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Testing for deterministic trend and seasonal components in time series models

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SUMMARY

A univariate time series model can be set up as the sum of trend, seasonal and irregular components. The trend and seasonal components will normally be stochastic, but deterministic components arise as a special case. This paper develops a test that the trend and seasonal components are deterministic using the approach of Lehmann. The procedure is then extended to test for deterministic components in a model formulated in first differences. Both tests are exact and critical values are tabulated.

Some key words: Deterministic component; Forecasting; Generalized least squares; Local and global trend; Most powerful invariant test; Time-varying parameter.

1. INTRODUCTION

A univariate time series model can be set up as the sum of trend, seasonal and irregular components. By allowing the trend and seasonal components to change slowly over time, more weight is put on the most recent observations in making predictions. A model of this kind is attractive for modelling time series because it has a natural interpretation. Furthermore, estimation is a feasible proposition, either in the time domain, using the Kalman filter, or in the frequency domain; see Kitagawa (1981) and Harvey & Todd (1983).

Although such a model will normally have stochastic trend and seasonal components, a model in which these components are deterministic emerges as a special case. The purpose of this paper is to develop a test of the deterministic, or global, model against the more general stochastic alternative. The global model we have in mind is

$$y_t = \alpha + \beta t + \Sigma_j \delta_j z_{jt} + \varepsilon_t \quad (t = 1, \ldots, T), \tag{1·1}$$

where $y_1, \ldots, y_T$ are the observations, α and β are the trend parameters, ε_t is a normally distributed white noise disturbance term with mean zero and variance σ^2, the z_{jt}'s are seasonal dummies and the δ_j's are their coefficients. If there are s seasons in the year there will normally be $s-1$ seasonal dummy variables. This makes a total of $s+1$ regression parameters and these parameters can be estimated efficiently by ordinary least squares. The corresponding stochastic model is

$$y_t = \mu_t + \gamma_t + \varepsilon_t \quad (t = 1, \ldots, T), \tag{1·2}$$

where μ_t and γ_t are the trend and seasonal components respectively. The trend is defined as

$$\mu_t = \mu_{t-1} + \beta_{t-1} + \eta_t, \quad \beta_t = \beta_{t-1} + \zeta_t \quad (t = 1, \ldots, T), \tag{1·3}$$

where η_t and ζ_t are normally and independently distributed white noise processes with zero means and variances σ_η^2 and σ_ζ^2 respectively. The seasonal component is

$$\sum_{j=0}^{s-1} \gamma_{t-j} = \omega_t \quad (t = 1, \ldots, T), \tag{1·4}$$

where ω_t is normally distributed white noise with variance σ_ω^2. The irregular component ε_t is normally distributed white noise with variance σ^2. A complete class of structural models can be defined by generalizing this specification, and it is argued by Harvey & Todd (1983) that models of this kind provide a useful alternative to the integrated autoregressive-moving average models proposed by Box & Jenkins (1976). However, for the purposes of this paper we will restrict attention to what we call the 'basic structural model'. A similar model forms the basis of the Bayesian forecasting procedure used by Harrison & Stevens (1976).

In the basic structural model, the trend has both its level, μ_t, and its slope, β_t, slowly changing over time. The seasonal pattern is also changing over time, but when $\sigma_\eta^2 = \sigma_\zeta^2 = \sigma_\omega^2 = 0$ both the trend and the seasonal components are deterministic and the model is equivalent to (1·1). It is the hypothesis that these three variances are all zero that we wish to test.

Various partially deterministic models also arise as special cases of the basic structural model. The most important of these is the seasonal random walk with drift, i.e.

$$\Delta y_t = \beta + \Sigma_j \delta_j z_{jt} + \eta_t \quad (t = 2, \ldots, T), \tag{1·5}$$

where η_t is normally distributed white noise with variance σ_η^2 and Δ is the first difference operator. This model has been found to fit many economic time series remarkably well; see, for example, Pierce (1978). The hypotheses to be tested are concerned with whether the mean and the seasonal pattern are indeed constant over time.

2. THE BASIC STRUCTURAL MODEL

The model defined by (1·2), (1·3) and (1·4) can be written as a regression model in which the parameters change over time. Suppose that $s = 4$. Definition of the vectors

$$\alpha_t = (\mu_t, \beta_t, \gamma_t, \gamma_{t-1}, \gamma_{t-2})', \quad \tau_t = (\eta_t, \zeta_t, \omega_t, 0, 0)'$$

enables us to write

$$\alpha_t = C\alpha_{t-1} + \tau_t \quad (t = 1, \ldots, T), \tag{2·1}$$

where C is an appropriately defined matrix; see Harvey & Todd (1983). Equation (1·2) can now be written as

$$y_t = \bar{x}_t' \alpha_t + \varepsilon_t \quad (t = 1, \ldots, T), \tag{2·2}$$

where $\bar{x}_t = (1\ 0\ 1\ 0\ 0)'$ for all t.

If α_0 is regarded as fixed, (2·1) can be used to express α_t in terms of α_0 for all $t = 1, \ldots, T$. Substitution into (2·2) then gives

$$y_t = x_t' \alpha_0 + w_t \quad (t = 1, \ldots, T), \tag{2·3}$$

where $x_t = C^{t\prime} \bar{x}_t$ and

$$w_t = \bar{x}_t' \sum_{j=1}^{t} C^{t-j} \tau_j + \varepsilon_t \quad (t = 1, \ldots, T). \tag{2·4}$$

The model is now in the form of a standard regression and if the covariance matrix of the

w_t's were known up to a scalar factor, the minimum variance unbiased estimator of α_0 could be computed by generalized least squares.

Model (2·3) can be expressed in matrix form as $y = X\gamma_0 + w$, where y is the $T \times 1$ vector $y = (y_1, \ldots, y_T)'$, X is the $T \times (s+1)$ matrix $X = (x_1, \ldots, x_T)'$, and $w = (w_1, \ldots, w_T)'$. The mean of w is the null vector while its covariance matrix, $\sigma^2 \Omega$, has as its element (s, t)

$$\omega_{st} = \delta_{st} + \bar{x}_t' \left(\sum_{j=1}^{\min(s,t)} C^{t-j} Q C'^{s-j} \right) \bar{x}_s$$

$$= \delta_{st} + \sum_{j=1}^{\min(s,t)} x_{t-j}' Q x_{s-j}, \tag{2·6}$$

where δ_{st} is the Kronecker delta and

$$Q = \sigma^{-2} E(\tau_t \tau_t') = \operatorname{diag}\{\bar{\sigma}_\eta^2, \bar{\sigma}_\zeta^2, \bar{\sigma}_\omega^2, 0, 0\}. \tag{2·7}$$

Given the relative variances, $\bar{\sigma}_\eta^2 = \sigma_\eta^2/\sigma^2$, $\bar{\sigma}_\zeta^2 = \sigma_\zeta^2/\sigma^2$ and $\bar{\sigma}_\omega^2 = \sigma_\omega^2/\sigma^2$, the generalized least squares estimator of $\gamma_0, \tilde{\gamma}_0$, can be computed. The corresponding generalized residual sum of squares is

$$S(\bar{\sigma}_\eta^2, \bar{\sigma}_\zeta^2, \bar{\sigma}_\omega^2) = (y - X\tilde{\gamma}_0)' \Omega^{-1} (y - X\tilde{\gamma}_0). \tag{2·8}$$

The generalized residual sum of squares is the main element in the test statistic proposed in the next section. However, it can be calculated without constructing and inverting the $T \times T$ matrix Ω. As expressed in (2·1) and (2·2) the model is effectively in state space form. It is therefore possible to run through the Kalman filter with starting values formed from the first $s+1$ observations; see Harvey (1981, p. 205) or Garbade (1977). The generalized residual sum of squares is then given by the sum of squares of the standardized one-step ahead prediction errors, i.e.

$$S(\bar{\sigma}_\eta^2, \bar{\sigma}_\zeta^2, \bar{\sigma}_\omega^2) = \sum_{t=s+2}^{T} \nu_t^2 / f_t, \tag{2·9}$$

where ν_t is the one-step ahead prediction error at time t and $f_t = \sigma^{-2} \operatorname{var}(\nu_t)$. The proof of the equivalence of (2·8) and (2·9) is along the lines of the proof set out by Harvey & Phillips (1979, pp. 54–5). The attraction of being able to use the Kalman filter algorithm to compute (2·9) is that this algorithm will need to be employed anyway, if it is decided to fit the more general local trend model and use it to make optimal predictions of future observations.

3. Test procedures

3·1. *General approach*

The three classical test procedures, i.e. likelihood ratio, Wald and Lagrange multiplier, all run into difficulties in the present context; compare with a similar situation analysed by Sargan & Bhargava (1983). The solution we propose is to develop a most powerful invariant test based on the theory of invariance (Lehmann, 1959, Chapter 5). The test is exact, but the price paid for this is that specific values must be assigned to $\bar{\sigma}_\eta^2, \bar{\sigma}_\zeta^2$ and $\bar{\sigma}_\omega^2$ under the alternative.

Given the regression interpretation of the model in (2·3), a most powerful invariant test can be derived by following the approach used, for example, by Berenblut & Webb (1973). In an unpublished paper L. Franzini extended this approach to develop tests for time-varying parameters in regression and the tests used here are of the same form as those given by Franzini. If the matrix Ω is understood to be evaluated at specific values of $\bar{\sigma}_\eta^2$, $\bar{\sigma}_\zeta^2$ and $\bar{\sigma}_\omega^2$, the critical region for testing the null hypothesis that $\bar{\sigma}_\eta^2 = \bar{\sigma}_\zeta^2 = \bar{\sigma}_\omega^2 = 0$ is of the form

$$\frac{(y-X\tilde{\gamma}_0)'\Omega^{-1}(y-X\tilde{\gamma}_0)}{(y-X\hat{\gamma}_0)'(y-X\hat{\gamma}_0)} < c, \tag{3·1}$$

where $\hat{\gamma}_0$ is the ordinary least squares estimator of γ_0. The denominator is therefore simply the residual sum of squares obtained by regressing y on X, while the numerator is the generalized residual sum of squares, (2·8). As already noted (2·8) can be most easily evaluated in the form (2·9).

Critical values for the test defined by (3·1) can be computed using the method of Imhof (1961). These values depend on T and s, and on the values assigned to the relative variances. If these relative variances can be fixed according to some suitable rule, critical values for the test procedure can be tabulated once and for all.

3·2. *Choice of test statistic*

The first step in deciding on suitable values of $\bar{\sigma}_\eta^2$, $\bar{\sigma}_\zeta^2$ and $\bar{\sigma}_\omega^2$ is to fix the relationship between them. Two possibilities will be considered. The first is to set them all equal, while the second is to set $\bar{\sigma}_\eta^2 = \bar{\sigma}_\omega^2$ but to have $\bar{\sigma}_\zeta^2 = 0$. The rationale behind the second choice is that if there is variation in the trend, of whatever sort, it will tend to show up in a test against $\bar{\sigma}_\eta^2 > 0$.

In terms of (2·7), the first test sets $Q = q\ \mathrm{diag}(1,1,1,0,0)$, while the second test has $Q = q\,\mathrm{diag}(1,0,1,0,0)$. Given these two tests the second step is to decide on suitable values of q. Let the proportion of total variance due to variation in the regression coefficients be denoted by θ. It can be shown that θ is related to q by the expression

$$\theta = \frac{q\,\mathrm{tr}(M\Omega^*)}{\mathrm{tr}(M)+q\,\mathrm{tr}(M\Omega^*)}, \tag{3·2}$$

where Ω^* is the $T\times T$ matrix defined implicitly by writing

$$\Omega = I+q\Omega^*, \quad M = I-X(X'X)^{-1}X'$$

(La Motte & McWhorter, 1978). The suggested procedure is therefore to fix q by solving (3·2) for a given choice of θ. In the special case when $\theta = q = 0$, the test statistic as given in (3·1) is inappropriate. However, as shown in the unpublished paper by Franzini, the use of L'Hôpital's rule gives a test of the form

$$\frac{(y-X\hat{\gamma}_0)'\Omega^*(y-X\hat{\gamma}_0)}{(y-X\hat{\gamma}_0)'(y-X\hat{\gamma}_0)} > c^*. \tag{3·3}$$

The sensitivity of the first test, the $b^\dagger(\theta)$ test, to three choices of θ, 0, 0·5 and 0·9, was examined for a range of true values of θ from 0·05 to 0·99. Exact powers were computed by the method of Imhof, and a full set of results can be found in our original research report which is available on request. Our conclusion is that setting $\theta = 0·5$ is a sensible choice since the resulting test has relatively high power for the full range of values of θ. Thus, for example, a comparison of $b^+(0)$ and $b^+(\frac{1}{2})$ for a sample size of 20 shows that $b^+(0)$ has a power of 0·110 for $\theta = 0·05$ while $b^+(\frac{1}{2})$ has a power which is only slightly lower at 0·105. On the other hand, the power of $b^+(0)$ at $\theta = 0·99$ is 0·774 while that of $b^+(\frac{1}{2})$ is 0·891. A comparison between $b^+(0{\cdot}9)$ and $b^+(\frac{1}{2})$ gives comparable results. The same exercise carried out for the $b(\theta)$ test leads to similar conclusions and in the power comparisons which follow θ will be set at $\frac{1}{2}$ in both cases. Bearing this in mind the two tests will be denoted by b^+ and b respectively.

3·3. *Comparison of powers*

The purpose of this subsection is to compare the power functions of the b^+ and b tests for a number of different quarterly models. The power functions of two other tests, the Durbin–Watson test and the 4th-order Durbin–Watson test (Wallis, 1972), are also examined. These tests, which throughout the paper are employed as one-sided tests against positive serial correlation, will be denoted by d_1 and d_4 respectively. Critical values for d_1 are tabulated by King (1981). All the powers reported were computed by the method of Imhof. The number of observations is $T = 20$ unless explicitly stated otherwise, and the level of significance is 5% in all cases.

The power functions shown in Fig. 1a are for a model in which there is equal variation in the level, slope and trend components, that is $Q = q\,\text{diag}(1, 1, 1, 0, 0)$. This is the situation for which the b^+ test is designed. However, although the power function of the b^+ test dominates that of the other tests both the b and d_1 tests perform quite well.

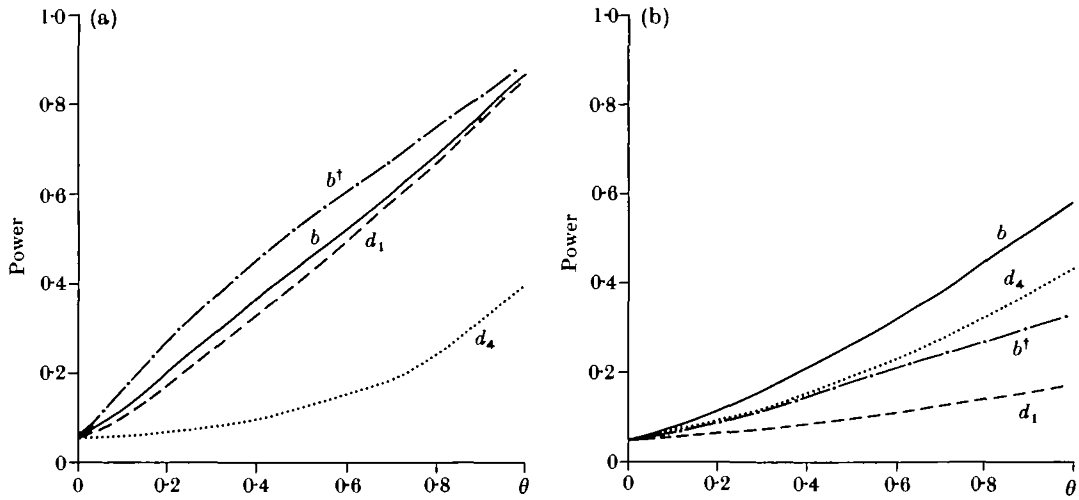

Fig. 1. Power functions (a) for variation in all components, (b) for trend, i.e. level and seasonal variation only.

Similar calculations for $T = 40$ indicate a tendency for the relative power of the b test to improve as the sample size is increased. For example, for $\theta = \frac{1}{2}$, the powers of the b^+ and b tests are 0·77 and 0·69 respectively.

Figure 1b shows the power functions when there is equal variation in the level and seasonal components, that is $Q = q\,\text{diag}(1, 0, 1, 0, 0)$. The b test is most powerful invariant when $\theta = \frac{1}{2}$, but it clearly dominates the other tests for all values of θ.

Two other cases were also examined. In the first, the variation was restricted to the level component in the trend, that is $Q = q\,\text{diag}(1, 0, 0, 0, 0)$, and the d_1 test performed rather well, as might be expected. However, its power function was dominated by that of the b^+ test for $\theta < 0{\cdot}54$. Both d_1 and b^+ had higher power than b for all values of θ, while the d_4 test was biased. The second case considered had only seasonal variation present, that is $Q = q\,\mathrm{diag}(0, 0, 1, 0, 0)$. The d_4 test was the most powerful test, although b still had relatively high power. Both d_1 and b^+ had decreasing power functions. This is because seasonal variation of the form (1·4) induces negative first-order autocorrelation in the residuals.

The main conclusion to be drawn from the above results is that b is the only statistic with relatively high power in all cases. The other three are all capable of performing extremely badly in certain situations. Furthermore a model in which there is both trend and seasonal variation, but σ_ζ^2 is relatively small, is one of the more likely cases to arise in practice. The recommended test procedure is therefore the b test. Critical values for $b(\frac{1}{2})$, the appropriate quarterly b-statistic, together with the values of q corresponding to $\theta = \frac{1}{2}$, are given in Table 1.

Table 1. *Critical values for b at 5% level of significance, $\theta = 0{\cdot}5$*

T	q	Critical value	T	q	Critical value	T	q	Critical value
12	0·384	0·437	44	0·136	0·570	76	0·082	0·651
16	0·314	0·447	48	0·126	0·583	80	0·078	0·659
20	0·265	0·465	52	0·117	0·595	88	0·072	0·672
24	0·229	0·485	56	0·109	0·606	96	0·066	0·684
28	0·201	0·504	60	0·102	0·616	104	0·061	0·695
32	0·180	0·523	64	0·096	0·626	112	0·057	0·705
36	0·162	0·540	68	0·091	0·635	120	0·053	0·714
40	0·148	0·555	72	0·087	0·643			

4. Test procedures for a partially deterministic model

Suppose that first differences are taken in the basic structural model and σ^2 is set equal to zero. The resulting model can then be written as

$$\Delta y_t = \beta_{t-1} + \gamma_t^* + \eta_t \quad (t = 2, \ldots, T), \tag{4·1}$$

where β_t is as defined in (1·3), and $\gamma_t^* = \Delta\gamma_t$, so that

$$\gamma_t^* = -\sum_{j=1}^{s-1} \gamma_{t-j}^* + \omega_t - \omega_{t-1}. \tag{4·2}$$

The model defined in (1·5) is a special case of (4·2) in which $\sigma_\zeta^2 = \sigma_\omega^2 = 0$. It is this hypothesis we wish to test.

If we proceed as before, (4·1) can be expressed in the form

$$\Delta y_t = \bar{z}_t' \delta_t + \eta_t \quad (t = 2, \ldots, T), \tag{4·3}$$

where $\bar{z}_t = (1, 1, 0, 0)'$ for all t and δ_t is a vector of time-varying regression parameters which obey the transition equation

$$\delta_t = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & -1 & 0 & 1 \\ 0 & -1 & 0 & 0 \end{bmatrix} \delta_{t-1} + \begin{bmatrix} \zeta_{t-1} \\ \omega_t \\ -\omega_t \\ 0 \end{bmatrix} \tag{4·4}$$

Compare this with the state space form of an autoregressive-moving average model (Harvey & Phillips, 1979). This model can be expressed in a form analogous to (2·5) and it follows once again that the most powerful invariant test against a specific alternative hypothesis is of the form (3·1). The recommended test procedure is to set $\sigma_\zeta^2 = \sigma_\omega^2 = q\sigma_\eta^2$ under the alternative hypothesis and to choose q in such a way that the θ defined for (4·3) is equal to $\frac{1}{2}$. This will be termed the b^* test. Table 2 shows the critical values for such a test with quarterly data at the 5% level of significance.

The denominator in the test statistic is simply the residual sum of squares from applying ordinary least squares to (1·5). The numerator can be computed by running

Table 2. *Critical values for b^* at 5% level of significance, $\theta = 0{\cdot}5$*

T	q	Critical value	T	q	Critical value	T	q	Critical value
12	0·156	0·421	44	0·051	0·611	76	0·030	0·692
16	0·125	0·453	48	0·047	0·624	80	0·029	0·699
20	0·103	0·486	52	0·044	0·637	88	0·026	0·712
24	0·088	0·514	56	0·041	0·648	96	0·024	0·724
28	0·077	0·538	60	0·038	0·658	104	0·022	0·734
32	0·068	0·560	64	0·036	0·668	112	0·021	0·743
36	0·061	0·579	68	0·034	0·677	120	0·020	0·752
40	0·056	0·595	72	0·032	0·685			

The number of differenced observations is $T-1$.

through the Kalman filter for the basic structural model with the covariance matrix of the disturbance term on the transition equation defined as $\sigma_\eta^2 Q^*$, where $Q^* = \text{diag}(1, q, q, 0, 0)$ and the variance of ε_t defined as $\sigma_\eta^2 h$ where $h = 0$. The scalar σ_η^2 does not appear in the filtering equations and (2·9) is applicable exactly as before.

The powers shown in Fig. 2a are for a model in which $Q^* = \text{diag}(1, q, q, 0, 0)$, that is there is equal variation in the slope and seasonal components. This is the situation for which the b^* test is designed and its power function dominates those of d_1 and d_4. However, while the power of d_1 is low, even for quite high values of θ, that of d_4 is relatively high. As previously, the tests are at the 5% level of significance, the sample size is 20 and the powers have been computed exactly by Imhof's method.

When there is trend variation only, i.e., $Q^* = \text{diag}(1, q, 0, 0, 0)$, d_1 dominates b^*, except for low values of θ; see Fig. 2b. The d_4 test is dominated by both d_1 and b^*. The figure for seasonal variation only, that is $Q^* = \text{diag}(1, 0, q, 0, 0)$, has not been reproduced

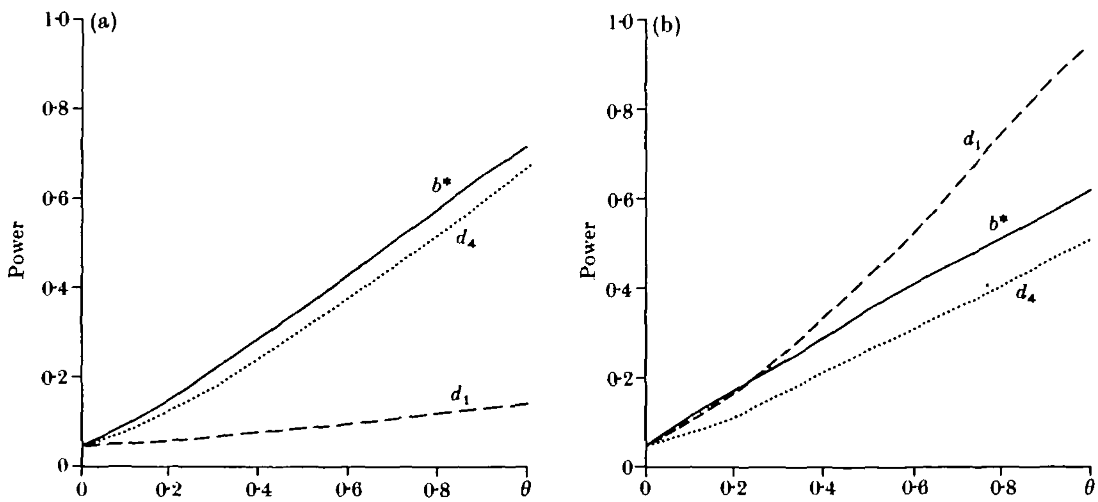

Fig. 2. Differenced observations: power functions (a) for trend, i.e. slope, and seasonal variation, (b) for trend variation only.

here. In this case the power functions of the b^* and d_4 tests are similar, although the b^* test dominates slightly for values of θ less than 0·73. Both tests attain a power of around 0·6 when $\theta = 0{\cdot}99$. The d_1 test, on the other hand, is biased for all values of θ.

The above models all have the variance of ε_t in (1·2) equal to zero. If this is not the case, the term $\varepsilon_t - \varepsilon_{t-1}$ must be added to the right-hand side of (4·3). We computed the power functions of the tests for $Q^* = \text{diag}(1, q, q, 0, 0)$ when $\sigma^2 = \sigma_\eta^2 = 1$. All three tests suffered some loss in power as compared with Fig. 2a, but for the b^* and d_4 tests this reduction was not too serious. Furthermore, both of these tests appear to be fairly robust to a nonzero variance for ε_t in the sense that when the null hypothesis $\sigma_\zeta^2 = \sigma_\omega^2 = 0$ is true, the probability that they reject it is still close to 0·05. For the b^* test this probability is 0·064 for $T = 20$ and 0·072 for $T = 40$. This robustness is an advantage because it establishes the b^* test more firmly as a test for nonstationarity in the disturbance term of (1·5).

Table 3. *Critical values for d_4 at 5% level of significance*, differenced observations*

$T-1$	Critical value	$T-1$	Critical value	$T-1$	Critical value
12	1·271	44	1·528	76	1·628
16	1·275	48	1·537	80	1·637
20	1·327	52	1·554	88	1·653
24	1·371	56	1·569	96	1·668
28	1·410	60	1·583	104	1·681
32	1·443	64	1·596	112	1·692
36	1·471	68	1·608	120	1·702
40	1·496	72	1·618		

* Statistic

$$d_4 = \sum_{t=5}^{T} (e_t - e_{t-4})^2 \Big/ \sum_{t=2}^{T} e_t^2,$$

where e_t is the tth residual obtained from fitting quarterly mean to Δy_t $(t = 2, \ldots, T)$.

While our results support the use of b^*, they also suggest that the power characteristics of d_4 are reasonable. Although it gives a less powerful test than b^* in most situations, it is easier to compute and so may sometimes be an attractive alternative. For this reason, critical values are provided in Table 3.

5. Example

The airline passenger data given by Box & Jenkins (1976, p. 531) consist of 144 monthly observations. Aggregation over quarters gives 48 observations and the logarithms of these observations were used to test whether the deterministic model in (1·1) is appropriate. This is a reasonable hypothesis since a plot of the data shows a fairly steady upward movement over time. However, (1·1) is easily rejected at the 5% level of significance since $b = 0{\cdot}353$ while the critical value is 0·583. A clear rejection is also obtained with the Durbin–Watson test; $d_1 = 0{\cdot}57$.

The partially deterministic model, (1·5), gives a much better fit to the airline data. Furthermore, it survives the Durbin–Watson test quite easily since $d_1 = 1{\cdot}87$ and the 5% critical vaue is 1·47. However, it is rejected by both b^* and the 4th-order Durbin–Watson test, d_4. In the case of b^*, the critical value is 0·624 while the sample test statistic is $b^* = 0{\cdot}458$.

6. Conclusions and extensions

When a model consisting of a linear trend and seasonal dummies has been fitted, the recommended test against stochastic variation in the trend and seasonal components uses b. Although this is the most powerful invariant test only against a particular case of the more general model, the test has relatively high power in a wide variety of circumstances. When the data have been differenced and a set of quarterly means fitted, the preferred test is based on b^*. Again this has a relatively high power over a wide range of alternatives. The d_4 test can also be used, but its power is, in general, somewhat below that of b^*.

Similar tests can be constructed for monthly and annual observations and for higher order polynomial trends (Harrison & Stevens, 1976, p. 217). Furthermore, explanatory variables can be introduced into (1·2) quite easily. If x_t denotes a $k \times 1$ vector of fixed explanatory variables, including lagged values, and δ is a $k \times 1$ vector of parameters, then

$$y_t = \mu_t + \gamma_t + x_t' \delta + \varepsilon_t \quad (t = 1, \ldots, T). \tag{6·1}$$

Tests of hypotheses concerning stochastic trend and seasonal components can be constructed in essentially the same way as was done in § 3. However, the distribution of each test statistic will depend on the explanatory variables and this suggests that the best way to proceed may be to form a bounds test. Similar considerations arise when the observations are in first differences and the tests are analogous to those in § 4.

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