratively weighted least squares (IWLS) algorithm.

If the adopted criterion is based on the sequence of likelihood values $r \geq 1$, $r \in N$, the E and M-steps are alternatively repeated until the relative difference

$$\frac{|\ell^{(r+1)} - \ell^{(r)}|}{\ell^{(r)}} < \varepsilon, \quad \varepsilon > 0 \tag{28}$$

changes by an arbitrarily small amount. Since $\ell^{(r+1)} \geq \ell^{(r)}$, convergence is obtained with a sequence of likelihood values which are upward bounded. Penalized likelihood criteria (such as AIC, CAIC or BIC) have been used to estimate the number of mixture components.

The use of finite mixtures has some significant advantages over parametric mixture models. First, it allows us to classify countries in clusters characterized by homogeneous values of the latent effects, where this kind of classification is possible only if country heterogeneity does exist. Second, since locations and corresponding probabilities are completely free to vary over the corresponding supports, the proposed approach can readily accommodate extreme and/or strongly asymmetric departures from the Gaussian assumption.

3. DATA DESCRIPTION AND RESULTS

To test whether heterogeneity bias affects conclusions drawn in the 'growth empirics' framework, we propose a reanalysis of the dataset used by Bernanke and Gürkaynank (2001) (data can be found at http://www.princeton.edu/∼bernanke/data.htm). The data are drawn from the Summers–Heston Penn World Tables (PWT) version 6.0, which extends the data through 1998 for most variables. For reasons of space we avoid comparison with older versions, and use PWT version 6.0 for years 1960–1995 for non-oil countries.

Bernanke and Gürkaynank (2001) discuss both cross-section and pooled cross-section data; for this reason, we start our reanalysis by fitting the univariate mixture model 7 to the cross-section data. Table I shows the results obtained. As can be easily seen, the approach based on the finite mixture parametrization finds four to five components (which can be interpreted as different groups of countries), thus pointing out that substantial heterogeneity is present. However, looking at countries (posterior) classification, we obtain non-sense clusters: for example, the USA and Uganda are in the same group together with Rwanda and Switzerland, etc. (see Table II).

Thus the finite mixture approach applied to cross-section data does not produce satisfactory results; in our perspective, this can be due to country-specific heterogeneity being masked by long-term averages (from 1960 to 1995) of per capita GDP. That is, country-specific heterogeneity cannot be captured if one does not look at between-countries variation which cannot be explained by observed covariates but remains persistent over the analyzed time period. For this reason, we turn to apply the proposed mixture model to the non-overlapping 5-year period from 1960 to 1995; the covariates have been averaged over the corresponding time period, while the dependent

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Table I. Cross-section estimates 1960–1995: OLS type and finite mixture models

Textbook Solow model: PWT 6 non oil sample

	OLS		FUME	
Textbook Solow model				
lnyl95	Coef.	SE	Coef.	SE
lnsk6095	1.11	0.14	1.13	0.07
lngd6095	−2.54	0.50	−2.21	0.35
cons	4.58	1.44	5.45	1.02
Obs.	90		90	
R^2	0.66			
ℓ			−71.09	
k			5	
Augmented Solow model				
lnsh6095	0.65	0.09	0.52	0.04
lnsk6095	0.54	0.11	0.67	0.05
lngd6095	−2.35	0.39	−2.05	0.22
cons	5.81	1.12	6.53	0.64
Obs.	90	90	$R2$	0.79
ℓ			−52.72	
k			4	
Restricted Solow model				
dskn6095	0.62	0.11	0.63	0.10
dshn6095	0.68	0.09	0.64	0.08
cons	8.84	0.10	8.81	0.10
$\hat{\alpha}$	0.27	0.04	0.28	0.04
$\hat{\beta}$	0.30	0.04	0.28	0.03

Table II. Country clusters: FUME cross-section estimates—augmented model

Groups	
1	Congo, Jamaica, Tanzania, Zaire, Zambia
2	Bangladesh, Bolivia, Ecuador, Ethiopia, Finland, Ghana, Greece, India, Kenya, Malawi, Mali, Nepal, Nigeria, Peru, Philippines, Sri Lanka, Togo, Zimbabwe
3	Algeria, Angola, Argentina, Austria, Belgium, Benin, Brazil, Burkina Faso, Burundi, Cameroon, Central Afr. R., Chile, Colombia, Costa Rica, Denmark, Dominican Rep., Egypt, France, Honduras, Indonesia, Ireland, Italy, Ivory Coast, Japan, Jordan, Korea Rep., Madagascar, Malaysia, Mauritania, Mexico, Morocco, Netherlands, New Zealand, Nicaragua, Niger, Norway, Pakistan, Panama, Portugal, S Africa, Senegal, Spain, Sweden, Syria, Thailand, Trinidad and Tobago, Tunisia, Turkey, UK, Uruguay, Venezuela
4	Australia, Botswana, Canada, El Salvador, Guatemala, Hong Kong, Israel, Mauritius, Mozambique, Papua N Guinea, Paraguay, Rwanda, Singapore, Switzerland, Uganda, USA

variables (level and growth rate of per capita GDP) are 5 years forward. The choice of 5-year periods is usually adopted in the panel growth literature to hold sufficient degrees of freedom while avoiding the negative effects of strong autocorrelation of dependent variables (Bond *et al.*, 2001). Moreover, leading dependent variables may reduce endogeneity bias.

Growth measures based on 5-year periods are adequate for studying the impact of traditional cyclical variables, but may not help in understanding the impact of long-run factors. For this purpose, we fit a similar finite-mixture model to 10-year intervals also. The parametric benchmark

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is represented by a feasible GLS (FGLS) approach. For the non-oil countries sample, Table III reports parameter estimates and model diagnostics for both FGLS and univariate finite mixture models 7, the latter applied to both 5- and 10-year periods. We present estimates for both textbook and human capital augmented Solow models and, finally, for the restricted model.

The human capital augmented model clearly outperforms the textbook Solow model, as can be evinced by looking at the log-likelihood of the augmented Solow model fitted via the FGLS or the proposed finite mixture approach (FUME in the following). FUME estimates for human and physical capital shares are lower than expected; however, parameter estimates for this approach should be considered as conditional on the unobservable latent component, and therefore have a different meaning from those obtained through the FGLS approach. FGLS estimation of the Solow model gives more reasonable values for labor and capital shares, due to

Table III. 5 years FGLS and 5 years and 10 years finite mixture models

	FGLS		FUME: 5 years		FUME: 10 years	
Textbook Solow model: PWT 6 non-oil sample						
lny	Coef.	SE	Coef.	SE	Coef.	SE
lnsk	0.592**	0.030	0.193**	0.023	0.193**	0.030
lngd	−1.005**	0.116	−0.188†	0.103	−1.126**	0.130
cons	7.487**	0.314	8.749**	0.295	6.279**	0.357
ℓ	−37.58	−124.49		−139.65		
σ^2			0.051	0.003	0.061	
$\sigma^2_{u_i}$			0.670	0.061		
K			7		5	
Obs.	538		538		353	
Augmented version: PWT 6 non-oil sample						
lnsk	0.399**	0.031	0.174**	0.020	0.180**	0.027
lnsh	0.350**	0.023	0.143**	0.014	0.086**	0.021
lngd	−1.238**	0.092	−0.418**	0.097	−0.728**	0.132
cons	7.537**	0.250	8.547**	0.260	7.57**	0.357
ℓ	22.580	−94.35		−115.1		
σ^2			0.046	0.003	0.046	
$\sigma^2_{u_i}$			0.513		0.061	
K			7		7	
Obs.	530		530		351	
Restricted model: PWT 6 non-oil sample						
dsk	0.376**	0.031	0.170**	0.023	0.201**	0.027
dsh	0.323**	0.024	0.142**	0.015	0.099**	0.020
cons	8.826**	0.033	8.811**	0.079	8.799**	0.086
ℓ	59.64	−104.87		−120.3		
σ^2			0.049		0.003	0.047
$\sigma^2_{u_i}$			0.524		0.061	
K			6		7	
$\hat{\alpha}$	0.221	0.129	0.017	0.155		
$\hat{\beta}$	0.190	0.108	0.013	0.076		
Obs.	530	530		351		

Significance levels: † 10%; * 5%; ** 1%.
Note: Dependent variable: log of per capita GDP levels (PWT data); lnsh, schooling of the working population (World Bank Report data); sk, Summers–Heston corrected investment/GDP ratio (PWT data); lngd, sum of the rates of change in population and in technological progress plus depreciation (PWT data); K, number of mixture components selected by BIC criteria; ℓ, log-likelihood; σ^2, within-country (residual) variance; $\sigma^2_{u_i}$, variance of the latent effects. Data are 5- or 10-year averages, dependent variable is forwarded.

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the marginal parametric interpretation given above. The assumption of conditional independence (given the latent effects) which is at the basis of the FUME approach implies that the *global* production function is obtained by weighting K different functions, each one corresponding to a different component (subgroup) of the analyzed sample. This means that the estimates for covariate effects can be shrunken with respect to a homogeneous model, since they come from a weighted sum of different production functions. Since the PWT 6 sample collects data also for poorest countries, measurement bias due to inefficiency of national statistical systems may be present.

Figure 1 reports the empirical density of the log per capita GDP levels and the estimated density obtained using, respectively, FGLS and FUME approaches. The empirical density clearly suggests that population heterogeneity is present; moreover, if we compare the estimated and empirical densities, the mixture model seems to better describe the data-generating process (DGP).

As it is well known, due to non-standard conditions we cannot use standard parametric tools to test the goodness of fit of a mixture model; as suggested by Aitkin (1997) and McLachlan and Peel (2000), we can use, for diagnostic purposes, a plot comparing the fitted mixture distribution with the empirical distribution function. Figure 2 shows the corresponding two (respectively based on FGLS and FUME) fitted CDFs, together with 95% confidence bands for the observed CDF based on the usual binominal interval. Note that the estimated CDF based on the finite mixture approach provides a closer fit to the observed data, the departure in right side reflecting the cluster composed by the USA and Switzerland. In contrast, the FGLS-based CDF shows substantial and significant departures from the observed CDF for several values of the log of per capita GDP.

Parametric univariate estimation correctly controls for heteroscedasticity, but it seems to be inadequate to account for individual heterogeneity influencing the DGP. The potential presence of heterogeneous subpopulations is adequately treated in the FUME approach; using penalized likelihood criteria such as BIC, AIC and CAIC (Tables IV and V report such values for 5-year and 10-year data) we choose seven components to estimate the unknown mixing distribution.

To provide a formal test for the null hypothesis that data are drawn from a seven-component finite mixture, we employed the bootstrap-based procedure detailed in Romano (1988). The

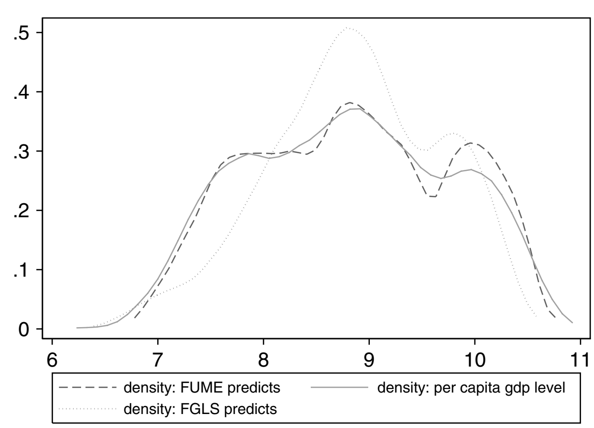

Figure 1. Observed and fitted kernel density distribution responses

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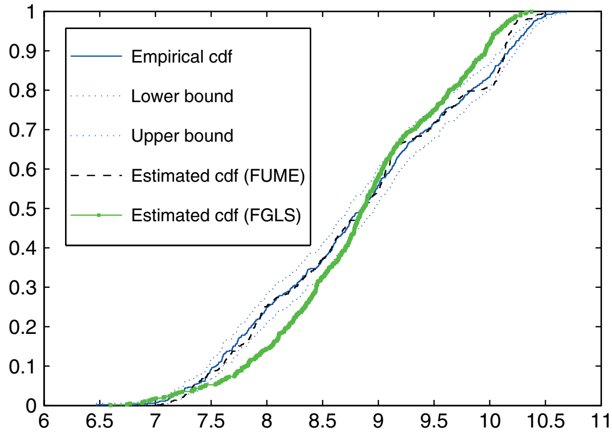

Figure 2. CDF limits, and FUME and FGLS CDFs. This figure is available in color online at www.interscience.wiley.com/journal/jae

Table IV. Penalized likelihood criteria for 5-year finite mixture models

K	2	3	4	5	6	7	8
Textbook Solow model: PWT 6 non-oil sample							
ℓ	−339.38	−265.18	−211.32	−173.78	−135.65	−124.49	−124.39
BIC	701.2587	561.8599	463.143	397.0662	329.802	316.486	325.284
AIC	683.7597	537.3612	431.6447	358.5683	284.3045	263.989	265.787
CAIC	706.2587	568.8599	472.143	408.0662	342.802	331.486	342.284
Augmented Solow model: PWT 6 non-oil sample							
ℓ	−262.08	−187.59	−146.38	−132	−126.47	−94.3491	−94.3487
BIC	546.6513	406.6872	333.263	313.4892	311.4471	256.1954	265.194
AIC	529.1523	382.1885	301.7647	274.9913	265.9496	203.6982	205.697
CAIC	551.6513	413.6872	342.263	324.4892	324.4471	271.1954	282.194
Restricted Solow model: PWT 6 non-oil sample							
ℓ	−277.25	−204.98	−160.15	−134.35	−104.87	−104.8	
BIC	576.9991	441.456	360.7984	318.1924	268.2336	277.0916	
AIC	559.5	416.9573	329.3001	279.6945	222.7361	224.5944	
CAIC	581.9991	448.456	369.7984	329.1924	281.2336	292.0916	

Notes: K: no. components; ℓ, log-likelihood.
BIC $= -2\ell(\cdot) + d\log(n)$
AIC $= -2\ell(\cdot) + d$
CAIC $= -2\ell(\cdot) + 2d\log(n)$
where d is the number of parameters and n is the sample size.

observed value of the (scaled) D statistic is equal to 0: 0566 with the corresponding (approximate) p-value $\cong 0.304$ ($B = 1000$ resamples). The same procedure has been adopted to test the null hypothesis that data are drawn from the homogeneous GLS model, obtaining an approximate p-value < 0.001 ($B = 1000$ resamples).

The proposed approach allows classification of countries on the basis of the posterior probabilities estimates $\hat{w}_{ik}$, which represent a potentially useful by-product of the adopted approach. According to a simple MAP rule, in fact, the ith country can be classified in the lth component if

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Table V. Penalized likelihood criteria for 10-year finite mixture models

K	2	3	4	5	6	7	8
Textbook Solow model: PWT 6 non-oil sample							
ℓ	−435.84	−194.8	−161.53	−139.66	−135.58	−124.83	
BIC	894.1874	421.1066	363.5675	328.809	329.6481	317.1496	
AIC	876.6883	396.6079	332.0692	290.3111	284.1506	264.6525	
CAIC	899.1874	428.1066	372.5675	339.809	342.6481	332.1496	
Augmented Solow model: PWT 6 non-oil sample							
ℓ	−198.62	−172.21	−148.84	−135.81	−127.46	−115.13	−114.71
BIC	424.2456	380.4198	342.6691	325.624	317.9204	302.2577	310.4087
AIC	403.2467	352.4213	307.671	283.6263	268.9231	246.2608	247.4121
CAIC	430.2456	388.4198	352.6691	337.624	331.9204	318.2577	328.4087
Restricted Solow model: PWT 6 non-oil sample							
ℓ	−211.04	−180.13	−157.42	−142.35	−141.99	−120.37	−120.37
BIC	444.5799	391.7541	355.3474	334.1907	342.4807	308.2419	317.2415
AIC	427.0809	367.2555	323.8491	295.6928	296.9831	255.7447	257.7447
CAIC	449.5799	398.7541	364.3474	345.1907	355.4807	323.2419	334.2415

Notes: K: no. components; ℓ, log-likelihood;
$\text{BIC} = -2\ell(\cdot) + d\log(n)$
$\text{AIC} = -2\ell(\cdot) + d$
$\text{CAIC} = -2\ell(\cdot) + 2d\log(n)$
where d is the number of parameters and n is the sample size.

Table VI. Location and probabilities of 5-year FUME

K	Textbook model		Augmented model		Restricted model	
	Loc.	Prob.	Loc.	Prob.	Loc.	Prob.
1	−1.580	0.045	−1.292	0.055	−1.307	0.053
2	−1.003	0.199	−0.848	0.212	−0.857	0.214
3	0.641	0.134	0.965	0.172	1.036	0.199
4	−0.528	0.105	0.565	0.124	−0.286	0.199
5	0.174	0.189	0.160	0.211	0.150	0.208
6	−0.172	0.130	−0.279	0.199	0.571	0.128
7	1.154	0.199	1.327	0.027		

Note: K, number of mixture components selected by penalized criteria; Loc, locations; Prob., prior probabilities to belonging to that local area.

$\hat{w}_{il} = \max(\hat{w}_{i1}, \ldots, \hat{w}_{iK})$. It is worth noting that each component is characterized by homogeneous values of estimated latent effects; i.e., conditionally on the observed covariates, countries from that group show a similar structure, at least in the steady state. The latent variables should capture the effect of missing covariates such as those related to institutions; estimated locations are shown in Table VI, while corresponding clusters are reported in Table VII.

For poor countries, we may note that the random terms negatively affect the level of the log of per capita GDP, while the effect is higher and positive for the richest ones such as the USA, Switzerland and other more industrialized countries. The conclusion is that the Solow model, conditionally on heterogeneous groups, helps us understand the differences among countries, while allowing for balanced growth paths; in other words, FUME is able to measure local variation in the observed data.

Table VII. Univariate posterior classification

Country	y	u_i	σ_{u_i}
Group 1			
Total	7.284	−1.274	0.041
Ethiopia	7.102	−1.292	0.001
Malawi	7.258	−1.292	0.001
Nepal	7.570	−1.210	0.172
Tanzania	7.149	−1.292	0.000
Zaire	7.353	−1.290	0.029
Group 2			
Total	7.765	−0.850	0.018
Bangladesh	7.767	−0.848	0.015
Benin	7.783	−0.847	0.001
Burkina Faso	7.473	−0.848	0.007
Burundi	7.457	−0.847	0.001
Congo	8.030	−0.848	0.007
Ghana	7.909	−0.847	0.000
India	7.804	−0.856	0.062
Indonesia	8.112	−0.847	0.005
Kenya	7.815	−0.847	0.000
Madagascar	7.718	−0.847	0.001
Mali	7.521	−0.865	0.086
Niger	7.713	−0.847	0.005
Nigeria	7.651	−0.847	0.003
Pakistan	7.989	−0.847	0.001
Rwanda	7.605	−0.847	0.004
Sri Lanka	8.234	−0.844	0.042
Togo	7.818	−0.847	0.000
Uganda	7.196	−0.873	0.104
Zambia	7.934	−0.847	0.000
Group 3			
Total	10.118	0.974	0.030
Australia	10.210	0.983	0.077
Austria	10.104	0.965	0.008
Belgium	10.165	0.965	0.004
Canada	10.251	1.114	0.178
Denmark	10.249	0.965	0.011
Finland	10.064	0.947	0.083
France	10.160	0.965	0.005
Hong Kong	9.902	0.965	0.014
Israel	9.914	0.961	0.041
Italy	10.057	0.965	0.008
Japan	10.080	0.965	0.010
Netherlands	10.135	0.965	0.003
New Zealand	10.082	0.965	0.003
Norway	10.219	0.965	0.003
Sweden	10.224	0.966	0.021
UK	10.075	0.965	0.003
Group 4			
Total	9.591	0.564	0.020
Argentina	9.622	0.565	0.007
Greece	9.676	0.564	0.009
Ireland	9.773	0.565	0.004
Mauritius	9.328	0.556	0.060
Mexico	9.486	0.565	0.005
Portugal	9.569	0.563	0.025
S Africa	9.486	0.573	0.059

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Table VII. (*Continued*)

Country	y	u_i	σ_{u_i}
Singapore	9.757	0.567	0.031
Spain	9.768	0.565	0.017
Trinidad and Tobago	9.572	0.565	0.010
Venezuela	9.456	0.565	0.004
Group 5			
Total	9.020	0.163	0.018
Algeria	9.167	0.168	0.057
Botswana	8.822	0.160	0.008
Brazil	9.188	0.162	0.026
Chile	9.128	0.160	0.003
Colombia	8.945	0.160	0.003
Costa Rica	9.095	0.161	0.022
El Salvador	8.956	0.160	0.016
Guatemala	8.872	0.160	0.014
Jordan	8.928	0.159	0.014
Korea, Rep.	9.127	0.160	0.011
Malaysia	9.073	0.160	0.002
Nicaragua	8.830	0.160	0.012
Panama	9.078	0.160	0.003
Papua N. Guinea	8.788	0.160	0.002
Paraguay	8.974	0.160	0.007
Peru	9.026	0.159	0.014
Tunisia	8.993	0.160	0.006
Turkey	9.011	0.160	0.005
Uruguay	9.344	0.201	0.122
Group 6			
Total	8.454	−0.279	0.020
Angola	8.252	−0.279	0.001
Bolivia	8.546	−0.279	0.001
Cameroon	8.275	−0.279	0.000
Central Afr. Rep.	8.072	−0.279	0.002
Dominican Rep.	8.622	−0.276	0.038
Ecuador	8.819	−0.268	0.069
Egypt	8.583	−0.279	0.006
Honduras	8.370	−0.279	0.001
Ivory Coast	8.469	−0.273	0.050
Jamaica	8.839	−0.279	0.002
Mauritania	8.092	−0.279	0.000
Morocco	8.669	−0.271	0.060
Mozambique	7.938	−0.279	0.000
Philippines	8.590	−0.279	0.000
Senegal	8.069	−0.300	0.108
Syria	8.719	−0.279	0.008
Thailand	8.667	−0.279	0.001
Zimbabwe	8.584	−0.279	0.003
Group 7			
Total	10.472	1.327	0.008
Switzerland	10.463	1.327	0.010
USA	10.482	1.327	0.005

Note: y, log of per capita GDP levels (PWT data); u_i, country random coefficient (mean values along time); σ_{u_i} standard deviation of country random coefficient (mean values along time).

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Table VIII. Bivariate GLM: augmented model

	5-year BGLM		10-year BGLM	
	Coef.	Bootstrap SE	Coef.	SE
lny				
lnsh	0.437**	0.064	0.578**	0.087
lnsk	0.443**	0.084	−2.253**	0.246
lngd	−1.986**	0.218	0.388**	0.067
cons	5.881**	0.626	5.251**	0.729
gy				
sk	0.076**	0.014	0.100**	0.015
sh	−0.068†	0.036	−0.076*	0.034
glf	−0.303**	0.100	−0.161	0.123
cons	0.011**	0.004	0.002	0.004
ℓ	−428.356		−303.298	
Obs.	1060		702	

Significance levels: † 10%; * 5%; ** 1%.
Note: lny, log of per capita GDP levels (PWT data); gy, log difference of lny; lnsh, schooling of the working population (World Bank Report data); lnsk, Summers–Heston corrected investment/GDP ratio (PWT data); lngd, sum of the rates of change in population and in technological progress plus depreciation (PWT data); ℓ, log-likelihood; data are 5- or 10-year averages, dependent variable is forwarded. Method of estimation GLM with adjusted SE for clustered data, cluster on country.

Following the path described by Bernanke and Gürkaynank (2001), we have estimated the bivariate regression model in equation (15), adopting either a fully parametric SURE model or a bivariate finite mixture approach. In the former case, we employed a maximum likelihood approach with correlated error terms. Looking at the bivariate results, we may note that the fully parametric model confirm the results obtained by Bernanke and Gürkaynank (2001) (see Table VIII), while the bivariate finite mixture approach does not find any significant parameter estimates in the growth equation (see Table IX).

Moreover, estimated locations for the latent component in the growth rate equation are close to zero; that is, we do find negligible differences in the rate of GDP growth for countries belonging to different groups. Instead, heterogeneity plays a relevant role in the levels equation. Results show that the latent component in the growth equation can be omitted; therefore, there is no empirical evidence of a *global* convergence process towards a single equilibrium path. To test whether the apparent homogeneity of growth rates across countries may be due to measurement error bias, we re-estimated a bivariate Solow model using as response in the first equation the residuals of the univariate FGLS model for GDP levels, and inserting in the corresponding linear predictor only a random component. The second equation for growth rates is the same as in (16): $j = 2$. Using this strategy, we implicitly impose that the parameter vector in the growth equation is equal to the one estimated by FGLS. Comparing parameter estimates obtained with this approach with those obtained from the unconstrained bivariate model, we found no empirical evidence of any significant difference; thus, we may guess that measurement error does not affect results obtained in the growth rate model, and that heterogeneity captured by the bivariate finite mixture model is due, rather, to unobservable covariables.

The values of penalized likelihood criteria as well as the estimated mass points (5-year data only) corresponding to the chosen number of components, $\hat{K}$, are reported in Tables X and XI.

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Table IX. Bivariate finite mixture models: augmented model

	5-year BFM		10-year BFM	
	Coef.	SE	Coef.	SE
lny				
lnsk	0.143**	0.013	0.174**	0.017
lnsh	0.149**	0.010	0.083**	0.015
lngd	−0.388**	0.074	−0.695**	0.082
cons	8.576**	0.210	7.64**	0.246
gy				
sk	0.075	0.102	0.085	0.132
sh	−0.100	0.220	−0.138	0.271
ngd	−0.428	0.800	−0.180	1.08
cons	0.016	0.025	0.009	0.033
ℓ	302.49		178.26	
σ^2	0.025**	0.0011	0.0215**	0.0011
$\sigma^2_{u_{\text{lny}}}$	0.528		0.614	
$\sigma^2_{u_{\text{gy}}}$	0.00003		0.00004	
$\text{cov}(u_{\text{lny}}, u_{\text{gy}})$	0.0012		0.004	
K	6		8	
Obs.	1060		702	

Significance levels: † 10%; * 5%; ** 1%.
Note: lny, log of per capita GDP levels (PWT data); gy, log difference of lny; lnsh, schooling of the working population (World Bank Report data); lnsk, Summers–Heston corrected investment/GDP ratio (PWT data); lngd, sum of the rates of change in population and in technological progress plus depreciation (PWT data); K, number of mixture components; ℓ, log-likelihood; σ^2, within-country (residual) variance; $\sigma^2_{u_{lny}}$ variance of the random intercept for y_1; $\sigma^2_{u_{gy}}$, variance of the random intercept for y_2; $\text{cov}(u_{\text{lny}}, u_{\text{gy}})$, covariance between random terms; data are 5- or 10-year averages; dependent variable is forwarded.

Using these estimates, we may classify countries on the basis of the posterior probability estimates $\hat{w}_{ik}$, which represent an important by-product of the adopted semiparametric approach. It is worth noting that each group is characterized by homogeneous values of the estimated random effects; i.e., conditionally on observed covariates, countries assigned to the same group have a similar structure (see Table XII for the bivariate classification). This data-driven clustering can be considered as an empirical, modeling, counterpart to multiple equilibria models discussed by Azariadis and Drazen (1990), Durlauf and Johnson (1995), Bernard and Durlauf (1996) and Galor (1996), among others. Should a multi-population density exist, we would find *significant* components for the random effects. Should this not be the case, we would face a spurious clustering problem and a single normal component distribution would be sufficient to correctly describe the DGP (see McLachlan and Peel, 2000).

The existence of a single equilibrium path in the long run is well supported if the correlation between the random effects in the two equations is significant and negative, thus indicating a progressive decrease in observed differences among countries. The bivariate results (see Table IX), however, show that the variance of the latent effect in the growth rate equation is near zero; also, the covariance between the latent effects for levels and growth rates is near zero, but still positive (Table IX). This result does not change with varying K (see Table X). In other words, the neoclassical relationship between levels and growth rates of per capita output does not statistically match the analyzed data. Looking at univariate and bivariate results we may suggest

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Table X. Penalized likelihood criteria and latent effect correlations for bivariate finite mixture models

K	2	3	4	5	6	7	8
5-year finite mixture models: PWT 6 non-oil sample							
$\rho(u_{\text{lny}}, u_{\text{gy}})$	0.999**	0.692**	0.438**	0.287**	0.308**	0.302**	
ℓ	−107.749	77.76735	189.5688	233.198	302.5	306.494	
BIC	285.1579	−58.0104	−253.749	−313.143	−423.883	−404.007	
AIC	235.4976	−127.535	−343.138	−422.395	−553	−552.988	
CAIC	295.1579	−44.0104	−235.749	−291.143	−397.883	−374.007	
10-year finite mixture models: PWT 6 non-oil sample							
$\rho(u_{\text{lny}}, u_{\text{gy}})$	0.999**	0.994**	0.834**	0.866**	0.868**	0.810**	0.804**
ℓ	−104.471	−15.7194	60.60506	100.069	133.098	171.611	178.257
BIC	260.8722	104.1401	−27.7369	−85.8934	−131.179	−187.433	−179.953
AIC	228.9427	59.43874	−85.2101	−156.138	−214.196	−283.221	−288.514
CAIC	270.8722	118.1401	−9.73689	−63.8934	−105.179	−157.433	−145.953

Significance levels: † 10%; * 5%; ** 1%
Notes: K, no. components; ℓ, log-likelihood; $\rho(u_{\text{lny}}, u_{\text{gy}})$, correlation among the two latent effects;
$\text{BIC} = -2\ell(\cdot) + d\log(n)$
$\text{AIC} = -2\ell(\cdot) + d$
$\text{CAIC} = -2\ell(\cdot) + 2d\log(n)$
where d is the number of parameters and n is the sample size.

Table XI. Location and probabilities of 5-year BFME

k	Level (lny)		Growth rate (gy)		
	Loc.	SE	Loc.	SE	Prob.
1	−1.253	0.088	−0.007	0.028	0.070
2	−0.845	0.079	−0.004	0.016	0.197
3	0.579	0.079	0.004	0.020	0.122
4	0.161	0.078	0.010	0.014	0.211
5	−0.288	0.078	−0.003	0.015	0.200
6	1.034		−0.004		0.200

Note: k, number of mixture components selected by penalized criteria; Loc, locations; SE, locations' standard errors; Prob., prior probability of belonging to that local area. The probabilities are for both equations in the bivariate model.

that countries do not converge towards a single path but that, rather, countries converge to cluster-specific paths, with clusters staying divergent. Convergence clubs may also emerge endogenously if there are multiple possible steady-state equilibria given fundamentals (see Galor, 1996). This means that the cluster structure cannot be exhaustively explained by the neoclassical model; the unobserved characteristics influencing per capita income levels (rather than growth rates) do not represent technological differences, since it is widely believed that technology in different countries converges. Rather, unobserved heterogeneity seems to be related to long-lasting differences in culture and institutions.

To test whether clusters converge, we employed the following strategy: a univariate finite mixture model has been fitted to data from 1960 to 1975, under the constraint that the number of locations is $K = 7$ and using as starting parameter vector the model parameter estimates reported, respectively, in Tables VI and III. Using the cluster structure obtained at the end of this path, we have estimated the corresponding cluster-specific mean of GDP levels for the out-of-sample (years

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Table XII. Bivariate posterior classification

Country	y	u_{lny}	$\sigma^2_{u_{\text{lny}}}$	gy	u_{gy}	$\sigma^2_{u_{\text{gy}}}$
Group 1						
Total	7.269	−1.252	0.009	0.003	−0.007	0.000
Ethiopia	7.102	−1.252	0.000	0.001	−0.007	0.000
Malawi	7.258	−1.252	0.000	0.010	−0.007	0.000
Nepal	7.570	−1.252	0.013	0.008	−0.007	0.000
Tanzania	7.149	−1.252	0.000	0.007	−0.007	0.000
Uganda	7.196	−1.248	0.040	−0.002	−0.007	0.000
Zaire	7.353	−1.252	0.000	−0.008	−0.007	0.000
Group 2						
Total	7.797	−0.852	0.016	0.005	−0.004	0.000
Bangladesh	7.768	−0.845	0.009	0.003	−0.004	0.000
Benin	7.783	−0.845	0.000	0.003	−0.004	0.000
Burkina Faso	7.473	−0.845	0.003	0.011	−0.004	0.000
Burundi	7.457	−0.845	0.000	0.005	−0.004	0.000
Congo	8.030	−0.845	0.001	0.009	−0.004	0.000
Ghana	7.909	−0.845	0.000	0.001	−0.004	0.000
India	7.804	−0.869	0.097	0.011	−0.004	0.001
Indonesia	8.112	−0.845	0.000	0.034	−0.004	0.000
Kenya	7.815	−0.845	0.000	0.012	−0.004	0.000
Madagascar	7.718	−0.845	0.000	−0.010	−0.004	0.000
Mali	7.521	−0.954	0.181	0.009	−0.005	0.001
Niger	7.713	−0.845	0.000	−0.013	−0.004	0.000
Nigeria	7.651	−0.845	0.001	−0.001	−0.004	0.000
Pakistan	7.989	−0.845	0.000	0.011	−0.004	0.000
Rwanda	7.605	−0.845	0.000	0.008	−0.004	0.000
Sri Lanka	8.234	−0.845	0.002	0.015	−0.004	0.000
Togo	7.818	−0.845	0.000	−0.006	−0.004	0.000
Zambia	7.934	−0.845	0.000	−0.017	−0.004	0.000
Group 3						
Total	9.591	0.579	0.002	0.019	0.004	0.000
Argentina	9.622	0.579	0.000	0.021	0.004	0.000
Greece	9.676	0.579	0.000	0.025	0.004	0.000
Ireland	9.773	0.579	0.000	0.024	0.004	0.000
Mauritius	9.328	0.578	0.016	0.010	0.004	0.000
Mexico	9.486	0.579	0.000	0.014	0.004	0.000
Portugal	9.569	0.579	0.001	0.028	0.004	0.000
S Africa	9.486	0.579	0.002	0.011	0.004	0.000
Singapore	9.757	0.579	0.001	0.040	0.004	0.000
Spain	9.768	0.579	0.000	0.020	0.004	0.000
Trinidad and Tobago	9.572	0.579	0.000	0.016	0.004	0.000
Venezuela	9.456	0.579	0.000	0.002	0.004	0.000
Group 4						
Total	9.020	0.161	0.004	0.018	0.010	0.000
Algeria	9.167	0.161	0.013	0.014	0.010	0.000
Botswana	8.822	0.161	0.000	0.038	0.010	0.000
Brazil	9.188	0.161	0.004	0.026	0.010	0.000
Chile	9.129	0.161	0.000	0.018	0.010	0.000
Colombia	8.945	0.161	0.000	0.016	0.010	0.000
Costa Rica	9.095	0.161	0.001	0.008	0.010	0.000
El Salvador	8.956	0.161	0.000	0.006	0.010	0.000
Guatemala	8.872	0.161	0.000	0.016	0.010	0.000
Jordan	8.928	0.161	0.002	0.016	0.010	0.000
Korea, Rep.	9.127	0.161	0.000	0.037	0.010	0.000
Malaysia	9.073	0.161	0.000	0.030	0.010	0.000

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Table XII. (*Continued*)

Country	y	u_{lny}	$\sigma^2_{u_{\text{lny}}}$	gy	u_{gy}	$\sigma^2_{u_{\text{gy}}}$
Nicaragua	8.831	0.161	0.001	−0.009	0.010	0.000
Panama	9.079	0.161	0.000	0.020	0.010	0.000
Papua N. Guinea	8.789	0.161	0.000	0.009	0.010	0.000
Paraguay	8.974	0.161	0.000	0.021	0.010	0.000
Peru	9.026	0.161	0.000	0.012	0.010	0.000
Tunisia	8.993	0.161	0.000	0.021	0.010	0.000
Turkey	9.011	0.161	0.000	0.017	0.010	0.000
Uruguay	9.344	0.167	0.050	0.020	0.010	0.001
Group 5						
Total	8.454	−0.289	0.007	0.006	−0.003	0.000
Angola	8.252	−0.288	0.000	−0.022	−0.003	0.000
Bolivia	8.547	−0.288	0.000	0.004	−0.003	0.000
Cameroon	8.275	−0.288	0.000	0.002	−0.003	0.000
Central Afr. Rep.	8.072	−0.288	0.000	−0.012	−0.003	0.000
Dominican Rep.	8.623	−0.288	0.003	0.022	−0.003	0.000
Ecuador	8.819	−0.286	0.029	0.021	−0.003	0.001
Egypt	8.583	−0.288	0.000	0.014	−0.003	0.000
Honduras	8.370	−0.288	0.000	0.011	−0.003	0.000
Ivory Coast	8.469	−0.288	0.004	0.009	−0.003	0.000
Jamaica	8.839	−0.288	0.000	−0.002	−0.003	0.000
Mauritania	8.092	−0.288	0.000	0.008	−0.003	0.000
Morocco	8.669	−0.288	0.013	0.013	−0.003	0.000
Mozambique	7.938	−0.288	0.000	−0.016	−0.003	0.000
Philippines	8.590	−0.288	0.000	0.012	−0.003	0.000
Senegal	8.070	−0.298	0.071	−0.007	−0.003	0.000
Syria	8.719	−0.288	0.000	0.015	−0.003	0.000
Thailand	8.668	−0.288	0.000	0.029	−0.003	0.000
Zimbabwe	8.584	−0.288	0.000	0.008	−0.003	0.000
Group 6						
Total	10.155	1.034	0.003	0.016	−0.003	0.000
Australia	10.210	1.034	0.000	0.015	−0.004	0.000
Austria	10.105	1.034	0.000	0.021	−0.004	0.000
Belgium	10.165	1.034	0.000	0.018	−0.004	0.000
Canada	10.251	1.034	0.000	0.010	−0.004	0.000
Denmark	10.249	1.034	0.000	0.014	−0.004	0.000
Finland	10.064	1.032	0.032	0.016	−0.003	0.001
France	10.160	1.034	0.000	0.017	−0.004	0.000
Hong Kong	9.902	1.034	0.002	0.032	−0.004	0.000
Israel	9.914	1.034	0.015	0.020	−0.003	0.000
Italy	10.058	1.034	0.000	0.022	−0.004	0.000
Japan	10.080	1.034	0.000	0.030	−0.004	0.000
Netherlands	10.135	1.034	0.000	0.015	−0.004	0.000
New Zealand	10.082	1.034	0.000	0.002	−0.004	0.000
Norway	10.219	1.034	0.000	0.024	−0.004	0.000
Sweden	10.224	1.034	0.000	0.013	−0.004	0.000
Switzerland	10.463	1.034	0.000	0.006	−0.004	0.000
UK	10.075	1.034	0.000	0.012	−0.004	0.000
USA	10.482	1.034	0.000	0.009	−0.004	0.000

Note: lny, log of per capita GDP levels (World Bank Report data); gy, rate of growth of lny; u_{lny}, country random coefficient for lny (mean values along time); u_{gy} country random coefficient for gy (mean values along time); $\sigma^2_{u_{\text{lny}}}$, variance of the random intercept for lny; $\sigma^2_{u_{\text{gy}}}$, variance of the random intercept for gy.

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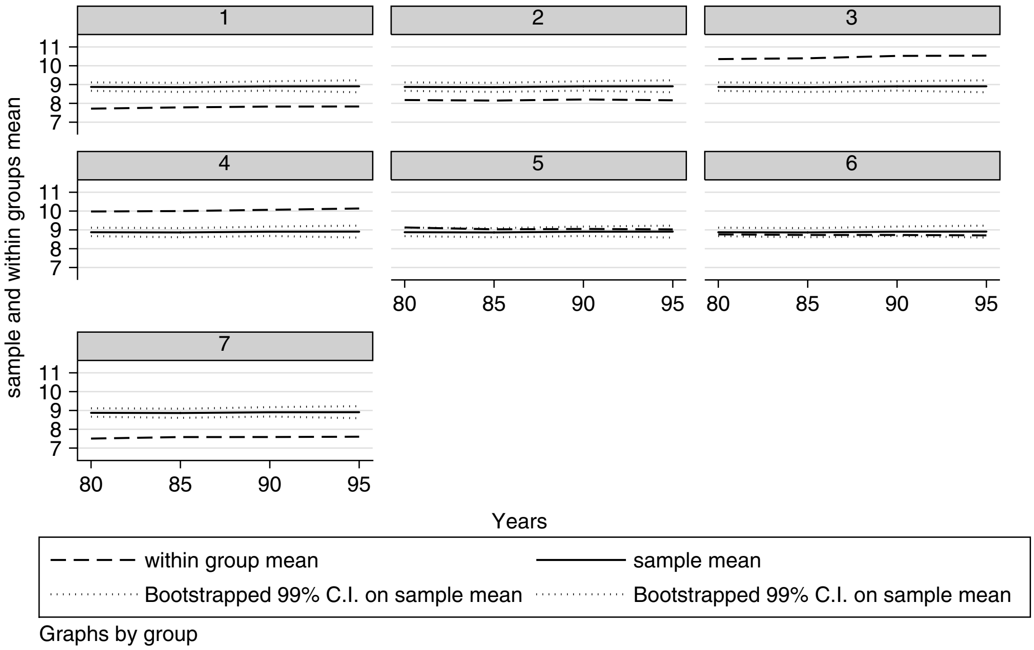

Figure 3. Overall and within-groups log per capita GDP mean. Country clusters conditioned to 1960–1975

- Group 1: Ghana, Indonesia, Kenya, Mali, Pakistan, Sri Lanka, Uganda, Zaire
- Group 2: Benin, Burkina Faso, Burundi, Egypt, Honduras, Jamaica, Korea Rep., Madagascar, Nigeria, Philippines, Rwanda, Senegal, Syria, Thailand, Togo, Zambia
- Group 3: Canada, Switzerland, USA
- Group 4: Argentina, Australia, Austria, Belgium, Denmark, Finland, France, Hong Kong, Israel, Italy, Japan, Mexico, Netherlands, New Zealand, Norway, S Africa, Sweden, Trinidad and Tobago, UK, Venezuela
- Group 5: Algeria, Brazil, Costa Rica, El Salvador, Greece, Guatemala, Ireland, Ivory Coast, Mauritania, Mauritius, Mozambique, Nicaragua, Papua N. Guinea, Paraguay, Peru, Portugal, Singapore, Spain, Uruguay
- Group 6: Angola, Bolivia, Botswana, Cameroon, Central Afr. Rep., Chile, Colombia, Dominican Rep., Ecuador, Jordan, Malaysia, Morocco, Niger, Panama, Tunisia, Turkey, Zimbabwe
- Group 7: Bangladesh, Congo, Ethiopia, India, Malawi, Nepal, Tanzania

1975–1995) period. In Figure 3 we present cluster-specific means plotted versus the overall mean, with corresponding 99% bootstrap-based confidence intervals. Convergence of cluster-specific means is not evident at all, except for clusters 5 and 6 with corresponding cluster-specific means laying inside the confidence interval. However, the dynamics of component specific mean for cluster 6 (see Figure 4) is divergent with respect to other components. Thus, our findings are in line with those discussed by Lee *et al*. (1997) using a stochastic Solow model.

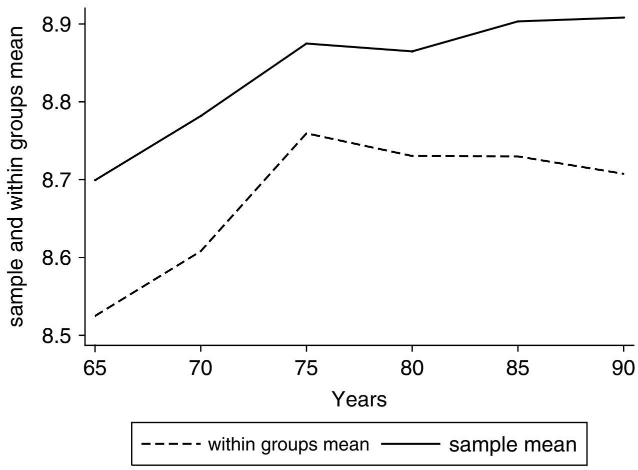

Figure 4. Overall and within-groups log per capita GDP mean. Country clusters conditioned to 1960–1975. Group 6

Our results are surprising given that several variables have been found in the growth empirics literature to have a significant effect on per capita real GDP growth (see, for example, Levine and Renelt, 1992; Sala-i-Martin, 1997). The corresponding models typically include in the linear predictor the initial level of per capita real GDP; given within-cluster convergence, this variable has a robustly negative coefficient in growth regressions. However, level variables and initial level of log GDP per capita are strongly correlated; therefore, the variance of the log difference of GDP per capita is almost entirely explained by the initial conditions (see, among others, Temple, 1998, 1999).

If we consider a standard cross-country growth model augmented by political rights variables and geographical dummies, we find that none of these variables is significant in the growth equation if initial per capita real GDP (see Table XIII) is not included.

We performed a factor analysis to check for multicollinearity between variables in the MRW Solovian growth model. Should the four regressors be orthogonal, we would find factors explaining not more than 20–25% of the total variance. In the present context, however, we find that the first factor accounts for approximately 64% of the total variance. The collinearity between regressors is confirmed also if we average capital inputs for the period 1965–1995 (see Figure 5). These results imply that cross-section tests on convergence are inflated by collinearity and, since initial per capita GDP is a good predictor for capital saving rates, additional covariate effects may be ill estimated. It is worth noting that a 'minimal cross-section growth regression', including only the initial level of per capita income, would not be a good choice owing to corresponding minimal economic insight.

We identify seven different clusters for the univariate model and six for the bivariate one. The number of clusters is apparently not consistent with those found in other proposals, where the number of groups is usually restricted to two, three or four groups (see, for example, Liu and Stengos, 1999; Kalaitzidakis *et al.*, 2001; Canova, 2004; Masanjala and Papageorgiou, (2004); Paap *et al.*, 2005). These differences are, however, reasonable if one considers that the estimation method, the analyzed time period and the number of countries do not coincide. Paap *et al.* (2005)

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Table XIII. Cross-section growth regression

	No initial level		Standard convergence model	
	Coef.	SE	Coef.	SE
ly60			-0.015^{**}	0.003
lnsh6095	0.0008	0.003	0.006^{*}	0.002
lnsk6095	0.0093^{*}	0.003	0.009^{**}	0.002
lngd6095	0.0001	0.016	$-0.025^{\dagger}$	0.014
Political right (1 = very free, 7 = no freedom)	0.0001	0.001	-0.002^{*}	0.001
East Asia and Pacific	0.049	0.046	0.135^{**}	0.037
Europe and Central Asia	0.0403	0.05	0.129^{**}	0.04
Latina America and Caribbean	0.0306	0.045	0.122^{**}	0.037
Middle East; North Africa	0.0408	0.045	0.132^{**}	0.037
North America	0.0322	0.048	0.136^{**}	0.04
South Asia	0.0447	0.046	0.118^{**}	0.037
Sub-Saharan Africa	0.0295	0.046	0.118^{**}	0.038
N	88			
BIC	−344.7445		−340.2705	
Deviance	0.0105		0.0071	

Significance levels: † 10%; * 5%; ** 1%.
GLM model (MQL Fisher scoring) without intercept to avoid aliasing.

identify three groups but the analysis is limited only to Africa, Asia and Latin-America. Becchetti *et al*. (2006) analyze the same dataset, associating a random parameter to the human capital variable; the estimated number of components is equal to five, perhaps due to a greater flexibility of their approach to unobserved heterogeneity. We believe that the unobserved characteristics which affect per capita income levels are not true technological differences, but rather represent the effects of long-lasting differences in culture and institutions. To better understand differences among groups, we report in Table XIV the values of the correlation coefficients between social and cultural indicators and the estimated set of latent effects. As can be seen, indicators of institutional quality and economic freedom are strictly and significantly correlated with the estimated locations u_k, representing the cluster-specific deviation from the sample average. Our results are also in line with those obtained by Canova (2004) and Jerzmanowski (2006), who find that the role of culture and institutions is making growth episodes persistent rather than ruling out growth take-offs.

The unobserved differences between countries are, however, innumerable and clearly multidimensional. For each component, the proposed estimator summarizes these differences using just two dimensions: estimated latent terms in the level of per capita GDP (pcgdp) and in the growth rate of pcgdp. The variance of the latent effects in the level equation is high when compared to both the variance in the growth equation and the covariance between the two latent effects. Thus, when we divide the countries into groups, the estimated differences between groups are roughly one-dimensional; still, we are sure that these estimated differences in pcgdp levels are functions of unobserved, multidimensional differences.

The almost complete absence of any sign of convergence across clusters gives us some reason to believe that three plausible unobserved differences are not, in themselves, major factors in explaining difference in pcgdp across countries. First, it seems unlikely that measurement error in estimated physical capital stocks is as important a factor as we would have guessed. If countries with the same observed characteristics (including measured physical capital per capita) differ only

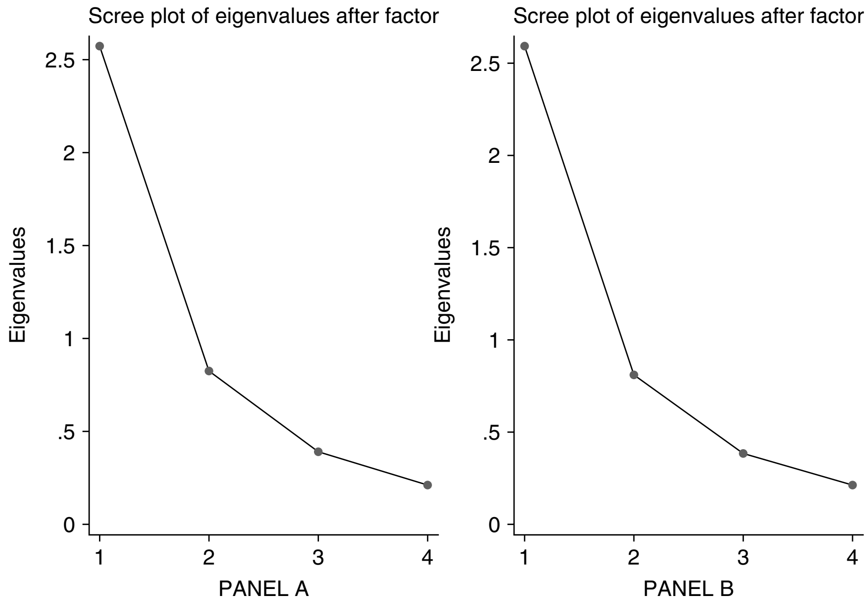

Figure 5. Factor analysis: plot of eigenvalues
- Method: principal-component factors
- Model for Panel A: ly60, lnsh6095, lnsk6095, lngd6095. Total variance explained first factor 0: 6482. Cumulate variance two factors 0.8507
- Model for Panel B: ly60, lnsh6595, lnsk6595, lngd6595. Total variance explained first factor 0: 6432. Cumulate variance two factors 0.8495

Table XIV. Spearman and pairwise correlation coefficients between selected country-specific indicators and the random terms in univariate (5-year) and bivariate mixtures

	u_i	u_{lny}	u_{gy}
Spearman correlation			
Political right (1 = very free, 7 = not free)	−0.7200***	−0.7191***	−0.1452**
Civil liberties (1 = very civil, 7 = uncivil)	−0.7465***	−0.7434***	
Freedom status (2 = free, 1 = partial, 0 = not free)	0.6814***	0.6782***	0.1923***
Pairwise correlation			
Perc. of sec. school attained, tot. pop.	0.4631***	0.4669***	−0.1474***
Average schooling years, tot.	0.4647***	0.4656***	−0.1290***
Fertility rate	−0.4714***	−0.4741***	
Openness in curr. prices ((exp +imp)/GDP)	0.0894**	0.0966**	

Significance levels: † 10%; * 5%; ** 1%.
Note: u_i is the random component in γ_{0i}, and u_{lny} and u_{gy} are those for y_1 and y_2 in the bivariate model.
Source of data: Freedom House. 2003. Available at http://www.freedomhouse.org/template.cfm?page=15.

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in the true level of physical capital per capita, a Solow model would imply that they converge to the same pcgdp. Second, the same argument applies to human capital for an augmented Solow model provided that the sum of the elasticities of GDP with respect to physical and human capital is less than one. Third, according to the consensus opinion of growth theorists, if the unobserved differences are differences in disembodied technology in use, pcgdp should converge.

In contrast, differences in institutions, laws, norms, customs, attitudes and aspirations may be persistent. For example, the Protestant ethic, which may be the spirit of capitalism may cause Protestant countries to be persistently richer than other countries (see Weber, 1930; Bagella *et al.*, 2002; Dobbin, 2004; Dudley and Blum, 2001). We believe that the estimated components may group together countries whose culture and institutions imply a similar level of pcgdp for a given population, capital, human capital and technology. Since these groupings collapse many dimensions into one, countries in the same group have very different levels of every single identifiable institutional and cultural factor.[1]

4. CONCLUSIONS

In this paper we discuss an empirical model with the aim of testing whether economic growth can be considered exogenous in the Solovian sense. Following Alfò and Trovato (2004), we define a bivariate mixture model for the Bernanke and Gürkaynak (2001) extension of the Solow model. Like Durlauf *et al.* (2001), we find that the explanatory power of the Solow growth model is enhanced when cross-country heterogeneity is considered. In this case, we find that growth rates are not significantly associated with investment rates. In particular, Bernanke and Gürkaynak (2001) reject the Solow growth model analyzing cross-country data on GDP, growth and investment. The evidence against the Solow model becomes statistically insignificant when a pooled cross-section model is applied to Bernanke and Gürkaynak's data allowing for unobserved heterogeneity across countries. We suggest that their rejection of the Solow model is caused by omitted variables bias; however, care is needed, since our tests may fail to reject the Solow model because of their reduced power. A more robust result of our analysis is an extremely strong evidence of unexplained heterogeneity in levels of per capita real GDP and an extremely weak evidence of heterogeneity in the rate of growth of per capita real GDP. This means that the data show no sign of convergence across classes of countries. The procedure could, in principle, have detected clusters of high-growth countries and of low-growth countries. It did not do so. The results suggest convergence clubs, that is, groups of countries with different levels of per capita real GDP within which countries converge to a group-specific growth path.

We find almost no evidence that different groups converge towards each other. This suggests that the unobserved factors which we model with a discrete mixture correspond to long-lasting characteristics not accounted for by the augmented Solow model.

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[1] We thank an anonymous referee for forcing us to confront this issue.

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